The Actuarial Profession
making financial sense of the future

GIRO Conference and Exhibition 2012

How to Outdo Your Adversaries While They are Trying to Outdo You

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Introduction

Roger B Myerson (1991)

"the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory provides general mathematical techniques for analysing situations in which two or more individuals make decisions that will influence one another's welfare"

Robert J Aumann (1987)

"the interactive behavior of Homo Rationalis – rational man ... [An] important function of game theory is the classification of interactive decision situations"

An insurance story

In the beginning …

- Two profit-maximizing firms *Broker B* and *Direct D*
  - *Direct D* underwrites and distributes policies itself
  - *Broker B* distributes policies underwritten by its panel of insurers
- Target segment is 25,000 consumers homogeneous in all aspects of risk and behavior. They all value the insurance policy at £400.
- Firms set price simultaneously when rates are loaded on the price comparison website *CompareThe Aardvark.com*
- Each firm expects to pay £145 per policy to meet claims and other costs
- Consumers show some preferences.
- Research shows a range of preferences, with some consumers willing to pay up to £80 more for a *Direct D* policy, and others willing to pay up to £80 more for a *Broker B* policy.

How should our firms decide on price?

- Both firms want to set price where individual profits are biggest
- Profits are function of demand, price and costs

$$\Pi_B = \ p_B - 145 \ \times q_B \text{ and } \Pi_D = \ p_D - 145 \ \times q_D$$

- But demand is a function of other firm’s price!

$$q_B = f_B(p_B, p_D) \text{ and } q_D = f_D(p_B, p_D)$$

Harold Hotelling had a useful idea

Transportation costs
(can consider this as utility cost)

Marginal consumer x is indifferent between firms at a certain price

Broker B | *Direct D*

0 Disutility cost of $80x$ if buy from *Broker B*

Disutility cost of $80(1\text{-}x)$ if buy from *Direct D* **1**

$\boldsymbol{x}$

- Marginal point $400 - p_B - 80x = 400 - p_D - 80(1-x)$
- Therefore $x = \dfrac{p_D - p_B + 80}{2 \times 80}$
- And $q_B = 25{,}000x$ and $q_D = 25{,}000(1-x)$

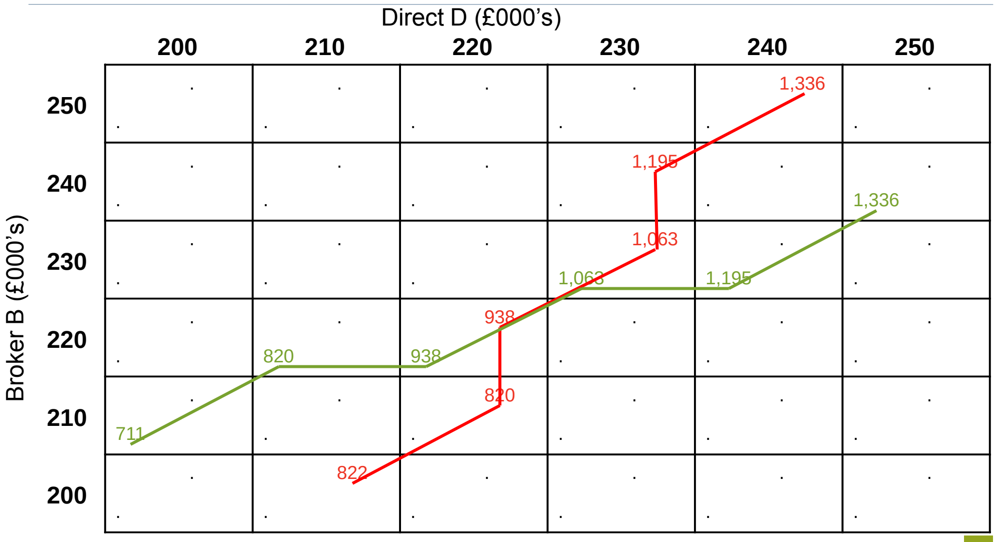
Constructing a payoff matrix of profits

$\Pi_B = (p_B - 145) \times q_B$ $\Pi_D = (p_D - 145) \times q_D$

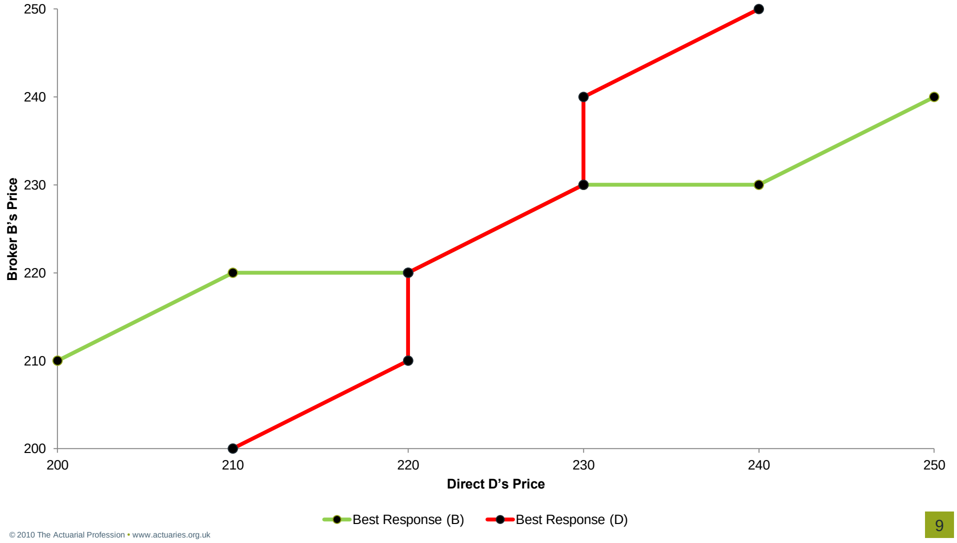
Direct D (£000's)

Broker B (£000's)	200	210	220	230	240	250
250	1,117 492	1,219 656	1,289 820	1,328 984	1,336 1,148	1,313 1,313
240	1,031 594	1,117 742	1,172 891	1,195 1,039	1,188 1,188	1,148 1,336
230	945 664	1,016 797	1,055 930	1,063 1,063	1,039 1,195	984 1,328
220	859 703	914 820	938 938	930 1,055	890 1,172	820 1,289
210	773 711	813 813	820 914	797 1,016	742 1,117	656 1,219
200	688 688	711 773	703 859	664 945	594 1,031	492 1,117

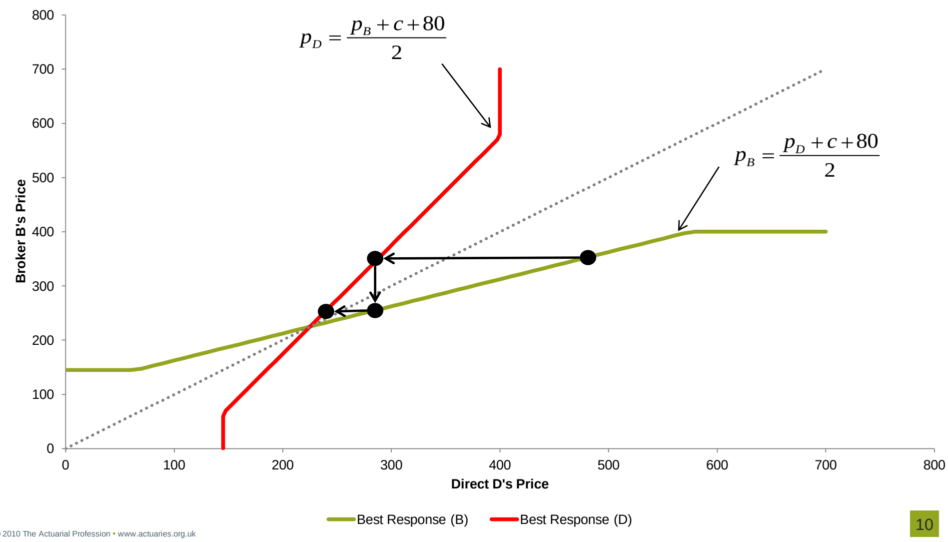
Constructing a payoff matrix of profits



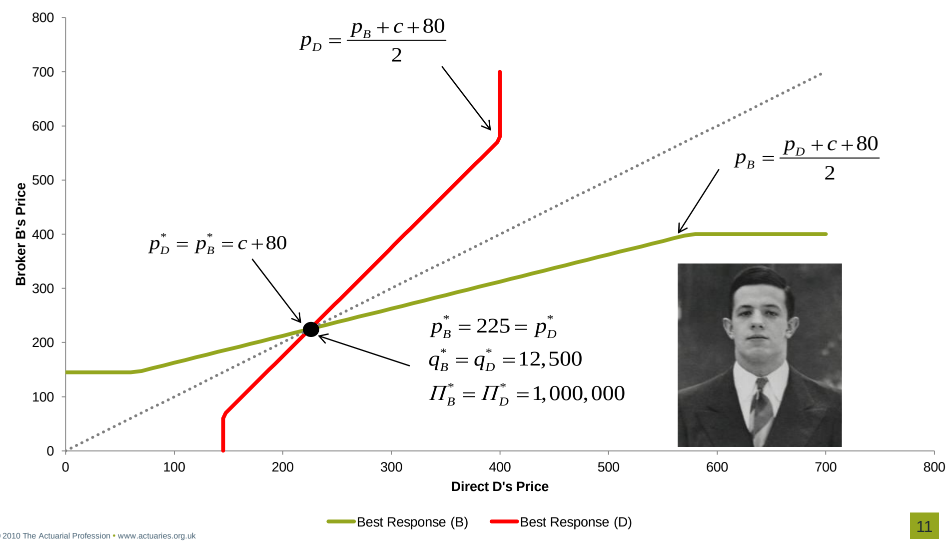
Constructing a payoff matrix of profits



Best response functions – each player can determine their best response given the pricing decision of the other



Nash equilibrium – no player can deviate profitably given other players choose their equilibrium



An insurance story

The broker's insurance panel

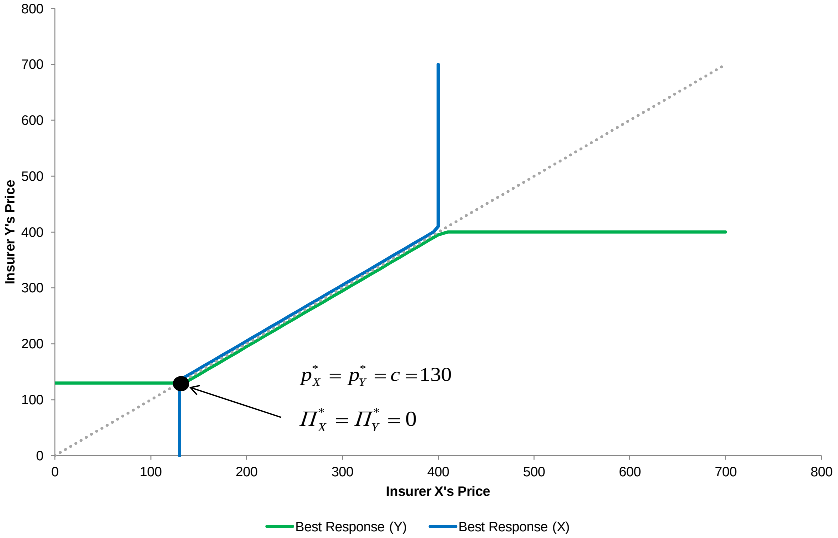
- *Broker B*'s panel has two profit-maximizing firms *Insurer X* and *Insurer Y*
- Both insurers quote for each policy simultaneously and *Broker B* selects the cheapest net price
- Each insurer expects to pay £130 per policy to meet claims and other costs
- *Broker B* incurs costs of £15 per policy over and above net price it pays to the cheapest insurer

Insurance panel competition

- *Broker B* will select cheapest price quoted by the insurers
- Each insurer will want to charge just less than the other insurer

- For *Insurer X* if it charges a price of p_X then
  - If $p_X < p_Y$ it wins all the business *Broker B* writes in the segment, i.e. $q_X = q_B$
  - If $p_X > p_Y$ it wins no business, i.e. $q_X = 0$
  - If $p_X = p_Y$ it shares the business with *Insurer Y*, $q_X = \frac{1}{2} q_B$

Bertrand paradox – It only takes two companies in undifferentiated market for perfect competition



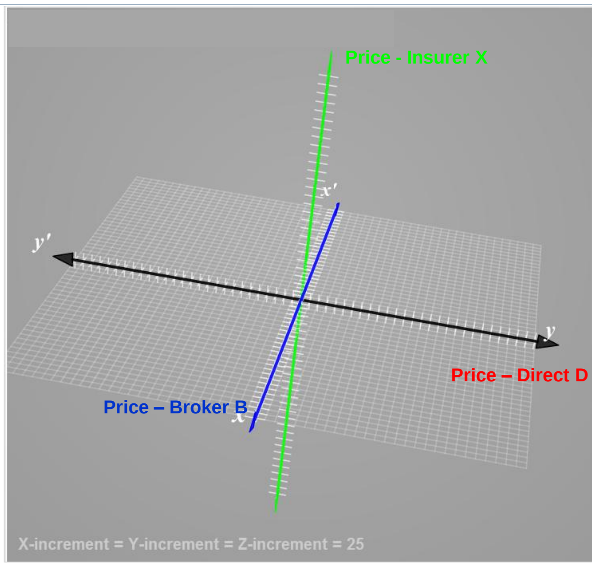
An insurance story

Insurer Y exits the market

- *Broker B* receives a call from *Insurer Y* who has decided to withdraw from the insurance market.
- *Broker B* decides that this could be a good opportunity to strengthen its relationship with *Insurer X* and offers *Insurer X* an exclusive arrangement.
- *Insurer X* is happy to proceed as the sole insurer on *Broker B*'s panel.

Pricing actions are dependent

- *So, Insurer X* is now the only insurer on the panel
- *Broker B* has only one net price for each quote
- *Insurer X* no longer competes with other insurers on the panel



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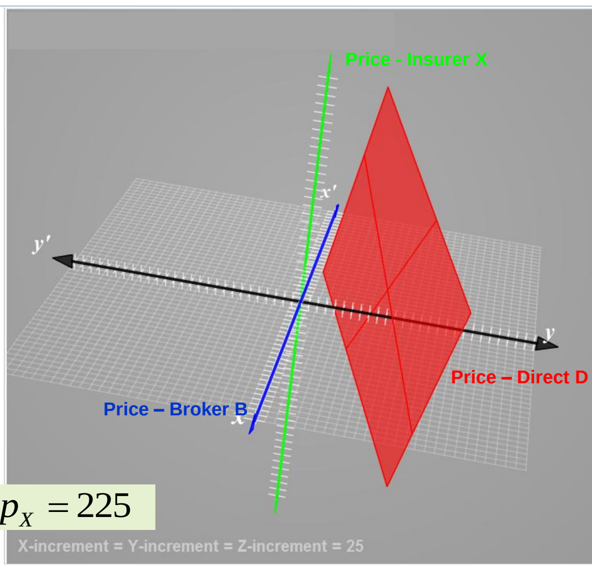
Direct D sets its price dependent on *Broker B*'s price

- *Direct D*'s demand will depend on its price and that of *Broker B*
- Best response is function of p_B and p_D
- Independent of *Insurer X*

$$BR_D : -p_B + 2p_D + 0p_X = 225$$



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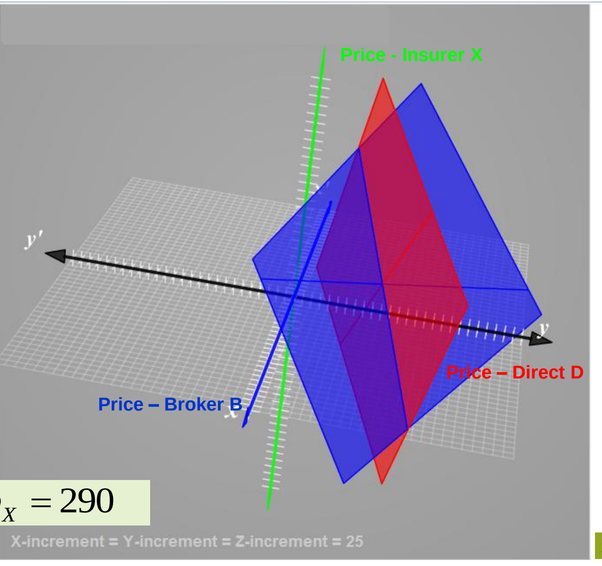
Broker B's best response is dependent on *Direct D* and *Insurer X*

- Broker B's demand depends on its price and that of *Direct D*
- Profits depend on price of *Insurer X*
- Best response is function of p_B, p_X and p_D



$$BR_B : 2p_B - 2p_D + p_X = 290$$

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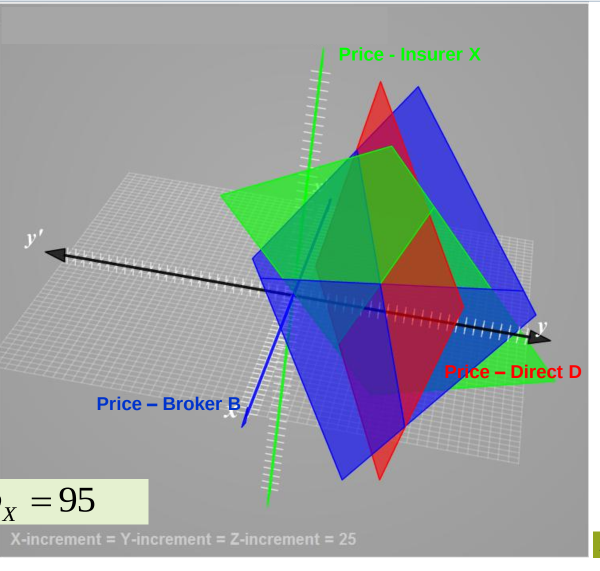
Insurer X's best response is dependent on *Broker B* and *Direct D*

- *Insurer X*'s demand will be equal to that of *Broker B*
- Best response is function of p_B, p_X and p_D



$$BR_X : 2p_B - p_D - p_X = 95$$

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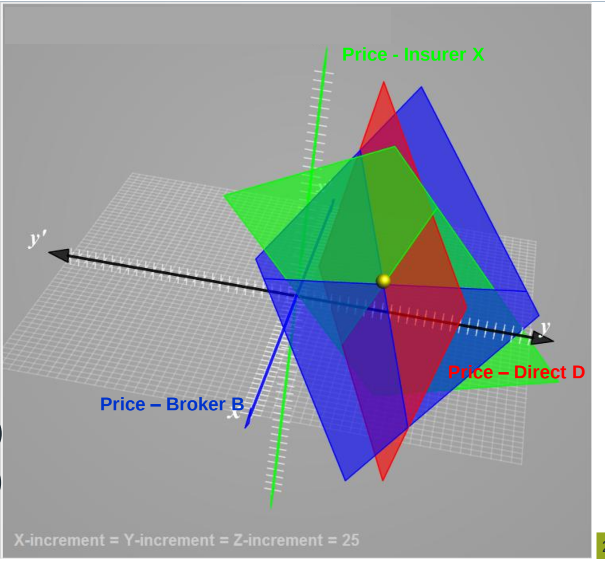
Equilibrium is at intersection of best response planes

- $p_B^* = 289$ (cf 225)
  $p_D^* = 257$ (cf 225)
  $p_X^* = 226$ (cf 130)
- $q_B^* = 7,500$ (30%)
  $q_D^* = 17,500$ (70%)
  $q_X^* = 7,500$
- $\Pi_B^* = 360,000$ (1m)
  $\Pi_D^* = 1,960,000$ (1m)
  $\Pi_X^* = 720,000$ (0)



Strategic effect of solus broker arrangement

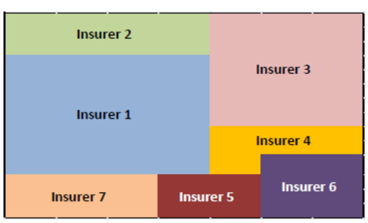
- Removal of competition increases power of *Insurer X* – no Bertrand Paradox and equivalent to local monopoly
- *Insurer X* increases its price …
- … increase in costs causes *Broker B* to increase its price …
- … *Direct D* responds to higher demand by increasing its price
- … resulting market equilibrium price is higher
- *Broker B* previously kept 100% of differentiation profit …
- … now retains only 33% and 66% goes to *Insurer X*

Insurer X's glory days are over

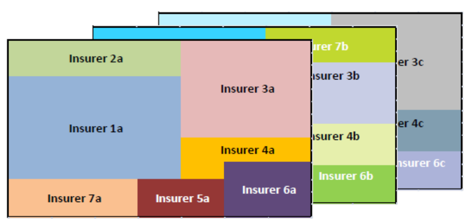
Broker B has some decisions to make!

- *Broker B's* Management is concerned about the significant loss of market share to *Direct D* and the drop in profits from £1m to £360,000
- *Broker B* arranges a board meeting to address this problem and the consensus is that the current solus arrangement with *Insurer X* is not working
- Some options are considered to rectify the situation and restore their bonuses

Option 1 - Revert to a competitive panel



- The broker should aim to have a perfectly complementary panel to ensure maximum quotability.
- The broker should also get at least 2 insurers quoting for each insurance application



Option 1 - Revert to a competitive panel

- Bertrand competition drives net premium to cost
- Panel insurers make zero profit
- *Broker B*'s costs reduce and it reduces its price …
- …. *Direct D* will respond to reduced demand by reducing its price

$$p_B^* = p_D^* = 225$$

$$q_B^* = q_D^* = 12,500$$

$$\Pi_B^* = \Pi_D^* = 1,000,000$$

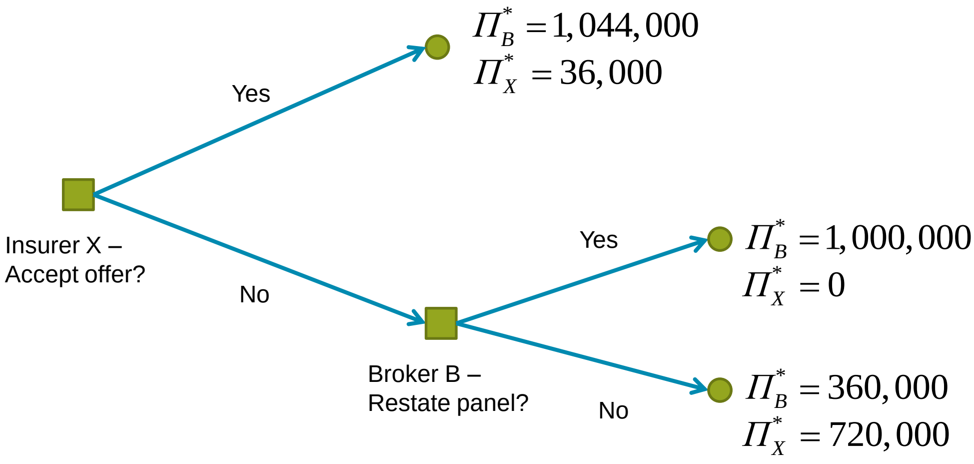
- However, building a competitive panel is expensive and time consuming

Option 2 - Profit Share

- Outcome of option 1 - both market price and total profits reduce
- Can *Broker B* have higher equilibrium price **and** increase profit?

- *Broker B* could contract with *Insurer X* for a share of its profits
  - *Broker B* offers a profit share agreement with *Insurer X* where any profits made by *Insurer X* are shared 5% to *Insurer X* and 95% to *Broker B*!
  - If *Insurer X* accepts, *Broker B* will maintain the solus panel
  - If *Insurer X* refuses, *Broker B* will retaliate by reintroducing a competitive panel

Option 2 - Profit Share



Strategic Moves – Threats, Promises, Commitments

- **Strategic Move** – manipulate rules to produce outcome to your own advantage
- Commitment vs Threat vs Promise
- Effectiveness depends on
  - Temptation to cheat
  - Ability or chance of other players observing a defection
  - Chance of other players being able to punish defector
  - Whether or not the game is repeated

An insurance story

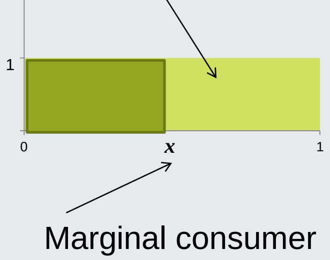
Return to competitive market

- Surprisingly, *Insurer X* rejects the offer (they never attended this workshop!)
- *Broker B* holds true to its threat and restates a competitive panel by entering an agreement with *Insurer Z*
- Various consumer media reports, including a high profile programme on BBC WatchDog, alters consumers preferences for different product brands
- Updated market analysis suggests that
  - Customers' preferences are no longer distributed uniformly - much more weighted to the middle of the interval
  - Some customers feel stronger about their favourite insurer and will now pay £140 more for one product over the other

Hotelling model assumes a uniform distribution of preferences

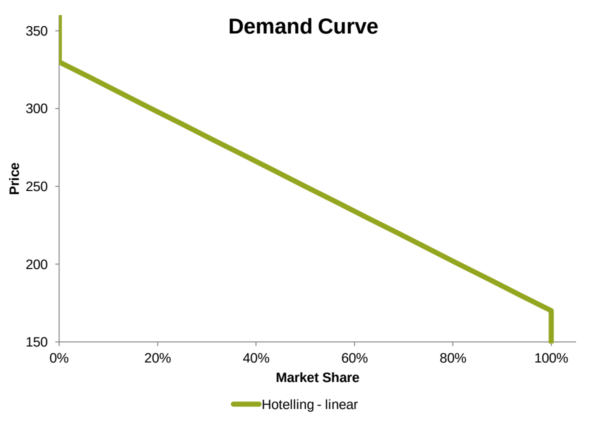
Mass of consumers

$$\int_0^1 1dz = z\Big|_0^1 = 1 - 0 = 1$$



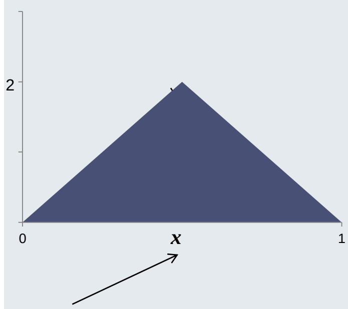
Marginal consumer

$$x_B = \frac{p_D - p_B + 80}{2 \times 80}$$



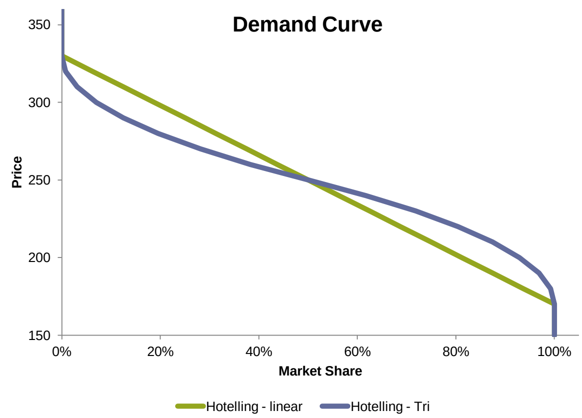
Demand curve for *Broker B* given $p_D = 250$

Hotelling model adjusted to a modal distribution of preferences



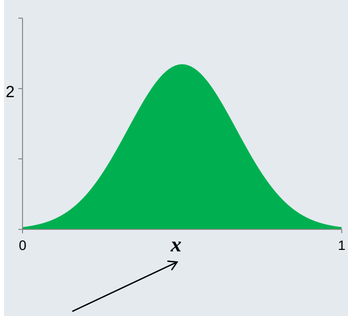
Marginal consumer

$$x_B = \frac{p_D - p_B + 80}{2 \times 80}$$



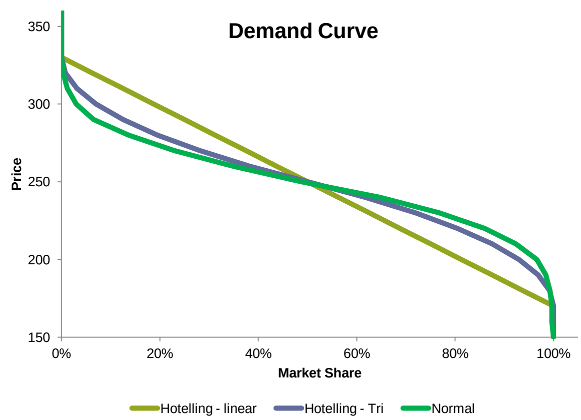
Demand curve for *Broker B* given $p_D = 250$

Hotelling model assuming a Normal distribution of preferences



Marginal consumer

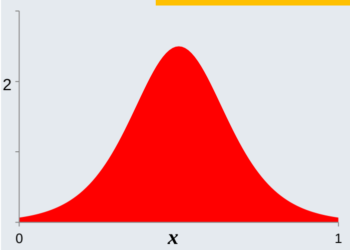
$$x_B = \frac{p_D - p_B + 80}{2 \times 80}$$



Demand curve for *Broker B* given $p_D = 250$

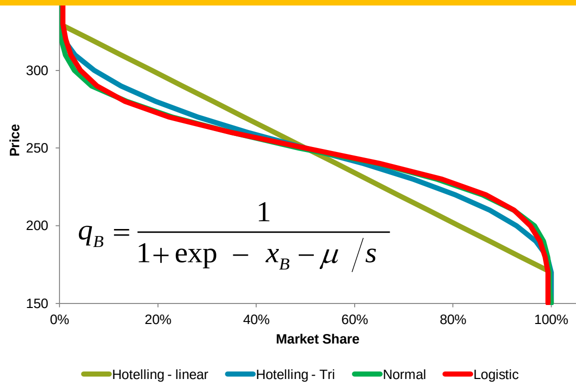
Hotelling model assuming a Logistic distribution of preferences

In practice, model directly $q_B = \dfrac{1}{1+\exp\left(\alpha\left(p_D - p_B\right)+\beta\right)}$



Marginal consumer

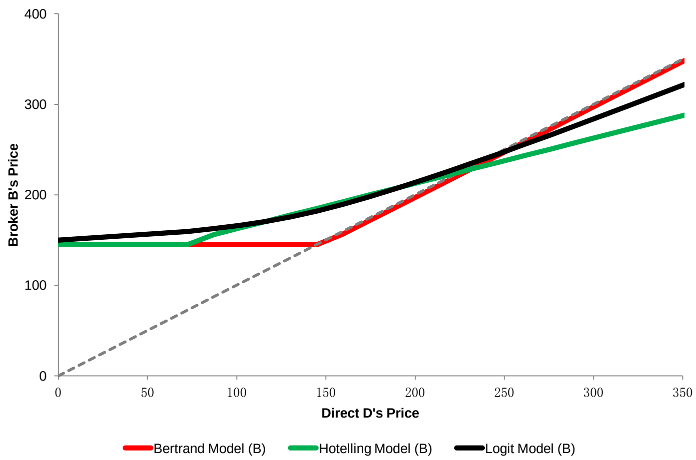
$$x_B = \frac{p_D - p_B + 80}{2\times 80}$$



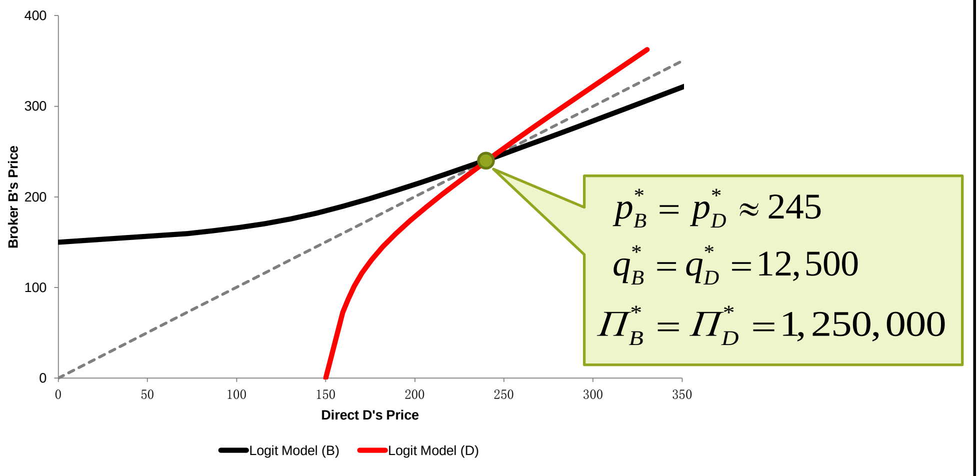
$$q_B = \frac{1}{1+\exp\left(-\left(x_B-\mu\right)/s\right)}$$

Demand curve for *Broker B* given $p_D = 250$

Best response functions for each of the different demand function forms – *Broker B*



Best response functions for *Broker B* and *Direct D* and resulting equilibrium price



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An insurance story

Allowing for customer lifetime

- *Direct D's* marketing department proposes that the price should be set for a customer so that it takes into account the expected profits that may arise from that customer in the future.
- The marketing team has commissioned the pricing department, which has estimated the discounted value of future profits for a new policy to be £43.50

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Customer Lifetime Value (CLV)

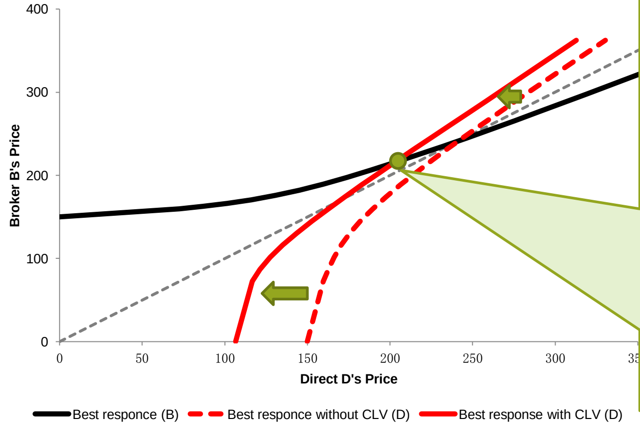
- CLV is usually used by marketers to assess the maximum amount which could be spent in acquiring a new customer
- *Direct D* could use the expected future profits (net of acquisition and other expenses) of a customer in its definition of profit:

$$\Pi_D = \; p_D - 145 + LV_D \; \times q_D$$

Expected future profit, which is specific to *Direct D*

$$\Pi_D = \; p_D - 145 + 43.50 \; \times q_D$$

Anticipated CLV reduces marginal cost which shifts best response to left and lowers equilibrium price



$$p_B^* \approx 220$$

$$p_D^* \approx 205$$

$$q_B^* \approx 9{,}000$$

$$q_D^* \approx 16{,}000$$

$$\Pi_B^* \approx 675{,}000$$

$$\Pi_D^* \approx 1{,}656{,}000$$

$$= 960{,}000 + \underbrace{696{,}000}_{\text{Future profit}}$$

CLV viability – Scenario where customer decides on insurer each year purely on price

	Year 1	2	3	4	...	n-2	n-1	n
Costs	*£10*	*£10*	*£10*	*£10*		*£10*	*£10*	*£10*

Equilibrium premium for an insurer quoting to win business with x number of years left of policy lifetim

	1	2	3	4	...	n-2	n-1	n
1 year left								£10
2 years left							£10	£10
3 years left						£10	£10	£10
n-4 years left				£10	...	£10	£10	£10
n-3 years left			£10	£10	...	£10	£10	£10
n-2 years left		£10	£10	£10	...	£10	£10	£10
n-1 years left	£10	£10	£10	£10	...	£10	£10	£10

CLV viability - Scenario where customer willing to pay £1 more to remain with current insurer each year

	Year 1	2	3	4	...	n-2	n-1	n
Costs	*£10*	*£10*	*£10*	*£10*		*£10*	*£10*	*£10*

Equilibrium premium for an insurer quoting to win business with x number of years left of policy lifetime

	1	2	3	4	...	n-2	n-1	n
1 year left								£10
2 years left							£9	£11
3 years left						£9	£10	£11
n-4 years left				£9	...	£10	£10	£11
n-3 years left			£9	£10	...	£10	£10	£11
n-2 years left		£9	£10	£10	...	£10	£10	£11
n-1 years left	£9	£10	£10	£10	...	£10	£10	£11

CLV viability – considerations

- Demonstrate that the insurer is able to charge more than other insurers for a customer in the future without that customer changing loyalty
- Discount the value of net profit to allow for the time value of money
- Acquiring business at a loss in the hope of generating profits in the future could be dangerous strategy
- Estimating future profits can be very difficult

An insurance story

Introduction of ancillary products

- *CompareTheAardvark.com* launches a new website that enables firms to sell an ancillary legal expenses product during the payment stage of the quotation process, i.e. after the customer has chosen the main product.
- *Broker B* is able to offer this product immediately.
- *Direct D* does not have such a product and is unable to offer something at this time.
- *Broker B* expects to net a profit of £29 for each legal expenses product it sells

Ancillary product profits have the net effect of reducing the marginal costs of the main product

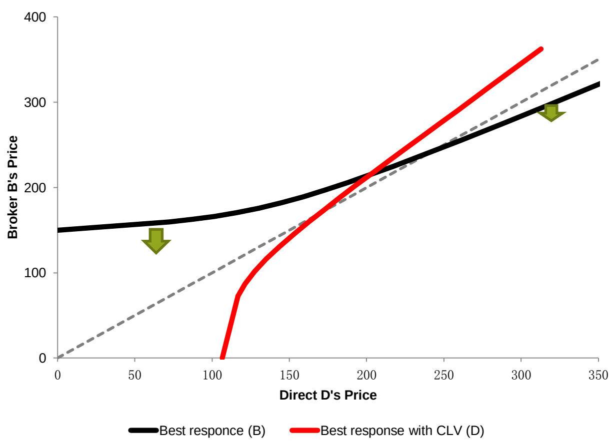
- *Broker B* is able to make a profit of £29 for each ancillary product sale
- Assuming that each main policy sale also leads to a legal expenses policy sale, then *Broker B*'s total profit becomes:

$$\Pi_B = \; p_B - 145 + AP_B \;\; \times q_B$$

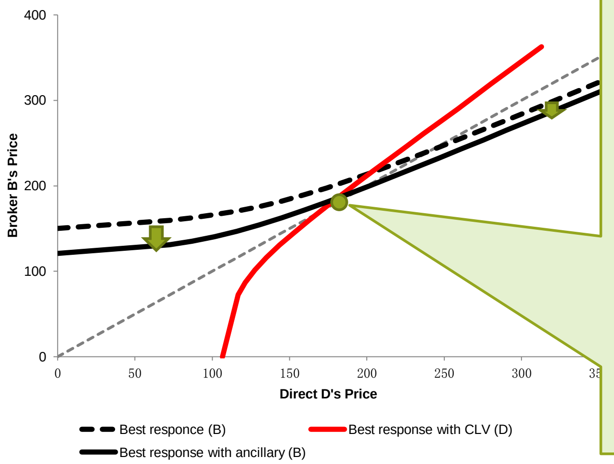
$$\Pi_B = \; p_B - 145 + 29 \;\; \times q_B$$

Expected profit of ancillary product sales, which is specific to *Broker B*

Best response and new equilibrium price following introduction of ancillary product profits



Best response and new equilibrium price following introduction of ancillary product profits



$$p_B^* \approx 185$$
$$p_D^* \approx 180$$
$$q_B^* \approx 11{,}500$$
$$q_D^* \approx 13{,}500$$
$$\Pi_B^* \approx 793{,}000$$
$$= 460{,}000 + \underbrace{333{,}000}_{\text{Ancillary Profit}}$$
$$\Pi_D^* \approx 1{,}060{,}000$$
$$= 473{,}000 + 587{,}000$$

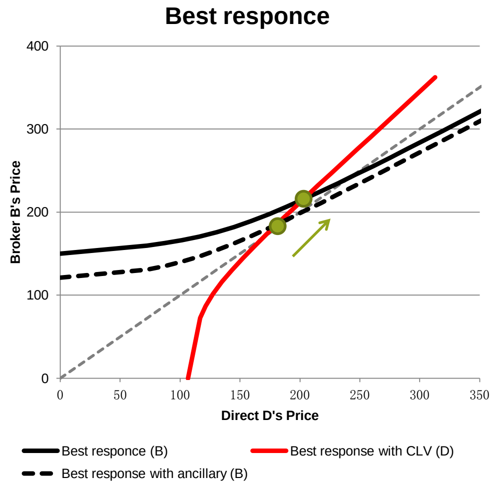
An insurance story

Regulator takes action

- The Regulator becomes aware of the large margins that insurers are making on the ancillary products and investigates
- The Regulator believes that these ancillary products are being sold in such a manner that customers are not provided with an opportunity to compare prices with other providers, i.e. insufficient competition.
- Legislation is introduced which now prevents the sale of ancillary products in such a non-competitive manner – CompareTheAardvark.com changes its website so that ancillary products must be quoted at the same time as the main product

Forcing the competitive sale of ancillary products will have no impact on final total price

- Competition will see price of ancillary product fall
- *Broker B's* responds to fall in ancillary profits by increasing price of core product
- *Direct D* will respond to increased demand by raising its price
- The core product's price increases in line with fall in ancillary profits



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An insurance story

EU Gender Directive

- European Court of Justice ruled to remove the ability of insurers to use gender as a factor in pricing and benefits from 21 December 2012

Before the legislation

- Target segment is made up of 12,500 men and 12,500 ladies. They all value the insurance policy at £400.
- Each insurer has expected marginal cost of
  £195 per policy for men and
  £95 per policy for ladies.
- Consumers have some preference for one insurer. Some consumers are willing to pay £80 more for a *Broker B* policy, with others are willing to pay £80 more for a *Direct D* policy. Men and women are both uniformly distributed between the extremes.
- Currently insurers are able to charge premiums which are different by gender ceteris paribus

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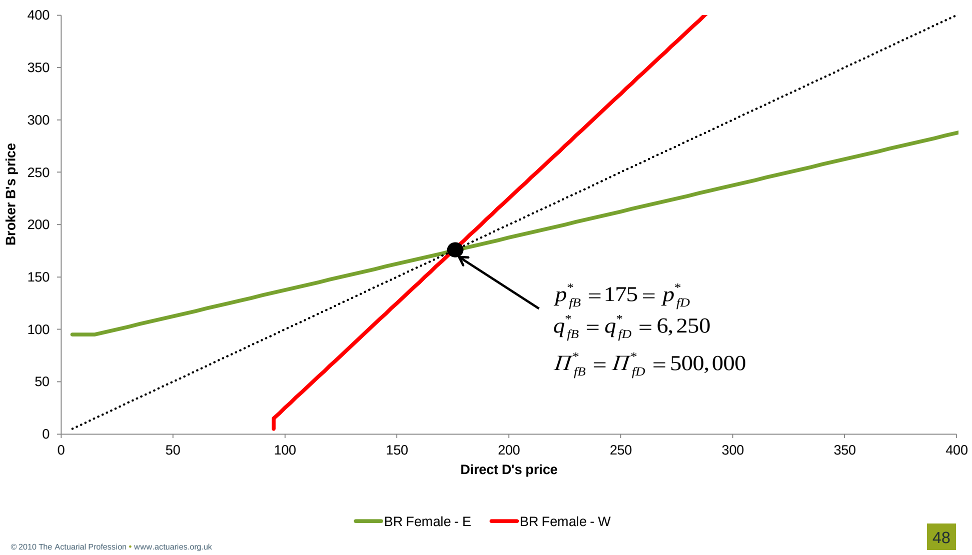
Solving the game – Pre-Gender Directive



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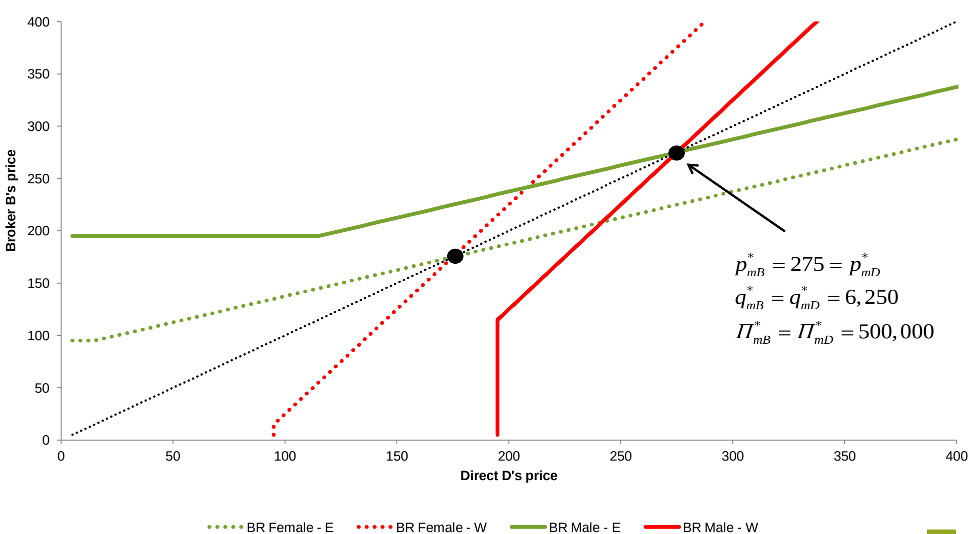
Solving the game – Pre-Gender Directive



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EU Gender Directive

After the legislation – post 21 December 2012

- Directive means each insurers must select a single premiums for both men and women ceteris paribus

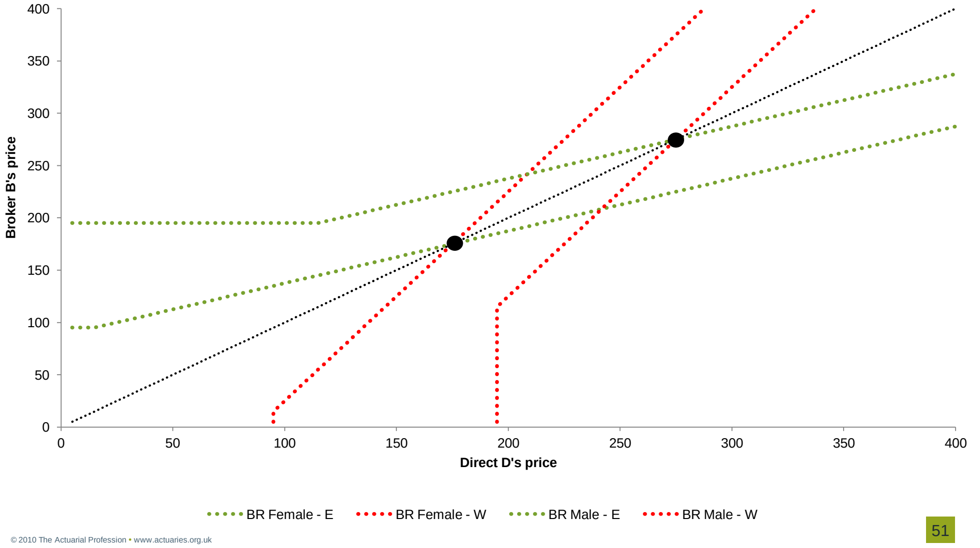
- Demand for *Broker B* will be:

$$Q_B(p_B, p_D) = q_{fB}(p_B, p_D) + q_{mB}(p_B, p_D)$$

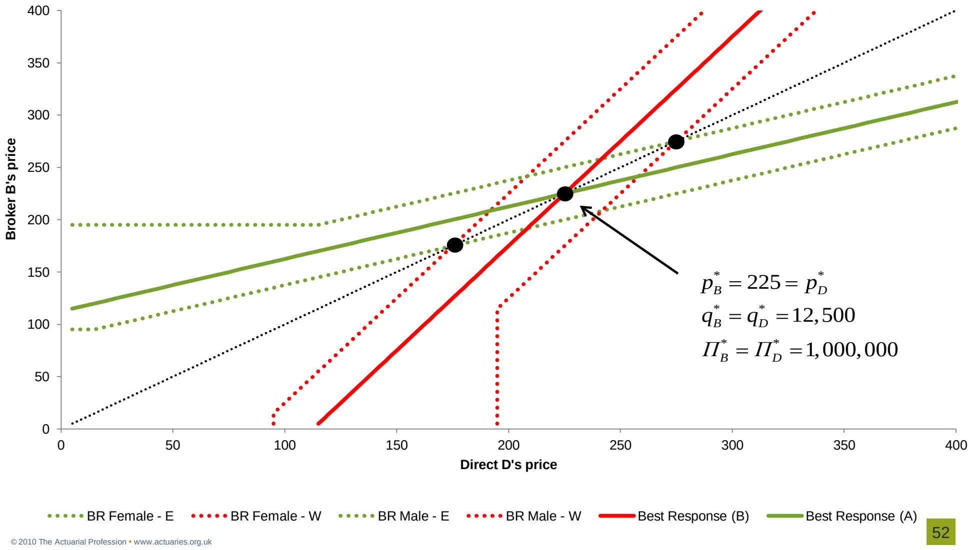
- Profit for *Broker B* will be:

$$\Pi_B(p_B, p_D) = \ p_B - 195 \ q_{mB}(p_B, p_D) + \ p_B - 95 \ q_{fB}(p_B, p_D)$$

Solving the game – Pre-Gender Directive



Solving the game – Post-Gender Directive



EU Gender Directive

Changing the game

- The marketing department of *Broker B* formulates a new plan to better target ladies through free pedicures and nail treatments. This dramatically increases the quality of the experience of *Broker B*'s customers.
- The new plan will cost a once off cost of £650,000.
- Their consultants estimate that the quality increase will increase willingness to pay of ladies by 15% with no change to that of men.

Should they go ahead with the new plan?

Solving the game – A new game

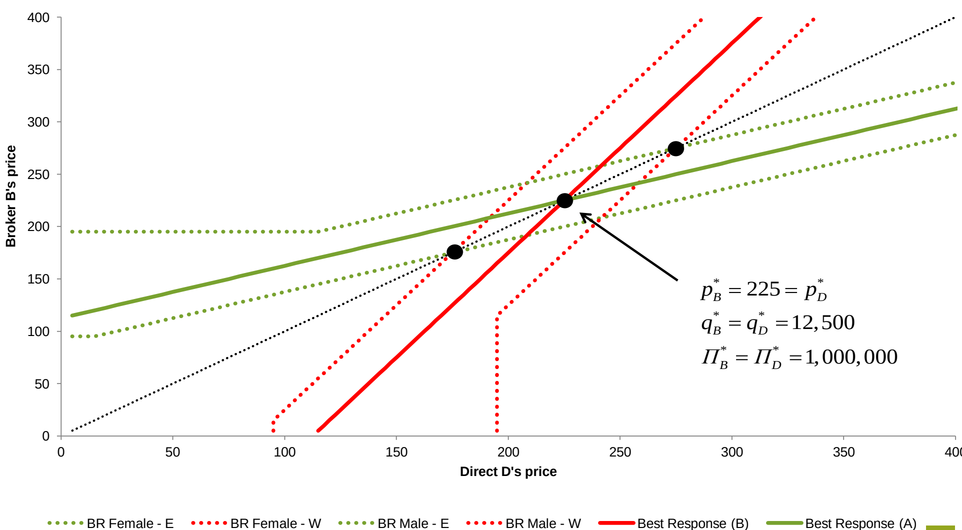
- Ladies willingness to pay increase by $400 \times 0.15 = 60$
- This has effect of moving *Broker B* to location $l_{fB} = 0.75$
- *Broker B*'s best reply is then

$$p_B = \frac{p_D \;\; 160 \;\; + 80c_f + 80c_m + 80 \;\; 80 \;\;\; 0 + 1 + 0.75 + 1}{2 \;\; 160}$$

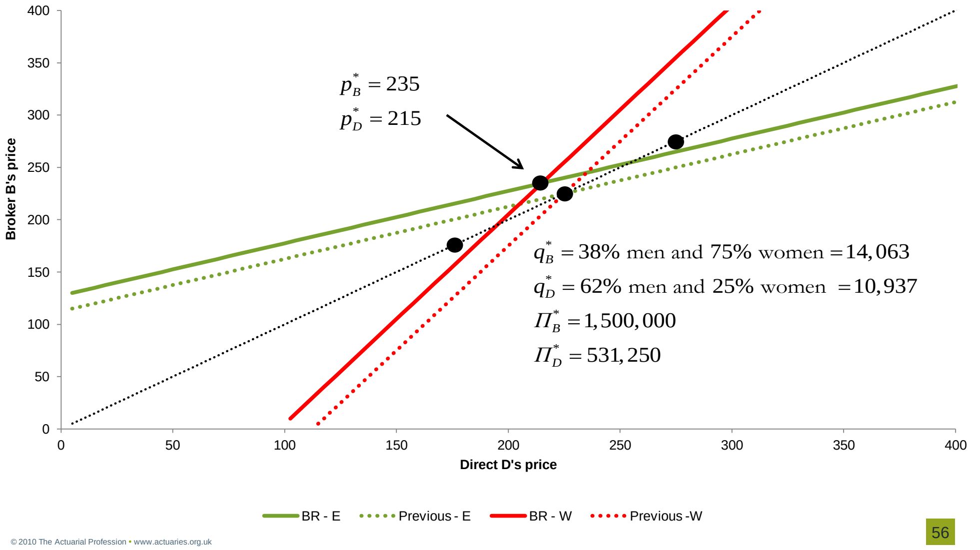
- Direct D's best reply is

$$p_D = \frac{p_B \;\; 160 \;\; + 80c_f + 80c_m + 80 \;\; 80 \;\;\; 4 - (0 + 1 + 0.75 + 1)}{2 \;\; 160}$$

Solving the game – Post-Gender Directive



Solving the game – Post-Gender Directive



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Solving the game – Post-Gender Directive
You must be attractive to women

- *Broker B* had 12,500 men and 12,500 women with profits of £1,000,000
- The 15% quality increase raises ladies' willingness to pay by £60 and not affecting men
- Other things equal, *Broker B* should benefit by £750,000
- But reduction in demand for *Direct D* prompts it to cut price …
- … resulting in *Broker B* having to reduce price in response
- Strategic effect is costly: final gain to *Broker B* is £500,000
- This doesn't cover the estimated cost of £650,000

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Conclusions (1)

- Ignoring potential competitor reactions when choosing strategic actions is suboptimal
- Models exist to allow you to overlay game thinking to insurance – but possibly not used extensively
- Bertrand Paradox in undifferentiated markets, p(A) = p(B) = c
- Product differentiation, the Bertrand paradox disappears and p(A) = p(B) = c + k
- An increase in k implies more product differentiation. Therefore, firms compete less vigorously (set higher prices) and obtain higher profits
- If k=0, then back to Bertrand

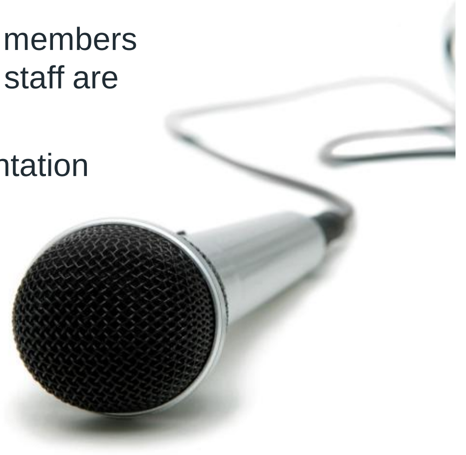
Conclusions (2)

- Ancillary product profits and CLV can reduce costs leading to lower equilibrium price. Removing ancillary product profits through legislation will raise price of underlying core product.
- CLV complex and dangerous – must get it right!
- Strategic moves (promises or threats) used to manipulate rules to a firm's own advantage
- EU Gender directive
  - Without any differentiation, price will just be the average between men and ladies
  - **You have to be attractive to women!**

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.



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