The Actuarial Profession
making financial sense of the future

Risk adjustment for loss reserving by a Cost of Capital technique

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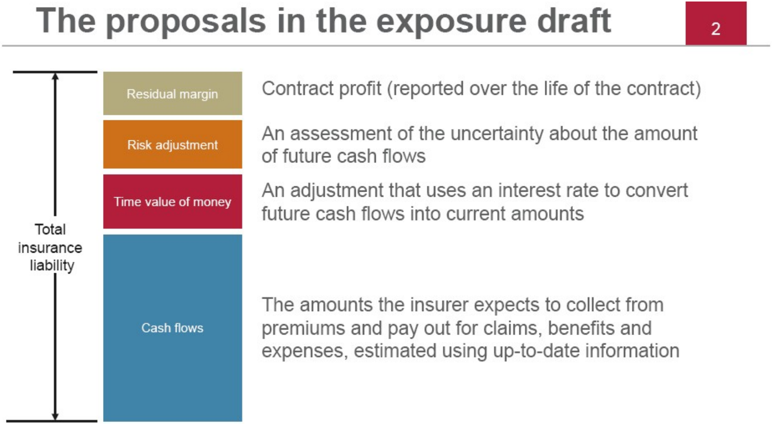
GIRO Conference and Exhibition 2011

11-14 October 2011

Outline

Introduction

IAS Board: state of the art on Risk Adjustment



IAS Board: state of the art on Risk Adjustment

Residual margin

Risk adjustment

Time value of money

Cash flows

ED proposal	Tentative decisions
Objective	
• "The ***maximum amount*** the insurer would ***rationally pay*** to be relieved of the risk that the ultimate cash flows exceed those expected."	• "The ***compensation*** the insurer ***requires to bear*** the risk that the ultimate cash flows could exceed those expected"
Risk adjustment vs composite margin	
• Include explicit estimate of the effects of uncertainty about future cash flows.	✓measurement of liability should include explicit risk adjustment
To complete	
? Revisit objective ? Whether to restrict permitted techniques ? Disclosures about risk adjustments ? Whether to take account of diversification benefits	

Fair value = market value (price)

or

Fair value =

best estimate future cash flow adjusted for time value of money

plus

Market Value Margin

Fair value as part of the subjective theory of value (Austrian school of economics)

Imagine two parties in a virtual exchange situation:

- The insurer who bears the risk taking
- A reinsurer who might take over that risk taking.

Fair value as part of the subjective theory of value (Austrian school of economics)

The *insurer* will consider the amount rationally being paid to be relieved of the total risk bearing (including the risk that the ultimate cash flows exceed those expected).

Fair value as part of the subjective theory of value (Austrian school of economics)

The *reinsurer* who is willing to take over that total risk within the regulated world of Solvency II.

Fair value as part of the subjective theory of value (Austrian school of economics)

For the *reinsurer* a minimum value is calculated taking into account:

- the objective prerequisites of Solvency II, usually meaning a minimal reserve the reinsurer has to keep (VaR 99.5 %)
- the subjective earning goal expressed in its Cost of Capital rate
- using Integral Financial Modelling on two loss triangles (paid and incurred) for estimates of future payments

What is Integral Financial Modelling (IFM)

A combined stochastic model for paid and incurred run-off tables that provides an excellent tool for modern risk and capital management.

The remainder of this presentation consists of two parts:

- We will first discuss how this Fair Value of liability might be determined
- Next, we will discuss how IFM estimates the probability distribution of future payments.

Fair Value of Liability

Let $B(t_1, t_2)$ denote the future payments in $[t_1, t_2]$ and suppose we have

- a function $b(t)$ such that

$$\mathbb{E}B(t_1, t_2) = \int_{t_1}^{t_2} b(s)\, ds$$

- a function $V(t)$ such that

$$\mathbb{P}(B(t, \infty) \geq V(t)) = 0.995.$$

Note At time t the insurer must have a minimal reserve equal to $V(t)$. This part of his capital cannot be used to yield more than the risk-free rate.

Now suppose the liability is tranferred to a reinsurer. Denote

- R_f risk free rate
- R_t total rate = $R_{coc} + R_f$

The reinsurer's capital $E(t)$ satisfies

$$\frac{dE}{dt} = (E(t) - V(t))R_t(t) + V(t)R_f(t) - b(t)$$

subject to the boundary condition $E(\infty) = 0$.
Solving this differential equation, we find

$$E(t) = \int_t^{\infty} (V(s)(R_t(s) - R_f(s)) + b(s)) \exp\left(-\int_t^s R_t(u)\,du\right) ds.$$

and in particular, we find the Fair Value $E(0)$.

Interpretation

$$E(t) = \int_t^\infty (V(s)R_{coc} + b(s)) \exp\left(-\int_t^s R_t(u)\,du\right) ds.$$

Since the reinsurer invests her capital at the risky rate $R_t(t)$, a payment made at time $s \geq t$ (so in our setup this would be $b(s)\,ds$), should be discounted using the rate R_t. This leads to a discounted total future payment at time t of

$$\int_t^\infty b(s) \exp\left(-\int_t^s R_t(u)\,du\right) ds.$$

Interpretation

$$E(t) = \int_t^{\infty} (V(s)R_{coc} + b(s)) \exp\left(-\int_t^s R_t(u)\, du\right) ds.$$

Reserving $V(s)$ means that the reinsurer can invest this capital only at the risk-free rate. This corresponds to a *loss* in the time interval ds equal to $V(s)R_{coc}\, ds$. This loss is then discounted at the risky rate, just like the payments:

$$\int_t^{\infty} V(s)R_{coc} \exp\left(-\int_t^s R_t(u)\, du\right) ds.$$

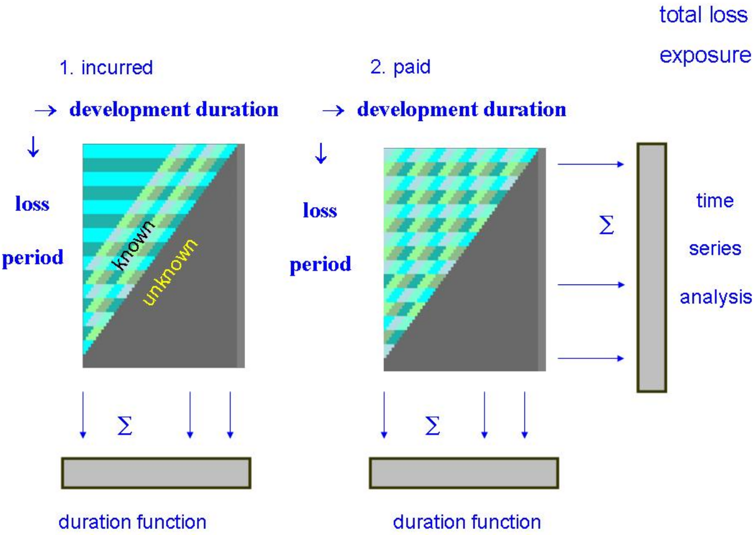
Liability Estimation

Suppose our data consist of *two* loss triangles, namely paid and reported incurred, together with a measure for exposure.

Many methods and models have been developed for analyzing a single loss triangle.

We model the paid and incurred *simultaneously*.

Picture of the model



Initial definitions

We will use the following notation to define our model for the paid and the incurred run-off tables.

- l indicates the loss period.
- k indicates the development period.
- $Y_{lk}^{(1)}$ indicates the incremental incurred.
- $Y_{lk}^{(2)}$ indicates the incremental paid.

Our goal is to model the vector $(Y^{(1)}, Y^{(2)})$, including all future values.

Auxiliary variables

We start with auxiliary independent Gaussian random variables:

$$Z_{lk}^{(1)} \sim N\left(\mu_{lk}^{(1)}, V_{lk}^{(1)}\right)$$

$$Z_{lk}^{(2)} \sim N\left(\mu_{lk}^{(2)}, V_{lk}^{(2)}\right)$$

Now, define the event

$$R = \left\{ \sum_k Z_{lk}^{(1)} = \sum_k Z_{lk}^{(2)} \quad (\forall l) \right\}.$$

This says that for each loss period, the total amount incurred equals the total amount paid.

Final step

Finally we define the incremental losses by

$$Y^{(1)} \sim Z^{(1)} \mid R$$
$$Y^{(2)} \sim Z^{(2)} \mid R$$

This means that $(Y^{(1)}, Y^{(2)})$ is normally distributed, and that the row sums of the two tables are always equal.

The parameters in this model are given by $\mu_{lk}^{(i)}$ and $V_{lk}^{(i)}$ for $i = 1, 2$. We should reduce the number of parameters.

Product structure of parameters

We choose the following product structure for our parameters:

$$\mu_{lk}^{(i)} = W_l \, e^{(X\alpha)_l} \, \Pi_k^{(i)} \quad (i = 1, 2)$$

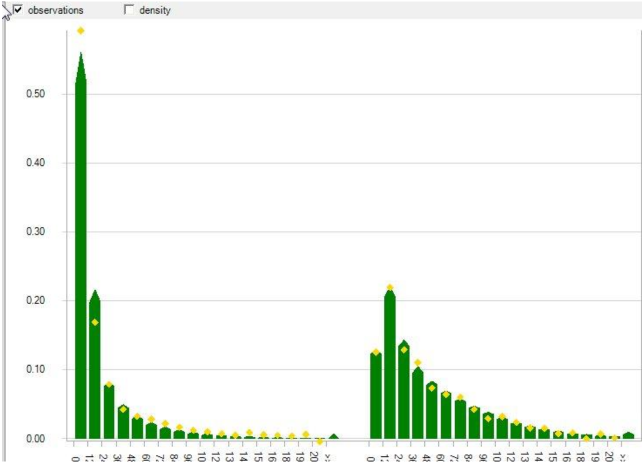
$$V_{lk}^{(i)} = \sigma^{(i)} \, W_l \, e^{(X\alpha)_l} \, \tilde{\Pi}_k^{(i)} \quad (i = 1, 2)$$

- W_l is the exposure measure for loss period l.
- $X\alpha$ is a linear model for the loss ratios, with design matrix X and parameter vector α.
- $\Pi^{(i)}$ and $\tilde{\Pi}^{(i)}$ distribute the total expectation, respectively variation, over all development periods:

$$\sum_k \Pi_k^{(i)} = \sum_k \tilde{\Pi}_k^{(i)} = 1 \quad (i = 1, 2).$$

- $\sigma^{(i)}$ $(i = 1, 2)$ are parameters used to tune the total variation.

Example of parametric development curves



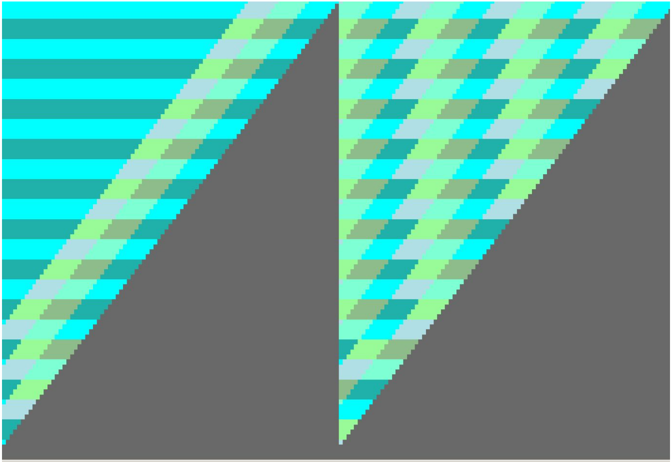
Data and estimation

The data usually consist of aggregated cells of the two run-off tables. We allow for any kind of aggregation given by some zero-one selection matrix S. So if $Y = (Y^{(1)}, Y^{(2)})$, our data is given by SY.

Note that SY is still normally distributed, with a known distribution, given the parameters. Therefore we can use Maximum Likelihood to estimate all the parameters.

Also prediction is relatively easy, since $Y \mid SY$ is again normally distributed. This structure makes our model very flexible. For a more detailed description, see “Combined analysis of Paid and Incurred Losses”, CAS e-forum, Fall 2008.

Example of aggregation



We have demonstrated that our joint model for paid and incurred losses performs very well on actual data.

Moreover, our model has several important advantages in managing “difficult” data:

1. It is flexible in aggregating various data sets and in handling aggregation levels of the input data.
2. It handles loss triangles with missing cells with ease.
3. It incorporates in a proper way negative data coming from negative adjustments to losses.

Implementation of Fair Value

We have the joined distribution of all future payments per loss period and per settlement period, conditioned on the data. If we denote $[t_{i-1}, t_i]$ as the i^{th} period after the current date $t = 0$, we know the distribution of

$$B(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} b(s)\, ds.$$

This directly leads to an estimate of $V(t_{i-1})$. We could then assume that V and b are constant functions on each interval $[t_{i-1}, t_i]$. Calculating the Fair Value will then be straightforward from

$$E(t) = \int_t^{\infty} (V(s)R_{coc} + b(s)) \exp\left(-\int_t^s R_t(u)\, du\right) ds.$$

Conclusion

Subjective theory of valuation together with our Integral Financial Modelling software leads straightforward to an Adjusted Loss Provision in agreement with the starting points of Solvency II and IFRS.

An example

Line of Business: Professional Liability (claims made)
Two loss Triangles: Incurred (reported) and Paid.

Data structure (accounting by loss period)

Incurred:

- year by year from 1995Q1
- quarter by year from 2004Q1

Paid:

- quarter by quarter from 1995Q1.

IFM one triangle analysis on paid to gain insight in:

- Loss ratio structure: Time series with regime changes at January 1999 and January 2003
- Structure Development Duration: Weibull-Gamma mixture

Chart Time Series on Paid

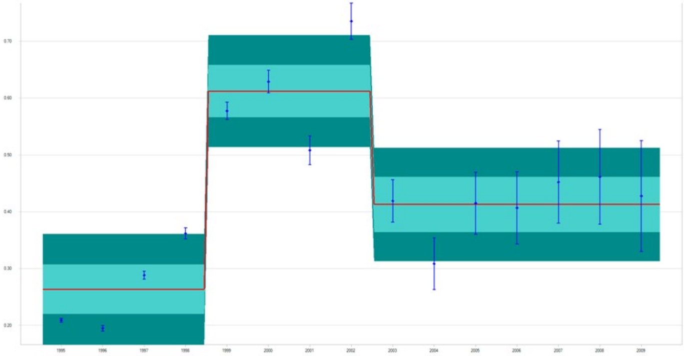
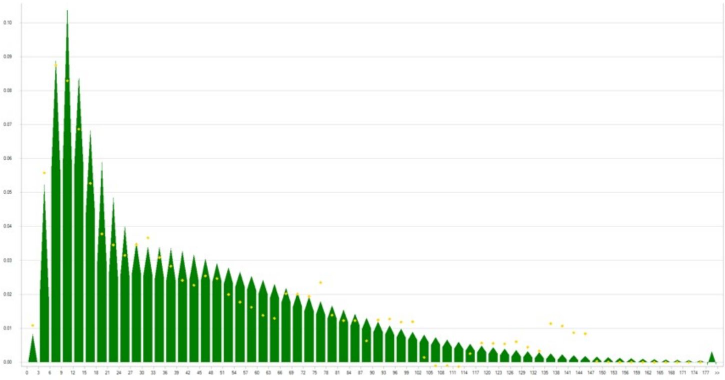
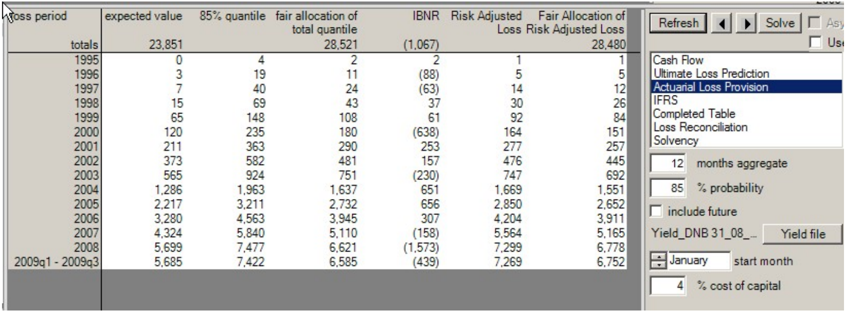


Chart Development Duration on Paid



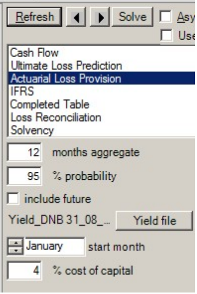
Outcomes Adjusted Loss on one triangle



loss period	expected value	85% quantile	fair allocation of total quantile	IBNR	Risk Adjusted Loss	Fair Allocation of Risk Adjusted Loss
totals	23,851		28,521	(1,067)		28,480
1995	0	4	2	2	1	1
1996	3	19	11	(88)	5	5
1997	7	40	24	(63)	14	12
1998	15	69	43	37	30	26
1999	65	148	108	61	92	84
2000	120	235	180	(638)	164	151
2001	211	363	290	253	277	257
2002	373	582	481	157	476	445
2003	565	924	751	(230)	747	692
2004	1,286	1,963	1,637	651	1,669	1,551
2005	2,217	3,211	2,732	656	2,850	2,652
2006	3,280	4,563	3,945	307	4,204	3,911
2007	4,324	5,840	5,110	(158)	5,564	5,165
2008	5,699	7,477	6,621	(1,573)	7,299	6,778
2009q1 - 2009q3	5,685	7,422	6,585	(439)	7,269	6,752

IFM projection on combined Incurred and Paid triangle

loss period	expected value	95% quantile	fair allocation of total quantile	IBNR	Risk Adjusted Loss	Fair Allocation of Risk Adjusted Loss
totals	23,343		27,578	(2,010)		27,878
1995	2	64	28	28	6	5
1996	43	170	96	(3)	62	56
1997	48	205	114	27	83	72
1998	34	217	111	105	86	69
1999	119	431	251	204	231	193
2000	396	754	547	(271)	572	508
2001	210	606	377	340	395	329
2002	384	845	579	255	632	540
2003	687	1,214	910	(72)	1,016	891
2004	1,079	1,839	1,400	414	1,571	1,381
2005	1,882	2,852	2,292	216	2,580	2,307
2006	2,922	4,104	3,421	(216)	3,856	3,487
2007	4,079	5,473	4,668	(601)	5,261	4,792
2008	5,792	7,417	6,478	(1,716)	7,294	6,697
2009q1 - 2009q3	5,668	7,175	6,305	(720)	7,125	6,552



Summary of outcomes (1)

loss period	expected value Loss Provision	expected value LP discounted by Yieldcurve	Risk Adjusted Loss	Fair Allocation of Risk Adjusted Loss
totals	25,908	23,851		28,480
1995	0	0	1	1
1996	3	3	5	5
1997	7	7	14	12
1998	16	15	30	26
1999	68	65	92	84
2000	126	120	163	151
2001	221	211	276	257
2002	392	373	476	445
2003	597	565	747	692
2004	1,366	1,286	1,669	1,551
2005	2,372	2,217	2,850	2,652
2006	3,539	3,280	4,204	3,911
2007	4,715	4,324	5,564	5,165
2008	6,255	5,699	7,299	6,778
2009q1 - 2009q3	6,233	5,685	7,269	6,752

IFM Projections on the paid triangle (Amounts in € 1,000)

Summary of outcomes (2)

loss period	expected value Loss Provision	expected value LP discounted by Yieldcurve	Risk Adjusted Loss	Fair Allocation of Risk Adjusted Loss
totals	25,766	23,343		27,878
1995	2	2	6	5
1996	44	43	62	56
1997	50	48	83	72
1998	36	34	86	69
1999	127	119	231	193
2000	434	396	572	508
2001	226	210	395	329
2002	419	384	632	540
2003	758	687	1,016	891
2004	1,186	1,079	1,571	1,381
2005	2,077	1,882	2,580	2,307
2006	3,232	2,922	3,856	3,487
2007	4,512	4,079	5,261	4,792
2008	6,404	5,792	7,294	6,697
2009q1 - 2009q3	6,262	5,668	7,125	6,552

IFM projections on Paid and Incurred (Amounts in € 1,000)

Summary of outcomes (3)

loss period	expected value Loss Provision	expected value LP discounted by Yieldcurve	Risk Adjusted Loss	Fair Allocation of Risk Adjusted Loss
totals	(141)	(509)	-	(602)
1995	2	2	5	4
1996	41	40	56	51
1997	43	41	69	60
1998	20	19	56	43
1999	59	54	139	109
2000	308	276	408	357
2001	5	(1)	119	72
2002	27	11	157	95
2003	161	122	269	199
2004	(179)	(207)	(98)	(169)
2005	(295)	(335)	(269)	(345)
2006	(307)	(358)	(348)	(424)
2007	(203)	(245)	(303)	(373)
2008	149	92	(5)	(81)
2009q1 - 2009q3	29	(18)	(143)	(200)
	Differences IFM projections (Amounts in € 1,000)			
	IFM[Incurred & Paid] minus IFM [Paid]			

Summary of outcomes (4)

In IAS Board terms:			
	IFM projected on		
	Paid triangle	Paid & Incurred combined	Difference
best estimate cash flow	25,908	25,766	(142)
time value of money	(2,057)	(2,423)	(366)
risk adjustment	4,629	4,535	(94)
Risk adjusted Loss (sum)	28,480	27,878	(602)

Conclusion

In this (well-managed) portfolio Risk Adjusted Loss amounts to about 17.5 % (expressed into the best estimate cash flow).