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FINANCE, INVESTMENT & RISK MANAGEMENT CONFERENCE

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HILTON DEANSGATE, MANCHESTER

Annuities and Aggregate Mortality Risk: Mountains out of Molehills

"Even actuaries recognize that longer life is a good thing – but, to the extent that it is unanticipated, it is also an enormous problem for the managers of annuity portfolios"
Mary Hardy 2005

- **How much of a problem need it be?**

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Conventional Annuities

- Annuitants are obliged to shed themselves of all risk except inflation risk
- With-profits annuities are an exception to this but have a small market share
- Suppose annuitants had to carry the risk for themselves
- Examine the pay-out of an annuity sold to a man aged sixty-five

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Two key issues

- The nature of the uncertainty about survival rates.
- Assumptions about attitudes to risk

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A Dynamic Context

- Uncertainty about the future induces precautionary saving.
- Uncertainty about mortality rates increases into the future.
- Discounting means that a small sacrifice today can provide a large insurance hedge for the future.

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- Consider a cohort which insures itself against individual mortality risk.
- Each year the assets of those who die accrue to the survivors.
- What is the pay-out profile which delivers the highest possible expected welfare?
- Expect to see precautionary saving.

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A Simple Model of Mortality Rates

- The problem in forecasting mortality is often described as uncertainty over the trend decline of log mortality.
- Assume that this trend is a random walk.
- But also assume that there are persistent shocks to the mortality rate.
- Assume a maximum mortality rate (80%).
- And a maximum age of 110.

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An Example

- The mortality rate when the cohort reaches age 80 is 5% below what had been expected when it was 79, and the rate of improvement is 1% p.a. compared to the estimate produced a year earlier of 0.5% p.a.
- Then the forecast mortality at age 81 is 5.5% below what had been forecast a year earlier.
- At 82 it is 6% below etc.

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The Process Algebraically

$$w_{1,t} = w_{1,t-1} + w_{2,t-1} + \varepsilon_{1,t}; \quad E(\varepsilon_{1,t}) = -\sigma_1^2/2,\ Var(\varepsilon_{1,t}) = \sigma_1^2;$$

$$w_{2,t} = w_{2,t-1} + \varepsilon_{2,t}; \quad E(\varepsilon_{2,t}) = -\sigma_2^2/2,\ Var(\varepsilon_{2,t}) = \sigma_2^2;$$

$$log\, m_t = log\, \mu_t + w_{1,t};$$

$$log\, m_{\tau,t} = log\, \mu_{\tau,t} + w_{1,t} + (\tau - t)w_{2,t};$$

$$Max(m_{\tau,\tau}) = 0.8$$

Assume processes are log-normal and correct drift generated by this. Expectations do not depend on variances.

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The Basis for the Annual Pay-out

- At the start of year *t* the actuary calculates the pay-out using
  $m_{\tau,t-1} \quad \tau > t-1$
- At the end of year the residual wealth is influenced by the actual mortality rate, m_t rather than the forecast rate

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No Allowance for Uncertainty

- The actuary running the scheme recalculates the pay-out in each year in the light of the present state of knowledge.
- This is given by the current forecasts of mortality for all future years.

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No Allowance for Risk

- The actuary calculates the payout in each year so as to keep the pay-out rate based on forecast mortality rates constant.
- Assume r=2.5 % per annum

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The Cohort of Men Aged 65 in 2005

- Current forecasts of mortality rates are derived from the GAD cohort life tables.
- Some smoothing is needed to remove effects of rounding errors.
- Set σ_1=0.01, σ_2 =0.003
- Life expectancy at 65 is 21.6 years
- The standard deviation is 1.2 Years.

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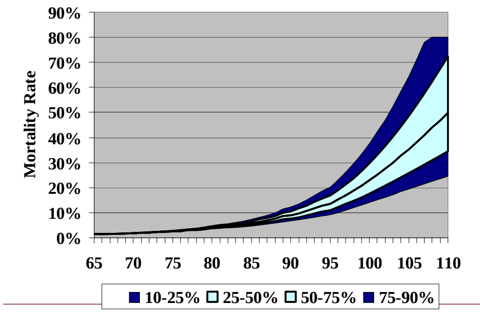
The Dispersion of Mortality Rates



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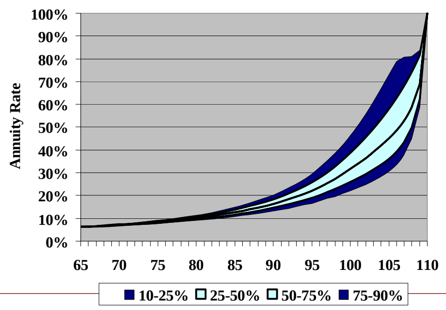
The Dispersion of Annuity Rates



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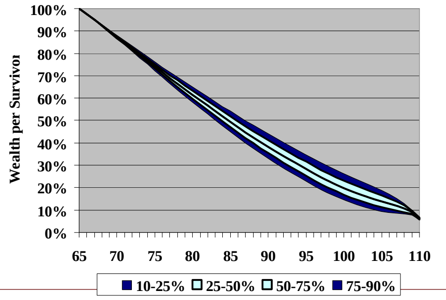
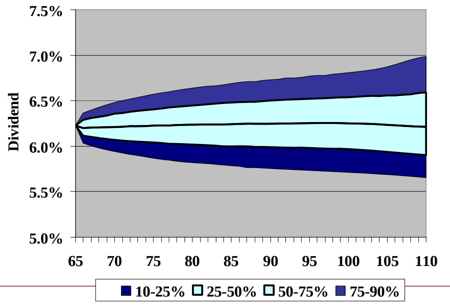
The Dispersion of Wealth



The Dispersion of Dividends



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The Meaning of Risk

- Outcomes may be different from their expected values

AND NOT

- That mortality rates are falling and there are good reasons- if only based on time-series models- to expect further falls.
- Consider only the uncertainty surrounding a given time-series model and not uncertainty about the model

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Constant Relative Risk Aversion

- Choose between carrying uncertainty about future consumption and
- Paying to avoid carrying the risk

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$$U(c(1-\theta)) = E\{U(c[1+\varepsilon])\};$$

$$E(\varepsilon) = 0;\ Var(\varepsilon) = \sigma^2$$

The proportionate risk premium is given as

$$\theta = -c\frac{u''(c)}{u'(c)}\frac{\sigma^2}{2}$$

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Suppose $U(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \gamma \neq 1, \quad U(c) = log(c) \quad \gamma = 1$

Then $\theta = \frac{\gamma\sigma^2}{2}$

Generally believed 0< γ<5

Consider γ=4, *r*=2.5% p.a.

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Dynamic Optimisation

- Solve problem recursively
- At age 110 all wealth is consumed
- At age 109 savings decision depends on the mortality parameters and initial wealth.
- Compute grid showing optimal pay-out as a function of these.
- In earlier years compute optimal payout over grid on the assumption that optimal behaviour is followed in subsequent years

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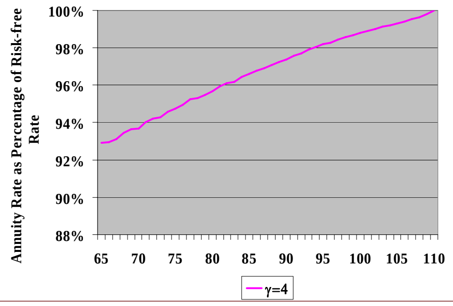
Annuity Rates for Risk-Averse Annuitants



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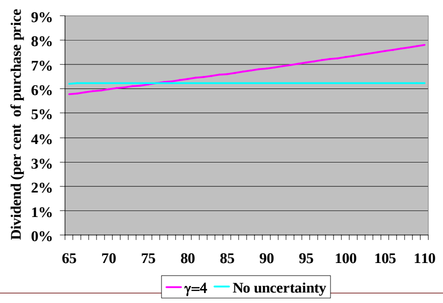
Dividends for Risk-averse Annuitants



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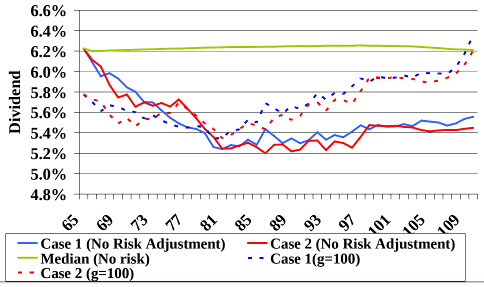
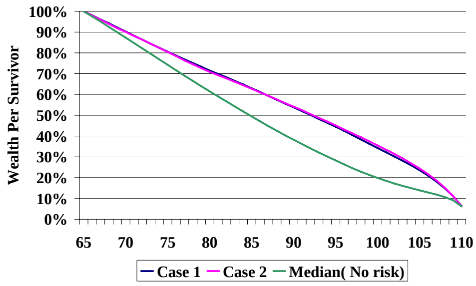
½ Percentile Cases at age 80 and 90



Wealth Profiles in ½ Percentile Cases



Giving People a Choice

- Always a good thing
- If there is a charge for carrying risk, then people will want to carry some risk for themselves
- If old people are protected from risk then in a market economy young people must be carrying the risk
- What is the market equilibrium?

- When young people decide how much to invest in underwriting conventional annuities sold to old people
- On retirement people allocate funds between

a) Conventional annuities (young people carry the aggregate mortality risk)

b) Mortality-adjusted annuities (annuitants carry the aggregate risk for themselves)

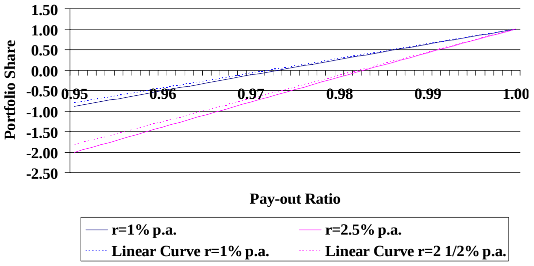
Market Equilibrium

Establish

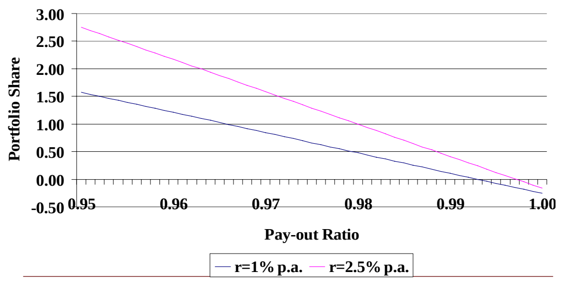
a) Market-clearing payout on conventional annuities

b) The proportion of wealth which old people invest in conventional rather than mortality-adjusted annuities

A Demand Curve (γ=4)



A Supply Curve (γ=4)



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Market Equilibrium

- Payout= 0.993 with r=2.5%
- 45% of retirement wealth is invested in conventional annuities- with the remainder in mortality-adjusted annuities.
- Note that public provision of annuities raises welfare because the government can spread the risk across all future generations.

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Conclusions

- Life insurance companies should make it easier for annuitants to carry their own aggregate mortality risk
- Aggregate mortality risk is probably lower than inflation risk
- Even with extreme risk aversion the reduction in initial payouts are not large.

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- The market premium for carrying aggregate mortality risk is unlikely to be large, particularly if people are allowed to carry risk for themselves.
- But the arithmetic involved in running DB pensions schemes is very different from selling annuities to old people.