

Life Conference and Exhibition 2011
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Using least squares Monte Carlo for capital calculation

21 November 2011

Agenda

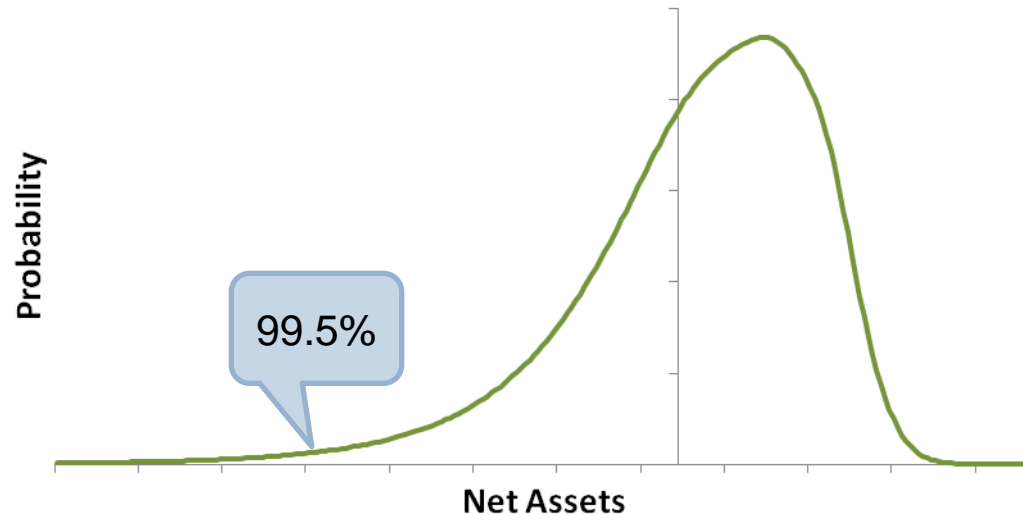
- SCR calculation
 - Nested stochastic problem
 - Limitations of the covariance matrix approach
- The least squares Monte Carlo technique
 - Theory
 - Practice
- Conclusions

What is capital?

- The amount of money we need today to ensure we will (probably) be solvent in the future
- | | | | |
|-----------|--------|------------|----------|
| Probably? | When? | Solvent? | Measured |
| 99.5% | 1 Year | Net Assets | VaR |
- Project distribution of our Net Assets in 1 year
- Difficult to do, especially where our liabilities have options and guarantees
 - Even calculating our Net Assets today requires many Monte Carlo simulations and days of calculation

How do we calculate capital?

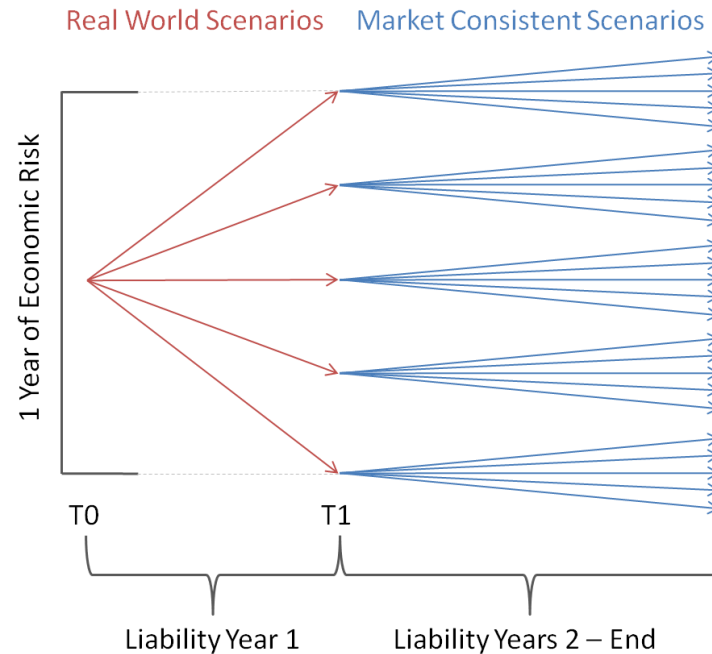
- Ideally want to know what the full probability distribution of Net Assets looks like in 1 year



- May settle for knowing the shape of the left tail which is where capital is calculated

Nested stochastic problem

- Need to do many thousands of real world scenarios
- With thousands of risk neutral scenarios for each real world scenario



Covariance matrix

- Determine the 99.5% instantaneous shock for each individual risk
- Calculate change in balance sheet for each risk
- Assume that risks follow a multivariate normal (elliptical) distribution
- Aggregate capital requirement = “root sum squares”

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}$$

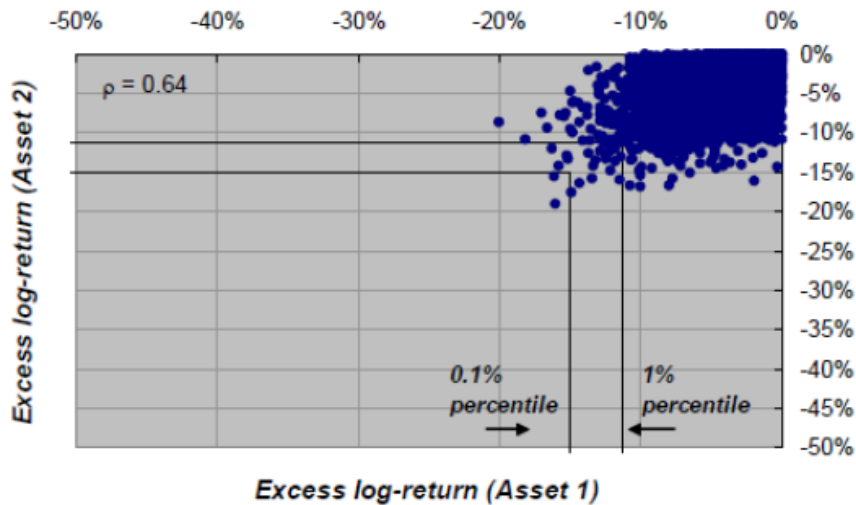
Covariance matrix

- Problem with this approach
- To calculate distribution of capital we need to know:
 - Joint probabilities of economic (and other) events occurring
 - What the balance sheet will look like under these events
- Covariance matrix assumes:
 - Multivariate normal (elliptical) distribution of risk factors
 - Losses are linearly dependent on risk factors
- Both are important for a capital calculation
- Often language can be confusing

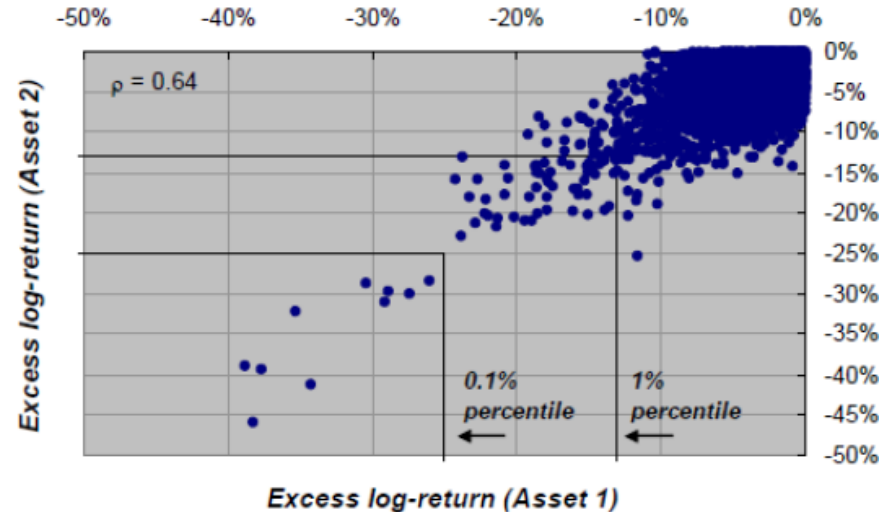
Non-linear dependence

- Correlation only explains linear dependence

Bivariate Lognormal
(linear correlation)



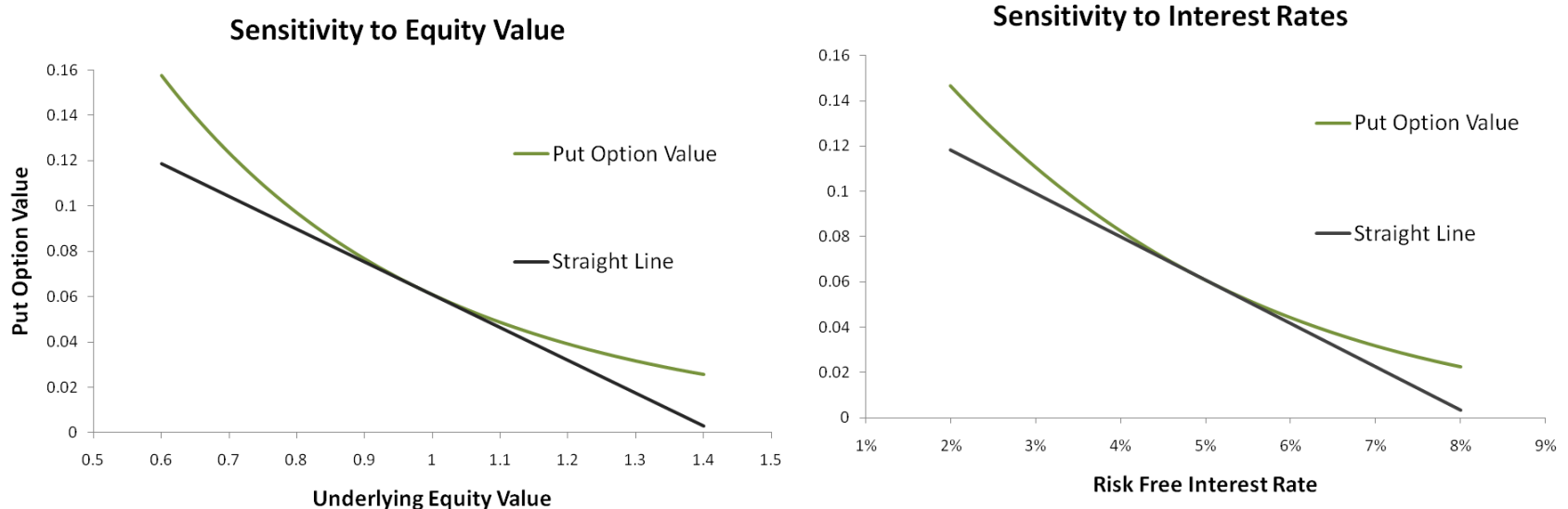
Fat tails and increased tail correlation
(non linear dependence)



- We may want to use more sophisticated real world models

Non-linear liabilities

- Liabilities are non-linear under changes in individual risk drivers



- And in joint stresses
 - When equity and interest rates change the loss is bigger than the sum of the two!

Proxy models

- We can solve these problems by using nested stochastic simulation
 - But this typically involves millions of scenarios which makes it too time consuming to run
- The use of proxy models has become a popular solution for this problem
 - Curve fitting
 - Replicating portfolios
 - Least squares Monte Carlo

Why use LSMC?

The alternatives

Options	Curve Fitting	Replicating Portfolios
Description	Accurate valuations are carried out for a smaller number of shocked scenarios. Then functions are fitted to these scenarios	A portfolio of assets is identified which responds to risk factors in a similar fashion to the liabilities. This can then be used to provide a rapid estimate of liability sensitivities
Runs Required	Maybe 100 * 2,000 simulations	10,000 - 20,000 simulations
Advantages	<ul style="list-style-type: none"> ▪ Relatively simple ▪ Easy to explain ▪ Consistent with methodology used for NP business ▪ Can include non-market risks 	<ul style="list-style-type: none"> ▪ Allows for well for complex dynamics of a With Profit Fund ▪ Efficient use of simulations ▪ Results can be explained in terms of well understood assets
Disadvantages	<ul style="list-style-type: none"> ▪ Requires a large number of results ▪ More likely to miss some of the complex behaviour 	<ul style="list-style-type: none"> ▪ Complex to implement requiring a large degree of asset modelling expertise ▪ Harder to incorporate non-market risks

Why use LSMC?

The solution

- Least Squares Monte Carlo offers the best of both of the previous options?
 - **Complex liabilities** – complex liabilities can be allowed for by introducing a number of non linear and cross terms to the functions used to estimate liabilities. Also the very wide range of shocks used ensure that all of the likely behaviour of the fund is allowed for.
 - **Simulations** – providing rapid updates of the solvency position of a fund. This could also feed into various reporting processes.
 - **Explanation** – while there is some complicated maths involved this is really just an extension of the simple curve fitting option.
 - **Output** – As the liabilities are described by polynomial functions these are easy to integrate with a capital aggregation tool.

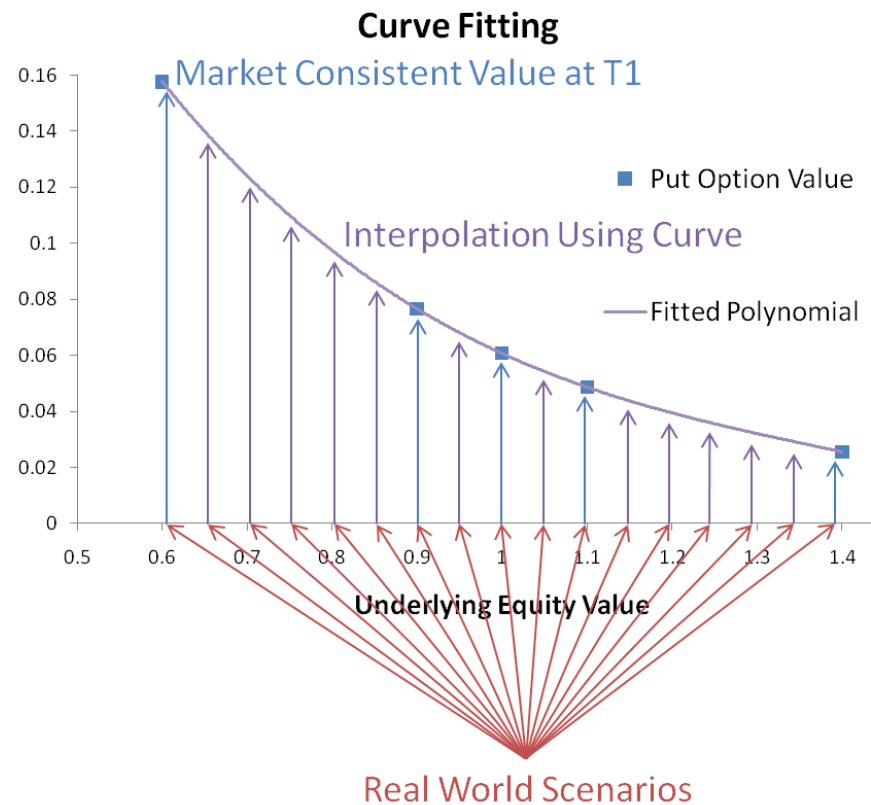
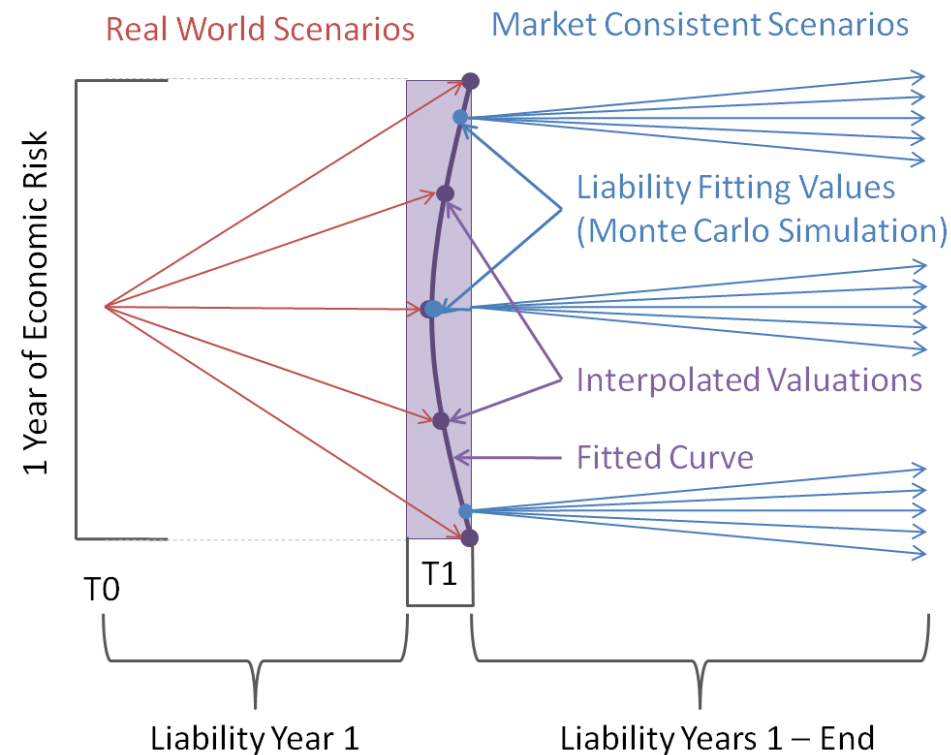
Least squares Monte Carlo background

- A technique originally applied in American option pricing
 - The decision to continue is a nested stochastic problem
 - Longstaff & Schwartz & others
- Uses a regression through Monte Carlo scenarios to approximate the continuation value
- The application to SCR calculation is slightly different and has parallels with curve fitting method and replicating portfolios

Curve Fitting

- Commonly used among UK life insurers today
- Perform a number of instantaneous shocks to balance sheet
- Fit a multi-dimensional formula to instantaneous shocks
- Use Monte Carlo simulation to get distribution of economic (and other) variables
- Calculate capital required from distribution of balance sheet

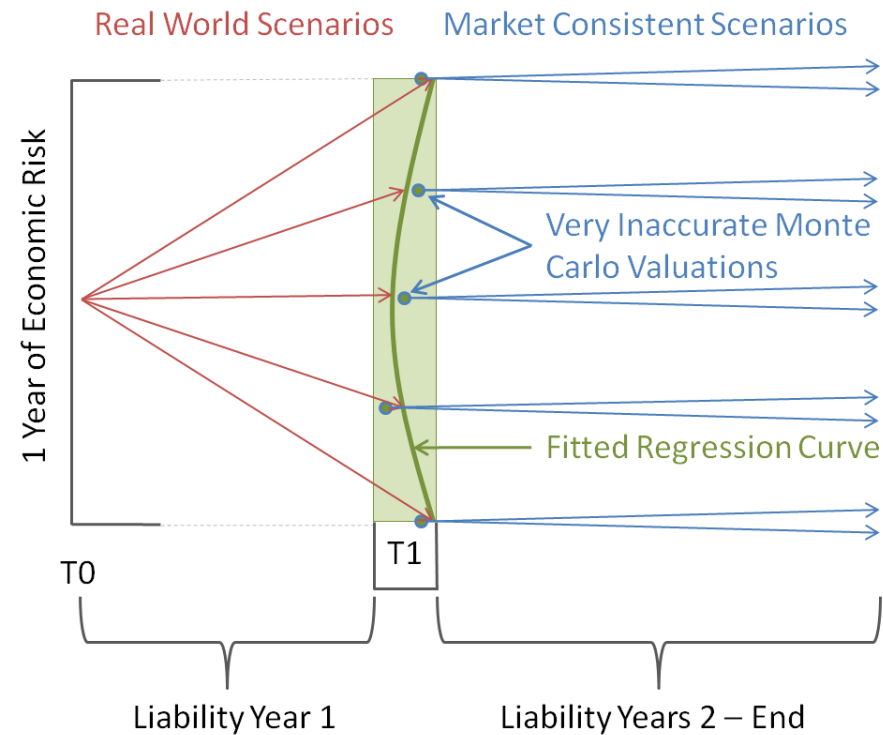
Curve fitting approach



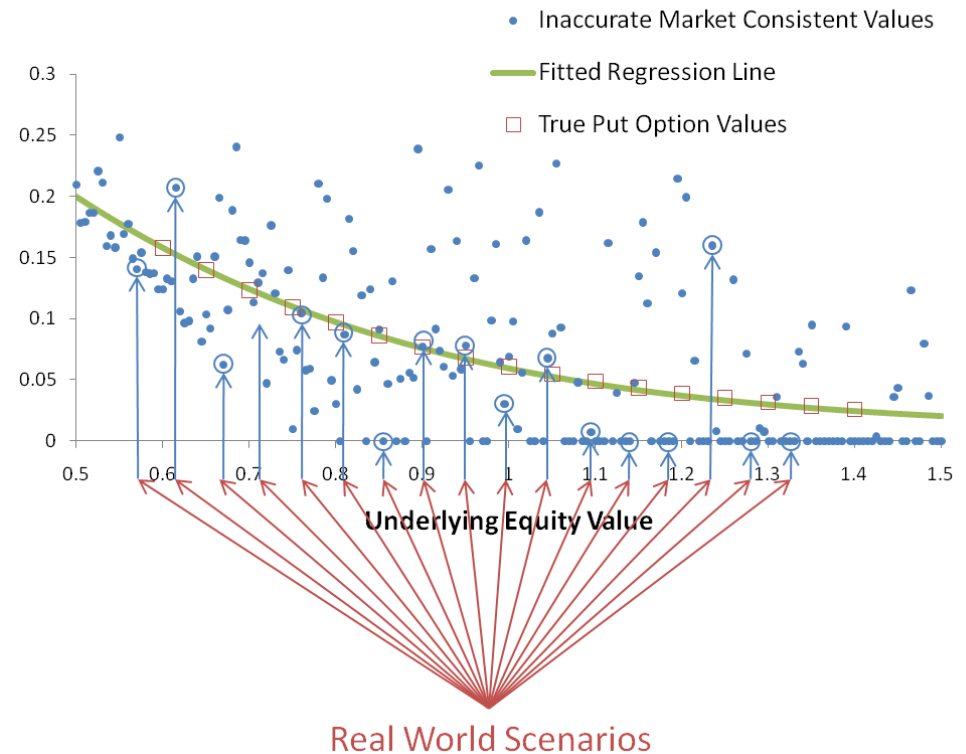
Curve Fitting Limitations

- Does not extend well to multiple dimensions
 - Using P points for R risk dimension requires P^R valuation points (with cross dimensions)
 - 4 fitting points in 5 risk dimensions requires 1024 valuations
 - Quickly becomes more onerous than full nested stochastic
- Difficult to choose small sub-set of points by hand when using multiple risk dimensions
- Inefficient use of scenarios
- Does not extend well to wider uses

LSMC Approach



Least Squares Monte Carlo Simulation



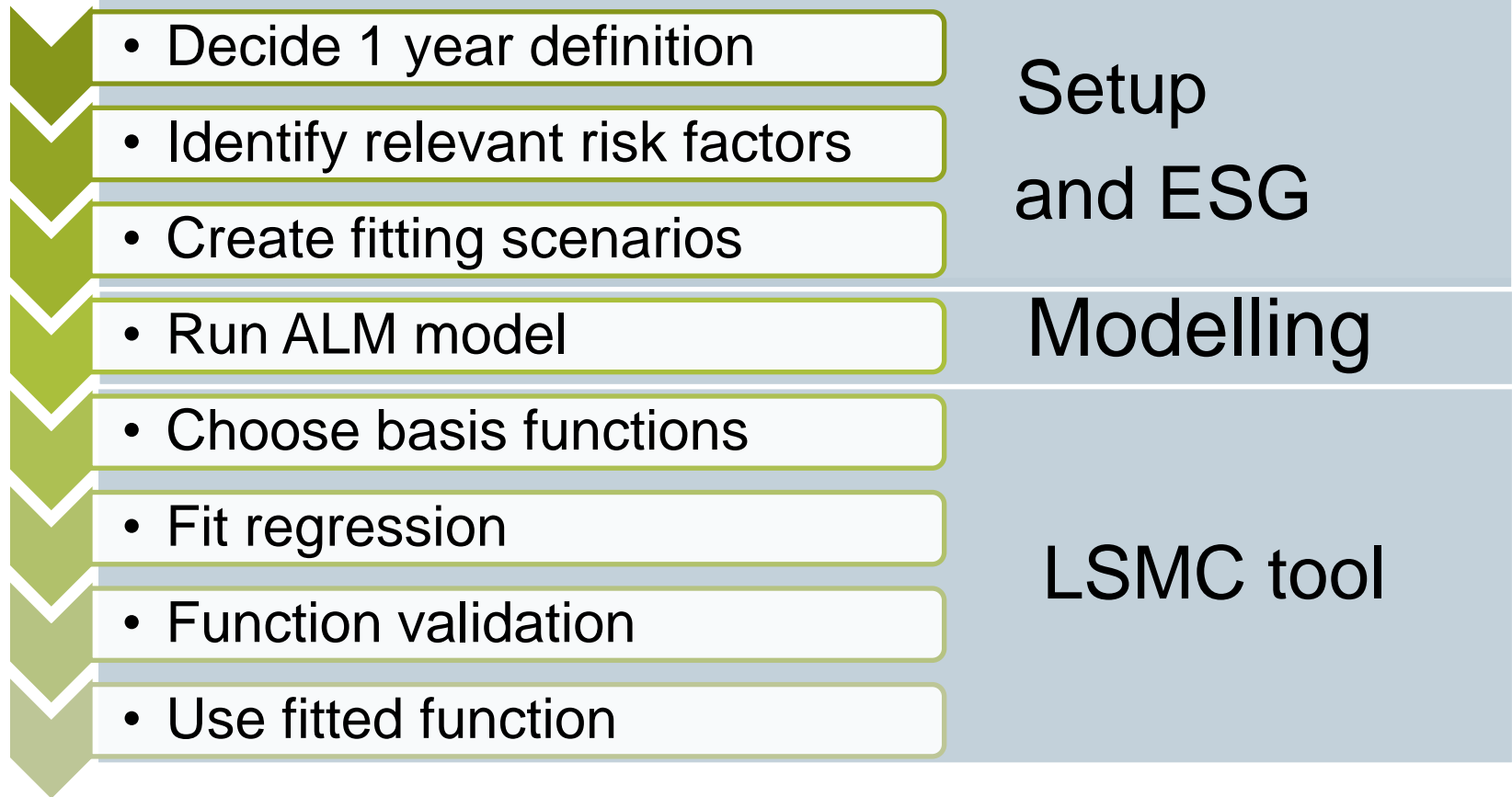
Least squares Monte Carlo

- Fit a multi-dimensional function
- E.g. a polynomial with cross terms
- Produces good fit for complex liabilities
- Works well with multiple risk drivers
- No need to fit to future cashflows as in replicating portfolios
 - Means that path dependency, dynamic behaviour are not a problem

Least Squares Monte Carlo

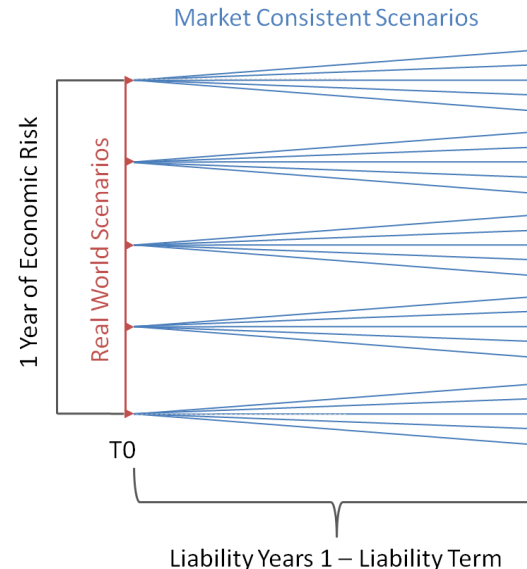
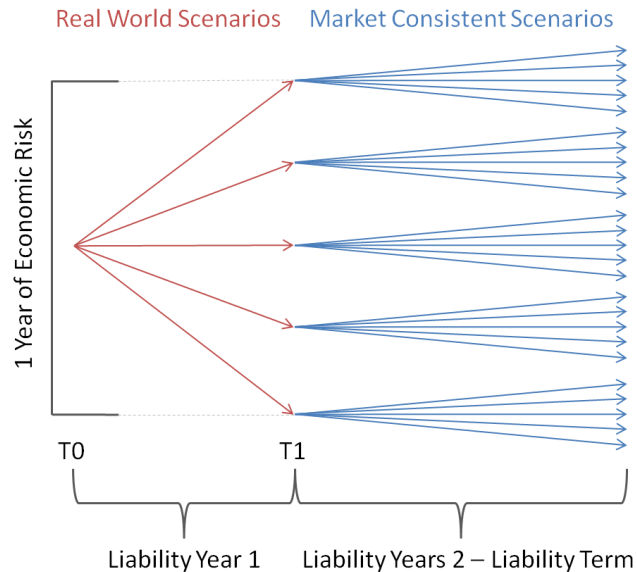
- Fast
 - Only one run, approx 20,000 – 50,000 scenarios
- Accurate
 - Converges to true liability function
 - Accuracy improves with number simulations
 - Potential sampling error can be estimated
- Can use with any real world economic scenarios
- Can use easily with current ALM models

The process



1 year definition

- Can use a true 1 year definition
 - Liabilities roll forward over the year
- Or a Time 0 definition with instantaneous shocks
 - As in Solvency II standard formula



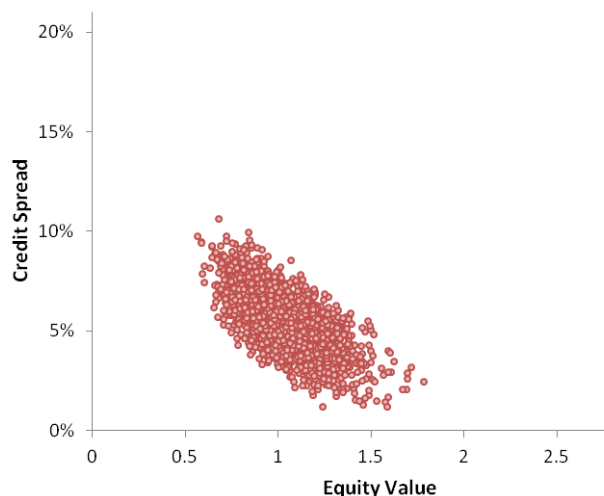
Identify risk drivers

- Parsimonious set of variables which describe risk over the year
 - Equity Return
 - 2 / 3 factors of yield curve movement?
 - Implied volatilities
- Also non-market risks
 - Mortality risk (shock to q_x)
 - Lapse risk

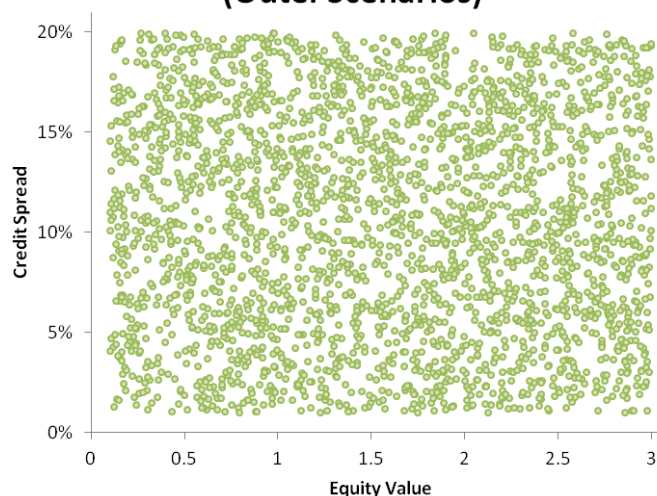
Create fitting scenarios

- Create a large set of year 1 stresses
 - Real world scenarios have too few points in tails
 - Multivariate uniform distribution fits well over wide range
- For each stress produce a small number of inner scenarios
 - 2 inner scenarios efficient if antithetics are used

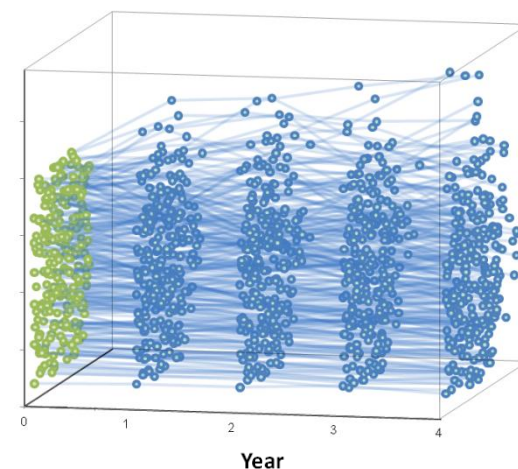
Real World Scenarios



1 Year Stresses
(Outer Scenarios)

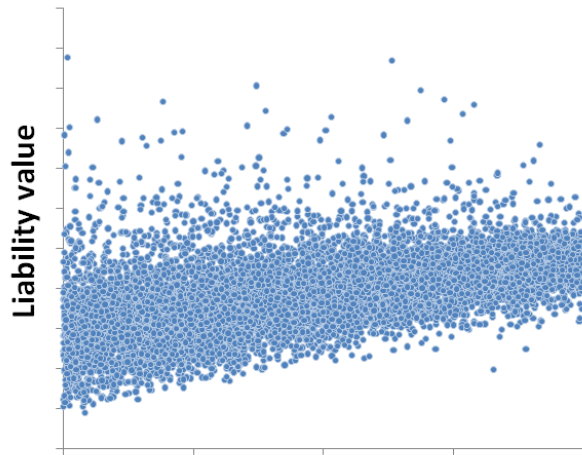


Fitting Scenarios

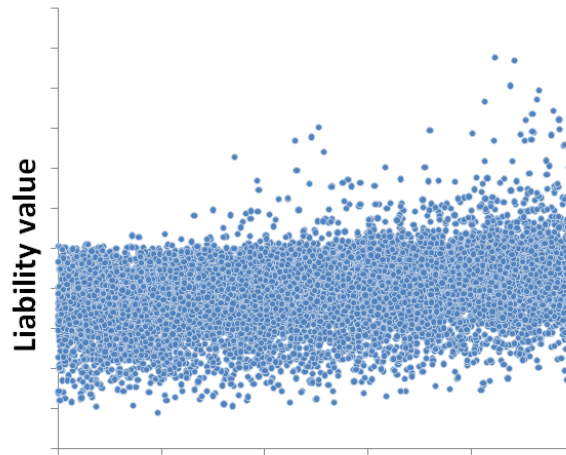


Run ALM model

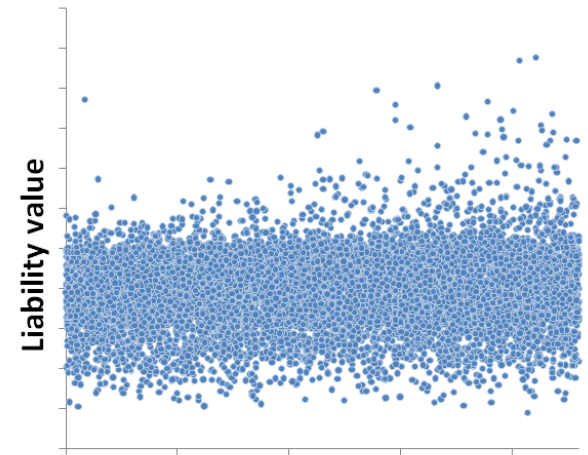
- Run fitting scenarios through ALM model
- Calculate the average PV of future liabilities
 - An inaccurate valuation for each fitting point
 - Can create PVs for total liability or split by component
- Create fitting data



Interest rate factor 1



Equity value



Equity implied volatility

Choose basis functions

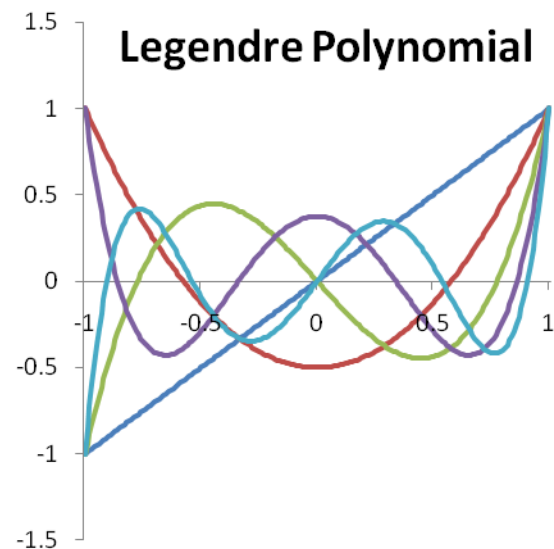
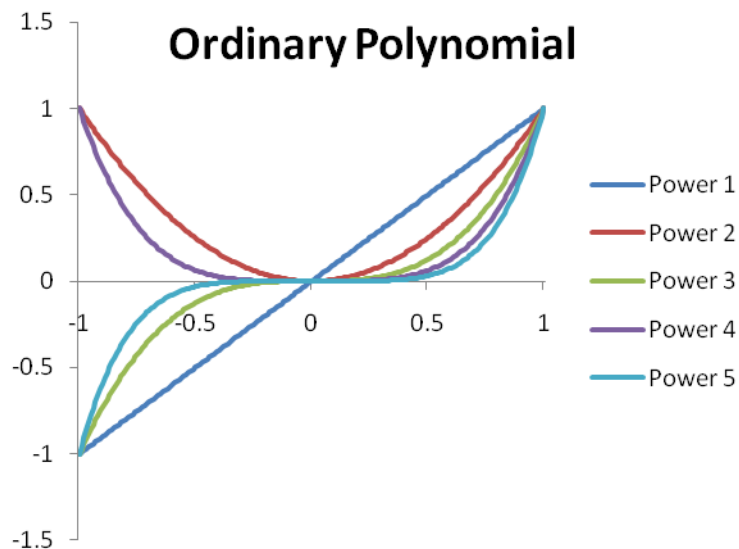
- Basis functions will be parameterized to fit the inaccurate fitting valuations
 - Eg polynomial terms
- Choice of basis functions related to regression method
 - Using an OLS (linear) regression: liability function is a linear combination of polynomial terms and cross terms
 - Polynomial basis in 2 risk drivers to power 3:

$$L(R_1, R_2) = \underbrace{a_1}_{\text{Constant}} + \underbrace{a_2 R_1 + a_3 R_1^2 + a_4 R_1^3}_{\text{Terms in risk 1}} + \underbrace{a_5 R_2 + a_6 R_2^2 + a_7 R_2^3}_{\text{Terms in risk 2}} + \underbrace{a_8 R_1 R_2}_{\text{Power 2 cross terms}} + \underbrace{a_9 R_1^2 R_2 + a_{10} R_1 R_2^2}_{\text{Power 3 cross terms}}$$

- Many other regression methods and bases possible

Choose basis functions

- Certain classes of polynomials are orthogonal under uniform sampling of fitting scenarios
- In this case orthogonal properties are advantageous for regression fitting



Fit regression

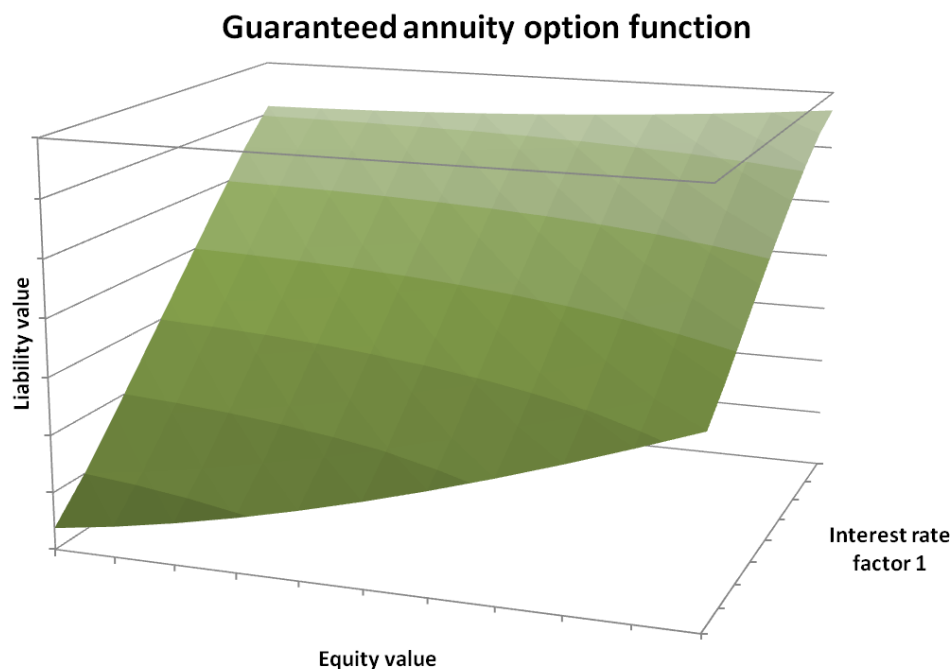
- Find best fit of basis functions to fitting data
- Fitting method is related to sampling method and choice of basis functions
- Typically least squares regression
- Find coefficients of polynomial function
- Also reduce set of possible terms
 - Statistical methods exist to select model that fits data well and avoids over fitting

Fit regression

- Final model typically polynomial of 10 – 100 terms
- Depends on number of fitting scenarios, complexity of liabilities
- Model selection procedures are automated
- Contrast to curve fitting
 - Fitting points must be chosen by hand
 - Human user must choose between competing models that fit data equally well but may differ significantly in un-sampled spaces
- Contrast to replicating portfolios
 - Human user must specify candidate assets by intuition
 - Large number of candidate assets required means far more prone to over fitting

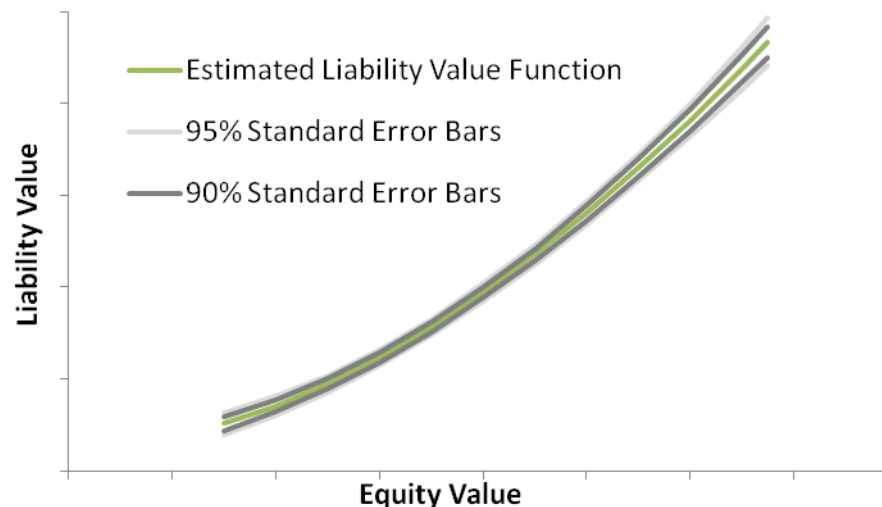
Liability function

- Can analyse shape of liability function in individual and joint dimensions
- We can see that GAO liability is sensitive to decreases in equity and interest rates



Model validation

- Estimate standard error bars around fitted function
 - Typically bootstrapping technique
 - Also gives estimate of standard error of SCR
- Choose a number of out of sample points of interest to test
 - 1 year single and joint risk stresses should be closely predicted by model



Use fitted function

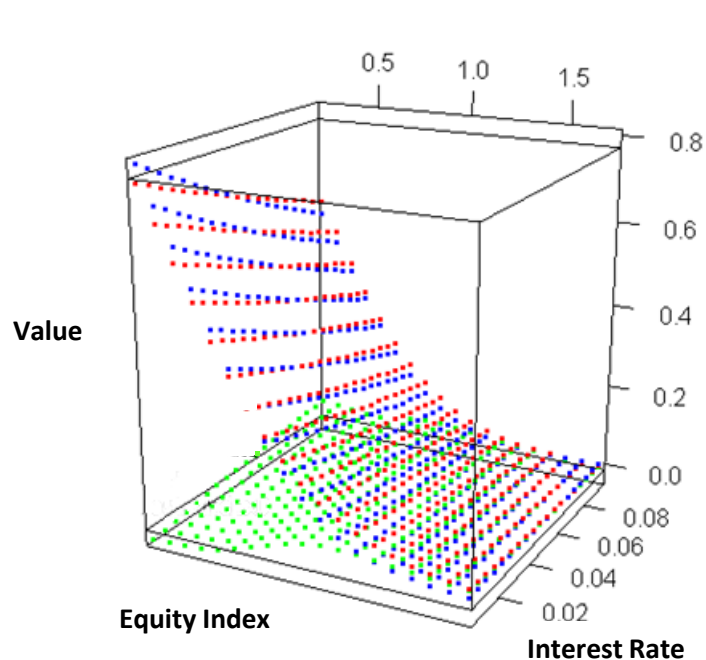
- Pass 1 year real world scenarios of market and non-market risk through liability function
- Calculate distribution of possible MCEV in 1 years time
- Calculate SCR, expected value etc
- Use function to roll forward estimate of MCEV & SCR between valuation periods
- Extend fitting & function to multiple timesteps
 - Multi-period capital, ORSA...

Control variates

- Instead of fitting to the PVs of the liabilities we can fit to the difference between the liabilities and some well known asset
 - E.g. if our liabilities are mostly like a put option then we can fit the difference between them
 - The total liability function is the sum of the control asset and the fitted function
- We can also think of this as a correction technique for RPs
 - The fitted function corrects for differences in the liabilities and the replicating portfolio
 - And for un-replicated risks like non-market risk
- Good control variates lead to more accurate fitted functions with fewer scenarios required

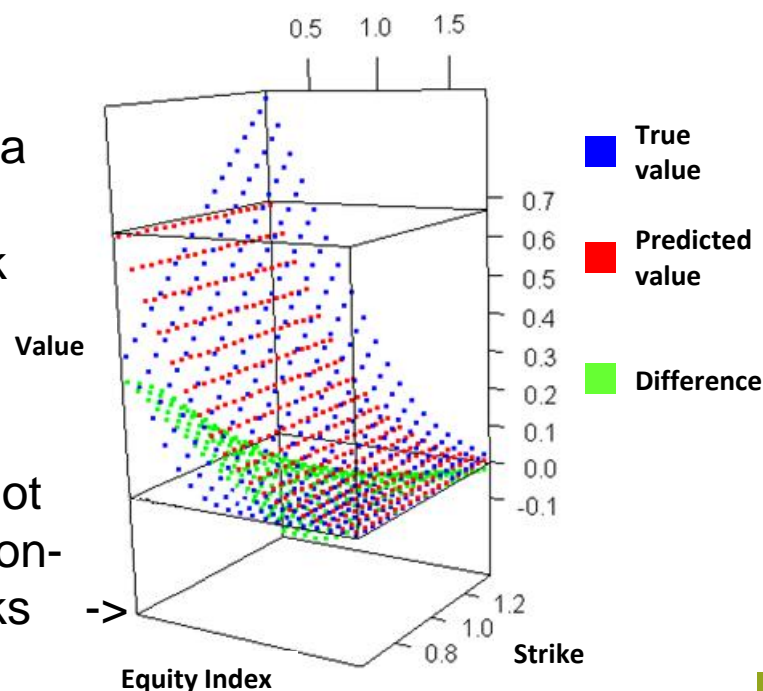
Control variates

- Example: fitting an Asian option with a set of European options
 - Treat the strike as a variable, non-market risk (similar to lapses)



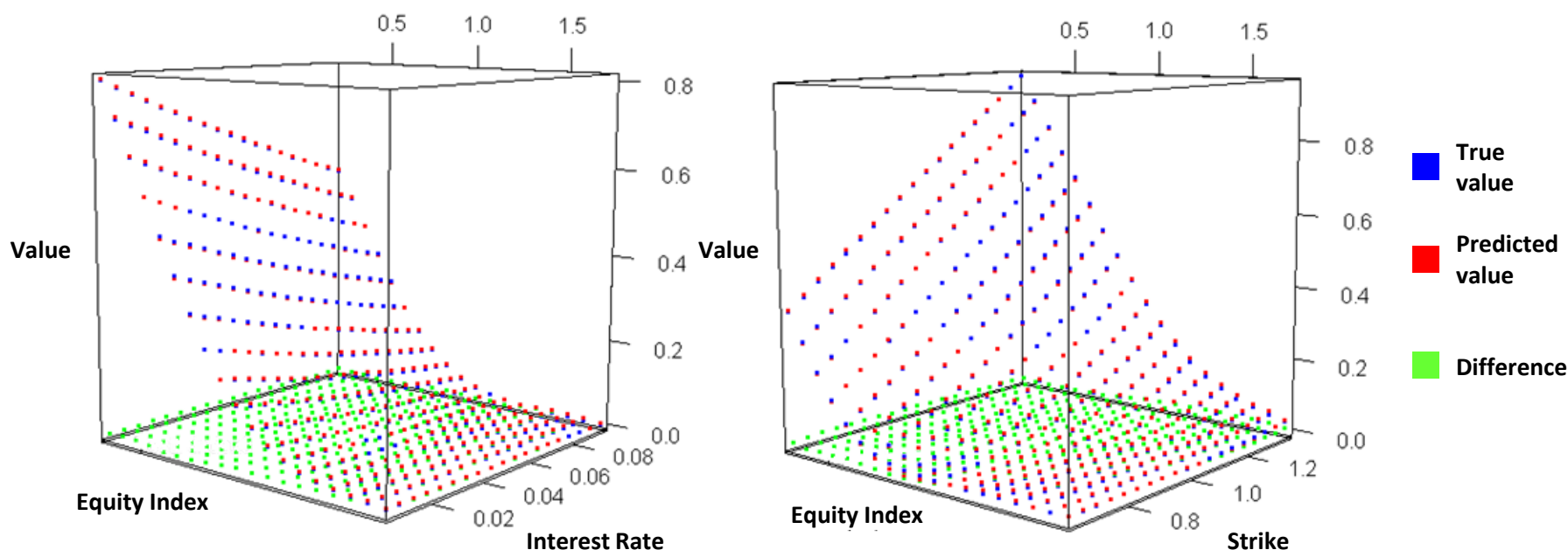
<- RP has a good fit for market risk variables

But does not describe non-market risks



Control variates

- Using LSMC to correct the differences leads to an accurate proxy for non-market risk
 - and also better prediction of market risks and joint risks



Benefits of the LSMC method

- Small number of scenarios
- Formal mathematical basis for convergence
- The choice of fitting points is methodically
- No human intuition, no accuracy trade-off
- Fitting can be automated
- Polynomial functions are very flexible
- The fitting is very fast to perform
- Evaluation of the proxy function is extremely fast
- All market and non-market risks and their joint behaviour can be modelled
- Confidence interval for the liability function and SCR can be estimated
- Easy to explain, chart, communicate, update

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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