INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

2 October 2013 (pm)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all nine questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Data are often subdivided when investigating mortality statistics.

(i) Explain why this is done. [2]

(ii) Discuss one potential problem with sub-dividing mortality data. [2]

(iii) List four factors which are commonly used to sub-divide mortality data. [2]

[Total 6]

2 The two football teams in a particular city are called United and City and there is intense rivalry between them. A researcher has collected the following history on the results of the last 20 matches between the teams from the earliest to the most recent, where:

U indicates a win for United;
C indicates a win for City;
D indicates a draw.

UCCDDUCDCUUDUDCCUDCC

The researcher has assumed that the probability of each result for the next match depends only on the most recent result. He therefore decides to fit a Markov chain to this data.

(i) Estimate the transition probabilities for the Markov chain. [3]

(ii) Estimate the probability that United will win at least two of the next three matches against City. [3]

[Total 6]

3 (i) Define a Poisson process. [2]

A bus route in a large town has one bus scheduled every 15 minutes. Traffic conditions in the town are such that the arrival times of buses at a particular bus stop may be assumed to follow a Poisson process.

Mr Bean arrives at the bus stop at 12 midday to find no bus at the stop. He intends to get on the first bus to arrive.

(ii) Determine the probability that the first bus will not have arrived by 1.00 pm the same day. [2]

The first bus arrived at 1.10 pm but was full, so Mr Bean was unable to board it.

(iii) Explain how much longer Mr Bean can expect to wait for the second bus to arrive. [1]

(iv) Calculate the probability that at least two more buses will arrive between 1.10 pm and 1.20 pm. [2]

[Total 7]

4 (i) State what needs to be assessed when considering the suitability of a model for a particular purpose. [5]

A city has care homes for the elderly. When an elderly person in the city is no longer able to cope alone at home they can move into a care home where they will be looked after until they die.

The city council runs some of the care homes and is reviewing the number of rooms it needs in all the council-run care homes. The following model has been proposed.

The age distribution of the city population over the age of 60 is taken from the latest census. Statistics from the national health service on the average age of entry into a care home and the proportion of elderly who go into council-run care homes are applied to the current population to give the rate at which people enter care homes. A recent national mortality table is then applied to give the rate at which care home residents die.

(ii) Discuss the suitability of this model. [4]

[Total 9]

5 A motor insurer offers a No Claims Discount scheme which operates as follows. The discount levels are {0%, 25%, 50%, 60%}. Following a claim-free year a policyholder moves up one discount level (or stays at the maximum discount). After a year with one or more claims the policyholder moves down two discount levels (or moves to, or stays in, the 0% discount level).

The probability of making at least one claim in any year is 0.2.

(i) Write down the transition matrix of the Markov chain with state space {0%, 25%, 50%, 60%}. [2]

(ii) State, giving reasons, whether the process is:

(a) irreducible.
(b) aperiodic.

[2]

(iii) Calculate the proportion of drivers in each discount level in the stationary distribution. [4]

The insurer introduces a "protected" No Claims Discount scheme, such that if the 60% discount is reached the driver remains at that level regardless of how many claims they subsequently make.

(iv) Explain, without doing any further calculations, how the answers to parts (ii) and (iii) would change as a result of introducing the "protected" No Claims Discount scheme. [3]

[Total 11]

6 (i) Explain what is meant by censoring in the context of a mortality investigation. [1]

A trial was conducted on the effectiveness of a new cream to treat a skin condition. 100 sufferers applied the cream daily for four weeks or until their symptoms disappeared if this happened sooner. Some of the sufferers left the trial before their symptoms disappeared.

(ii) Describe two types of censoring that are present and state to whom they apply. [2]

The following data were collected.

Number of sufferers	*Day symptoms disappeared*	*Number of sufferers*	*Day they left the trial*
2	6	3	2
1	7	1	10
1	10	3	13
2	14		

(iii) Calculate the Nelson-Aalen estimate of the survival function for this trial. [5]

(iv) Sketch the survival function, labelling the axes. [2]

(v) Estimate the probability that a person using the cream will still have symptoms of the skin condition after two weeks. [1]

[Total 11]

7 (i) Explain why the Gompertz model is commonly used in investigations of human mortality. [1]

The following model of mortality was used in an investigation of the effects of where someone lives and income on the risk of death.

$$\log_e \mu_x = \alpha + \beta_0 x + \beta_1 U + \beta_2 I ,$$

where μ_x is the force of mortality at age x, U takes the value 1 if the person lives in an urban area and 0 if the person lives in a rural area, I is annual income in US dollars, and α, β_0, β_1 and β_2 are parameters.

(ii) Show that the model is both a Gompertz model and a proportional hazards model. [3]

The estimates of the parameters were $\alpha = -9.0$ $\beta_0 = 0.09$, $\beta_1 = 0.3$ and $\beta_2 = -0.0001$.

(iii) Calculate the predicted force of mortality for an urban resident aged 40 years with an annual income of \$20,000. [2]

(iv) Calculate the additional income that an urban resident must have in order to have the same force of mortality as a rural resident of the same age. [2]

(v) Calculate the 10-year survival probability for an urban resident aged 40 years whose annual income is \$20,000. [2]

(vi) Determine the age of a rural resident with the same income as an urban resident aged 40 years, who has the same chance of surviving for the next 10 years. [4]

[Total 14]

8 Outside an apartment block there is a small car park with three parking spaces. A prospective purchaser of an apartment in the block is concerned about how often he would return in his car to find that there was no empty parking space available. He decides to model the number of parking spaces free at any time using a time homogeneous Markov Jump Process where:

- The probability that a car will arrive seeking a parking space in a short interval dt is $A.dt + o(dt)$.
- For each car which is currently parked, the probability that its owner drives the car away in a short interval dt is $B.dt + o(dt)$.

where $A, B > 0$.

(i) Specify the state space for the above process. [1]

(ii) Draw a transition graph of the process. [2]

(iii) Write down the generator matrix for the process. [2]

(iv) Derive the probability that, given all the parking spaces are full, they will remain full for at least the next two hours. [2]

(v) Explain what is meant by a jump chain. [1]

(vi) Specify the transition matrix for the jump chain associated with this process. [2]

Suppose there are currently two empty parking spaces.

(vii) Determine the probability that all the spaces become full before any cars are driven away. [1]

(viii) Derive the probability that the car park becomes full before the car park becomes empty. [3]

(ix) Comment on the prospective purchaser's assumptions regarding the arrival and departure of cars. [3]

[Total 17]

9 (i) (a) State three different methods of graduating crude mortality data.

(b) Give, for each method, one advantage and one disadvantage.

[5]

An insurance company has graduated the experience of one block of its life business against a standard table, the following is an extract of the data.

Age x	*Exposed to risk*	*Observed deaths*	*Graduated rates*
30	36,254	26	0.000590
31	37,259	20	0.000602
32	28,057	23	0.000617
33	31,944	23	0.000636
34	30,005	26	0.000660
35	28,389	12	0.000689
36	36,124	31	0.000724
37	28,152	22	0.000765
38	24,001	25	0.000813
39	30,448	31	0.000870

(ii) Carry out a test for overall goodness of fit. [5]

(iii) Carry out two other statistical tests to check the validity of the graduation. [6]

(iv) Discuss, with reference to the tests you have performed, whether it would be reasonable for the company to use the graduated rates to price life insurance policies. [3]

[Total 19]

END OF PAPER