
Applying EVT and alternatives to portfolio construction and the management of risk

Presentation to Institute and Faculty of Actuaries
Open Forum on Extreme Value Theory

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Using EVT and alternatives

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- Why are return series often 'fat tailed'?
- Strengths and weaknesses of Extreme Value Theory (EVT)
- Interaction with portfolio construction
- See also:
 - Kemp, M.H.D. (2010). *Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails*. John Wiley & Sons
 - Toolkit, charts etc. on www.nematrian.com/extremeevents.aspx



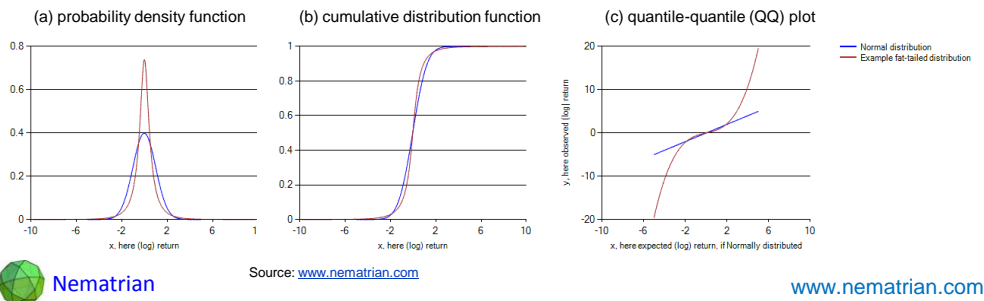
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Modelling fat-tailed behaviour for *individual* risks

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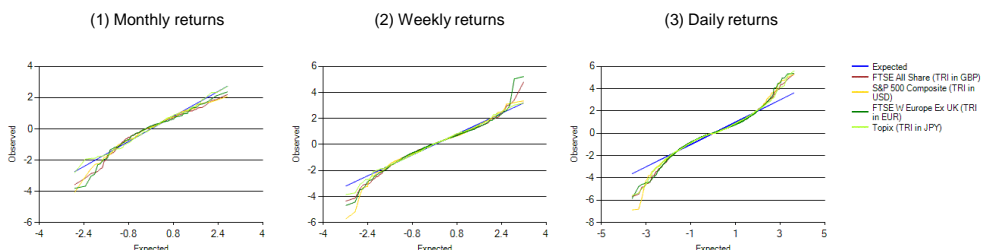
- 'Fat-tailed' means probability of extreme-sized outcomes seems to be higher than if coming from (usually) a (log) Normal distribution
- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below, i.e. QQ-plots
- Note: portfolio construction usually involves *multiple* assets / risk exposures



Many (most?) investment return series are 'fat-tailed'

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- Some instrument types intrinsically skewed (e.g. high-grade bonds, options)
- Others (e.g. equities) still exhibit fat-tails, particularly higher frequency data



Why are return series often fat-tailed?

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- Time-varying nature of the world in which we live
 - Market / sector / instrument volatility (and maybe other distributional characteristics) change through time
 - Heteroscedasticity, GARCH, regime switching
 - Returns may be (*conditionally*) Normal over short time periods, but data series still (*unconditionally*) non-Normal when viewed over longer time periods
- Selection effects, e.g. manager behaviour may (consciously or unconsciously) bias towards fat-tailed behaviour, see Kemp (2010)
- Crowded trades and leverage
- As well as intrinsically skewed behaviour such as for individual bonds



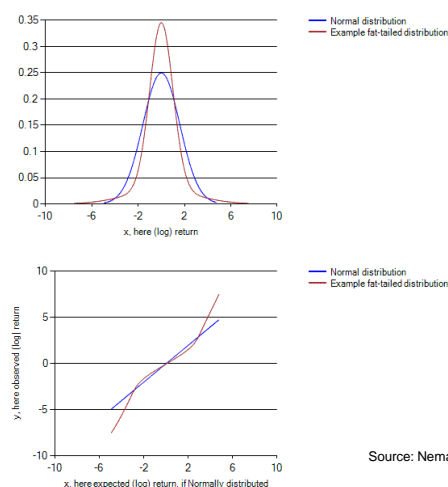
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Distributional mixtures of Normal distributions

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- E.g. draw X with prob p from N_1 and prob $(1-p)$ from N_2
 - Quite different behaviour to *linear combination mixtures*, i.e. $a.X_1 + b.X_2$
- If N_1 and N_2 have **same** mean but **different** s.d.'s then distributional mixture fat-tailed (if $p \neq 0$ or 1) but linear combination mixture isn't.
- Time-varying volatility is similar, involves draws from different distributions at different times



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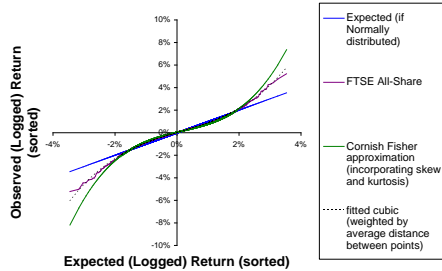
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Explains some equity index fat tails, particularly upside

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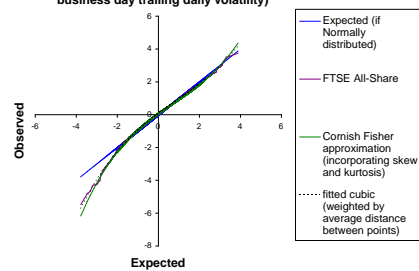
Raw Data

Daily returns (End Jun 1994 to end Dec 2007)



With Short-term Volatility Adjustment

Daily returns (end Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)



Average extent to which tail exceeds expected level (average of 6 most extreme outcomes)				
	Downside (%)		Upside (%)	
	Unadj	Adj for vol	Unadj	Adj for vol
FTSE All-Share (in GBP)	54	41	42	3
S&P 500 (in USD)	68	70	50	7
FTSE Eur ex UK (in EUR)	48	53	54	-3
Topix (in JPY)	54	72	42	39

Source:
Threadneedle, FTSE
Thomson Datastream



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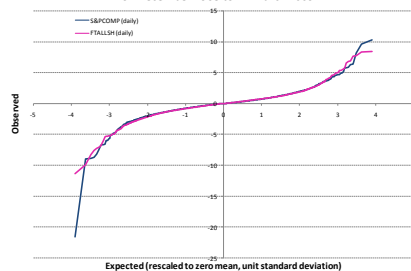
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And over longer time periods

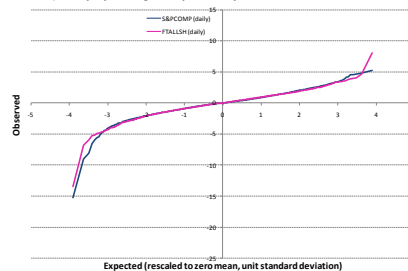
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Raw Data

Tail analysis for S&P 500 and FTSE All-Share price movements
31 December 1968 to 24 March 2009



Tail analysis for S&P 500 and FTSE All-Share price movements
(vol adj, by trailing 50 day vol, early 1969 to 24 March 2009)



Source: Threadneedle, S&P, FTSE, Thomson Datastream



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Crowded trades and leverage

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- Some fat tails still seem to come “out of the blue”
 - E.g. Quant funds in August 2007
 - Too many investors in the same *crowded trades*? Behavioural finance implies potentially unstable
 - For less liquid investments, impact may be via an apparent shift in price basis
- System-wide equivalents via leverage?
 - Leverage introduces/magnifies *liquidity* risk, *forced unwind* risk and *variable borrow cost* risk
 - Like selection, involves behavioural finance effects



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Extreme Value Theory (EVT)

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- EVT an enticing prospect
 - Appears to offer a mathematically sound way of identifying shape of the ‘tail’ of a distribution, and hence identifying likelihood of extreme (i.e. rare) events
 - Capital adequacy seeks to protect against (we hope) relatively rare events
 - Insurance and credit risk pricing can be dominated by potential magnitude and likelihood of large losses, which are also (we hope) rare
- But bear in mind
 - Inherent unreliability of extrapolation – including extrapolation into the tails of a probability distribution
 - Possibility (indeed probability) that the world is not time stationary
 - We may need to consider a multivariate analogue for portfolio construction



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Restatement of EVT results

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- Suppose interested in risk measures relating to losses, x_j
- EVT aims to supply two closely related results:
 1. Distributions of 'block maxima' (or 'block minima'), i.e. maximum value of x_j in blocks of m observations of x (more traditional use of EVT, wasteful of data):

$$m_j \equiv \max_{j-1-m+1 \leq k \leq jm} x_k$$

2. Distributions of 'threshold exceedances' (i.e. 'peaks-over-thresholds'), where u is a predetermined high threshold and we focus on realisations of x_j that exceed u (more relevant to, e.g. computation of Value at Risk i.e. VaR), i.e.:

$$y_i \equiv x_i - u \quad \text{for } i \text{ s.t. } x_i - u > 0$$



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Main result for block maxima

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- Suppose that $x_1, x_2, \dots, x_n, \dots$ are independent random variables possessing same cumulative distribution function, F , and that there exist sequences a_n and b_n such that the following tends in distributional form to Q , a non-degenerate probability distribution from which random variable y is drawn

$$\frac{\max_{1 \leq j \leq n} x_j - b_n}{a_n} \xrightarrow{D} y$$

- Then Q is equal to $H(\xi)$ for some ξ (if a_n and b_n appropriately scaled) where $H(\xi)$ is the generalised extreme value (GEV) distribution. F is then said to be in the maximum domain of attraction of $H(\xi)$

value of $\xi = 1/\alpha$	GEV sub-type	(cumulative) distribution function
$\xi = 0$	Gumbel	$\exp(-\exp(-x))$ for $-\infty < x < \infty$
$\xi > 0$	Fréchet	$\exp(-(1 + \xi x)^{-1/\xi})$ for $1 + \xi x > 0$, otherwise 0
$\xi < 0$	Weibull	$\exp(-(1 - \xi x)^{1/\xi})$ for $1 - \xi x > 0$, otherwise 1



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Main result for excesses

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- Let F_u be defined as follows: $F_u(z) \equiv \Pr(x - u < z | x > u)$

- Then under same hypotheses as applied to block maxima we have:

$$\lim_{u \rightarrow x_f} \sup_{0 < z < x_f - u} |F_u(z) - G_{\mu, \sigma, \xi}(z)| = 0$$

- Where $G_{\mu, \sigma, \xi}(z)$ has the form:

$$G_{\mu, \sigma, \xi}(z) = \begin{cases} 1 - \left(1 + \xi \frac{z - \mu}{\sigma}\right)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{z - \mu}{\sigma}\right), & \xi = 0 \end{cases}$$
- Here ξ has the same type of meaning as before, e.g. $G_{\mu, \sigma, \xi}(z)$ is in the maximum domain of attraction of $H(\xi)$



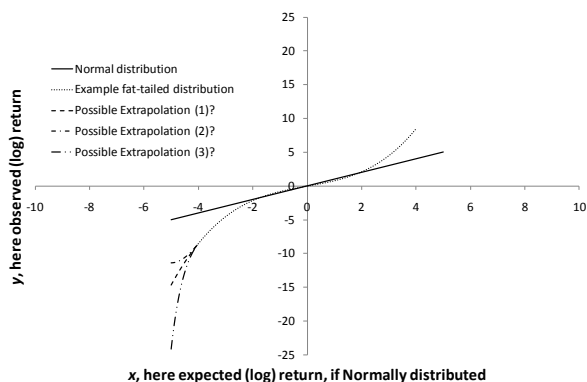
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Potential weaknesses

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- EVT seems very helpful
 - Characterises limiting distributions very succinctly
 - But requires regularity conditions that may not be satisfied
 - Relies on existence of a limiting distribution but this is not guaranteed
- At issue is potential unreliability of **extrapolation**
 - E.g. Press et al. (2007)



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Using EVT and alternatives to Estimate VaRs

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- Assume limiting distribution of tail is fat-tailed GPD
 - Thus use approximation: $F_u(y) \approx G_{\mu, \sigma, \xi}(y)$
 - Problem of estimating F and its (tail) quantiles then reduces to problem of estimating μ , σ and ξ for the approximating generalised Pareto distribution
 - Can be done using mean excess functions, maximum likelihood (ML) estimation, method of moments etc.
- But equally we could fit to the relevant part of the QQ-plot using any other reasonable form of curve fitting approach
 - E.g. polynomial curve fit such as a cubic (see earlier), as long as the resulting extrapolation is credible



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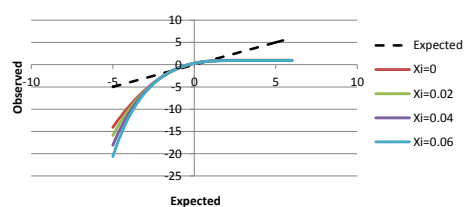
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Some subtleties of EVT

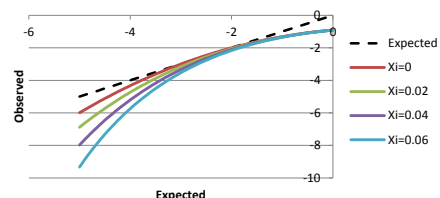
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- QQ-plot of GPD is convex upwards (if $\xi \geq 0$)
- If data is Normal then fitting a GPD may overstate size of extreme events somewhat
 - Since Normal has same tail characteristics as $\xi = 0$ GPD
- In real life often have multiple series / loss types
 - Can construct *multivariate* EVT theory, but more complex
 - E.g. McNeil, Frey & Embrechts (2005)

QQ-plot, entire GPD (versus Normal)



QQ-plot versus $N(x)$ fitted to -2 and -3



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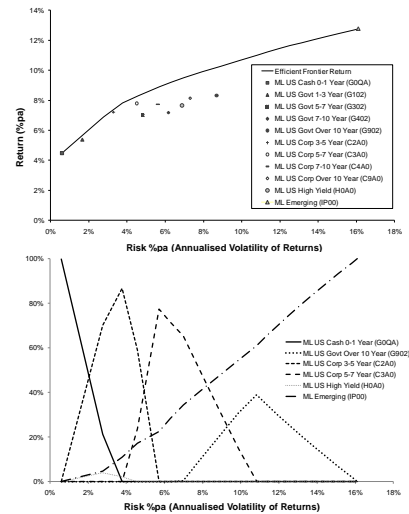
Portfolio construction

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- Traditional (quantitative) approach involves **portfolio optimisation**
 - Typically mean-variance optimisation
 - Identify expected return ('alpha') from each position
 - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
 - Maximise $a.r - \lambda.a^T V a$
- Intrinsically **multivariate**



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Portfolio construction – sensitivities

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- Output results are notoriously sensitive to input assumptions
- Possible responses:
 - Treat quant models with scepticism (the fundamental manager's approach?)
 - Use robust approaches, Bayesian priors/anchors, e.g.
 - Black-Litterman
 - 'Shrinkage'
 - Position limit 'priors' (e.g. 1/N, long-only etc.)
 - Resampling
 - Focus on reverse optimisation



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Incorporating fat tails – Solution A – simplest

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- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
 - Calculate covariance matrix between return series after stripping out effect of time-varying volatility
 - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
 - Adjust risk aversion/risk budget appropriately
 - Then unravel time-varying volatility adjustment
 - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed



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Incorporating fat tails – Solution B – more sophisticated

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- Model with a mixture of multivariate Normal distributions (or GPDs, ...)
 - Time-stationary? Maybe not realistic?
 - Time-varying?
 - (Discrete) regime switching, and/or
 - (Continuous) parameterisation (and continuous time?)
- However:
 - Even a mixture of just two multivariate Normal distributions involves estimation of twice as many parameters
 - Results even more sensitive to input assumptions
 - Time varying => dynamic => sensitivity to transaction costs



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Summary

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- Fat-tailed behaviour
 - Very common in practice
 - Several intrinsic reasons for its existence, including time-varying world
- Extreme Value Theory (EVT)
 - Enticingly simple (at least in concept)
 - But subject to same underlying issues as any other form of extrapolation
- Portfolio construction can be refined to cater better for extreme events
 - Adjust for (global) time-varying volatility
 - Any further refinements become very complex



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