EXAMINATION

21 April 2010 (am)

Subject CT4 — Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List four factors often used to subdivide life insurance mortality statistics. [2]

2 Write down integral equations for the mean and variance of the complete future lifetime at age x, T_x. [2]

3 For each of the following processes:

counting process;
general random walk;
compound Poisson process;
Poisson process;
Markov jump chain.

(a) State whether the state space is discrete, continuous or can be either.

(b) State whether the time set is discrete, continuous, or can be either.

[5]

4 A Markov Chain with state space {A, B, C} has the following properties:

- it is irreducible
- it is periodic
- the probability of moving from A to B equals the probability of moving from A to C

(i) Show that these properties uniquely define the process. [4]

(ii) Sketch a transition diagram for the process. [1]

[Total 5]

5 Ten years ago, a confectionery manufacturer launched a new product, the Scrummy Bar. The product has been successful, with a rapid increase in consumption since the product was first sold. In order to plan future investment in production capacity, the manufacturer wishes to forecast the future demand for Scrummy Bars. It has data on age-specific consumption rates for the past ten years, together with projections of the population by age over the next twenty years. It proposes the following modelling strategy:

- extrapolate past age-specific consumption rates to forecast age-specific consumption rates for the next 20 years
- apply the forecast age-specific consumption rates to the projected population by age to obtain estimated total consumption of the product by age for each of the next 20 years
- sum the results to obtain the total demand for each year

Describe the advantages and disadvantages of this strategy. [5]

6 An oil company has discovered a vast deposit of oil in an equatorial swamp. The area is extremely unhealthy and inhabited by venomous spiders. There is an antidote to bites from these spiders but it is expensive. The antidote acts instantly but does not provide future immunity. The company commissions a study to estimate the rate of being bitten by the spiders among its employees, in order to determine the amount of antidote to provide.

Employees of the company are posted to the swamp for six month tours of duty starting on 1 January, 1 April, 1 July or 1 October. The first employees to be posted arrived on 1 January 2008. The swamp is so inaccessible that no employees are allowed to leave before their six month tours of duty are completed.

Accidental deaths are common in this dangerous location.

The table below gives some data from the study.

Quarter beginning	*Number of new arrivals at start of quarter*	*Number of accidental deaths during quarter*	*Number of spider bites during quarter*
1 January 2008	90	10	15
1 April 2008	80	8	25
1 July 2008	114	10	30
1 October 2008	126	13	40

(i) Estimate the quarterly rate of being bitten by a spider for each quarter of 2008, stating any assumptions you make. [7]

(ii) Suggest reasons why the assumptions you made in (i) might not be valid. [1]

[Total 8]

7 A government has introduced a two-tier driving test system. Once someone applies for a provisional licence they are considered a Learner driver. Learner drivers who score 90% or more on the primary examination (which can be taken at any time) become Qualified. Those who score between 50% and 90% are obliged to sit a secondary examination and are given driving status Restricted. Those who score 50% or below on the primary examination remain as Learners. Restricted drivers who pass the secondary examination become Qualified, but those who fail revert back to Learner status and are obliged to start again.

(i) Sketch a diagram showing the possible transitions between the states. [2]

(ii) Write down the likelihood of the data, assuming transition rates between states are constant over time, clearly defining all terms you use. [3]

Figures over the first year of the new system based on those who applied for a provisional licence during that time in one area showed the following:

Person-months in Learner State	1,161
Person-months in Restricted State	1,940
Number of transitions from Learner to Restricted	382
Number of transitions from Restricted to Learner	230
Number of transitions from Restricted to Qualified	110
Number of transitions from Learner to Qualified	217

(iii) (a) Derive the maximum likelihood estimator of the transition rate from Restricted to Learner.

(b) Estimate the constant transition rate from Restricted to Learner.

[3]

[Total 8]

8 A certain profession admits new members to the status of student. Students may qualify as fellows of the profession by virtue of passing a series of examinations. Normally student members sit the examinations whilst working for an employer. There are two sessions of the examinations each year.

An employer provides study support to student members of the profession. It wishes to assess the cost of providing this study support and therefore wishes to know the average time it can expect to take for its students to qualify.

The employer has maintained records for 23 of its students who all sat their first examination in the first session of 2003. The students' progress has been recorded up to and including the last session of 2009. The following data records the number of sessions which had been held before the specified event occurred for a student in this cohort:

Qualified	6, 8, 8, 9, 9, 9, 11, 11, 13, 13, 13
Stopped studying	4, 5, 8, 11, 14

The remaining seven students were still studying for the examinations at the end of 2009.

(i) Determine the median number of sessions taken to qualify for those students who qualified during the period of observation. [2]

(ii) Calculate the Kaplan-Meier estimate of the survival function, $S(t)$, for the "hazard" of qualifying, where t is the number of sessions of examinations since 1 January 2003. [5]

(iii) Hence estimate the median number of sessions to qualify for the students of this employer. [2]

(iv) Explain the difference between the results in (i) and (iii) above. [2]

[Total 11]

9 (i) Write down the hazard function for the Cox proportional hazards model defining all the terms that you use. [2]

A farmer is concerned that he is losing a lot of his birds to a predator, so he decides to build a new enclosure using taller fencing. This fencing is expensive and he cannot afford to build a large enough area for all his birds. He therefore decides to put half his birds in the new enclosure and leave the others in the existing enclosure. He is convinced that the new enclosure is an improvement, but has asked an actuarial student to determine whether the new enclosure will result in an increase in the life expectancy of his birds. The student has fitted a Cox proportional hazards model to data on the duration until a bird is killed by a predator and calculated the following figures relating to the regression parameters:

		Parameter estimate	*Variance*
Bird	Chicken	0	0
	Duck	–0.210	0.002
	Goose	0.075	0.004
Enclosure	New	0.125	0.0015
	Old	0	0
Sex	Male	0.2	0.0026
	Female	0	0

(ii) State the features of the bird to which the baseline hazard applies. [1]

(iii) For each regression parameter:

(a) Define the associated covariate.

(b) Calculate the 95% confidence interval based on the standard error. [3]

(iv) Comment on the farmer's belief that the new enclosure will result in an increase in his birds' life expectancy. [2]

(v) Calculate, using this model, the probability that a female duck in the new enclosure has been killed by a predator at the end of six months, given that the probability that a male goose in the old enclosure has been killed at the end of the same period is 0.1 (all other decrements can be ignored). [4]

[Total 12]

10 An airline runs a frequent flyer scheme with four classes of member: in ascending order Ordinary, Bronze, Silver and Gold. Members receive benefits according to their class. Members who book two or more flights in a given calendar year move up one class for the following year (or remain Gold members), members who book exactly one flight in a given calendar year stay at the same class, and members who book no flights in a given calendar year move down one class (or remain Ordinary members).

Let the proportions of members booking 0, 1 and 2+ flights in a given year be p_0, p_1 and p_{2+} respectively.

(i) (a) Explain how this scheme can be modelled as a Markov chain.

(b) Explain why there must be a unique stationary distribution for the proportion of members in each class.

[3]

(ii) Write down the transition matrix of the process. [1]

The airline's research has shown that in any given year, 40% of members book no flights, 40% book exactly one flight, and 20% book two or more flights.

(iii) Calculate the stationary probability distribution. [5]

The cost of running the scheme per member per year is as follows:

Ordinary members	£0
Bronze members	£10
Silver members	£20
Gold members	£30

The airline makes a profit of £10 per passenger for every flight before taking into account costs associated with the frequent flyer scheme.

(iv) Assess whether the airline makes a profit on the members of the scheme. [4]

[Total 13]

11 A reinsurance policy provides cover in respect of a single occurrence of a specified catastrophic event. If such an event occurs, future cover is suspended. However if a reinstatement premium is paid within one time period of occurrence of the event then the insurance coverage is reinstated. If a second specified event occurs it is not permitted to reinstate the cover and the policy will lapse.

The transition rate for the hazard of the specified event is a constant 0.1. Whilst policies are eligible for reinstatement, the transition rate for resumption of cover through paying a reinstatement premium is 0.05.

(i) Explain whether a time homogeneous or time inhomogeneous model would be more appropriate for modelling this situation. [2]

(ii) (a) Explain why a model with state space {Cover In Force, Suspended, Lapsed} does not possess the Markov property.

(b) Suggest, giving reasons, additional state(s) such that the expanded system would possess the Markov property. [3]

(iii) Sketch a transition diagram for the expanded system. [2]

(iv) Derive the probability that a policy remains in the Cover In Force state continuously from time 0 to time t. [2]

(v) Derive the probability that a policy is in the Suspended state at time $t > 1$ if it is in state Cover In Force at time 0. [5]

[Total 14]

12 (i) State three different methods of graduating raw mortality data and for each method give an example of a situation when the method would be appropriate. [3]

A life insurance company last priced its whole of life contract 30 years ago using a standard mortality table. The company wishes to establish whether recent mortality experience in the portfolio of business is in line with the pricing basis. These are the data:

Recent Experience			*Extract from the standard table used for pricing the product*	
Age last birthday	*Exposed to Risk during 2009*	*Deaths during 2009*	*x*	*Number of survivors to age x*
50	2,381	16	50	32,669
51	3,177	21	51	32,513
52	3,460	22	52	32,338
53	1,955	15	53	32,143
54	3,122	24	54	31,926
55	3,485	29	55	31,685
56	2,781	26	56	31,417
57	3,150	31	57	31,121
58	3,651	39	58	30,795
59	3,991	48	59	30,435
			60	30,039

(ii) Test the goodness of fit of these data with the pricing basis and comment on your results. [8]

(iii) (a) State, with reasons, one further test which you would deem appropriate to perform on these data.

(b) Carry out that test.

[4]
[Total 15]

END OF PAPER