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A Systematic Approach to Event Modelling & Clash Pricing

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25 August 2016

Does my tail look big in this?

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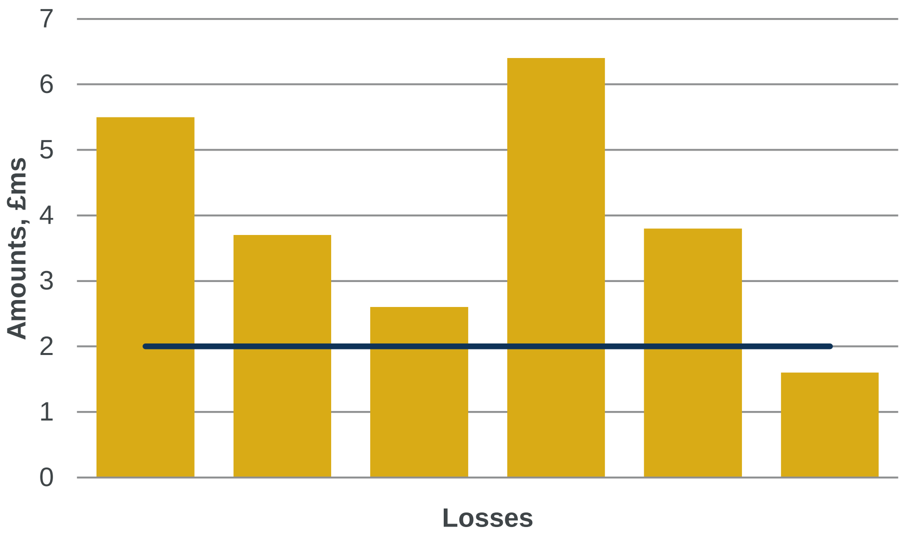
Agenda

- What is clash
- A brief overview
- Methodology
- Results
- Issues and future improvements

What is Clash?

- Reinsurance
- Protects the insured against multiple losses from the same event



Overview

- Losses are usually modelled on an individual risk basis using a frequency-severity approach
- Unfortunately this approach doesn't allow Clash treaties to be modelled, and generates tails that are too thin for Capital modelling
- We modify the usual simulation methodology to simulate events, which enables us to:
  - Correlate losses within an event
  - Model risk and clash treaties on a coherent basis
  - Price Clash treaties
  - Generate thicker tails to get a more "realistic" view of capital requirements and probability of risk XL horizontal failure

Methodology

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Data

The premium for the excess book would be presented in a typical Limit/Attachment Profile:

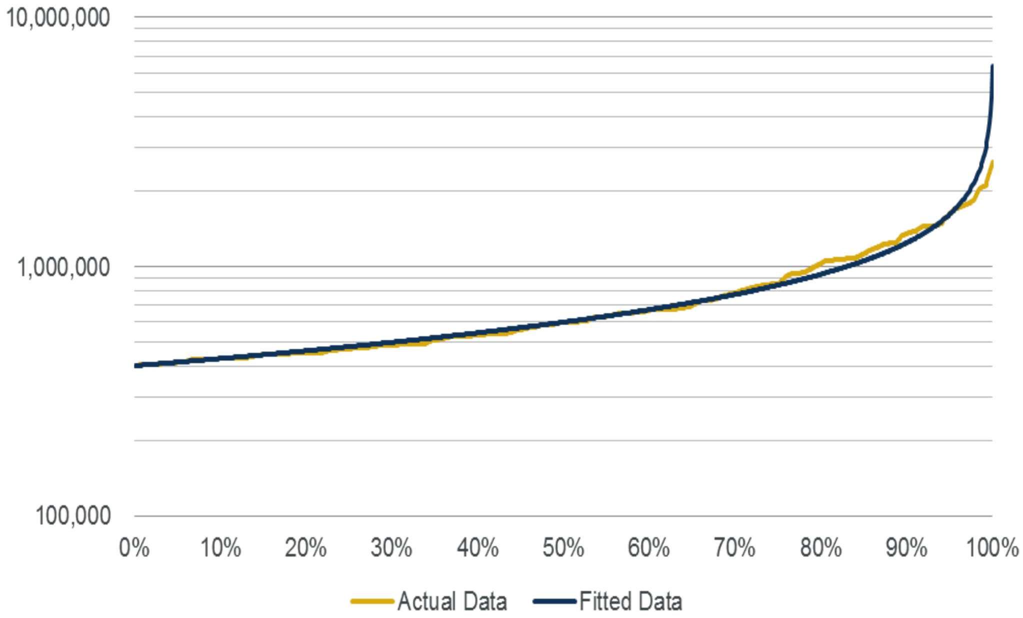
Attachment

Limit	A_1	A_2	A_3
L_1	x_{11}	x_{12}	x_{13}
L_2	x_{21}	x_{22}	x_{23}
L_3	x_{31}	x_{32}	x_{33}

Severity Assumptions

Need to assume a distribution for the severity of losses:

- Back solving market ILFs
- Fitting to client's own experience



Limited Expected Values (LEVs)

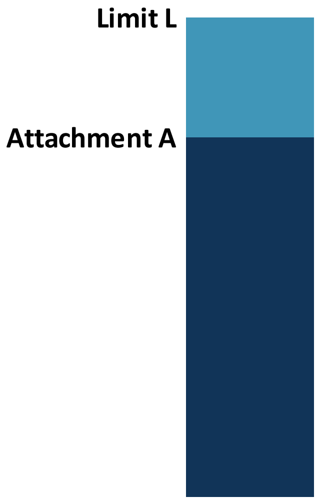
We are going to assume that losses are correlated fgu, so we first estimate the equivalent fgu premium in each cell by assuming that premium is distributed pro-rata to expected loss.

The Limited Expected Value (LEV) is defined as:

$$LEV(u) = \int_0^u xf(x)dx + u(1 - F(u))$$

where F(x) & f(x) are the cumulative and probability density functions of the severity distributions respectively.

Obtain the FGU Premium



The premium for the excess layer is

$$x_{ij} = \frac{y_{ij} * \left(LEV(L_i + A_j) - LEV(A_j)\right)}{LEV(L_i + A_j)}$$

The equivalent fgu premium y_{ij} in each cell is

$$y_{ij} = \frac{x_{ij} * LEV(L_i + A_j)}{LEV(L_i + A_j) - LEV(A_j)}$$

Expected number of FGU Losses

Given the assumed loss ratio LR, and a cell frequency of λ_{ij} we have:

$$\lambda_{ij} * LEV(L_i + A_j) = y_{ij} * LR$$

Re-writing this gives: $\lambda_{ij} = \frac{LR * x_{ij}}{LEV(L_i + A_j) - LEV(A_j)}$

Number of losses per cell is:

Attachment

Limit	A_1	A_2	A_3
L_1	λ_{11}	λ_{12}	λ_{13}
L_2	λ_{21}	λ_{22}	λ_{23}
L_3	λ_{31}	λ_{32}	λ_{33}

Adding up the individual cell frequencies, we get the total frequency:

$$\Lambda = \sum_i \sum_j \lambda_{ij}$$

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Obtain the Conditional Distribution

Remember that $\Lambda = \sum_i \sum_j \lambda_{ij}$

Attachment

Limit

	A_1	A_2	A_3
L_1	$\frac{\lambda_{11}}{\Lambda}$	$\frac{\lambda_{12}}{\Lambda}$	$\frac{\lambda_{13}}{\Lambda}$
L_2	$\frac{\lambda_{21}}{\Lambda}$	$\frac{\lambda_{22}}{\Lambda}$	$\frac{\lambda_{23}}{\Lambda}$
L_3	$\frac{\lambda_{31}}{\Lambda}$	$\frac{\lambda_{32}}{\Lambda}$	$\frac{\lambda_{33}}{\Lambda}$

Obtain the Conditional Distribution

$$c_{11} = \frac{\lambda_{11}}{\Lambda}$$

$$c_{12} = \frac{\lambda_{11}}{\Lambda} + \frac{\lambda_{12}}{\Lambda}$$

$$c_{13} = \frac{\lambda_{11}}{\Lambda} + \frac{\lambda_{12}}{\Lambda} + \frac{\lambda_{13}}{\Lambda}$$

$$c_{21} = \frac{\lambda_{11}}{\Lambda} + \frac{\lambda_{12}}{\Lambda} + \frac{\lambda_{13}}{\Lambda} + \frac{\lambda_{21}}{\Lambda}$$

Attachment

Limit

	A_1	A_2	A_3
L_1	c_{11}	c_{12}	c_{13}
L_2	c_{21}	c_{22}	c_{23}
L_3	c_{31}	c_{32}	c_{33}

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Specify an Event Distribution

If we assume that the distribution for the number of losses coming from an event is

Number of Losses (k)	Probability
1	p_1
2	p_2
3	p_3
..	..
..	..
n	p_n

Then the expected number of losses given an event has occurred is:

$$N = \sum_k k \, p_k$$

And the expected number of events is:

$$E = \Lambda / N.$$

Simulation Process

- Sample from the distribution for the total number of events, E, to determine how many events have occurred
- For each event , sample off the Number of losses distribution to determine how many losses have occurred, n
- Sample n independent numbers from a Standard Normal distribution
- Correlate using the Cholesky algorithm and generate losses from the severity distribution
- Use the appropriate conditional c_{ij} distribution to determine the particular excess(j) and limit(i) points for the loss sampled, and therefore the net loss to the insurer

Simulation Process Continued

- Apply any Risk XL terms to the individual losses to determine these recoveries
- Aggregate claims (capped at max contribution) for each loss, and when all losses from a particular event have been sampled, apply the Clash Excess of loss terms
- Repeat for all Events
- Repeat for each Simulation

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Detour — Correlations

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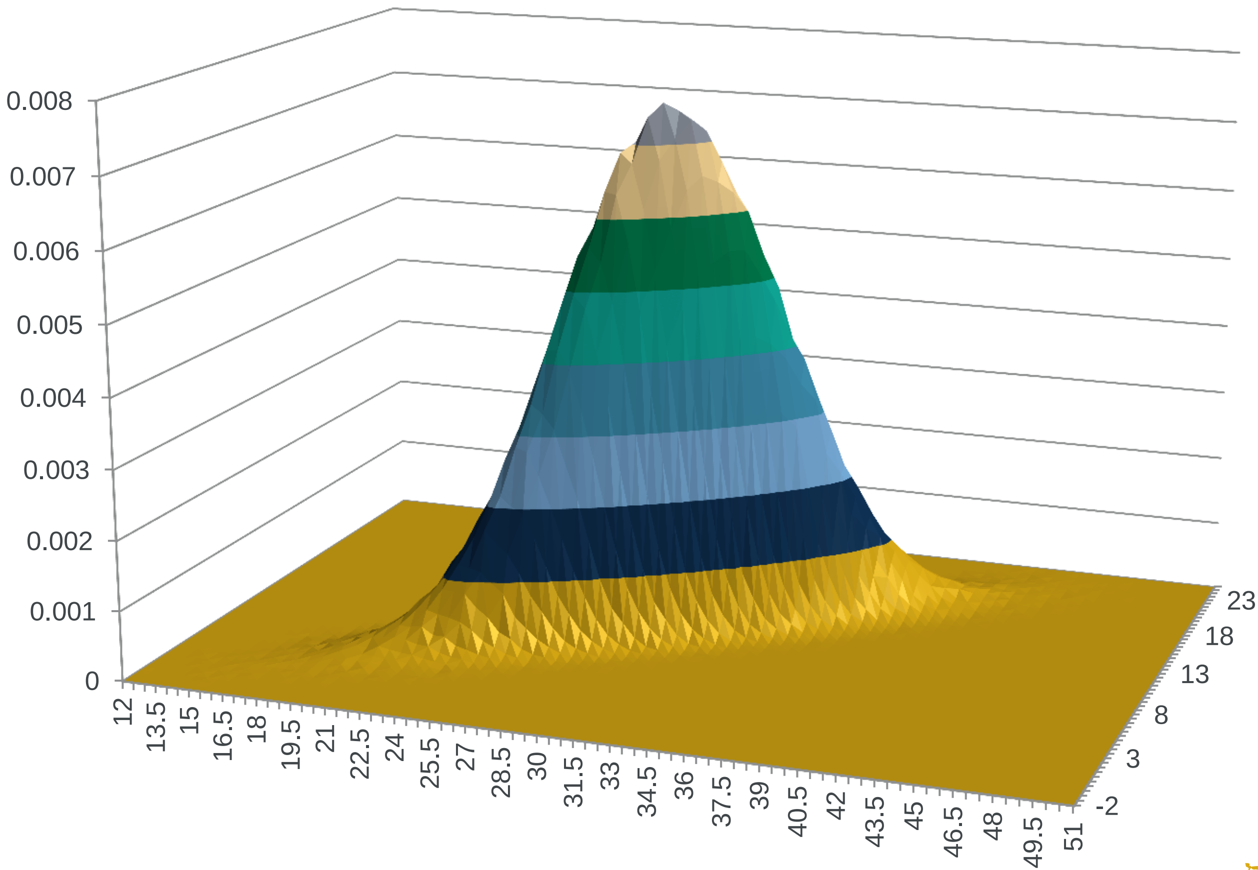
What is a Copula?



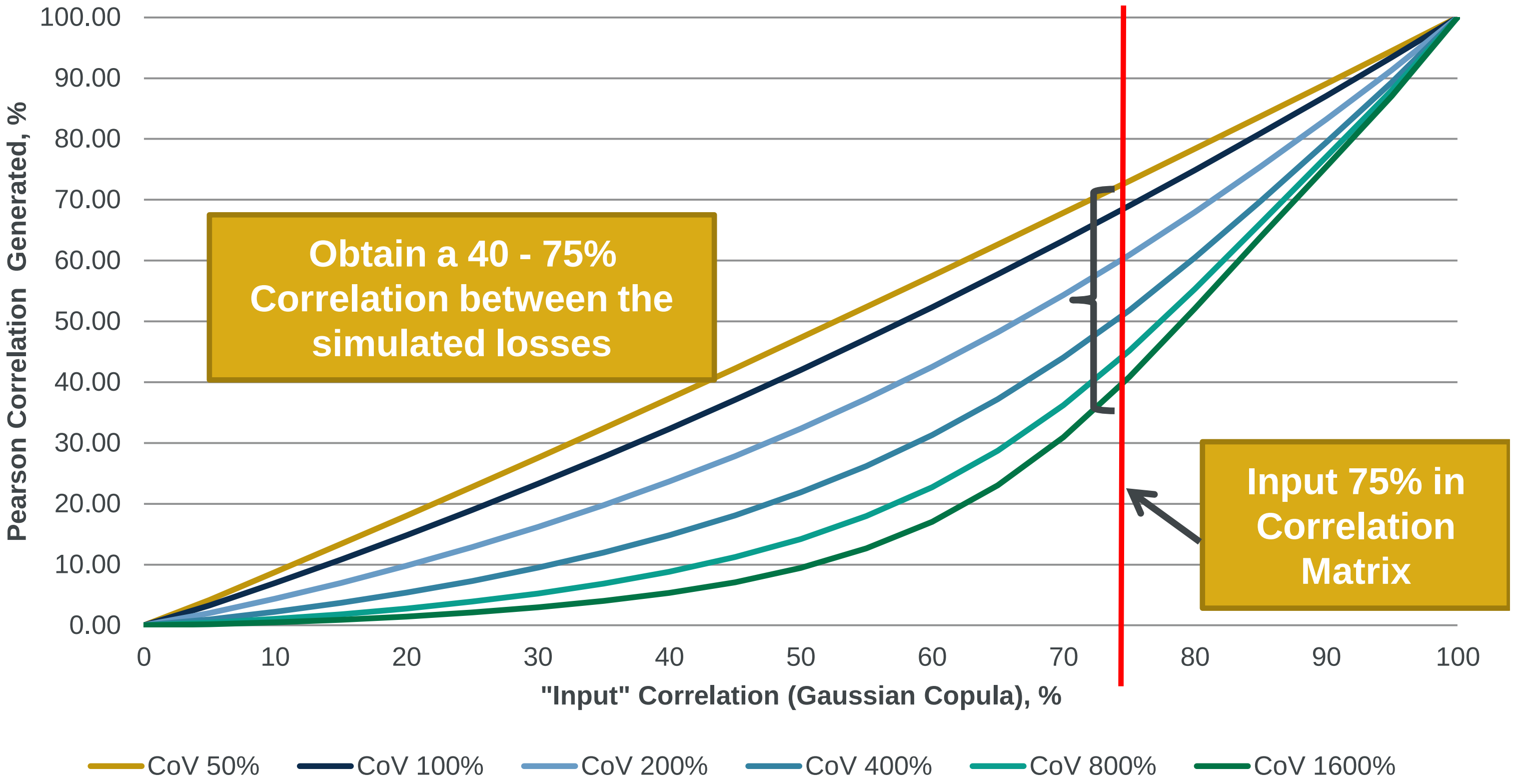
Correlation

It should be noted that some of the loss distributions modelled are extremely right skew.

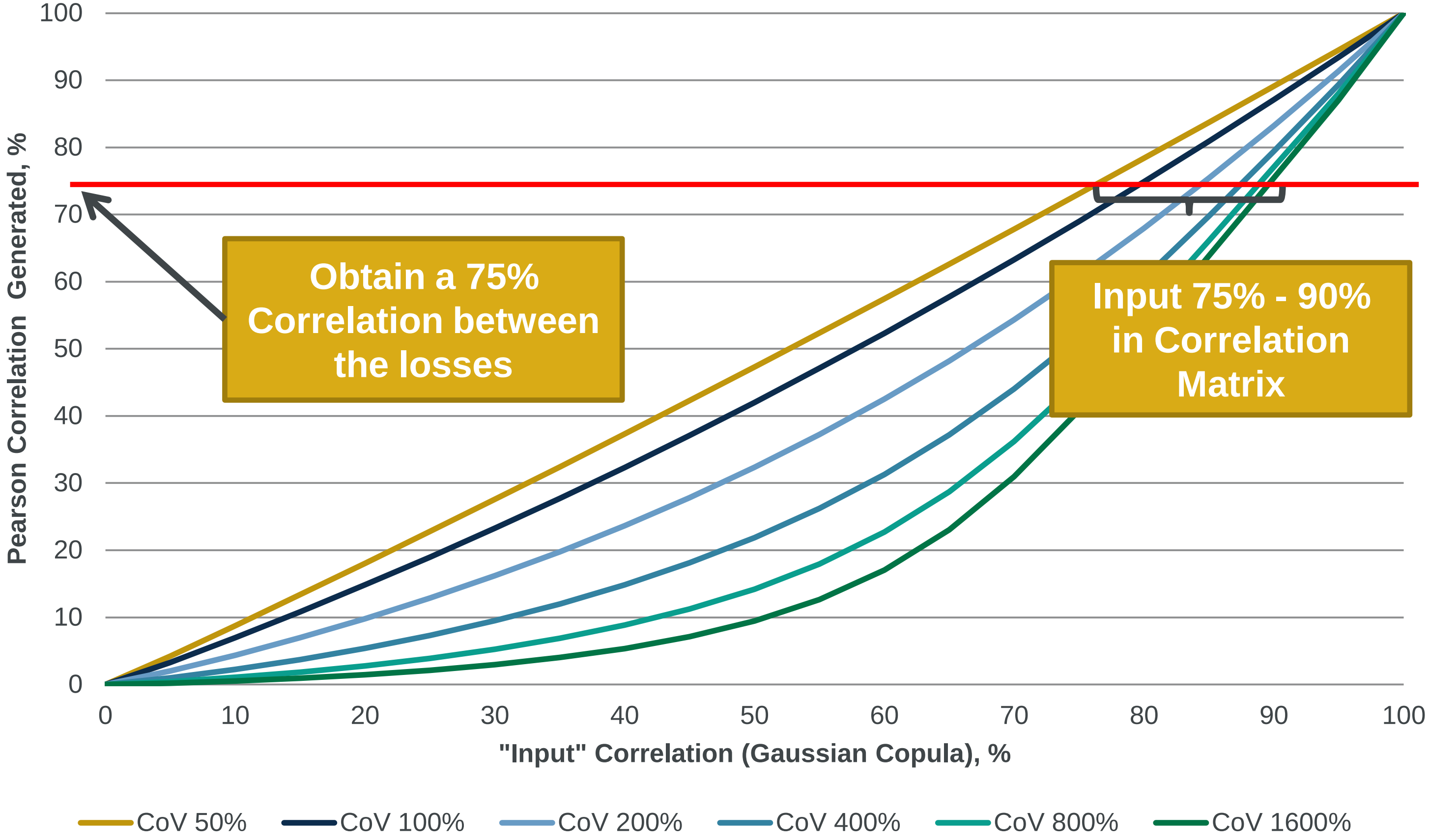
A Gaussian copula will generate Normal losses with the required correlation.

Using a Gaussian copula to correlate losses from a right skew distribution will generate lower Pearson correlations than those embedded in the correlation matrix

Correlation



Correlation



Results

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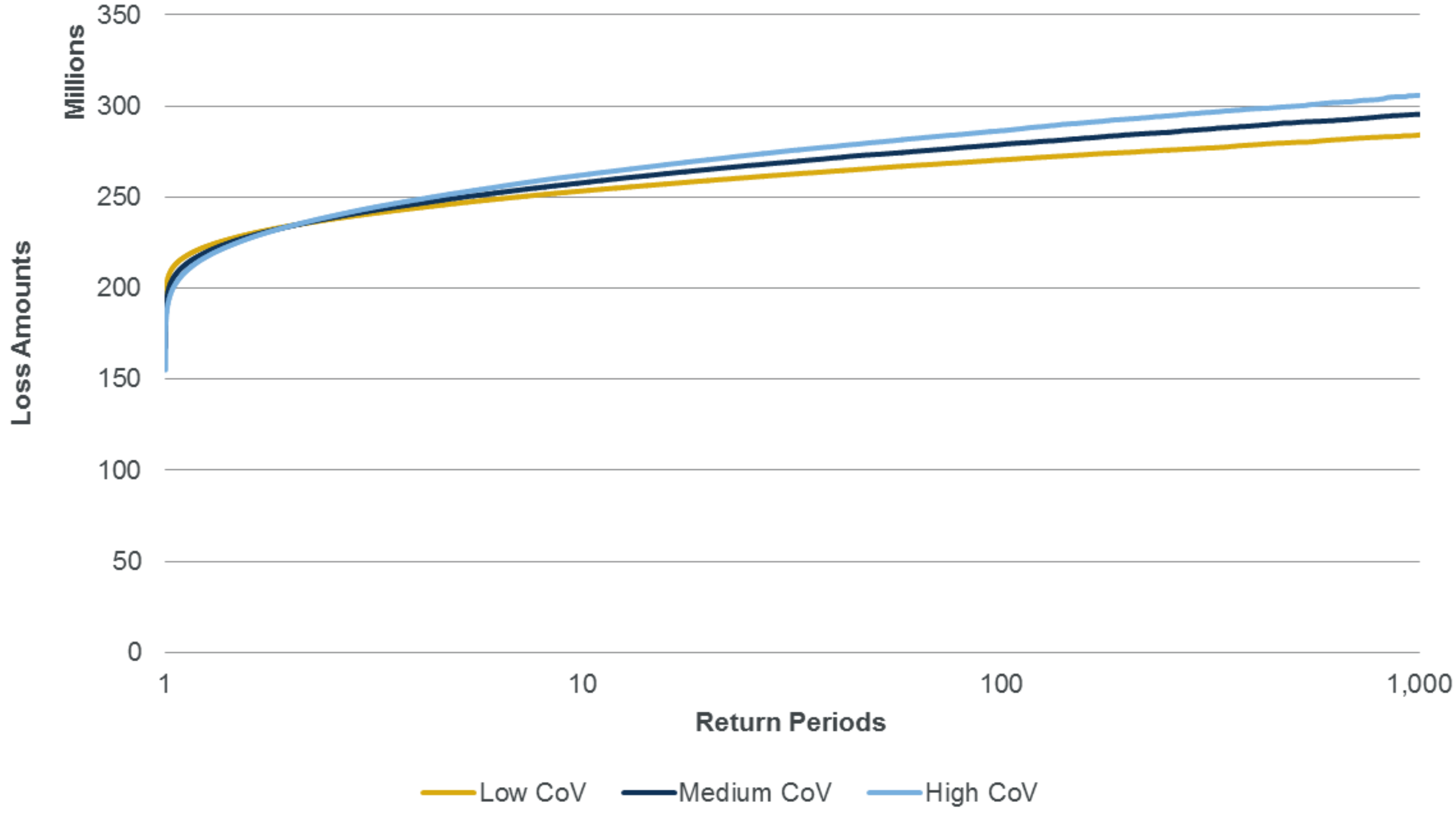
Results

- Coefficient of variance on severity
- Clash Assumptions
- 12 runs

Clash

CoV		No Clash	Low	Medium	High
	Low				
	Medium				
	High				

Impact of Different CoVs on a Risk Model



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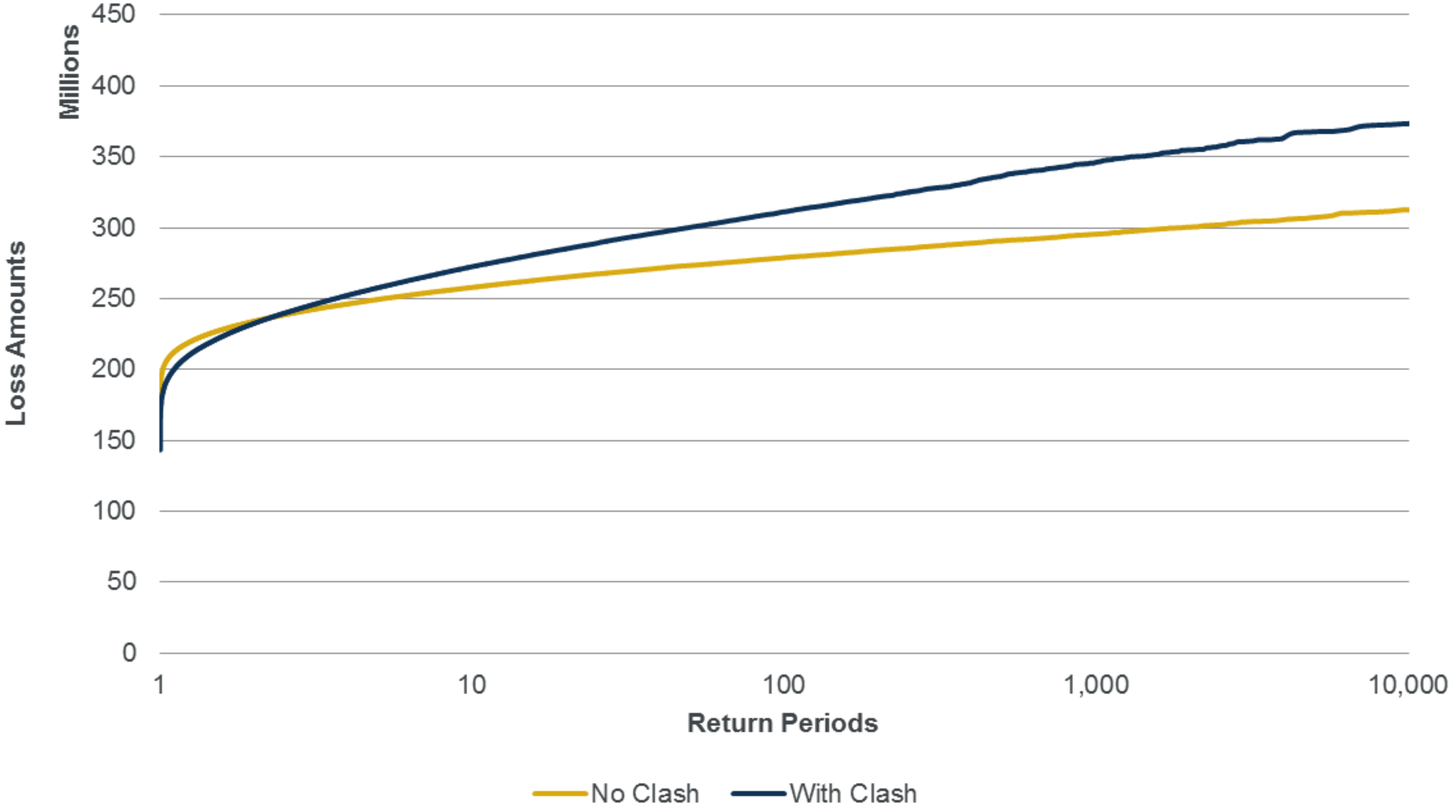
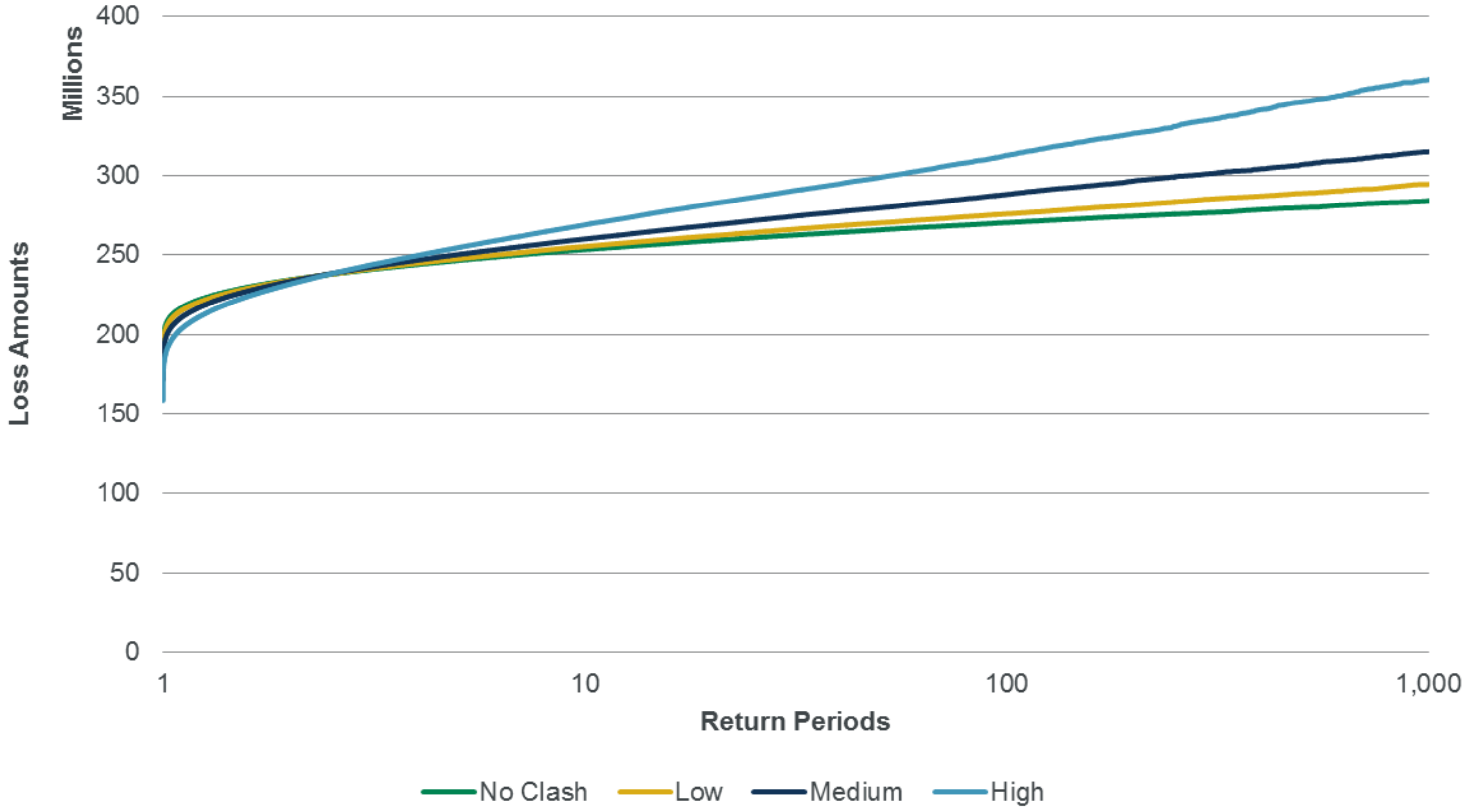
Impact of Clash

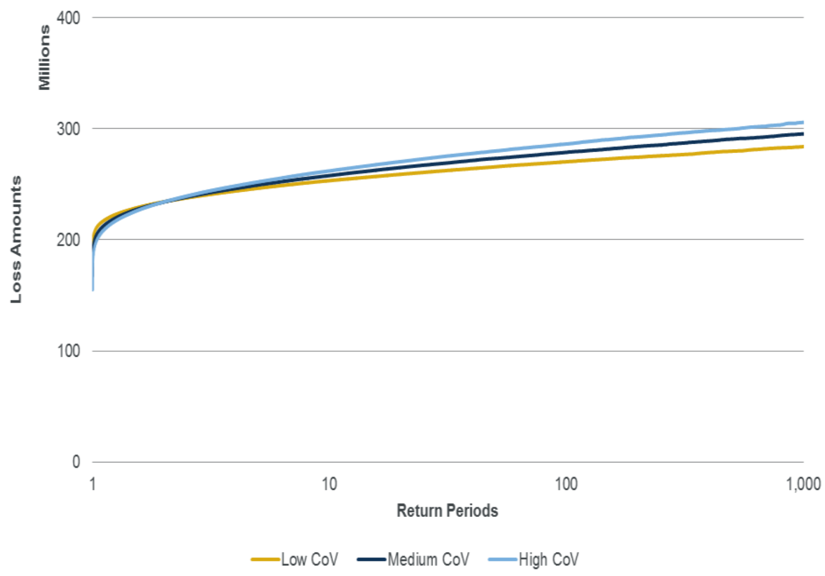


Impact of Different Clash Assumptions

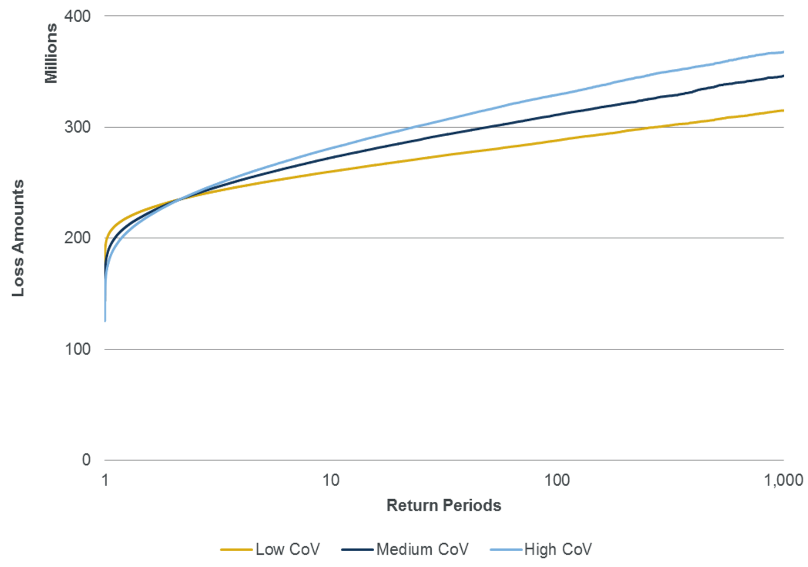


Comparing a Risk Model to an Event Model

Risk



Event



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Issues

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Issues

- Lack of data available to estimate parameters
  - Can cross check your assumptions against those implied by the market price
- Significantly slower than a pure risk XL run because the model simulates losses fgu as opposed to excess of an attachment point
- Assumes same severity distribution for the individual losses that arise as part of a systemic event verses a "non-systemic" losses
- Clash reinsurance varies materially in types of events covered; the "events" considered in determining the distribution in number of losses may not fully match the event definition that will trigger clash reinsurance

Possible Improvements

1. Vary exposure by underwriting year (implicitly assuming exposure flat)

2. Vary loss ratio by Attachment / Limit / Year

3. Explicitly model the different loss processes

Questions Comments

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Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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