Institute and Faculty of Actuaries

Validating Dependencies: Considerations & Pitfalls

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Agenda

- Introduction [Eric Pizarro]
- Simplifying the validation [Eric Pizarro]
  - Validation structure
  - Validation tests
- Three tips for dependency validation [Dimitris Papachristou]
  - Identifying material dependencies and estimating their impact
  - Intuitive interpretation of correlations
  - Understanding the impact of structure

Caveat

- This presentation reflects the views of the presenters and not necessarily those of their employers

Part I: Simplifying the Validation

Making dependencies simpler

High uncertainty
Many alternatives
Small amount of data
Make it simple

Low uncertainty
Few alternatives
Large amount of data
Make it complex

Source: *Risk Savvy*, Gigerenzer (2014)

Agenda: Making it simpler (but not too simple)

- Expert judgements: fewer alternatives & more structure
- Validating the dependencies: simple tests
  - Causal explanations
  - Data
  - Simple rules
- Considerations and pitfalls of each of the above
- We will use premium risk for two classes to illustrate the approaches
- All data, assumptions, etc. selected for convenience NOT realism

Reduce, re-use, recycle

- First step to making it simple: reduce the number of expert judgements
- Recommendation: split the qualitative and quantitative judgements
  - Often different people (UW vs. actuaries) responsible for each
  - One may vary without the other
- **Qualitative** alternatives: shape and strength
  - Shape: stronger in common or extreme outcomes?
  - Strength: number of joint occurrences over a career (never, once, multiple)

Standardise the structure

- **1st step:** (Partial) list of considerations to be evaluated for each class

Loss ratio component	Volatility driver	Loss ratio impact: Common vs. extreme
Premium rates	UW cycle; UW strategy	Both
Attritional claims	Risk type	Common
Large claims		
frequency	Risk type	Both
severity	Risk type; legal	Both
Cats/ extreme events	Region and peril	Extreme

- **2nd step:** qualitative assessment of shape and strength of dependency
- Example: EL and PL have shared UW cycle, policies on same risks, no cats
  - ⇒ non-tail dependence of medium strength

Standardise the structure (2)

- **Quantitative** alternatives: considerations
  - Materiality: e.g. if attritional is small, level of dependency less material
  - Map to the combinations of strength and shape (6 in our example)
  - Get the relativities right: it will be simpler to move dependencies up/down later
- **Advantages**
  - Transparency: easier to explain and validate internally and externally
  - Relatively easy to update: e.g. strength weakens if fewer shared risks
- **Pitfalls**
  - Still need to quantify expert judgements
  - Still need to decide on dependency structure

Data based tests: considerations

- Data is not often the best validation tool but it could be utilised more

- **Rule #1**: You can't ignore the past
- **Rule #2**: The data will rarely be credible for parameterisation

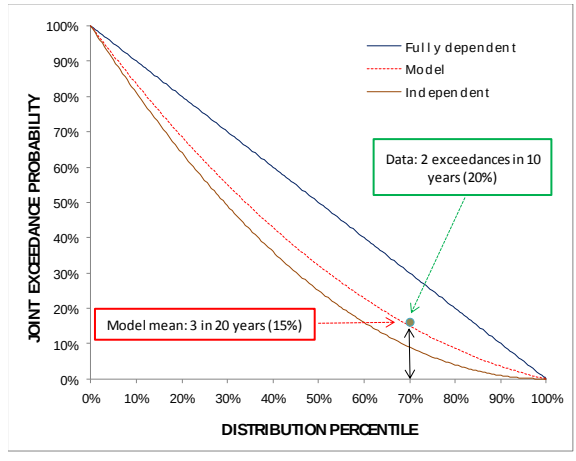
- **Q**. So how can we use data in the validation of dependencies?
- **A**. It can be used to rule out/falsify/invalidate a dependency structure

Data based tests: Examples

- Two simple approaches for using data to invalidate a dependency structure
- Data:
  - 10 years of ULRs for both EL and PL
  - Both ULRs exceed their 70th percentiles in **two** of the 10 years
- Model dependencies:
  - The probability of both exceeding their 70th percentile ("JEP") is **15.00%**
  - Compares to 9% assuming independence and 30% for full dependence
- Two simple approaches for using data
  - Hypothesis test and a simple Bayesian approach

Example 1: Hypothesis testing

- Does the data invalidate the model at the 70th percentile?



Example 1: Hypothesis testing (cont'd)

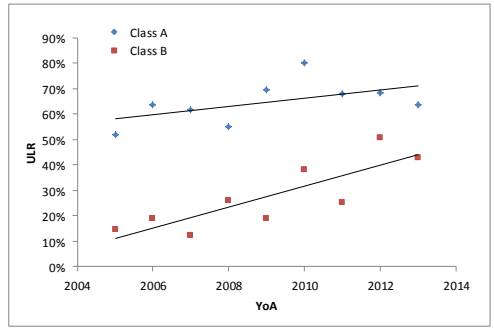
- The "hypothesis" is our modelled JEP of 15%
- Test: model's probability of observing **more** than the number of joint exceedances in the data must be **at least** 10%
  - 10% is the **confidence level**
  - 1 – BINOM.DIST(2,10, 0.15, TRUE) = 18% > 10%
- Result: Pass
- Independence (JEP = 9%) result is 5.4% (fail)
- Remark: nothing new here – this a standard statistical test

Data based tests: some pros & cons

- **Advantages**
  - No ambiguity: straightforward calculation and explicit criteria
  - Ties back to history
  - Easy to document!
- **Pitfall #1**: Distributions change over time
- Mitigating Pitfall #1: "On-level" your data for rate changes, inflation, etc.

Working around Pitfall #1

- The (realistic) data below shows a high correlation – or does it?
- Correlation is based on movements relative to the means – but these are changing
- Movements relative to a projected mean may give a more realistic answer – in this case near nil



Data based tests: some pros & cons

- **Pitfall #2**: Skewed distributions - Linear correlations will be distorted
- Mitigating Pitfall #2: Use rank correlations (and map to linear) or JEP
- **Pitfall #3**: Lack of data (introduces parameter uncertainty)
- Managing Pitfall #3
  - Bootstrap data for confidence interval around JEP (only works if Pitfall #1 addressed)
  - "Augment" the data: e.g. if have 5 pairs of classes with same model JEP, pool the data from all 5 pairs to test the JEP
- **Pitfall #4**: The JEP percentile and the confidence interval are chosen by judgement
- Managing Pitfall #4: There's a range of reasonable values

Example 2: Bayesian alternative

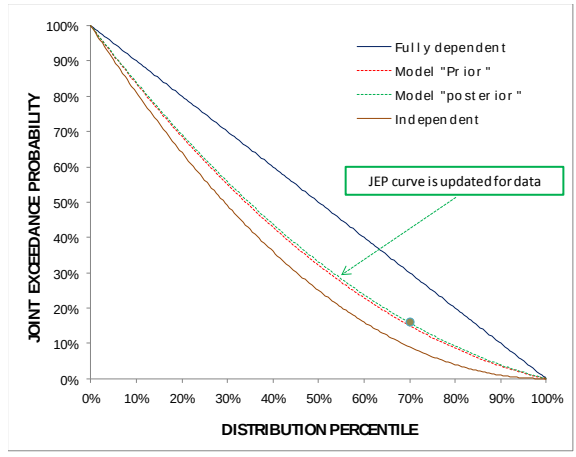
- Not a validation tool but it does update assumptions for new data
- Recent guidance related to model approval:
  - Expert judgement must be updated to for new information (EIOPA "Guidelines" Sep 2013)
  - Allowance for parameter uncertainty where it is material (PRA SS5/14 April 2014)
- A Bayesian approach lends itself to satisfying the above
- Simple Bayesian model at p=0.70
  - k = number of years with both distributions exceeding the percentile p
  - θ = JEP
  - Model distribution for k is binomial: $Bin(k \mid n, \theta) = [n!/(n-k)!k!]\theta^k (1-\theta)^{n-k}$
  - Prior distribution for θ: $g(\theta = 9\%) = 0.25$; $g(\theta = 15\%) = 0.50$; $g(\theta = 21\%) = 0.25$
  - Posterior distribution is $g(\theta \mid k) \propto Bin(k \mid n, \theta) * g(\theta)$

Example 2: Bayesian alternative (cont'd)

- In our example, n = 10, k =2
- The mean of the prior distribution $E(\theta) = \sum \theta * g(\theta) = 15.0\%$ (*as in Example 1*)
- The mean of the **posterior** $E(\theta \mid k=2) = \sum \theta * g(\theta \mid k=2) =$ **15.8%**
- This is our updated view of the model JEP
- The prior and posterior reflect our uncertainty in the JEP (θ)
- Next step? One option is to update the JEP curve e.g. by a parallel shift
- Normally we would update the posterior mean each year

Example 2: Bayesian alternative (cont'd)

- Expected JEP from model vs. data at the 70th percentile.



Bayesian alternative: some pros & cons

- **Advantages**
  - Formally combines expert judgement and data
  - Explicitly incorporates parameter uncertainty
  - Easy to document!
- **Pitfalls**
  - Still need to choose the prior (expert judgement)
  - Data emerges slowly (but see earlier suggestion about augmenting data)
  - Validating Bayesian models with strong priors – it's an open question

The Turkey illusion

- A farmer visits his turkey every day and feeds him.
  - The turkey uses Laplace's ***rule of succession*** to calculate the probability that the farmer will feed him again on day n = (n+1)/(n+2)
  - Works well until the day before Thanksgiving
- The ***turkey illusion*** **"**mistakes uncertainty for known or calculable risks. The result is an illusion of certainty…[I]t stems from the mistaken belief that every problem should be solved with probability theory, such as Bayes' rule." (Gigerenzer, 2014)
- Are simple rules sometimes better?
- Some proposals from finance:
  - Banking leverage: Don't exceed10:1 (Mervyn King)
  - Investments: Allocate your money equally to each of N funds (Harry Markowitz)

Are we all just deluded turkeys?

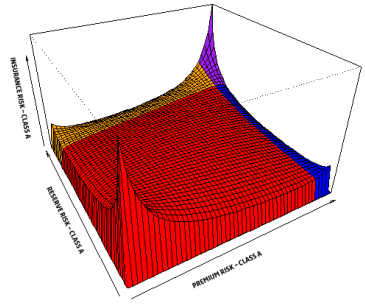
- Is there space for simple rules on dependencies under S2?
  - Truly "simple" ⇒ no probabilities, distributions or stats
  - BUT dependencies are defined by probabilities…and we need to validate tail dependencies
- Hopefully yes…but we may need to be a bit "creative"

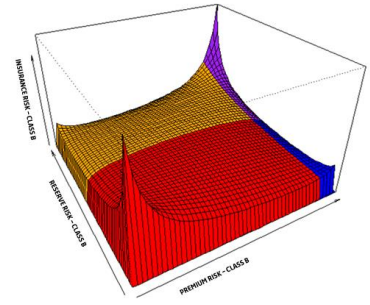
Rule based test: Risk and profit

- Require a loss on the second (smaller) risk given an extreme loss on the larger risk
- Example: There must be a loss on reserve risk given a 1-in-200 or worse outcome on premium risk
- **Advantages**
  - Easy to communicate
  - Easy to apply
  - Prevents capital drift
- **Pitfalls**
  - May favour/discriminate against some pairs of risks
  - Usually sets a minimum/maximum for dependencies – but not an optimum
  - Still need to justify the rule

Risk and profit: Pitfall

- **Purple area** (far corner) shows the joint probability of a loss on reserve risk (**y-axis**) and 1:200 loss on premium risk (**x-axis**)
- Easier to pass test if reserve risk distribution is <u>less</u> skewed (right graph)





Part II
Three Helpful Dependency Validation Tips

Identifying Key Correlations
Estimate their Impact

Identifying key correlations

- An Internal Model may have thousands of correlations.

  Which ones have the biggest impact?
  Can I quantify the impact easily and quickly?

- Simple method for identifying key correlations: Assume
  - Multivariate Elliptical - For example, multivariate normal
  - If capital requirement is $C = \sqrt{\sum C_i^2 + 2\sum\sum_{j<i} \rho_{ij} C_i C_j}$
  - C = 99.5th percentile – mean (could use some other percentile)
- Then
- $\frac{\frac{\vartheta C}{\vartheta \rho_{ij}}}{C} = \frac{C_i}{C} \cdot \frac{C_j}{C}$ or $\frac{\Delta C}{C} \approx \frac{C_i}{C} \cdot \frac{C_j}{C} \cdot \Delta\rho_{ij} = a_i \cdot a_j \cdot \Delta\rho_{ij}$

$$\frac{\Delta C}{C} \approx \frac{C_i}{C} \cdot \frac{C_j}{C} \cdot \Delta\rho_{ij} = a_i \cdot a_j \cdot \Delta\rho_{ij}$$

Example

- Assume: Lognormal loss distributions

	A	B	C	D
mean	100	40	200	150
st. dev	30	20	30	40

	A	B	C	D
99.th %ile	204	121	290	285
Ci=99.th - mean	104	81	90	135
alpha	0.37	0.29	0.32	0.48

C=99.th - mean	280

0.37=104/280

0.48=135/280

LoB	LoB	a_i*a_j
A	D	0.18
C	D	0.16
B	D	0.14
A	C	0.12
A	B	0.11
B	C	0.09

0.18=0.37 * 0.48

- If Cor(A, D) increases by 10% then
  - Approximate method above: capital will increase by ~1.8% (= 0.18 x 0.10)
  - Based on actual distributions and 100K sims capital increases by ~2% $\pm$ 0.5%

Comments

Uses

- Identify key correlations
- Perform approximate sensitivity analysis

Observations

- Often only a few individual correlations move the capital significantly
- …or systematic mis-estimation of a group of correlations
- Often correlations between key risk drivers are close to 0

Some Limitations

- The method does not capture directly the effect of underlying common drivers
- The key correlations are not necessarily those which are mis-estimated
- The approximation is often not bad -depends on skewness, copula, profit, etc.

Intuitive Link between Drivers and Correlations

Link between Common Drivers and Correlations

- Broadly speaking, correlation between two portfolios is the ratio

$$\frac{\textit{VOLATILITY DUE TO THE COMMON DRIVERS}}{\textit{AVERAGE OF TOTAL VOLATILITY OF THE TWO PORTFOLIOS}}$$

Volatility is measured by variance

The average is a geometric average

Example 1: Intuitive Calculation of Correlation

- The only common driver (and only source of dependency) between the reserves of portfolios X and Y is assumed to be inflation.
- The Coefficient of Variation of the reserves of X is 11%
- The Coefficient of Variation of the reserves of Y is 9%
- The Standard Deviation of the inflation factor is 6%
- ***Question***: *What is the correlation between the reserves of X and Y?*

$$\frac{\textit{VOLATILITY DUE TO THE COMMON DRIVERS}}{\textit{AVERAGE OF TOTAL VOLATILITY OF THE TWO PORTFOLIOS}}$$

$$\frac{0.06^2}{\sqrt{0.11^2 * 0.09^2}} = 36\%$$

- In practice guess-estimation of correlation is more complicated

Example 2: Which correlation is higher?

- Consider
  - COMPANY A
    - A1 small aviation – airlines book
    - A2 small marine liability book

    } Volatile Portfolios
  - COMPANY B
    - B1 large motor book
    - B2 large household book in a country with very rare small storms and very weak earthquakes

    } Not Volatile Portfolios
- The only common driver between portfolios is assumed to be inflation and is the same for LoBs and size of losses!!
- Which correlation you expect to be higher?
  - Between A1 and A2 or
  - Between B1 and B2

Example 3: Which correlation is higher?

- A1, A2 and B1, B2 are LoBs with the same total risk profile
- All policies are independent, apart from a possible catastrophe
- A catastrophe occurs every year

Cat A affects both A1 and A2

Cat A ~ U(100, 300)

Same Cat for A1 and A2

Cat B affects both B1 and B2

Cat B ~ U(300, 400)

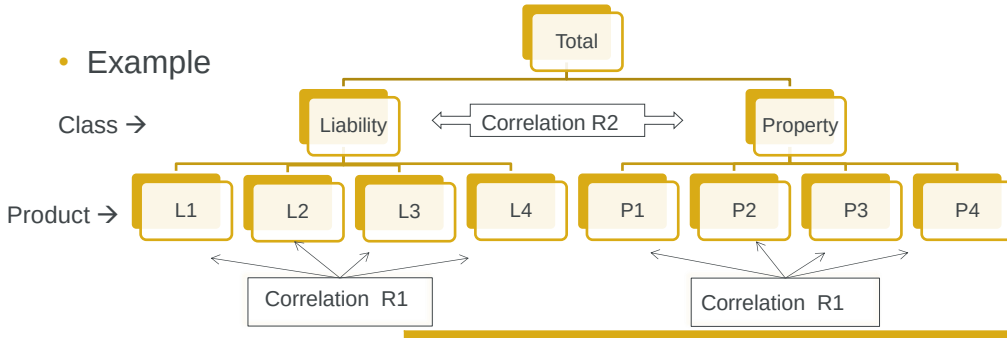
Same Cat for B1 and B2

Dependencies cannot be assessed independently

At what level?

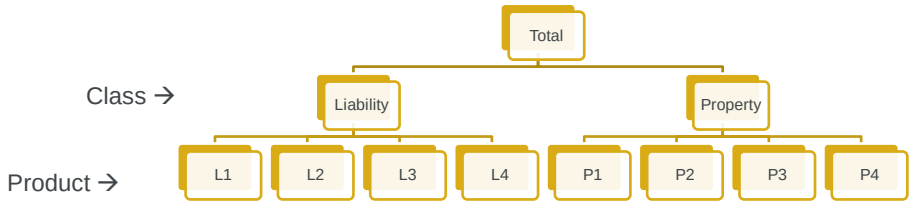
Assumption – all products follow N(0,1)

- Example



R1 – correlation between products within a class	R2 – correlation between classes	Total Capital
30%	0%	?
10%	40%	?
0%	60%	?

At what level?



R1 – correlation between products within a class	R2 – correlation between classes	Capital for each class	Total Capital
30%	0%	7.10	10.04
10%	40%	5.87	9.83
0%	60%	5.15	9.22

Dependencies can not be assessed independently

- Comparison between A and C
  - If someone examines correlations at class level (R2) then correlations in case A appear weak and in case C strong
  - However, case A gives a higher capital requirement
- Dependencies affect the marginal distributions at higher level
- Similar features appear if instead of correlations we were examining copulae.
  - This is something to bear in mind when we assess tail dependencies

Part II Summary

- One can use straightforward methods to
  - Identify key correlations and estimate their impact
  - Carry out approximate correlation sensitivity analysis easily and quickly
- Using intuitive methods to interpret correlations can provide a helpful framework for expert judgement and validation of dependencies
- Dependencies can not be assessed independently of the marginal distributions and the whole dependency structure

References

- Tsanakas A., Millossovich Pietro (2014) *Sensitivity Analysis Using risk Measures*, working paper
- Papachristou D. (2011) *Insights into Correlations and Dependencies*, GIRO Conference

Any other ideas?

"A designer knows he has achieved perfection not when there is nothing left to add, but when there is nothing left to take away."

- Antoine de Saint-Exupery