INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2013 (pm)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the differences between deterministic and stochastic models. [4]

2 In the context of a survival model:

(i) Define right censoring, Type I censoring and Type II censoring. [3]

(ii) Give an example of a practical situation in which censoring would be informative. [2]

[Total 5]

3 For both of the following sets of four stochastic processes, place each process in a separate cell of the following table, so that each cell correctly describes the state space and the time space of the process placed in it. Within each set, all four processes should be placed in the table.

		Time Space	
		Discrete	*Continuous*
State Space	*Discrete*		
	Continuous		

(a) General Random Walk, Compound Poisson Process, Counting Process, Poisson Process

(b) Simple Random Walk, Compound Poisson Process, Counting Process, White Noise

[5]

4 The mortality of a certain species of furry animal has been studied. It is known that at ages over five years the force of mortality, μ, is constant, but the variation in mortality with age below five years of age is not understood. Let the proportion of furry animals that survive to exact age five years be ${}_5p_0$.

(i) Show that, for furry animals that die at ages over five years, the average age at death in years is $\dfrac{5\mu+1}{\mu}$. [1]

(ii) Obtain an expression, in terms of μ and ${}_5p_0$, for the proportion of all furry animals that die between exact ages 10 and 15 years. [3]

A new investigation of this species of furry animal revealed that 30 per cent of those born survived to exact age 10 years and 20 per cent of those born survived to exact age 15 years.

(iii) Calculate μ and ${}_5p_0$. [3]

[Total 7]

5 Population censuses in a certain country are taken each year on the President's birthday, provided that the President's astrological advisor deems the taking of a census favourable. Censuses record the age of every inhabitant in completed years (that is, *curtate* age). Deaths in this country are registered as they happen, and classified according to age nearest birthday at the time of death.

Below are some data from the three most recent censuses.

Age in completed years	*Population 2006 (thousands)*	*Population 2009 (thousands)*	*Population 2010 (thousands)*
64	300	320	350
65	290	310	330
66	280	300	320

Between the censuses of 2006 and 2009 there were a total of 3,000 deaths to inhabitants aged 65 nearest birthday, and between the censuses of 2009 and 2010 there were a total of 1,000 deaths to inhabitants aged 65 nearest birthday.

(i) Estimate, stating any assumptions you make, the death rate at age 65 years for each of the following periods:

- the period between the 2006 and 2009 censuses
- the period between the 2009 and 2010 censuses [6]

(ii) Explain the exact age to which your estimates apply. [1]

[Total 7]

6 (i) State the form of the hazard function for the Cox Regression Model, defining all the terms used. [2]

(ii) State two advantages of the Cox Regression Model. [2]

Susanna is studying for an on-line test. She has collected data on past attempts at the test and has fitted a Cox Regression Model to the success rate using three covariates:

Employment	Z_1 = 0 if an employee, and 1 if self-employed
Attempt	Z_2 = 0 if first attempt, and 1 if subsequent attempt
Study time	Z_3 = 0 if no study time taken, and 1 if study time taken

Having analysed the data Susanna estimates the parameters as:

Employment	0.4
Attempt	–0.2
Study time	1.15

Bill is an employee. He has taken study time and is attempting the test for the second time. Ben is self-employed and is attempting the test for the first time without taking study time.

(iii) Calculate how much more or less likely Ben is to pass, compared with Bill. [3]

Susanna subsequently discovers that the effect of the number of attempts is different for employees and the self-employed.

(iv) Explain how the model could be adjusted to take this into account. [2]
[Total 9]

7 The Shining Light company has developed a new type of light bulb which it recently tested. 1,000 bulbs were switched on and observed until they failed, or until 500 hours had elapsed. For each bulb that failed, the duration in hours until failure was noted. Due to an earth tremor after 200 hours, 200 bulbs shattered and had to be removed from the test before failure.

The results showed that 10 bulbs failed after 50 hours, 20 bulbs failed after 100 hours, 50 bulbs failed after 250 hours, 300 bulbs failed after 400 hours and 50 bulbs failed after 450 hours.

(i) Calculate the Kaplan-Meier estimate of the survival function, $S(t)$, for the light bulbs in the test. [6]

(ii) Sketch the Kaplan-Meier estimate calculated in part (i). [2]

(iii) Estimate the probability that a bulb will not have failed after each of the following durations: 300 hours, 400 hours and 600 hours. If it is not possible to obtain an estimate for any of the durations without additional assumptions, explain why. [3]
[Total 11]

8 During a football match, the referee can caution players if they commit an offence by showing them a yellow card. If a player commits a second offence which the referee deems worthy of a caution, they are shown a red card, and are sent off the pitch and take no further part in the match. If the referee considers a particularly serious offence has been committed, he can show a red card to a player who has not previously been cautioned, and send the player off immediately.

The football team manager can also decide to substitute one player for another at any point in the match so that the substituted player takes no further part in the match. Due to the risk of a player being sent off, the manager is more likely to substitute a player who has been shown a yellow card. Experience shows that players who have been shown a yellow card play more carefully to try to avoid a second offence.

The rate at which uncautioned players are shown a yellow card is 1/10 per hour.

The rate at which those players who have already been shown a yellow card are shown a red card is 1/15 per hour.

The rate at which uncautioned players are shown a red card is 1/40 per hour.

The rate at which players are substituted is 1/10 per hour if they have not been shown a yellow card, and 1/5 if they have been shown a yellow card.

(i) Sketch a transition graph showing the possible transitions between states for a given player. [2]

(ii) Write down the compact form of the Kolmogorov forward equations, specifying the generator matrix. [3]

A football match lasts 1.5 hours.

(iii) Solve the Kolmogorov equation for the probability that a player who starts the match remains in the game for the whole match without being shown a yellow card or a red card. [2]

(iv) Calculate the probability that a player who starts the match is sent off during the match without previously having been cautioned. [3]

Consider a match that continued indefinitely rather than ending after 1.5 hours.

(v) (a) Derive the probability that in this instance a player is sent off without previously having been cautioned.

(b) Explain your result. [2]

[Total 12]

9 A life office compared the mortality of its policyholders in the age range 30 to 60 years inclusive with a set of mortality rates prepared by the Continuous Mortality Investigation (CMI). The mortality of the life office policyholders was higher than the CMI rates at ages 30–35, 38–41, 45–50 and 54–59 years inclusive, and lower than the CMI rates at all other ages in the age range.

(i) Perform two tests of the null hypothesis that the underlying mortality of the life office policyholders is represented by the CMI rates. [7]

(ii) Comment on your results from part (i). [2]

(ii) Explain the problem which duplicate policies cause in the context of the CMI mortality investigations. [3]

[Total 12]

10 (i) State the Markov property. [1]

A certain non-fatal medical condition affects adults. Adults with the condition suffer frequent episodes of blurred vision. A study was carried out among a group of adults known to have the condition. The study lasted one year, and each participant in the study was asked to record the duration of each episode of blurred vision. All participants remained under observation for the entire year.

The data from the study were analysed using a two-state Markov model with states:

1. not suffering from blurred vision.
2. suffering from blurred vision.

Let the transition rate from state i to state j at time $x+t$ be μ_{x+t}^{ij}, and let the probability that a person in state i at time x will be in state j at time $x+t$ be ${}_t p_x^{ij}$.

(ii) Derive from first principles the Kolmogorov forward equation for the transition from state 1 to state 2. [5]

The results of the study were as follows:

Participant-days in state 1	21,650
Participant-days in state 2	5,200
Number of transitions from state 1 to state 2	4,330
Number of transitions from state 2 to state 1	4,160

Assume the transition intensities are constant over time.

(iii) Calculate the maximum likelihood estimates of the transition intensities from state 1 to state 2 and from state 2 to state 1. [1]

(iv) Estimate the probability that an adult with the condition who is presently not suffering from blurred vision will be suffering from blurred vision in 3 days' time. [6]

[Total 13]

11 (i) Explain what is meant by a time inhomogeneous Markov chain and give an example of one. [2]

A No Claims Discount system is operated by a car insurer. There are four levels of discount: 0%, 10%, 25% and 40%. After a claim-free year a policy holder moves up one level (or remains at the 40% level). If a policy holder makes one claim in a year he or she moves down one level (or remains at the 0% level). A policy holder who makes more than one claim in a year moves down two levels (or moves to or remains at the 0% level). Changes in level can only happen at the end of each year.

(ii) Describe, giving an example, the nature of the boundaries of this process. [2]

(iii) (a) State how many states are required to model this as a Markov chain.

(b) Draw the transition graph. [2]

The probability of a claim in any given month is assumed to be constant at 0.04. At most one claim can be made per month and claims are independent.

(iv) Calculate the proportion of policyholders in the long run who are at the 25% level. [6]

(v) Discuss the appropriateness of the model. [3]

[Total 15]

END OF PAPER