HOW A SINGLE-FACTOR CAPM WORKS IN A MULTI-CURRENCY WORLD: RESULTS FROM THE LATEST RESEARCH

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This work is a joint effort

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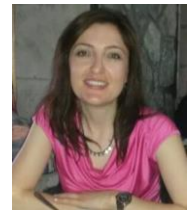
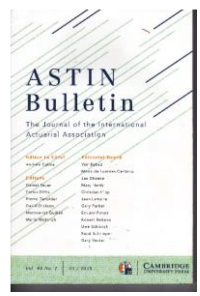


We have come a long way

- AFIR Colloquium 2013
- "Why the capital asset pricing model fails in a multi-currency world"

We have come a long way



Why use the CAPM?

- It's an equilibrium model.
- It assumes homogenous expectations.
- It's a simple model.
- Nobody has ever proved it's wrong.

Global CAPM (GCAPM)

$$E\{R_i\} = R_F + \beta_i^W \left[E\{R_W\} - R_F \right]$$

International CAPM (ICAPM)

$$E\{R_i\} = R_F + \beta_i^W \left[E\{R_W\} - R_F\right] + \gamma_i^1 \left[E\{R_M^1\} - R_F^1\right] + \ldots + \gamma_i^C \left[E\{R_M^C\} - R_F^C\right]$$

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What do we want?

- A model that:
  - Treats variance as the measure of risk, regardless of source, and
  - That produces the same price.

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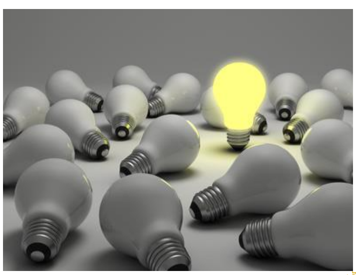
We had an idea



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But we ran into some problems

- Convergence problems
  - Number of constraints > number of unknowns

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So back to the drawing board!

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Single-factor multi-currency CAPM (SFM-CAPM) assumptions

- (1) Investors who measure their investment returns in currency *c* (i.e. ‘currency-*c* investors’) have indifference curves in mean–variance space, the means and variances being those measured in that currency.

- (2) All investors, regardless of the currency in which they measure returns, have homogeneous expectations of the means, variances and covariances of:

  (a) the returns in each currency on assets issued in that currency; and

  (b) rates of strengthening of each currency.

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The SFM-CAPM

$$E\left\{R_{di}^{c}\right\} = R_{\mathrm{F}}^{c} + \beta_{di}^{c}\left[E\left\{R_{\mathrm{M}}^{c}\right\} - R_{\mathrm{F}}^{c}\right]$$

SFM-CAPM constraint

If the SFM-CAPM applies in a multi-currency world then, for any currencies c and e:

$$\kappa_{di}^{c} = \kappa_{di}^{e}$$

where:

$$\kappa_{di}^{c} = \frac{\sigma_{di,\mathrm{M}}^{c} - \sigma_{d1,\mathrm{M}}^{c}}{\sigma_{\mathrm{M,M}}^{c}}\left(\mu_{\mathrm{M}}^{c} - r_{c}\right)$$

SFM-CAPM mark 1

Minimise:

$$D_{\mu}^{2} = \frac{1}{Q_{\mu}}\left[\sum_{c=1}^{C}\left\{\sum_{i=2}^{n_c}\left(\hat{\mu}_{ci}^{(\mathrm{S})} - \hat{\mu}_{ci}^{(\mathrm{G})}\right)^{2}\right\} + \sum_{c=2}^{C}\left(\hat{\mu}_{c}^{(\mathrm{S})} - \hat{\mu}_{c}^{(\mathrm{G})}\right)^{2}\right]$$

subject to the constraints:

$$\kappa_{di}^{c} = \frac{\hat{\sigma}_{di,\mathrm{M}}^{c} - \hat{\sigma}_{d1,\mathrm{M}}^{c}}{\hat{\sigma}_{\mathrm{M,M}}^{c}}\left(\hat{\mu}_{\mathrm{M}}^{c(\mathrm{S})} - r_{c}\right) = \frac{\hat{\sigma}_{di,\mathrm{M}}^{e} - \hat{\sigma}_{d1,\mathrm{M}}^{e}}{\hat{\sigma}_{\mathrm{M,M}}^{e}}\left(\hat{\mu}_{\mathrm{M}}^{e(\mathrm{S})} - r_{e}\right) = \kappa_{di}^{e}$$

SFM-CAPM mark 2

Minimise:

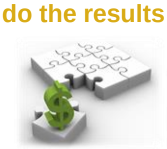
$$D^{2} = D_{\mu}^{2} + hD_{\kappa}^{2}$$

where:

$$D_{\mu}^{2} = \frac{1}{Q_{\mu}}\left[\sum_{c=1}^{C}\left\{\sum_{i=2}^{n_c}\left(\hat{\mu}_{ci}^{(\mathrm{S})} - \hat{\mu}_{ci}^{(\mathrm{G})}\right)^{2}\right\} + \sum_{c=2}^{C}\left(\hat{\mu}_{c}^{(\mathrm{S})} - \hat{\mu}_{c}^{(\mathrm{G})}\right)^{2}\right]$$

$$D_{\kappa}^{2} = \frac{1}{Q_{\kappa}}\sum_{c,e=1}^{C}\sum_{(d,i)\in\Psi_c}\left(\kappa_{di}^{c} - \kappa_{di}^{e}\right)^{2}$$

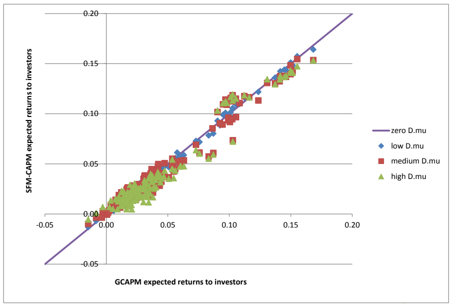
What do the results look like?



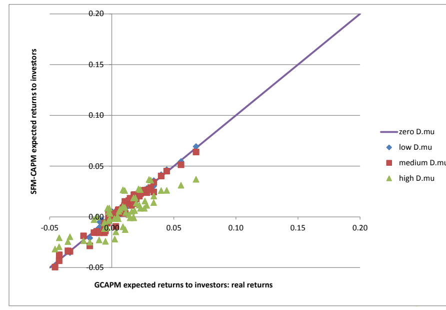

Data

Data set	Period		USA		UK		SA		TR	
			e, cb	ilb	e, cb	ilb	e, cb	ilb	e, cb	ilb
nominal returns										
1	1975Q2	1985Q4	√		√		√			
2	1986Q1	1995Q4	√		√		√		√	
3	1996Q1	2005Q2	√		√		√		√	
4	2005Q3	2012Q1	√		√		√		√	
5	2005Q3	2012Q1	√	√	√	√	√	√		
6	2009Q4	2012Q1	√	√					√	√
7	1975Q2	2012Q1	√		√		√			
8	1986Q1	2012Q1	√		√		√		√	
real returns										
1	2003Q2	2009Q3	√	√	√	√				
2	2005Q3	2009Q3	√	√	√	√	√	√		
3	2009Q4	2012Q1	√	√					√	√
4	2005Q3	2012Q3	√	√	√	√	√	√		
5	2003Q2	2012Q3	√	√	√	√				

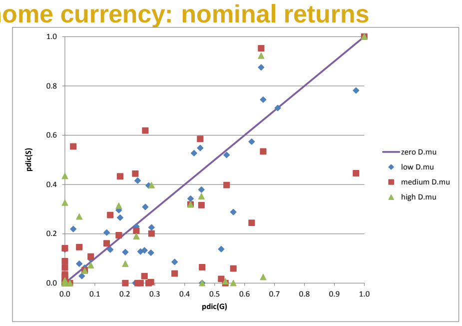
Expected returns to investors: nominal



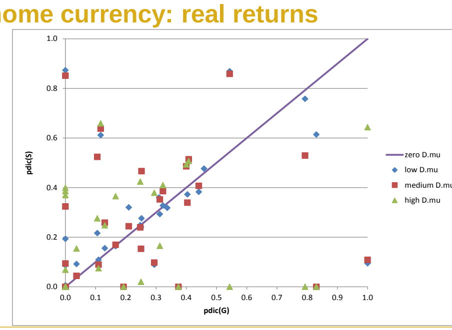
Expected returns to investors: real



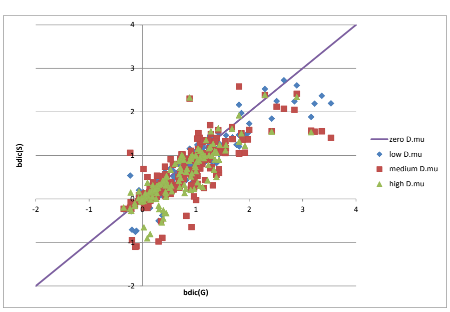
Optimal portfolios: home currency: nominal returns



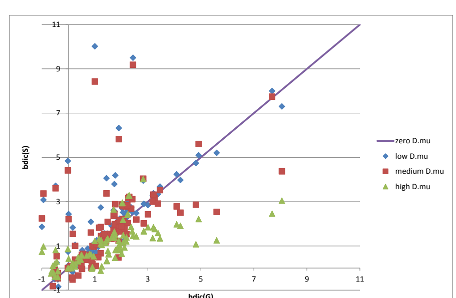
Optimal portfolios: home currency: real returns



Betas: nominal returns



Betas: real returns



The SFM-CAPM

$$E\left\{R_{di}^{c}\right\} = R_{\mathrm{F}}^{c} + \beta_{di}^{c}\left[E\left\{R_{\mathrm{M}}^{c}\right\} - R_{\mathrm{F}}^{c}\right]$$

Why adopt the SFM-CAPM?

- It's better than the ICAPM.
- It's better than the GCAPM.
- The difference is material.

A word of advice

- Use real returns rather than nominal returns:
  - It's closer to the GCAPM, so the adjustments required are smaller.
  - The GCAPM is supposed to be about optimising consumption.
  - Financial mathematicians can't use real returns; actuaries can.

Teşekkürler

Thank you