INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 September 2012 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe two benefits and two limitations of using models in actuarial work. [4]

2 A large company wishes to construct a model of sickness rates among its employees to use in evaluating the present and future financial health of its sick pay scheme. Outline factors which the company should take into consideration when developing the model. [5]

3 (i) State the principle of correspondence as it applies to mortality rates. [1]

A life insurance company has the following data:

	Number of policies in force on			
Age last birthday	*1 January 2009*	*1 January 2010*	*1 July 2010*	*1 January 2011*
49	2,000	2,100	2,300	2,500
50	2,100	2,200	2,300	2,400
51	2,300	2,400	2,500	2,600

Number of deaths classified by age next birthday and calendar year

Age next birthday	*2009*	*2010*
49	175	200
50	200	225
51	225	235

(ii) Estimate, using these data, the force of mortality at age 50 next birthday for the period 1 January 2009 to 1 January 2011. [5]

(iii) State the exact age to which your answer to part (ii) relates. [1]
[Total 7]

4 (i) State one advantage of a semi-parametric model over a fully parametric one. [1]

(ii) Write down a general expression for the Cox proportional hazards model, defining all the terms you use. [2]

A life office is trying to understand the impact of certain factors on the lapse rates of its policies. It has studied the lapse rates on a block of business subdivided by:

- sex of policyholder (Male or Female)
- policy type (Term Assurance or Whole Life)
- sales channel (Internet, Direct Sales Force or Independent Financial Adviser)

The office has fitted a Cox proportional hazards model to the data and has calculated the following regression parameters:

Covariate	*Regression parameter*
Female	0.2
Male	0
Term Assurance	−0.1
Whole Life	0
Internet	0.4
Independent Financial Adviser	−0.2
Direct Sales Force	0

(iii) State the sex/sales channel/policy type combination to which the baseline hazard relates. [1]

A Term Assurance is sold to a Female by an Independent Financial Adviser.

(iv) Calculate the probability that this Term Assurance is still in force after five years given that 60% of Whole Life policies bought on the Internet by Males have lapsed by the end of year five. [4]
[Total 8]

5 A no claims discount system operates with three levels of discount, 0%, 15% and 40%. If a policyholder makes no claim during the year he moves up a level of discount (or remains at the maximum level). If he makes one claim during the year he moves down one level of discount (or remains at the minimum level) and if he makes two or more claims he moves down to, or remains at, the minimum level.

The probability for each policyholder of making two or more claims in a year is 25% of the probability of making only one claim.

The long-term probability of being at the 15% level is the same as the long-term probability of being at the 40% level.

(i) Derive the probability of a policyholder making only one claim in a given year. [4]

(ii) Determine the probability that a policyholder at the 0% level this year will be at the 40% level after three years. [2]

(iii) Estimate the probability that a policyholder at the 0% level this year will be at the 40% level after 20 years, without calculating the associated transition matrix. [3]
[Total 9]

6 (i) Define the stationary distribution of a Markov chain. [2]

A baseball stadium hosts a match each evening. As matches take place in the evening, floodlights are needed. The floodlights have a tendency to break down. If the floodlights break down, the game has to be abandoned and this costs the stadium \$10,000. If the floodlights work throughout one match there is a 5% chance that they will fail and lead to the abandonment of the next match.

The stadium has an arrangement with the Floodwatch repair company who are brought in the morning after a floodlight breakdown and charge \$1,000 per day. There is a 60% chance they are able to repair the floodlights such that the evening game can take place and be completed without needing to be abandoned. If they are still broken the repair company is used (and paid) again each day until the lights are fixed, with the same 60% chance of fixing the lights each day.

(ii) Write down the transition matrix for the process which describes whether the floodlights are working or not. [1]

(iii) Derive the long run proportion of games which have to be abandoned. [3]

The stadium manager is unhappy with the number of games being abandoned, and contacts the Light Fantastic repair company who are estimated to have an 80% chance of repairing floodlights each day. However Light Fantastic will charge more than Floodwatch.

(iv) Calculate the maximum amount the stadium should be prepared to pay Light Fantastic to improve profitability. [4]
[Total 10]

7 The volatility of equity prices is classified as being High (H) or Low (L) according to whether it is above or below a particular level. The volatility status is assumed to follow a Markov jump process with constant transition rates $\varphi_{LH} = \mu$ and $\varphi_{HL} = \rho$.

(i) Write down the generator matrix of the Markov jump process. [1]

(ii) State the distribution of holding times in each state. [1]

A history of equity price volatility is available over a representative time period.

(iii) Explain how the parameters μ and ρ can be estimated. [2]

Let ${}_t p_s^{ij}$ be the probability that the process is in state j at time s+t given that it was in state i at time s ($i, j = H, L$), where $t \geq 0$. Let ${}_t p_s^{\overline{ii}}$ be the probability that the process remains in state i from time s to time s+t.

(iv) Write down Kolmogorov's forward equations for $\frac{\partial}{\partial t}\, {}_tP_s^{\overline{LL}}$, $\frac{\partial}{\partial t}\, {}_tP_s^{LL}$ and $\frac{\partial}{\partial t}\, {}_tP_s^{LH}$. [2]

Equity price volatility is Low at time zero.

(v) Derive an expression for the time after which there is a greater than 50% chance of having experienced a period of high equity price volatility. [2]

(vi) Solve the Kolmogorov equation to obtain an expression for ${}_tP_0^{LL}$. [4]
[Total 12]

8 (i) Describe a situation when graduation of raw mortality data using a parametric formula might be appropriate and explain why. [2]

(ii) (a) State another method of graduation.
(b) Suggest a situation in which its use may be appropriate. [1]

A large insurance company has graduated the mortality experience of part of its business. The original data and the graduated rates are as follows.

Age	*Exposed to risk*	*Number of deaths*	*Graduated rates* ($\hat{q}_s$)
40	1284	4	0.00240
41	2038	4	0.00266
42	1952	12	0.00297
43	2158	7	0.00332
44	2480	11	0.00371
45	1456	7	0.00415
46	2100	12	0.00464
47	1866	16	0.00519
48	1989	15	0.00577
49	1725	10	0.00642

(iii) Test this graduation for overall goodness of fit. [5]

(iv) Discuss whether it may be necessary to test for smoothness. [2]

(v) Test the data for individual outliers. [3]
[Total 13]

9 A certain town runs a training course for traffic wardens each year. The course lasts for 30 days, but the examination which enables someone to qualify as a traffic warden can be sat any day during the course. In 2011 there were 13 participants who started the training course. The following table has been compiled to show the day each candidate qualified or the day each candidate who did not qualify left the course.

Candidate	*Day qualified*	*Day left without qualifying*
A		30
B	5	
C		21
D	19	
E	12	
F		30
G	1	
H		19
I	12	
J		30
K	15	
L		10
M	24	

(i) Explain whether the following types of censoring are present:

- interval censoring
- right censoring
- informative censoring

[3]

(ii) Calculate the Kaplan-Meier estimate of the non-qualification function. [6]

(iii) Sketch a graph of the Kaplan-Meier estimate, labelling the axes. [2]

When the data were gathered, the reasons for exit of candidates *D* and *H* were accidentally transposed, and those for candidates *B* and *L* were also accidentally transposed.

(iv) Explain how your answer to part (ii) would change if you had access to the correct (i.e. untransposed) data for candidates *D*, *H*, *B* and *L*. [3]

[Total 14]

10 On a small distant planet lives a race of aliens. The aliens can die in one of two ways, either through illness, or by being sacrificed according to the ancient custom of the planet. Aliens who die from either cause may, some time later, become zombies.

(i) Draw a multiple-state diagram with four states illustrating the process by which aliens die and become zombies, labelling the four states and the possible transitions between them. [2]

(ii) Write down the likelihood of the process in terms of the transition intensities, the numbers of events observed and the waiting times in the relevant states, clearly defining all the terms you use. [4]

(iii) Derive the maximum likelihood estimator of the death rate from illness. [3]

The aliens take censuses of their population every ten years (where the year is an "alien year", which is the length of time their planet takes to orbit their sun). On 1 January in alien year 46,567, there were 3,189 live aliens in the population. On 1 January in alien year 46,577 there were 2,811 live aliens in the population. During the intervening ten alien years, a total of 3,690 aliens died from illness and 2,310 were sacrificed, and the annual death rates from illness and sacrifice were constant and the same for each alien.

(iv) Estimate the annual death rates from illness and from sacrifice over the ten alien years between alien years 46,567 and 46,577. [2]

The rate at which aliens who have died from either cause become zombies is 0.1 per alien year.

(v) Calculate the probabilities that an alien alive in alien year 46,567 will, ten alien years later:

(a) still be alive
(b) be dead but not a zombie

[7]
[Total 18]

END OF PAPER