INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

18 April 2012 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Define a general random walk. [1]

(ii) State the conditions under which a general random walk would become a simple random walk. [1]
[Total 2]

2 (i) Explain the reasons why data are subdivided when conducting mortality investigations. [2]

(ii) Describe the problems which can arise with subdividing data. [2]
[Total 4]

3 A graduation of a set of crude mortality rates is tested for goodness-of-fit using a chi-squared test.

Discuss the factors to be considered in determining the number of degrees of freedom to use for the test statistic.
[4]

4 A new drug treatment for patients suffering from a chronic skin disease with visible symptoms was tested. The drug was administered through a daily dose for the duration of the trial. As soon as the drug regime started, the symptoms disappeared in all patients, but after some time had a tendency to reappear as the agent causing the disease developed resistance to the drug. The trial lasted for six months.

The data below show the number of patients experiencing a return of their symptoms in each month after the drug regime started.

Month	*Number of patient-months exposed to risk*	*Number of patients experiencing a return of their symptoms*
1	200	5
2	190	8
3	175	15
4	150	10
5	135	6
6	125	3

(i) Calculate the hazard of symptoms returning in each month. [2]

As part of the investigation, it is desired to assess the impact of certain risk factors on the hazard of symptoms returning. It is suggested that to achieve this, the hazard could be modelled using either a Gompertz model or a semi-parametric model.

(ii) Comment on the use of each of these models in this situation. [4]
[Total 6]

5 For a particular investigation the hazard of mortality is assumed to take the form:

$$h(t) = A + Bt$$

where A and B are constants and t represents time.

For each life i in the investigation (i = 1, …, n) information was collected on the length of time the life was observed t_i and whether the life exited due to death ($\delta_i = 1$ if the life died, 0 otherwise).

(i) Show that the likelihood of the data is given by:

$$L = \prod_{i=1}^{n} (A + Bt_i)^{\delta_i} \exp[-At_i - \frac{1}{2}Bt_i^2].$$ [3]

(ii) Derive two simultaneous equations from which the maximum likelihood estimates of the parameters A and B can be calculated. [3]
[Total 6]

6 (i) List the advantages and disadvantages of using models in actuarial work. [4]

A new town is planned in a currently rural area. A model is to be developed to recommend the number and size of schools required in the new town. The proposed modelling approach is as follows:

- The current age distribution of the population in the area is multiplied by the planned population of the new town to produce an initial population distribution.
- Current national fertility and mortality rates by age are used to estimate births and deaths.
- The births and deaths are applied to the initial population distribution to generate a projected distribution of the town's population by age for each future year, and hence the number of school age children.

(ii) Discuss the appropriateness of the proposed modelling approach. [5]
[Total 9]

7 Mr Bunn the baker made 12 pies to sell in his shop. He placed the pies in the shop at 9 a.m. During the rest of the day the following events took place.

Time	*Event*
10 a.m.	A boy bought two pies
11 a.m.	A man bought three pies
12 noon	Mr Bunn accidentally sat on one pie and squashed it so it could not be sold
1 p.m.	A woman bought two pies
2 p.m.	A dog from across the street ran into Mr Bunn's shop and stole two pies
3 p.m.	A girl on the way home from school bought one pie
5 p.m.	Mr Bunn closed for the day and the remaining pie was still in the shop

(i) Estimate the time it takes Mr Bunn to sell 40% of the pies he makes, using the Nelson-Aalen estimator. [6]

(ii) Comment on whether you think this estimate would be a good basis for Mr Bunn to plan his future production of pies. [3]
[Total 9]

8 The mortality experience of a large company pension scheme is to be tested to see if the experience of males aged 65–72 years is consistent with a standard table. The results were collated by the firm conducting the analysis on a computer spreadsheet, with positive and negative standardised deviations being distinguished only by being in a different coloured font. Unfortunately the results have been supplied to the company in the form of a printout produced on a black-and-white printer from which it is not possible to tell the signs of the deviations.

The values of the standardised deviations shown are as follows:

0.052
0.967
2.528
0.328
1.234
0.250
1.023
0.756

(i) Suggest two tests which could be conducted from the information given. [2]

(ii) Carry out the tests you suggested in your answer to part (i). [8]
[Total 10]

9 (i) List four factors other than age and smoker status by which life insurance mortality statistics are often subdivided. [2]

Two offices in different towns of the same life insurance company write 25-year term assurance policies. Below are data from these two offices relating to policyholders of the same age. Both deaths and policies in force are on an age last birthday basis.

	Gasperton	*Great Hawking*
Policies in force on 1 January 2009	2,000	1,770
Policies in force on 1 January 2010	2,100	1,674
Deaths in calendar year 2009	25	21

(ii) Calculate the central death rate for the calendar year 2009 at this age for the offices in Gasperton and Great Hawking. [2]

A detailed examination of the records shows that 50% of the policyholders in Gasperton at both censuses were smokers, and 20% of policyholders in Great Hawking at both censuses were smokers. National death rates at this age for smokers in 2009 were 40% higher than those for non-smokers.

(iii) Estimate the central death rates for smokers and non-smokers in Gasperton and Great Hawking. [4]

The life insurance company charges policyholders in Gasperton and Great Hawking the same premiums for the 25-year term assurance policies. It charges smokers in both towns 40% more than non-smokers.

(iv) Comment on the company's pricing structure in the light of your results from parts (ii) and (iii) above. [3]

[Total 11]

10 An investigation was conducted into the effect marriage has on mortality and a model was constructed with three states: 1 Single, 2 Married and 3 Dead. It is assumed that transition rates between states are constant.

(i) Sketch a diagram showing the possible transitions between states. [2]

(ii) Write down an expression for the likelihood of the data in terms of transition rates and waiting times, defining all the terms you use. [3]

The following data were collected from information on males and females in their thirties.

Years spent in Married state	40,062
Years spent in Single state	10,298
Number of transitions from Married to Single	1,382
Number of transitions from Single to Dead	12
Number of transitions from Married to Dead	9

(iii) Derive the maximum likelihood estimator of the transition rate from Single to Dead. [4]

(iv) Estimate the constant transition rate from Single to Dead and its variance. [2]
[Total 11]

11 The series Y_i records, for each time period i, whether a car driver is accident free during that period ($Y_i = 0$) or has at least one accident ($Y_i = 1$).

Define $X_i = \sum_{j=1}^{i} Y_j$ with state space $\{0,1,2,\ldots.\}$.

An insurer makes an assumption about the driver's accident proneness by considering that the probability of a driver having at least one accident is related to the proportion of previous time periods in which the driver had at least one accident as follows:

$$P(Y_{n+1} = 1) = \frac{1}{4}\left(1 + \frac{X_n}{n}\right), \quad \text{for} \quad n \geq 1$$

with $P(Y_1 = 1) = \frac{1}{2}$

(i) Demonstrate that the series X_i satisfies the Markov property, whilst Y_i does not. [2]

(ii) Explain whether the series X_i is:

(a) irreducible
(b) time homogeneous

[3]

(iii) Draw the transition graph for X_i covering all transitions which could occur in the first three time periods, including the transition probabilities. [4]

(iv) Calculate the probability that the driver has accidents during exactly two of the first three time periods. [2]

(v) Comment on the appropriateness of the insurer's assumption about accident proneness. [2]
[Total 13]

12 A company operates a sick pay scheme as follows:

- Healthy employees pay a percentage of salary to fund the scheme.

- For the first two consecutive months an employee is sick, the sick pay scheme pays their full salary.

- For the third and subsequent consecutive months of sickness the sick pay is reduced to 50% of full salary.

To simplify administration the scheme operates on whole months only, that is for a particular month's payroll an employee is either healthy or sick for the purpose of the scheme.

The company's experience is that 10% of healthy employees become sick the following month, and that sick employees have a 75% chance of being healthy the next month.

The scheme is to be modelled using a Markov Chain.

(i) Explain what is meant by a Markov Chain. [1]

(ii) Identify the minimum number of states under which the payments under the scheme can be modelled using a time homogeneous Markov Chain, specifying these states. [2]

(iii) Draw a transition graph for this Markov chain. [2]

(iv) Derive the stationary distribution for this process. [4]

(v) Calculate the minimum percentage of salary which healthy employees should pay for the scheme to cover the sick pay costs. [2]

(vi) Calculate the contributions required if, instead, sick pay continued at 100% of salary indefinitely. [2]

(vii) Comment on the benefit to the scheme of the reduction in sick pay to 50% from the third month. [2]
[Total 15]

END OF PAPER