Institute and Faculty of Actuaries

# Consequences of Picking the Wrong Model

Andrew Smith
andrewdsmith8@deloitte.co.uk

## Failures in Pricing Models

News

### Quant Congress: Gaussian copula "failing dramatically" in pricing CDOs

Author: Peter Madigan
Source: Risk magazine | 08 Jul 2008
Categories: Credit Derivatives, Structured Products

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**Gaussian copula distribution models are an overly simplistic and inadequate means of valuing tranches of collateralised debt obligations (CDOs) and other structured products, warned a senior quant yesterday.**

Speaking at the Quant Congress USA in New York, Jon Gregory, formerly global head of credit quantitative analytics at Barclays Capital in London, told delegates that the Gaussian copula "fails quite dramatically when applied in practical terms to the credit market" and does not legislate for the possibility of idiosyncratic or systemic defaults.

Related articles
- A multifactor bottom-up model for pricing credit derivatives
- Cutting Edge introduction: The collateral currency convexity problem
- Cutting Edge introduction: Wrong-way risk and the limits of correlation
- Risk software survey 2012

Focusing particularly on super senior tranches, Gregory illustrated that, under a Gaussian model, a super senior tranche of a CDO referencing 125 investment-grade assets should theoretically be able to withstand 46 individual default events before the super senior tranche experienced any loss of principal.

"So the result is that the holder of one of these tranches, with spreads having blown out massively over the past 12 months, would be able to receive as much as 50 basis points for holding a CDO that, on this basis, apparently has no risk. That is opposed to the three or four basis points an investor would have received on this kind of structure in early 2007," said Gregory.

Gregory also pointed out flaws in the assumptions Gaussian models make on the maturity of losses on a super senior tranche, given that a Gaussian distribution draws a straight upward-sloping line between the expected loss on an equity tranche of a CDO and the maturity of the structure.

WIRED MAGAZINE: 17.03

TECH BIZ : IT

### Recipe for Disaster: The Formula That Killed Wall Street

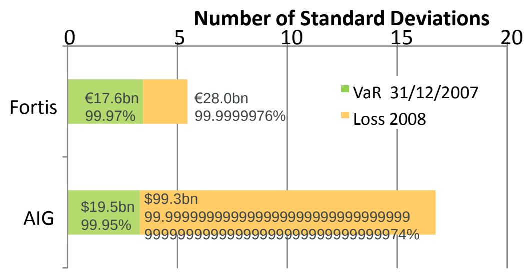
By Felix Salmon 02.23.09

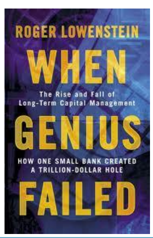
In the mid-'80s, Wall Street turned to the quants—brainy financial engineers—to invent new ways to boost profits. Their methods for minting money worked brilliantly... until one of them devastated the global economy.
*Photo: Jim Krantz/Gallery Stock*

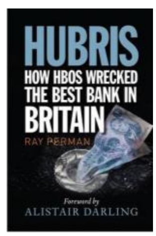
Road Map for Financial Recovery: Radical Transparency Now!

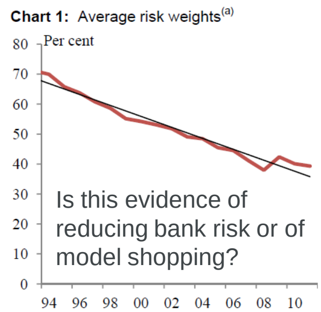
**A year ago,** it was hardly unthinkable that a math wizard like David X. Li might someday earn a Nobel Prize. After all, financial economists—even Wall Street quants—have received the Nobel in economics before, and Li's work on measuring risk has had more impact, more quickly, than previous Nobel Prize-winning contributions to the field. Today, though, as dazed bankers, politicians, regulators, and investors survey the wreckage of the biggest financial meltdown since the Great Depression, Li is probably thankful he still has a job in finance at all. Not that his achievement should be dismissed. He took a notoriously tough nut—determining correlation, or how seemingly disparate events are related—and cracked it wide open with a simple and elegant mathematical formula, one that would

## Failures of Value-at-Risk Models




Source: The Banker and Bank calculations.
(a) Weighted average risk weights of Deutsche Bank, HSBC, BNP Paribas, Barclays, Citigroup, UBS, BAML, BONY, Commerzbank, ING, JPM, LBG, RBS, Santander, State Street, UniCredit, Wells Fargo. Data are not available for the remaining G-SIBs.
http://www.bankofengland.co.uk/publications/Documents/speeches/2013/speech657.pdf

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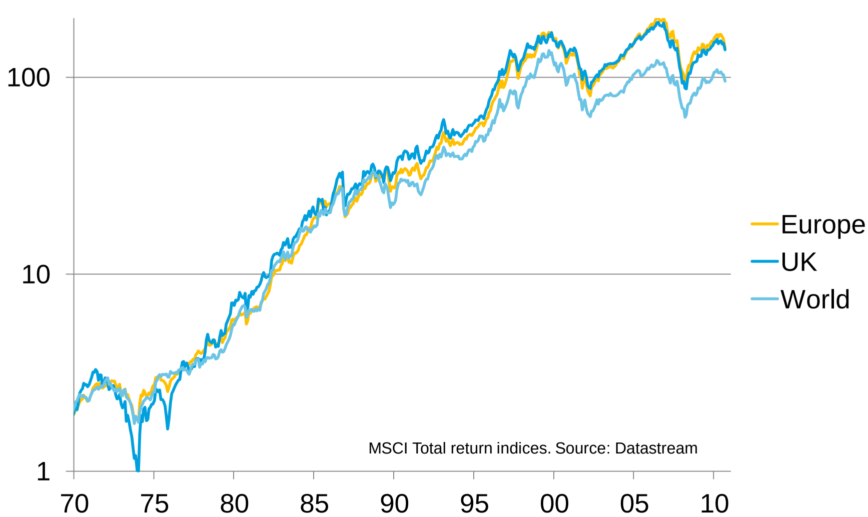
## Historic Equity Returns in €

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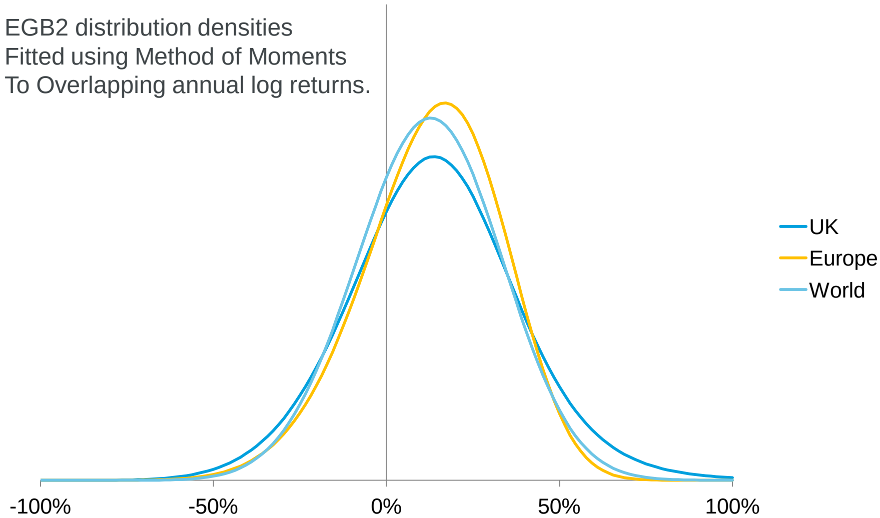
## Will yesterday's fit work tomorrow?

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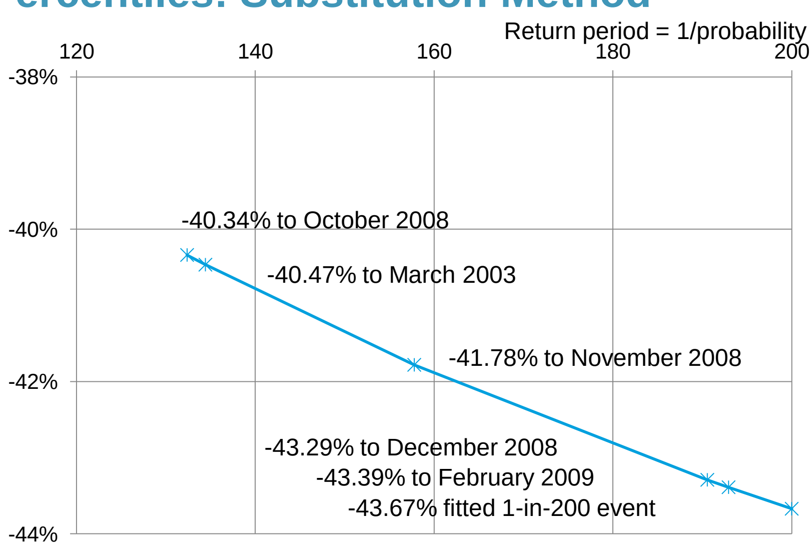
## Percentiles: Substitution Method

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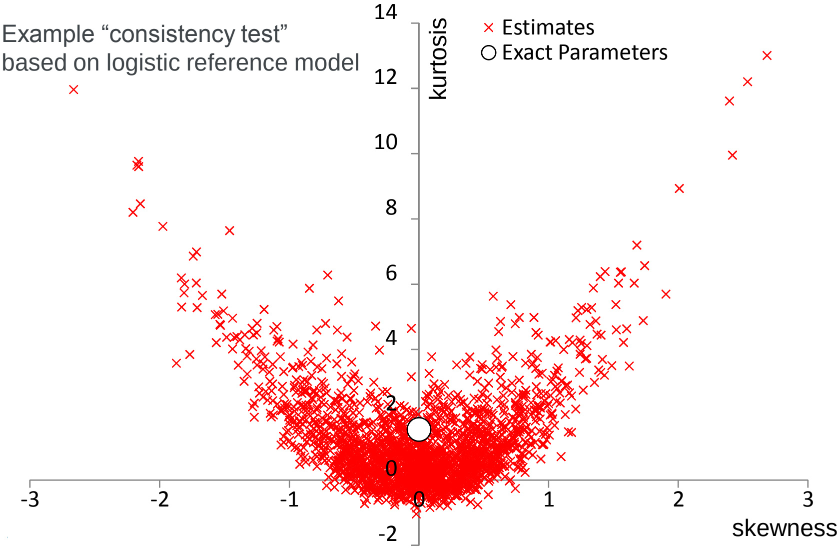
## What About Parameter Estimation Error?

Example “consistency test”
based on logistic reference model

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## Four Possible Questions

What is the weakest stress test I could possibly justify, to use as an opening gambit in negotiations but leaving some wiggle room to strengthen if forced to do so?

What is the 0.5% percentile asset return using substitution (ie my best estimate assumptions and ignoring any possibility that the model or parameters may be mis-specified?)

How can I construct a prediction interval that has at least a 99.5% probability of containing the next observation in the presence of model & parameter uncertainty?

How can I construct an interval that has at least a 99.5% chance of including the true 0.5%-ile in the presence of model & parameter uncertainty?

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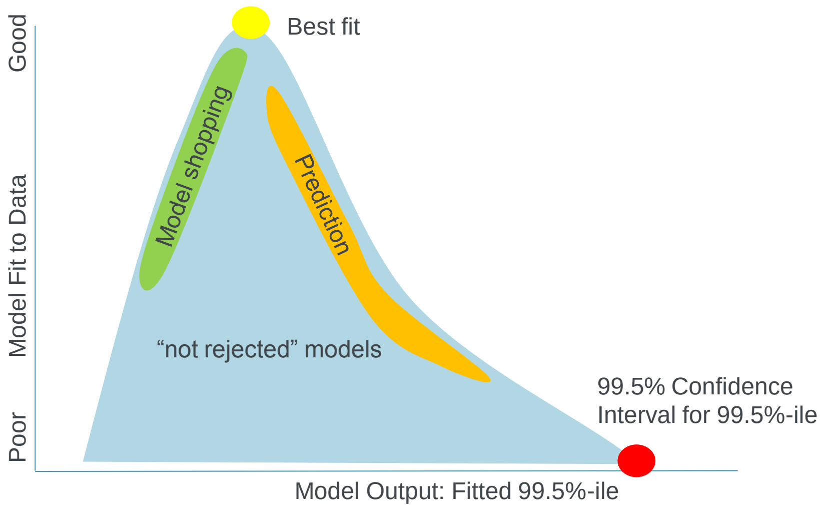
## Understanding a Range of Models

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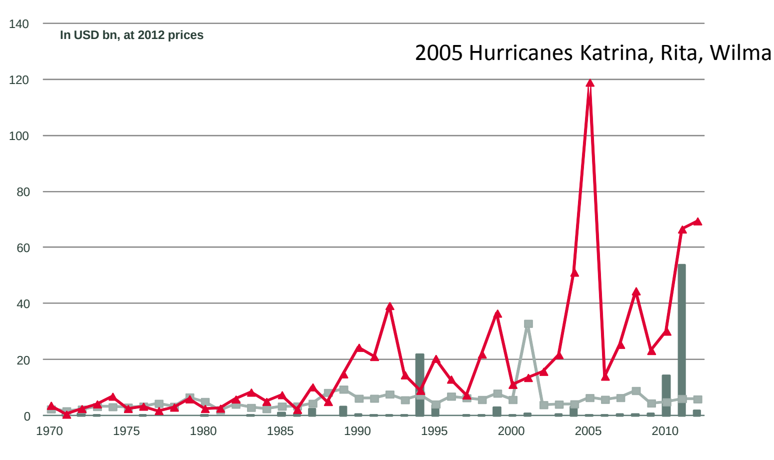
## Swiss Re Catastrophe History




Source: Sigma reports

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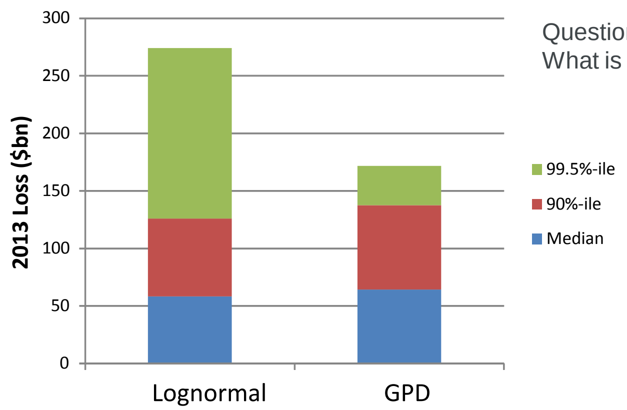
## Fitted 2013 Distributions (GLM + MOM)




$500bn = 1-in-5000   $500bn = impossible

Question:
What is **Prob**{loss > $500bn}?

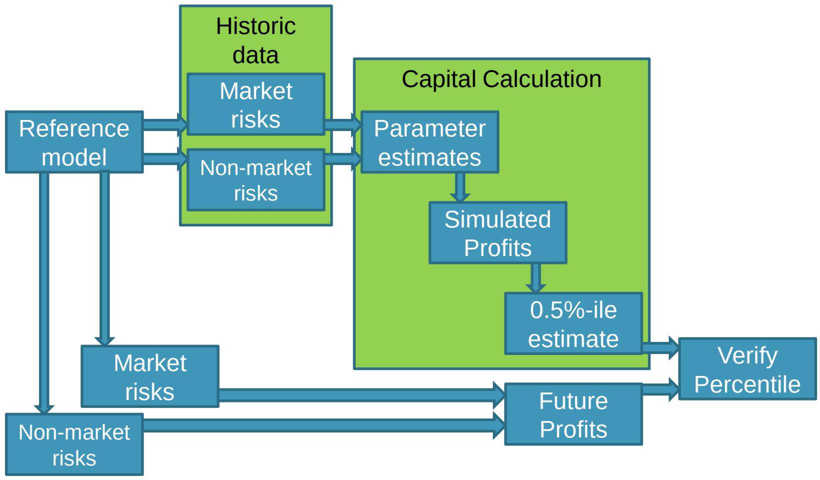


## A Range of Model Risk Tools

| | Benchmarking Approach | Consistency Tests | Robustness Tests |
|---|---|---|---|
| **Description** | Comparing the outputs of different models calibrated to the same data. | Generating random data from a model, and feeding that data back into the calibration process to see if you recover the parameters you started with. | Taking random data from one model, using it to fit a different model, and seeing how good the predictions are relative to the first model. |
| **What it tells you** | The range of different experts' estimates given the data. | The likely accuracy of parameter estimates, both in terms of bias and variability. | How wrong your inference could be if you pick the wrong model. |
| **What it doesn't tell you** | How much the results might be distorted by random fluctuations in the observed history. | What happens if the model specification is incorrect? | How your fitting techniques behave on real data. |

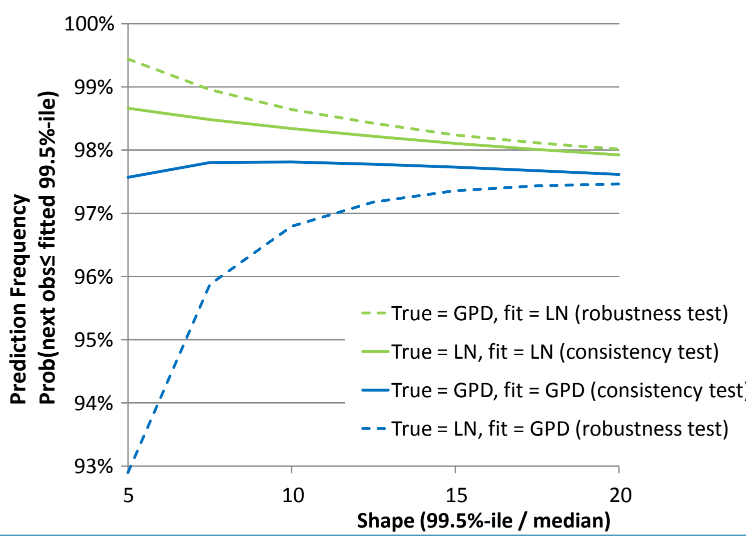
For more details, see http://www.theactuary.com/features/2013/10/gi-prepare-for-the-worst/

## Testing Prediction Intervals






## Lognormal fit is statistically robust



# FSA Persistency Survey 2012

### Single Premium

| Anniversary Start year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1998 | 1000 | 987 | 966 | 933 | 906 |
| 1999 | 1000 | 989 | 966 | 938 | 906 |
| 2000 | 1000 | 987 | 965 | 932 | 894 |
| 2001 | 1000 | 987 | 964 | 929 | 870 |
| 2002 | 1000 | 983 | 953 | 892 | 836 |
| 2003 | 1000 | 975 | 950 | 909 | 865 |
| 2004 | 1000 | 981 | 946 | 908 | 856 |
| 2005 | 1000 | 976 | 949 | 901 | 843 |
| 2006 | 1000 | 971 | 937 | 895 | 841 |
| 2007 | 1000 | 976 | 940 | 896 | 855 |
| 2008 | 1000 | 972 | 939 | 901 | |
| 2009 | 1000 | 976 | 949 | | |
| 2010 | 1000 | 980 | | | |

### RP – Tied Agent

| Anniversary Start year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1998 | 1000 | 899 | 811 | 720 | 630 |
| 1999 | 1000 | 894 | 790 | 685 | 583 |
| 2000 | 1000 | 879 | 762 | 648 | 561 |
| 2001 | 1000 | 869 | 742 | 635 | 550 |
| 2002 | 1000 | 877 | 777 | 645 | 569 |
| 2003 | 1000 | 885 | 737 | 648 | 465 |
| 2004 | 1000 | 883 | 771 | 646 | 517 |
| 2005 | 1000 | 885 | 784 | 710 | 622 |
| 2006 | 1000 | 893 | 799 | 688 | 582 |
| 2007 | 1000 | 897 | 781 | 669 | 574 |
| 2008 | 1000 | 889 | 798 | 695 | |
| 2009 | 1000 | 903 | 829 | | |
| 2010 | 1000 | 876 | | | |

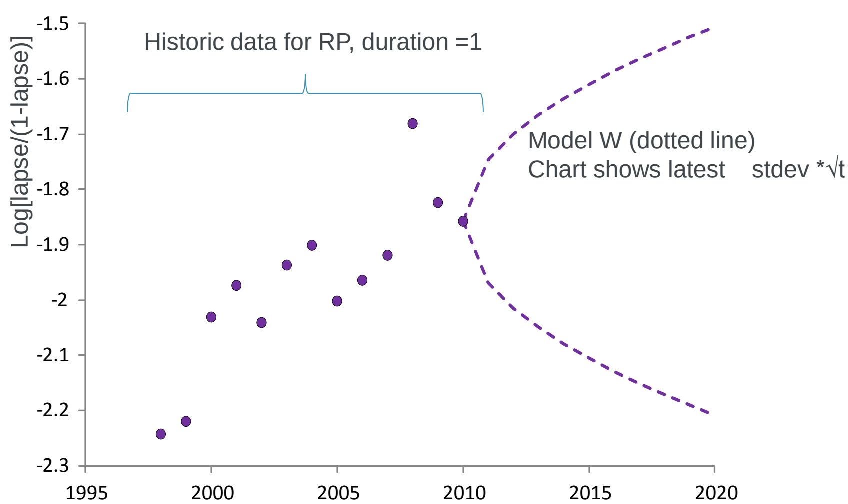
### RP – IFA

| Anniversary Start year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1998 | 1000 | 918 | 829 | 744 | 663 |
| 1999 | 1000 | 915 | 811 | 715 | 638 |
| 2000 | 1000 | 879 | 758 | 666 | 567 |
| 2001 | 1000 | 866 | 765 | 638 | 548 |
| 2002 | 1000 | 881 | 742 | 640 | 554 |
| 2003 | 1000 | 860 | 748 | 640 | 551 |
| 2004 | 1000 | 849 | 720 | 605 | 530 |
| 2005 | 1000 | 856 | 733 | 620 | 518 |
| 2006 | 1000 | 863 | 737 | 607 | 523 |
| 2007 | 1000 | 865 | 711 | 612 | 518 |
| 2008 | 1000 | 830 | 715 | 590 | |
| 2009 | 1000 | 854 | 713 | | |
| 2010 | 1000 | 856 | | | |

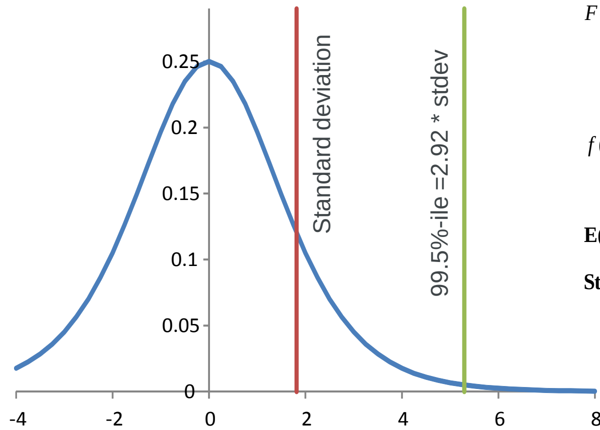
The study contains numerous other data sets, but there are concerns over accuracy (for example, negative lapse rates).



# Random Walk Forecasts (Model W)



## Assume Logistic Distribution for Increments




$$F(x) = \frac{1}{1 + \exp\left(-\frac{x}{\beta}\right)}$$

$$f(x) = \frac{\exp\left(\frac{x}{\beta}\right)}{\beta\left\{1 + \exp\left(\frac{x}{\beta}\right)\right\}^2}$$

$$\mathbf{E}(X) = 0$$

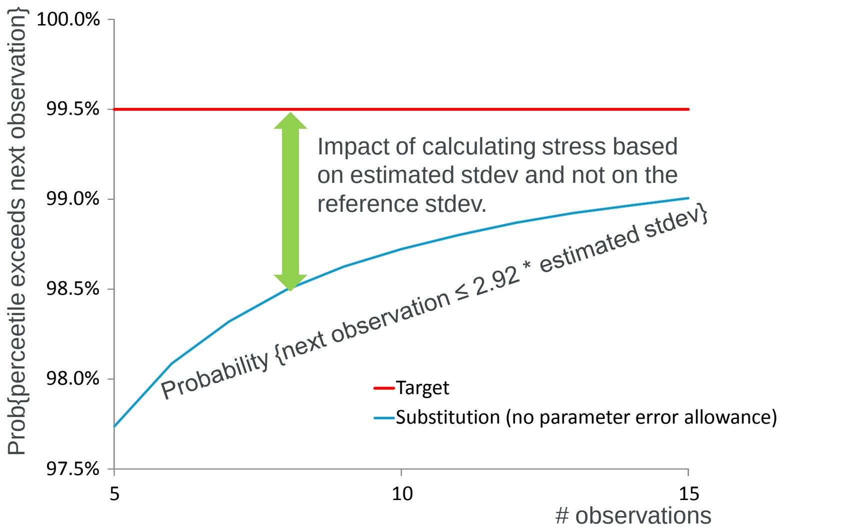
$$\mathbf{Stdev}(X) = \frac{\pi\beta}{\sqrt{3}}$$



## Some Unrealistic Assumptions

| Assumption | Response |
|---|---|
| Log[lapse rate / (1-lapse rate) ] performs a random walk | ??? |
| Increments have a logistic distribution | ??? |
| Sample standard deviation is a good way to measure dispersion of a logistic distribution. | ??? |
| We know the standard deviation of the increments | ??? |
| The same model applies to the future as to the past | ??? |

## Prediction Test : Substitution Method

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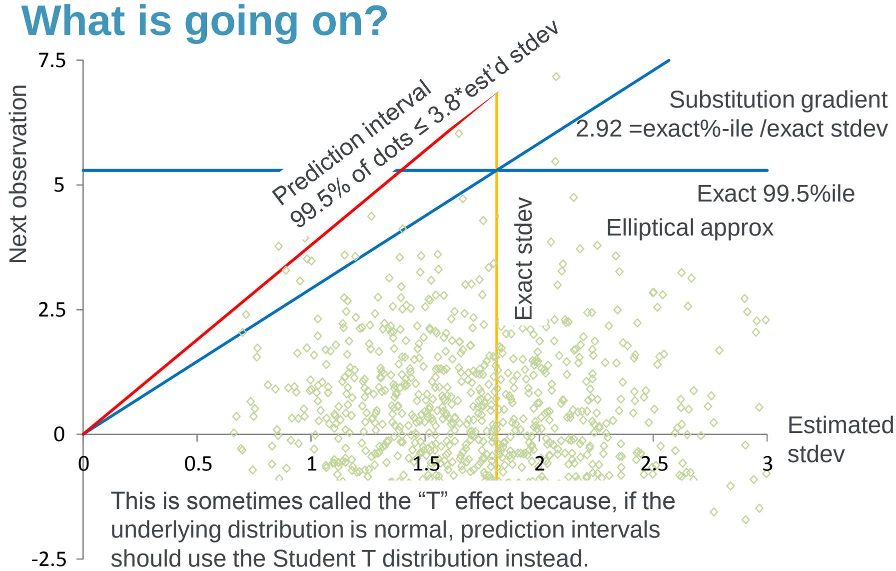
## What is going on?




This is sometimes called the "T" effect because, if the underlying distribution is normal, prediction intervals should use the Student T distribution instead.

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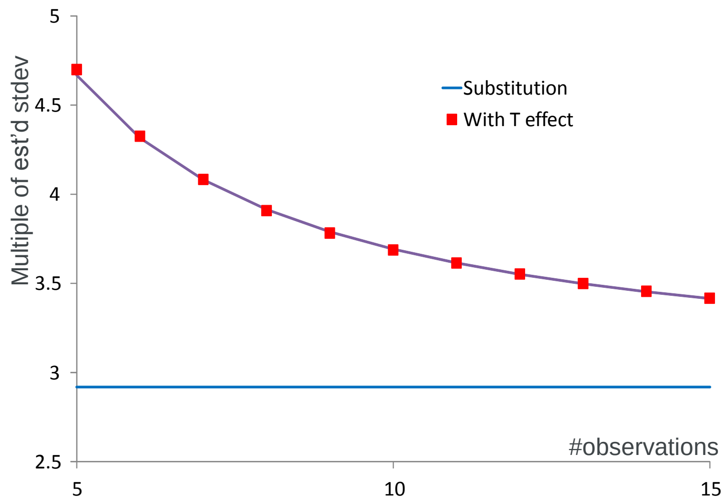
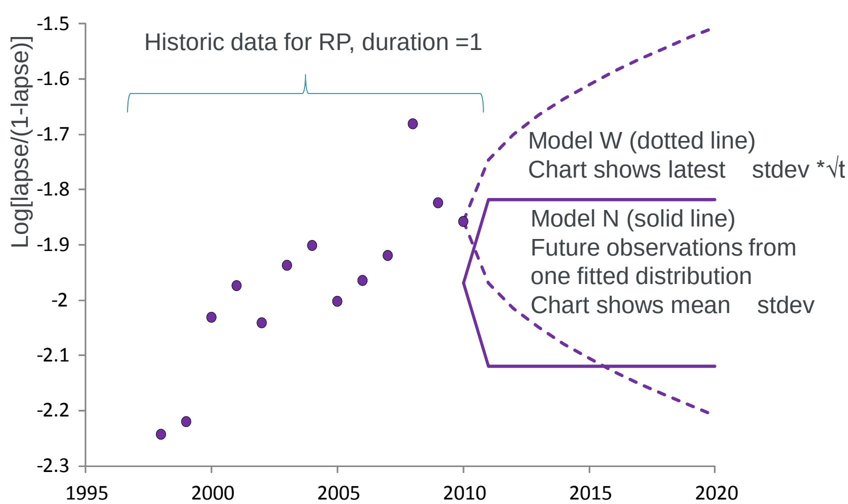
## The T effect Disappears for Large Samples

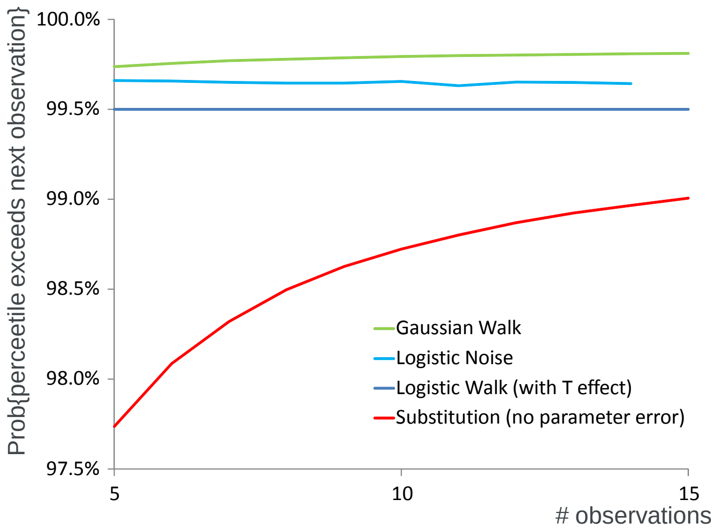





## Alternative Models: Noise & Walk



## Impact of Mis-specified Models

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## Unrealistic Assumptions Revisited

| Assumption | Response |
|---|---|
| Log[lapse rate / (1-lapse rate) ] performs a random walk | Prediction interval is cautious if the lapse rates are independent. |
| Increments have a logistic distribution | Prediction interval is cautious if we assume normal distributions instead, |
| Sample standard deviation is a good way to measure dispersion of a logistic distribution. | The prediction test is evidence that the method works; how we derived the estimates is irrelevant. |
| We know the standard deviation of the increments | Use a larger multiple of estimated standard deviation |
| The same model applies to the future as to the past | You cannot get rid of all limitations and exclusions with clever statistics. |

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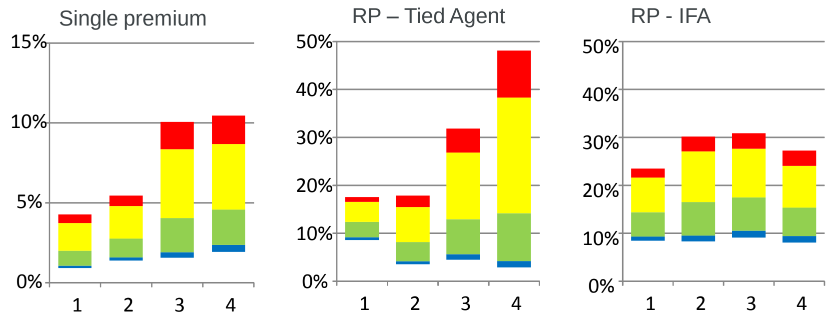
# Implied Lapse Stresses

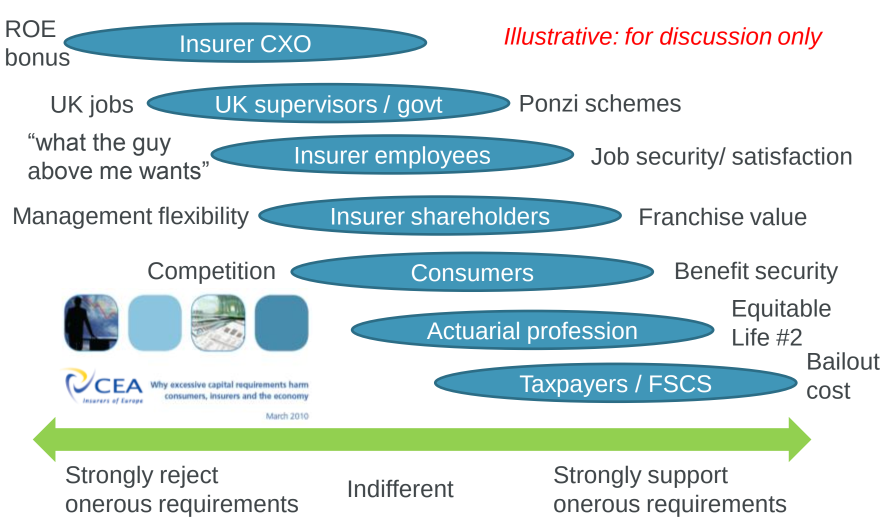



Key:
- 99.5%-ile incorporating parameter error
- 99.5%-ile ignoring parameter error
- Latest lapse rate
- 0.5%-ile ignoring parameter error
- 0.5%-ile incorporating parameter error

Source: Dickson & Smith
Life Convention 2013



# Serious about Model Risk: Who Wins?



## Conclusions

- A regulatory requirement to validate a single "best fit" internal model may address model shopping more than it addresses model error.
- "Picking the right model" is not a practical solution to model error; inevitably there are many possible models capable of passing validation and we cannot know which (if any) is correct.
- A theoretical approach to model and parameter risk is to randomise data sets using reference models in order to test prediction intervals, but this is not yet market practice in financial firms. There is a limit to statistical methods which inevitably make some form of "future will be like the past" assumption.
- There is a need for reflexivity: ability or willingness by employees within an organisation to question its dominant beliefs, norms and expectations (Spicer & Alvesson)
- Commercial incentives, functional stupidity and personal integrity.



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