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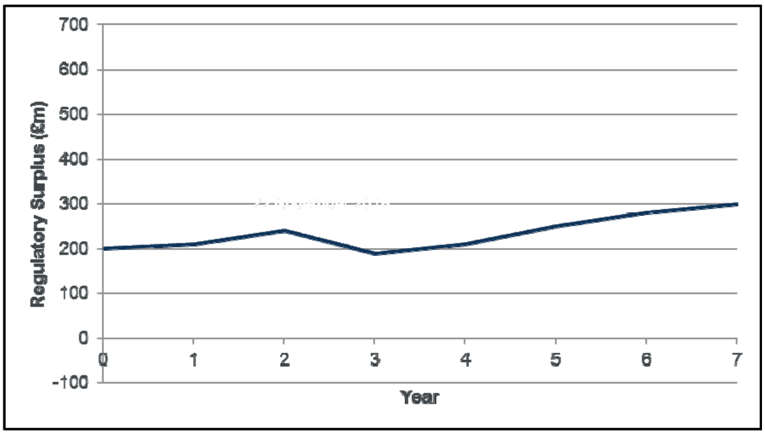
The Stochastic Full Balance Sheet Model

Bill Curry

22 November 2018

Surplus projections



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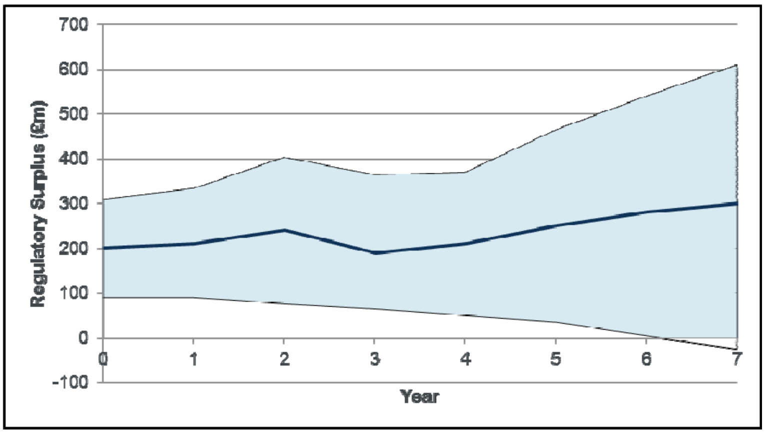
Surplus projections



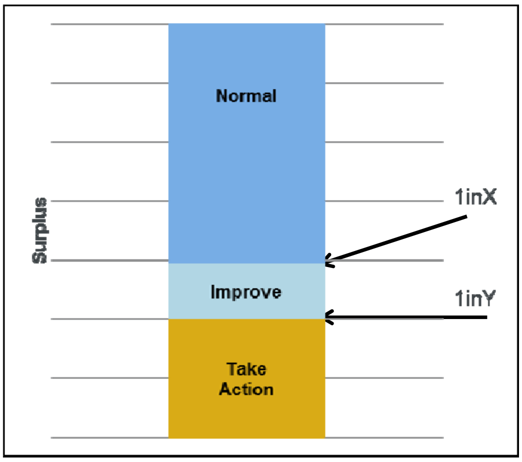
1in10 1yr surplus changes

Key questions in risk management

- What is the cost to our solvency position of a 20% equity fall?
- What fall in solvency is a 1in10 loss?
- What is the probability our capital coverage ratio goes below 120%?
- What is the probability we breach SCR?
- What event is most likely to cause a breach of SCR?

Risk appetite

- Risk appetite buffers take into account the stability of the capital position



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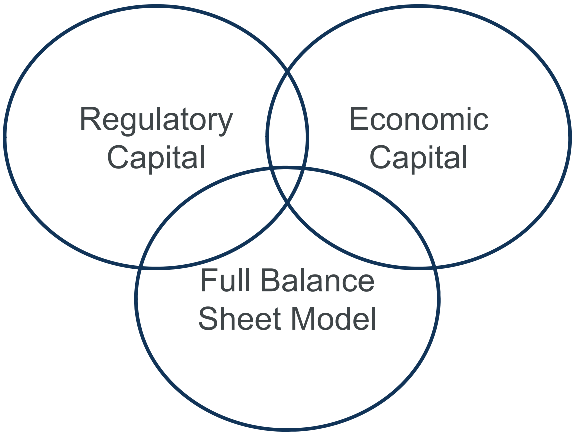
Types of group model



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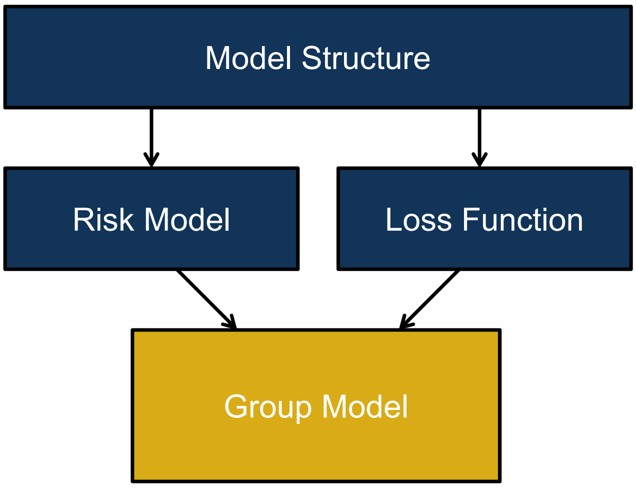
Group model components



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Group model components

Regulatory capital – SII Internal Model

Model Structure

- Model structure part prescribed under SII

Risk Model

Loss Function

- Majority of firms use copula simulation models
- Individual distributions specified for each risk

- Losses to assets and liabilities normally estimated through proxy functions

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Group model components

Regulatory capital – SII Standard Formula

Model Structure

- Model structure mostly prescribed under SII

Risk Model

Loss Function

- No model actually specified. May be thought of as multivariate normal

- Losses represented by linear loss functions fitted to the 1in200 points
- No cross terms to represent interaction between risks

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Group model components

Economic capital

Model Structure

- Model structure not prescribed, however commonly similar to the regulatory capital model

Risk Model

Loss Function

- May be similar to a firm's regulatory model
- Other risks could be included
- Different calibrations could be used.

- Commonly similar to the regulatory model

Differences could be in:

  - MA or VA
  - Pension valuation
  - Contract boundaries
  - Etc.

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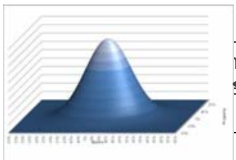
Group model components

Stochastic full balance sheet model

Model Structure

- Structure may be close to economic capital model

Generate Sims → Estimate Losses → Analysis



ns best view of risks
ed to economic capital

- Losses represent changes in the full SII balance sheet rather than just assets and liabilities
- Need to allow for realistic ch… discount rates (VA, MA, IAS19)

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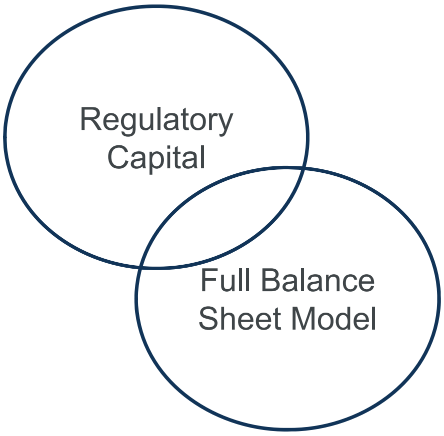
Types of group model



Strength of the Capital Position

Stability of the Capital Position

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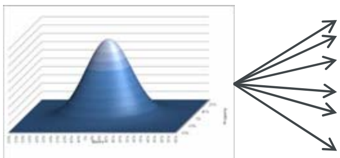
Model Features

Simulation generation

- Simulation generation may use standard copula modelling techniques



We should consider

- Should risk calibrations be Point In Time or Through The Cycle?

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Proxy models

- The purpose of a proxy model is to enable fast estimation of balance sheet changes as a function of risk movements.

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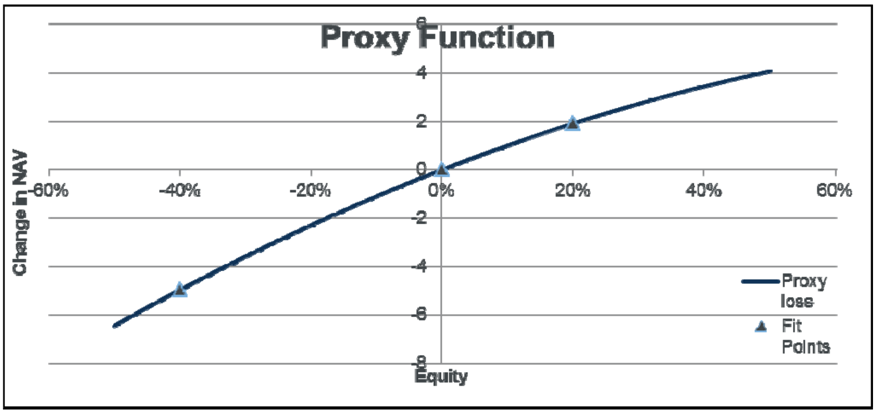

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Proxy models

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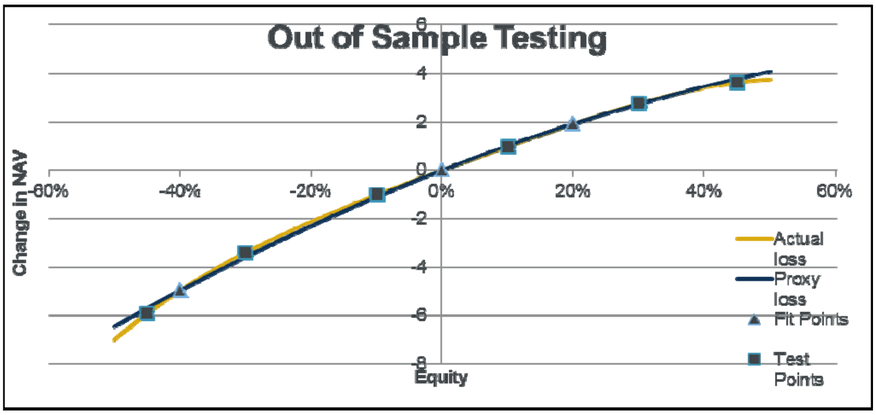
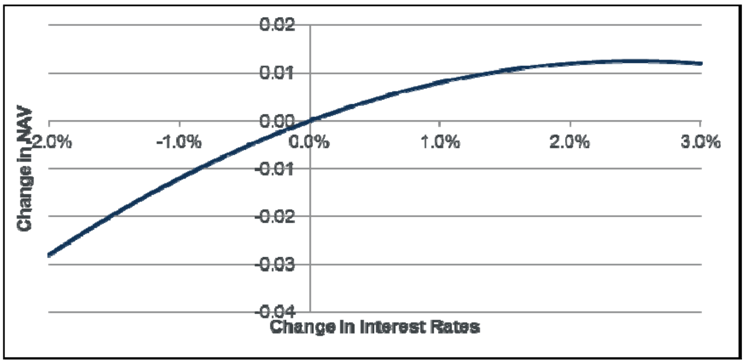


Roll forwards

- Roll forwards techniques are used to estimate how loss functions change
- Interest rate example,

  Change in NAV = $X - 20X^2$

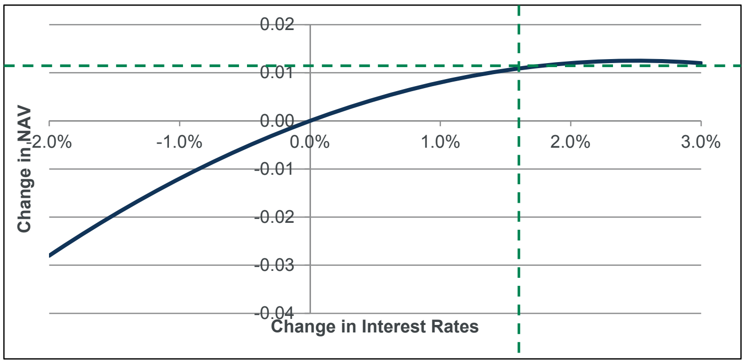


Roll forwards

- Roll forwards techniques are used to estimate how loss functions change
- Interest rate example,

Change in NAV = $X - 20X^2$

Say we have a 1.6% interest rate increase



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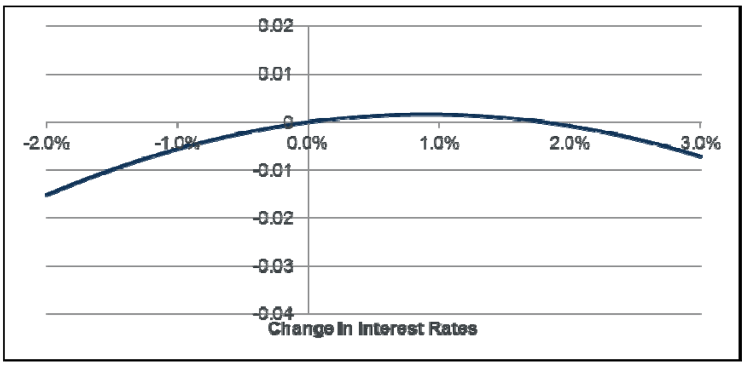
Roll forwards

New Change in NAV = $(X+1.6\%) - 20(X+1.6\%)^2$

$= 0.36X - 20X^2 +$ ~~0.01088~~



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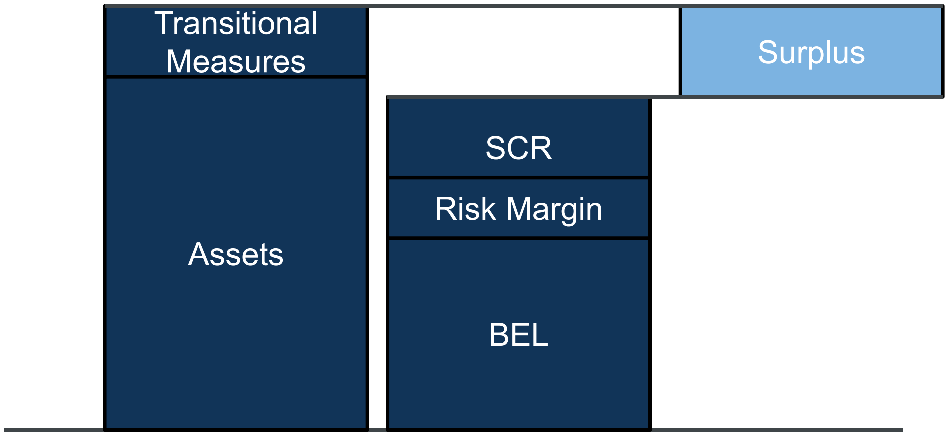
The solvency II balance sheet



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Asset and liability modelling

- Asset and liability models typically the same as used in an Internal Model or Economic Capital model

Difficulties may arise over discount rates used for:

- Volatility Adjustment (VA) business
- Matching Adjustment (MA) business
- Pension liabilities

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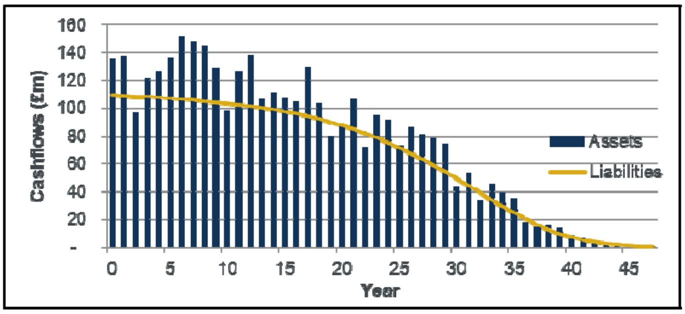
Example model

Annuity example - loss model

- 100,000 60 year old annuitants
- Annuity amount £1000 p.a.
- Expenses of £100, inflating at 1% p.a.
- Mortality as per an example mortality table
- Yield curve flat at 2%
- Risk free fixed interest cash-flows to broadly match the liability run off



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Annuity example - risk model

- Normally distributed risks assumed for
  - Longevity
  - Expense
  - Inflation
  - Interest Rate PC1
  - Interest Rate PC2
  - Interest Rate PC3
  - Credit
- Risks aggregated using a Gaussian copula with specified correlations

	Longevity	Inflation	Expense	PC1	PC2	PC3	Credit
Longevity	100%	0%	0%	0%	0%	0%	0%
Inflation	0%	100%	0%	50%	0%	0%	-20%
Expense	0%	0%	100%	0%	0%	0%	0%
PC1	0%	50%	0%	100%	0%	0%	-25%
PC2	0%	0%	0%	0%	100%	0%	0%
PC3	0%	0%	0%	0%	0%	100%	0%
Credit	0%	-20%	0%	-25%	0%	0%	100%

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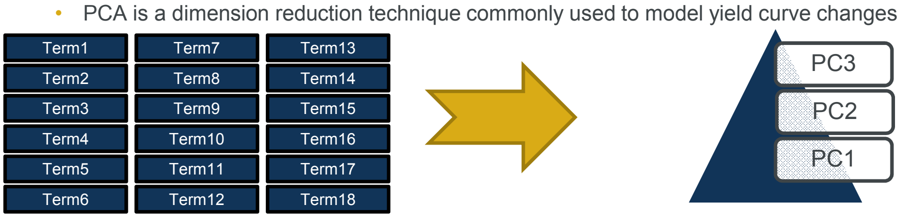
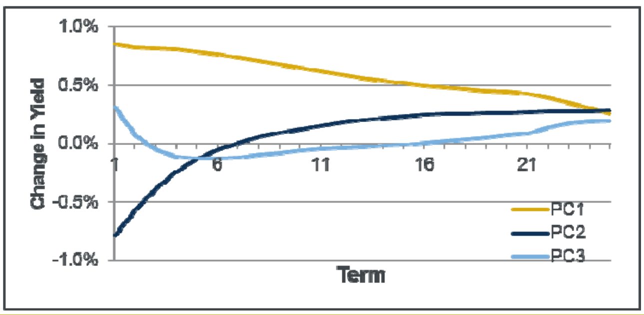
Annuity example - principal components analysis

- PCA is a dimension reduction technique commonly used to model yield curve changes





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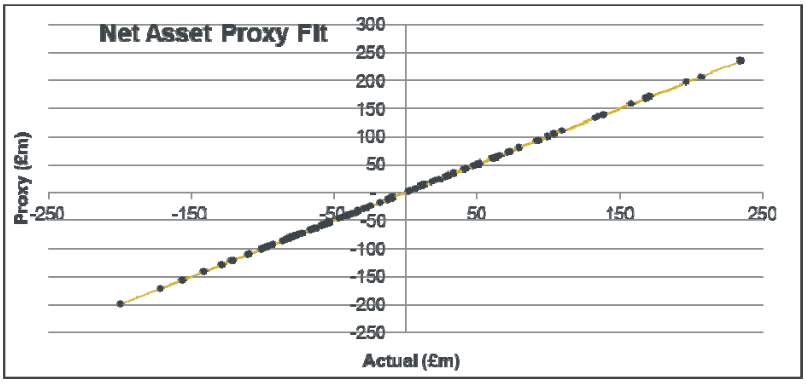
Annuity example – fitting approach

- Use Sobol sequence of 1023 fitting points
- Polynomial terms up to order 4 used to fit function to net assets
- Step regression applied to fit the proxy functions
- Out Of Sample (OOS) testing carried out using 100 random points from the risk model.



Proxy change in NAV = f(L,I,E,PC1,PC2,PC3)

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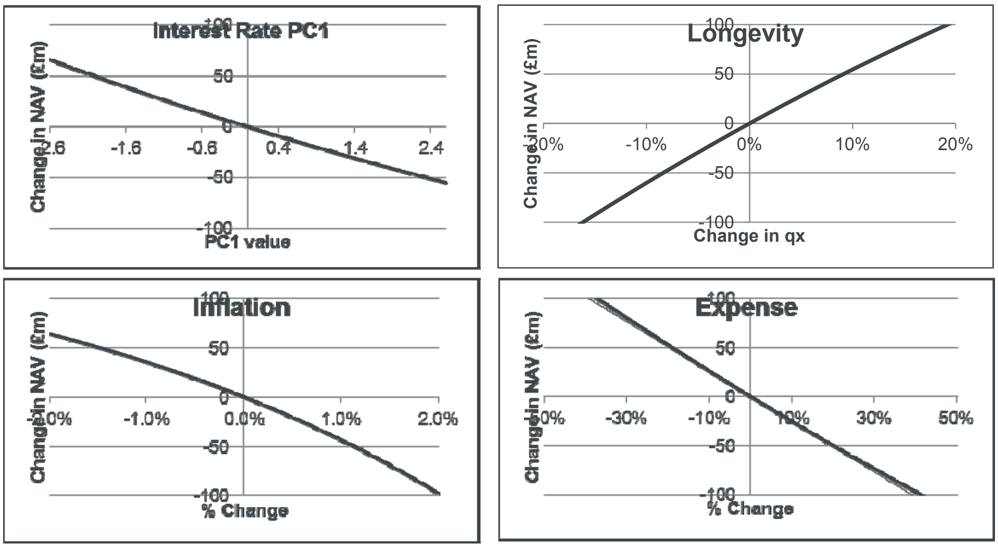
Annuity example – key exposures



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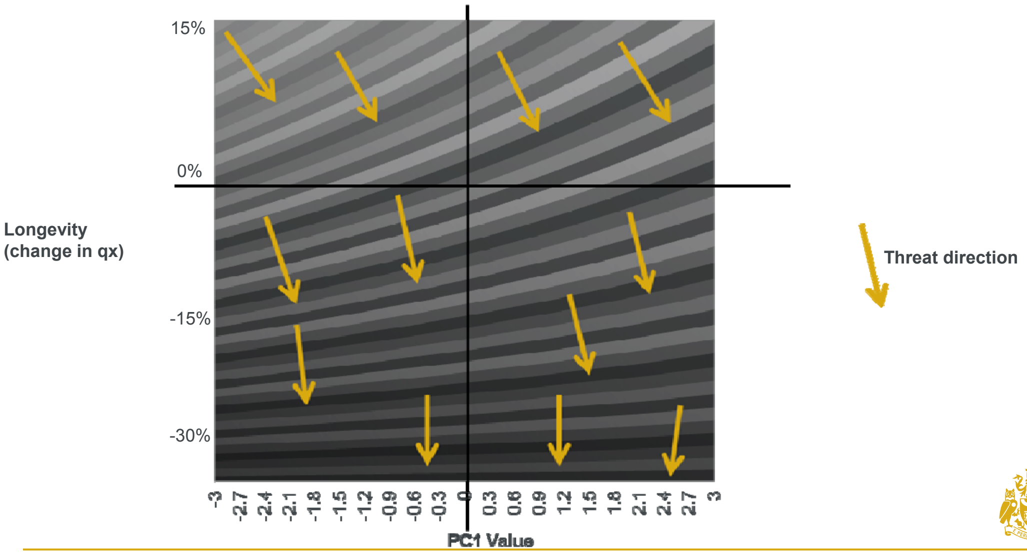
Annuity example – joint exposure PC1 vs longevity



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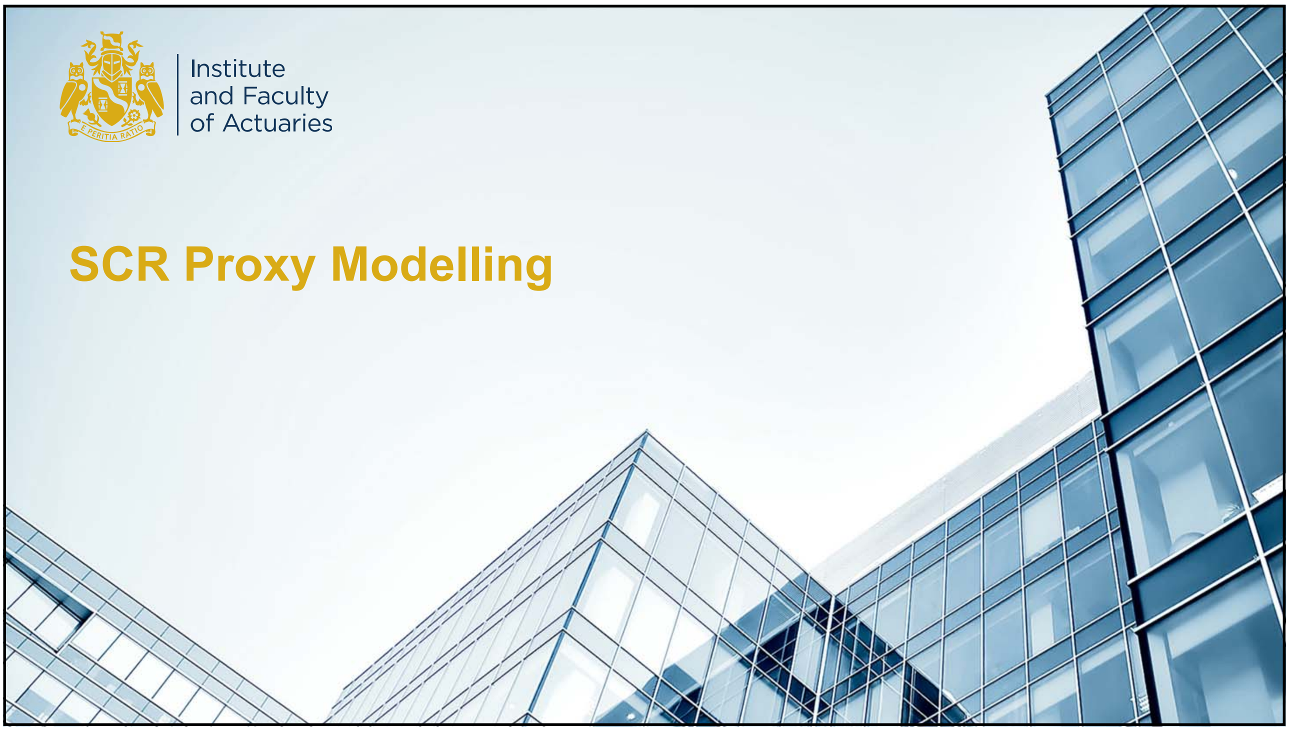

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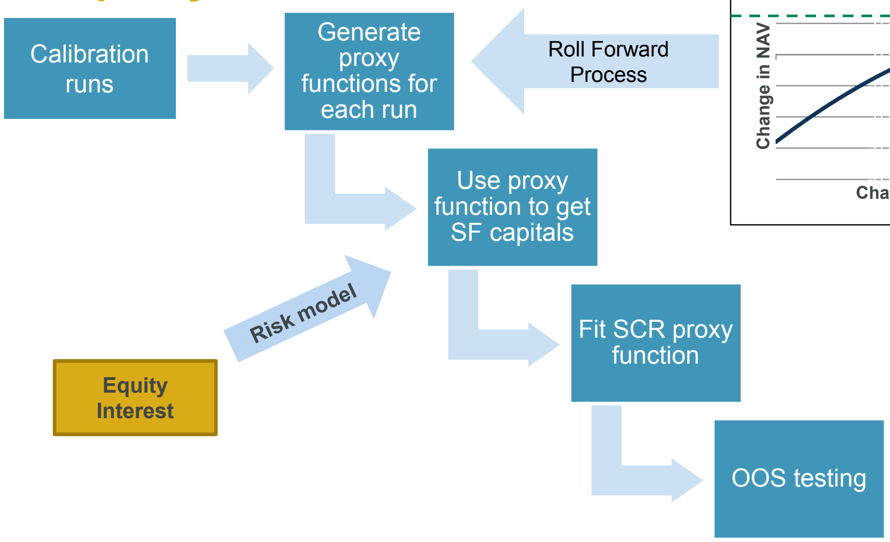
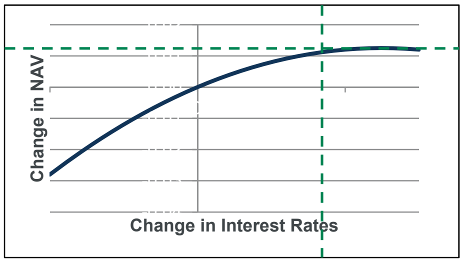
SCR Proxy Modelling

SCR proxy models – SF





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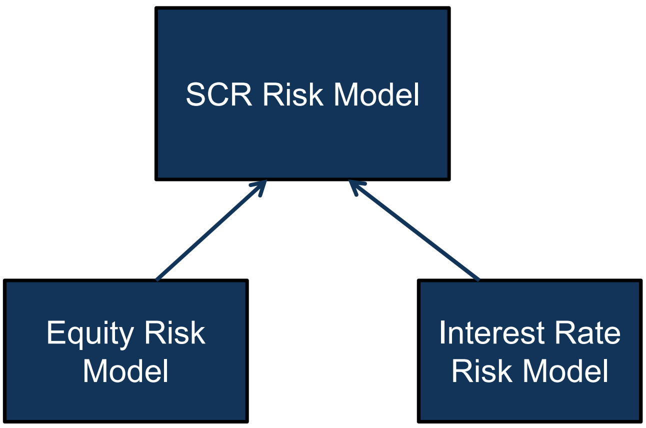
SCR proxy models – Standard Formula



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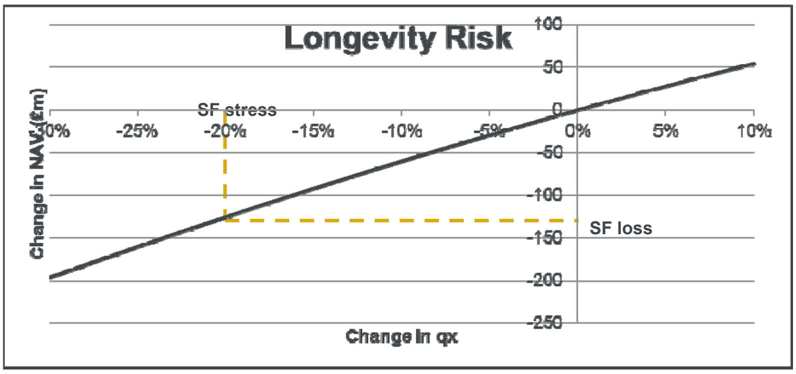
SCR proxy models – Standard Formula - longevity

- SF Longevity stress is a 20% fall in q_x



SF Longevity Capital Estimate = -f(-20%,0,0,0,0,0)

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SCR proxy models – Standard Formula - expense

- SF Expense stress is a 10% increase in expenses, together with a 1% increase in expense inflation
- Our example model uses a separate expense level risk and inflation risk
- Estimate SF expense capital using a combined expense and inflation event

SF Expense Capital Estimate = -f(0,1%,10%,0,0,0)

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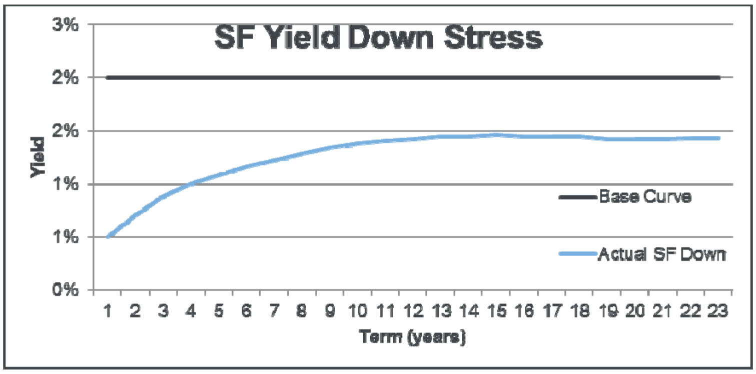
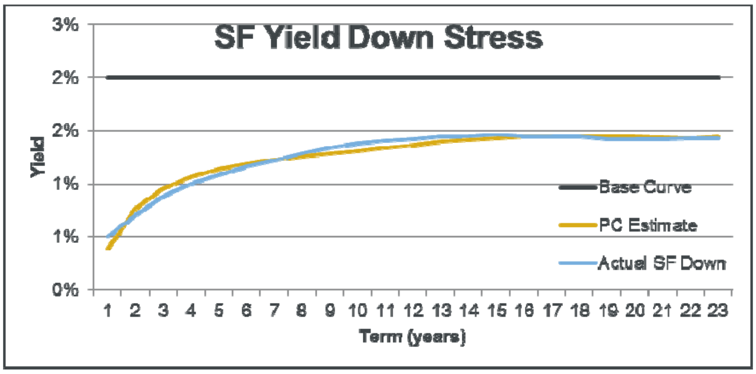
SCR proxy models – Standard Formula – interest rates

- SF Interest Rate up and down stresses are a function of the current yield curve



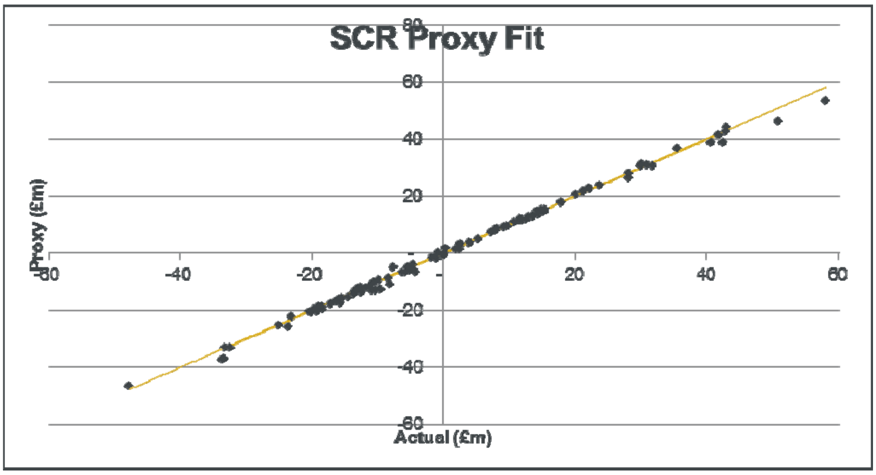
SCR proxy models – Standard Formula – interest rates

- SF Interest Rate up and down stresses are a function of the current yield curve
- We may estimate any change in yield curve as a linear combination of our principle components



Example, at yields of 2%, SF Yield down Capital Estimate = -f(0,0,0,-1.23,0.25,-1.16)

SCR proxy models – Standard Formula



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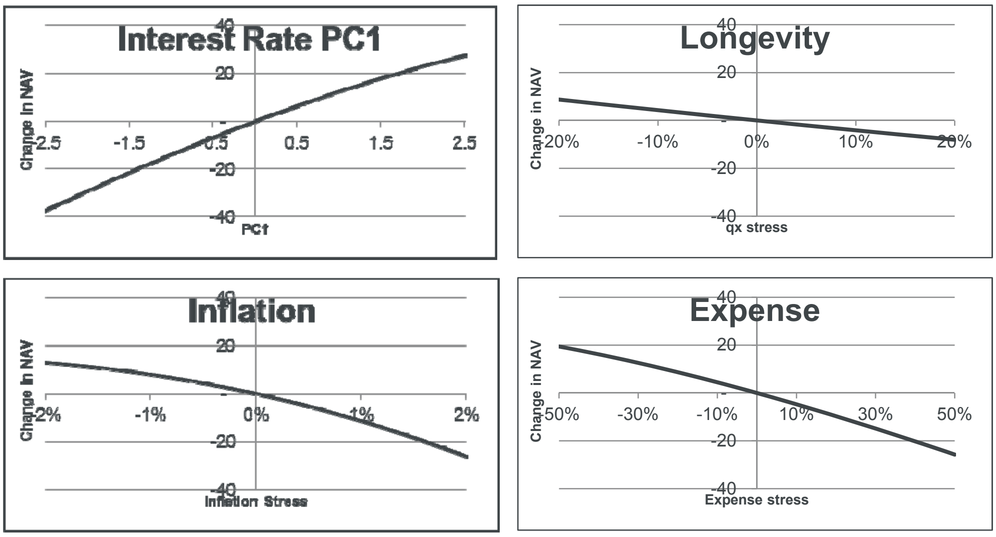
SCR proxy models – Standard Formula – key exposures



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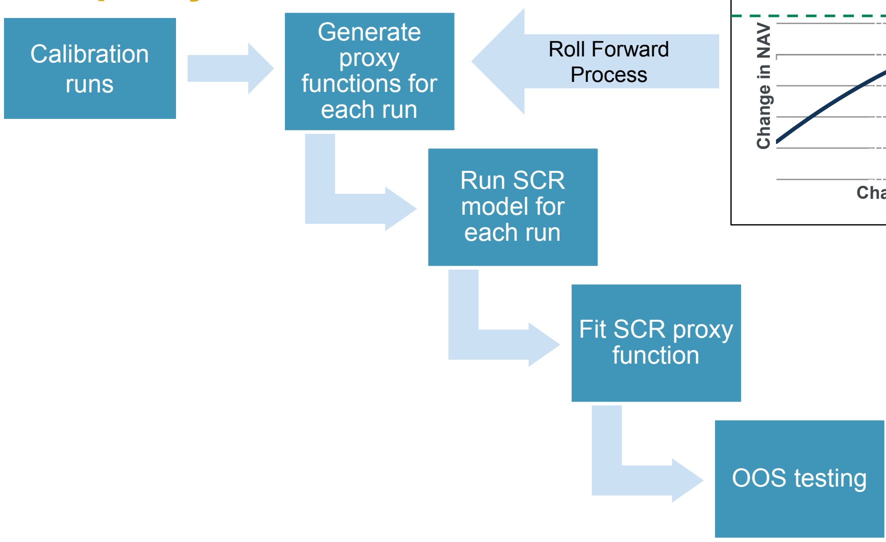
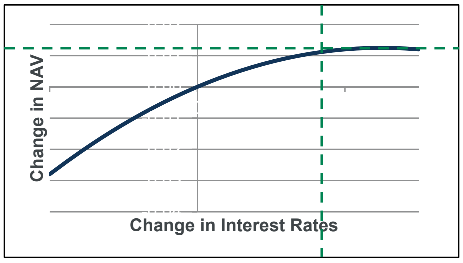
SCR proxy models – Internal Model





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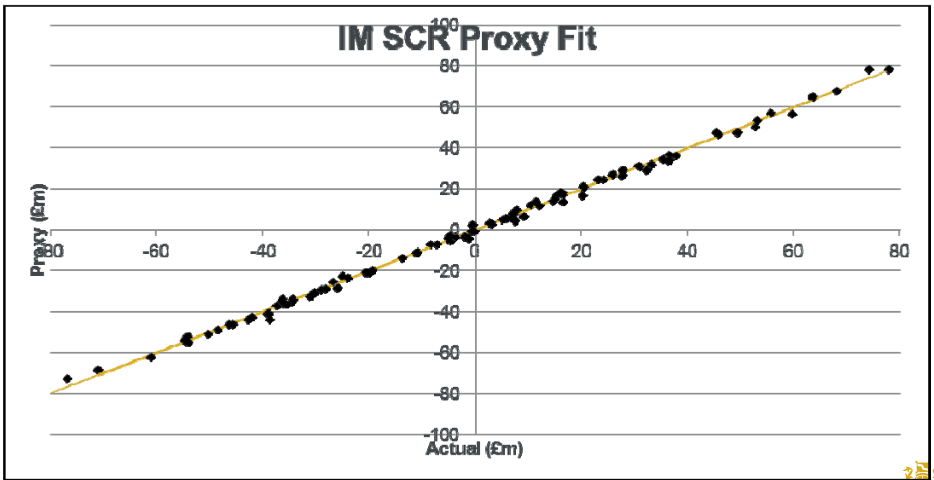
SCR proxy models – Internal Model



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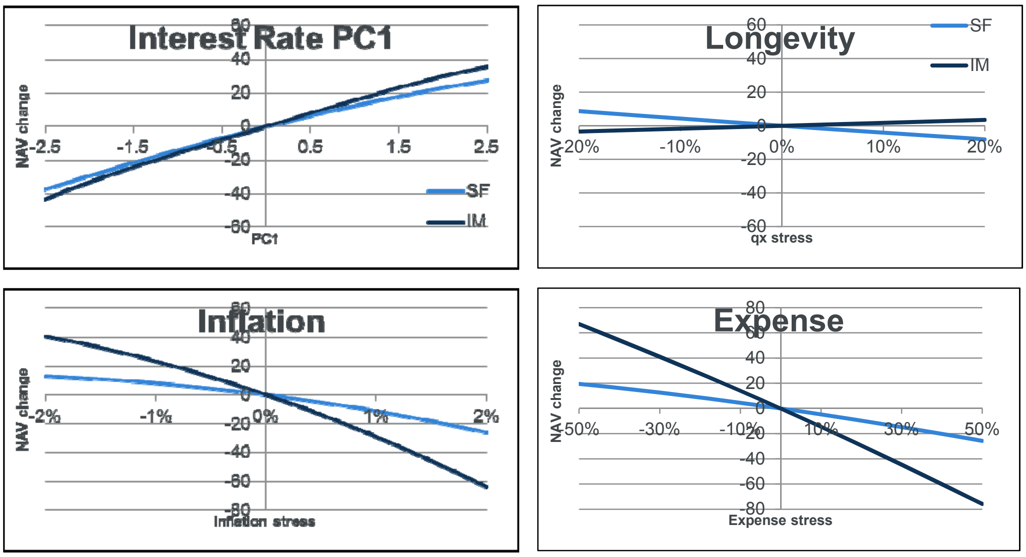
SCR proxy models – Internal Model key exposures



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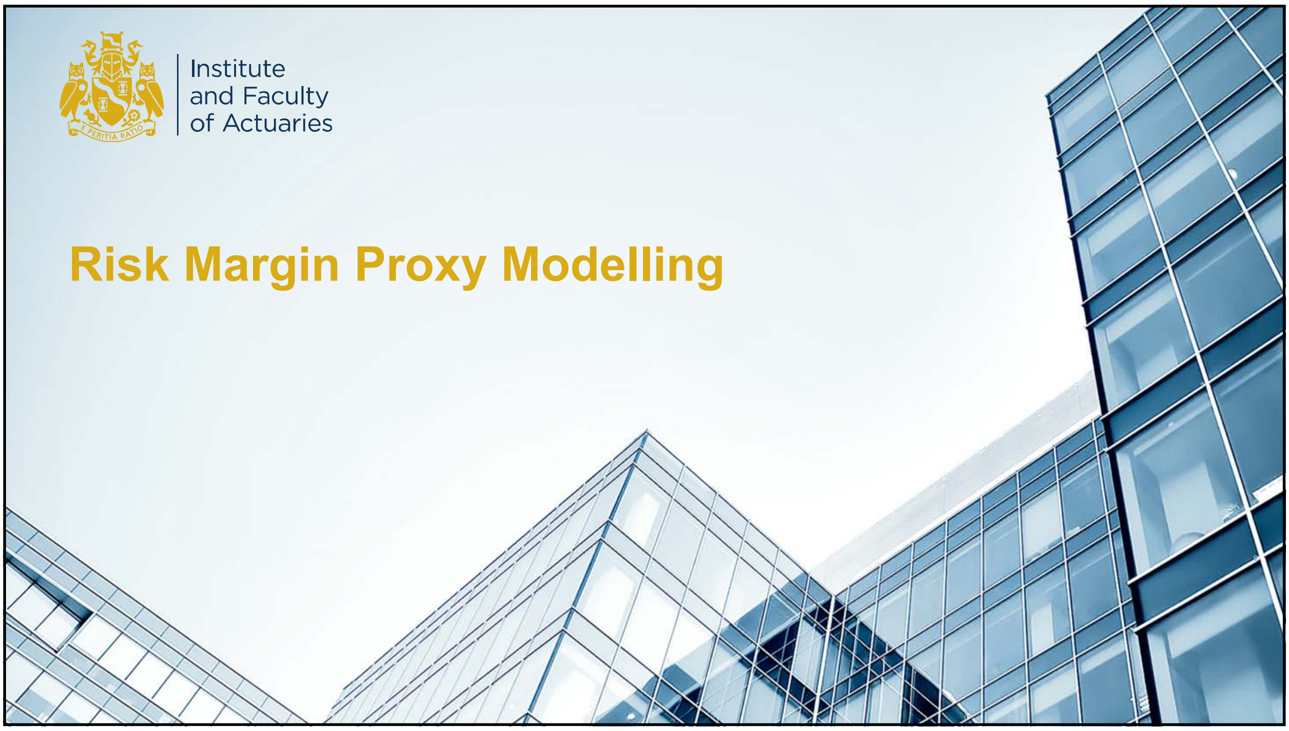

Risk Margin Proxy Modelling

Risk Margin proxy models

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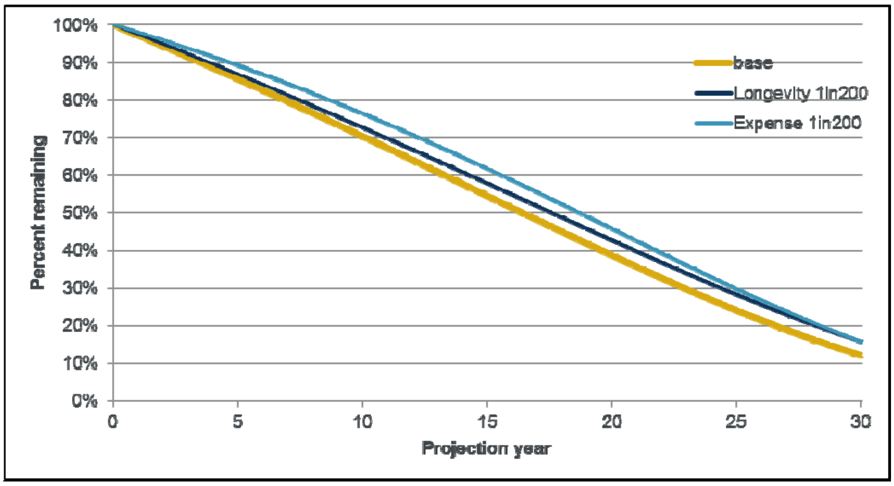
Risk Margin proxy models – run off example

- Annuity BEL run off under stress



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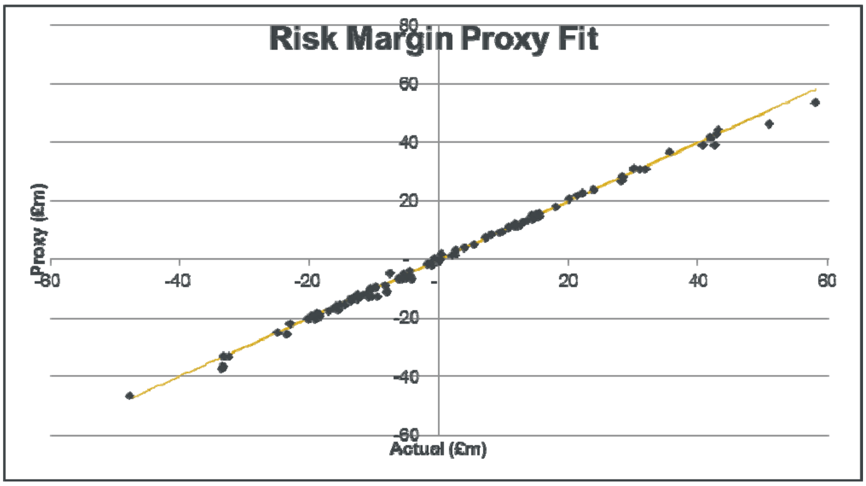
Risk Margin proxy models – Standard Formula



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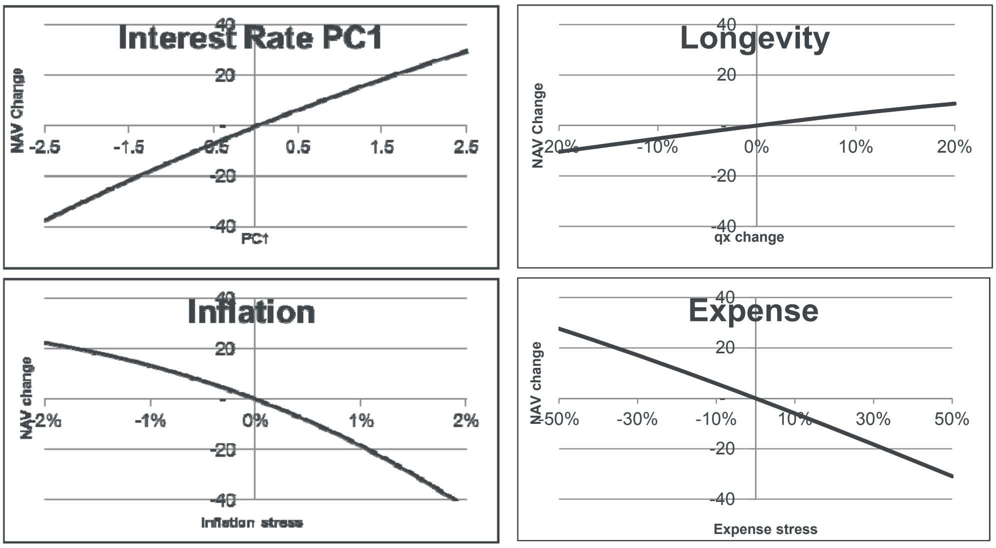
RM proxy models – Standard Formula – key exposures



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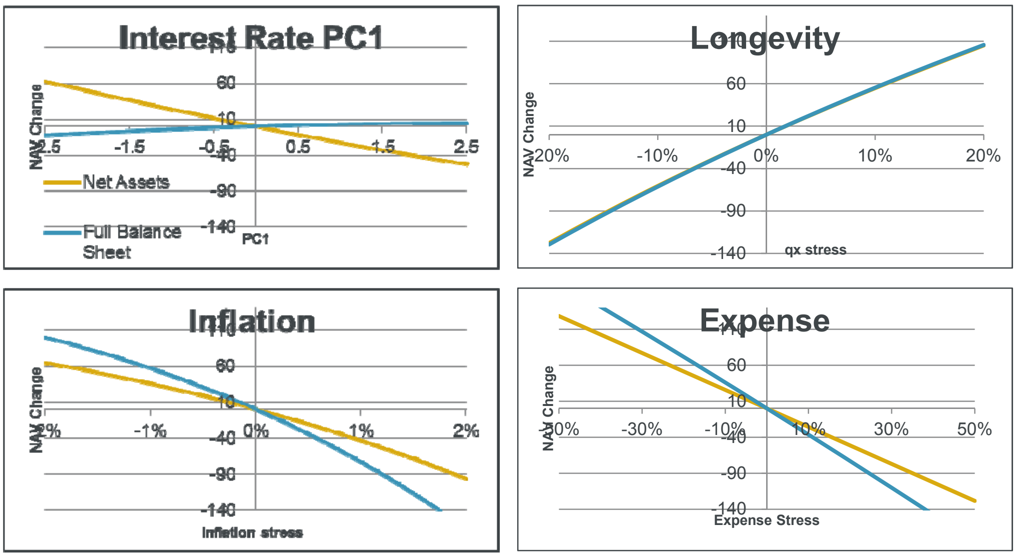
Standard Formula example – key exposures



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Discount Rates modelling

Liability modelling – Volatility Adjustment

- VA represents a flat addition to the discount curve for applicable long term liabilities
- Designed to protect insurers from the impact of volatility on the insurer's solvency position
- Calculated as 65% of the spread between the interest rate of the assets in a reference portfolio and the risk free rate, allowing for a fundamental spread
- Published monthly by EIOPA
- Permitted to change under SCR stress by some European supervisors.

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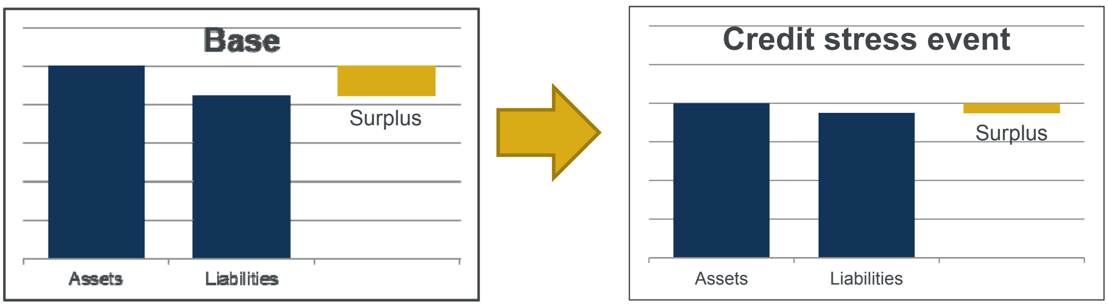
Liability modelling – Volatility Adjustment

- How it works in practice



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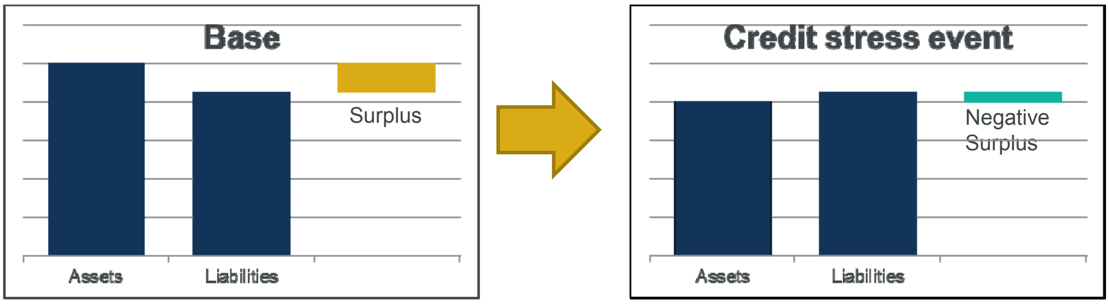
Liability modelling – Volatility Adjustment

- How it works under (UK) SCR calculations



Liabilities unchanged

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Liability modelling – dynamic VA model

- VA ~ 65% x (Spread – Fundamental Spread) calculated by rating, maturity

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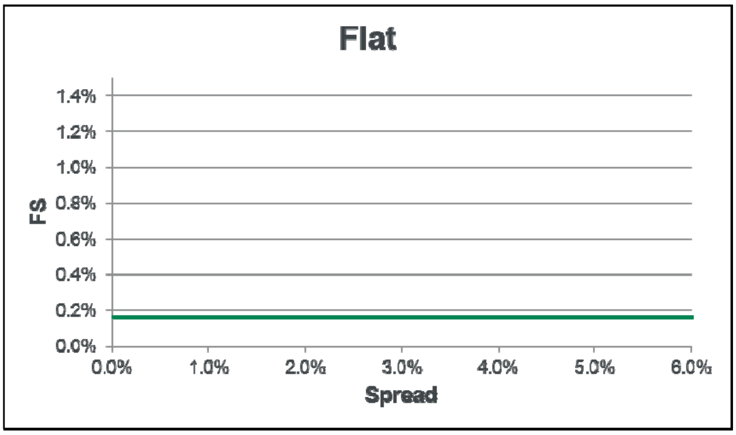
Liability modelling – dynamic VA model

- VA ~ 65% x (Spread – Fundamental Spread) calculated by rating, maturity



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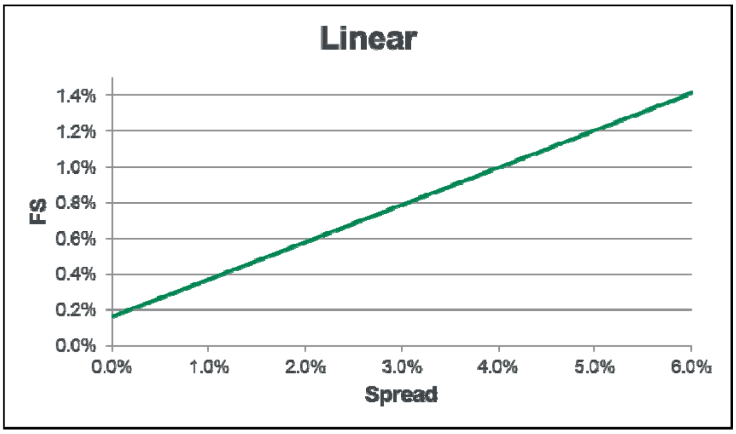

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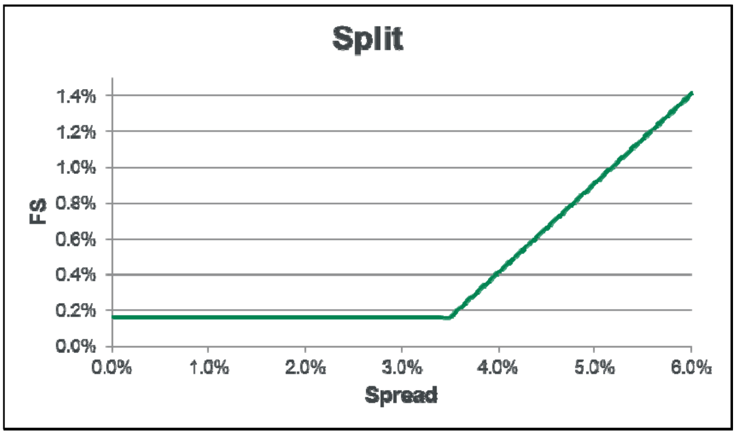

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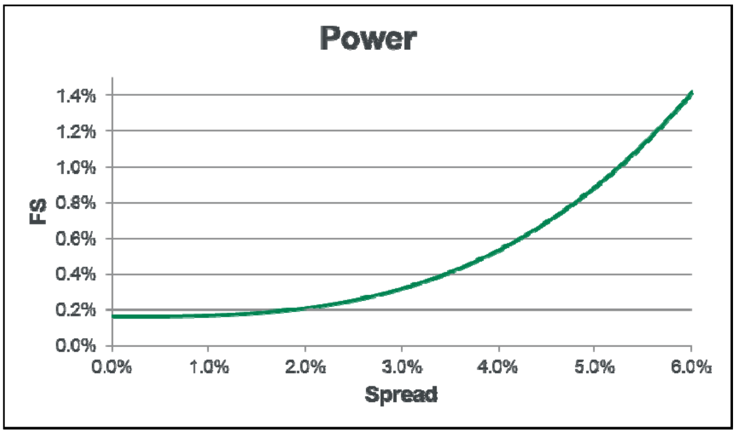

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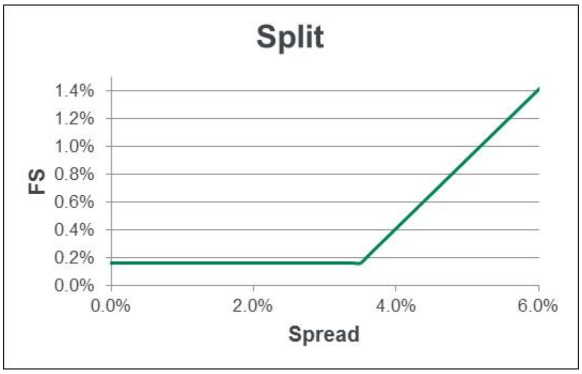
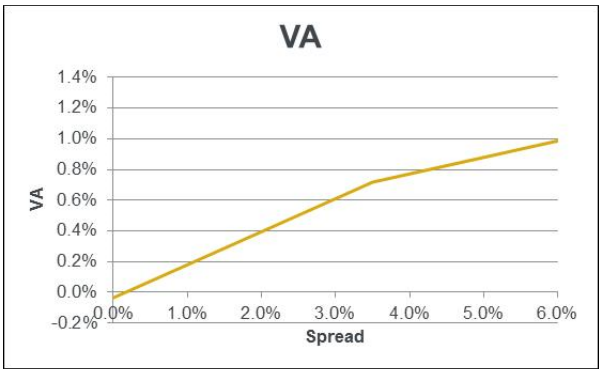
Liability modelling – dynamic VA model





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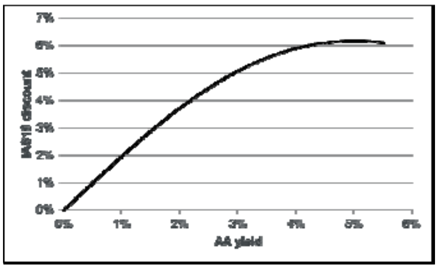
Liability modelling

Matching Adjustment

IAS19 Discount Rates

- We need realistic models to take into account the movement of these under stress



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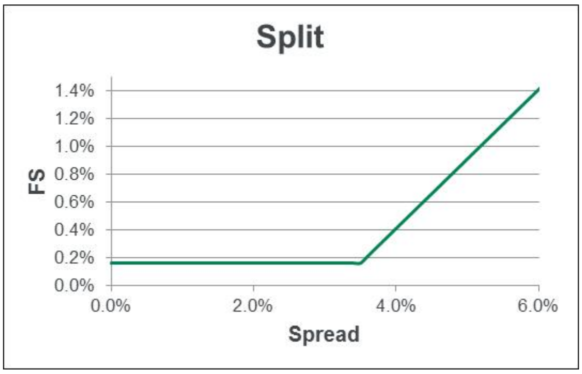
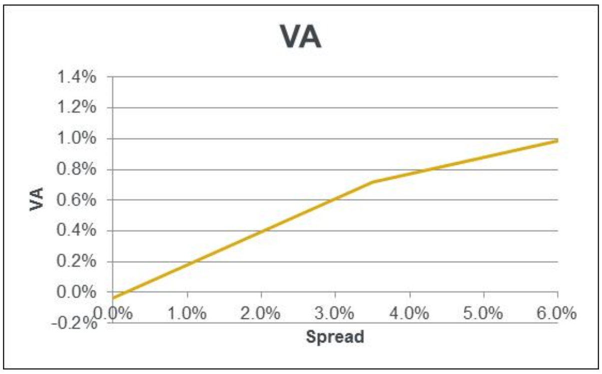
Annuity example - dynamic VA, SF model





Include credit risk as an additional normally distributed risk in the model.

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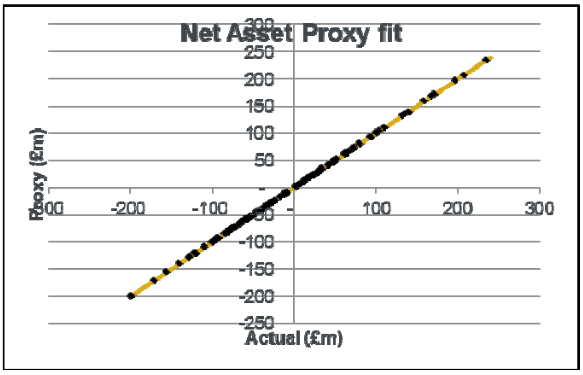
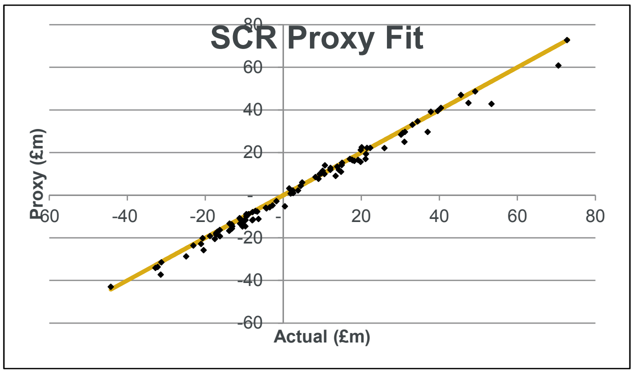
Annuity example - dynamic VA, SF model

Repeat curve fit process for net assets and SCR, RM unchanged





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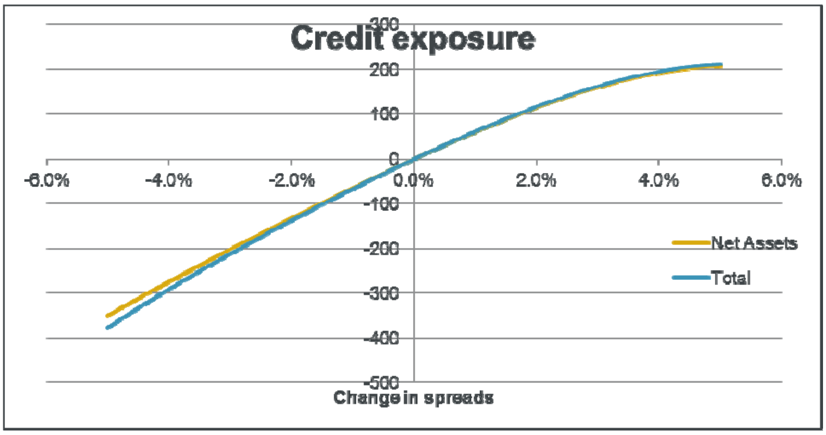
Annuity example - dynamic VA, SF model



Low materiality changes in other risk exposures

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Using the Example Model

More assumptions

- Starting surplus = £250m
- Risk Appetite thresholds (based on a one year time frame):
  - Plan to be able to withstand a 1in30 shock
  - We take urgent action if our surplus is unable to withstand a 1in10 shock

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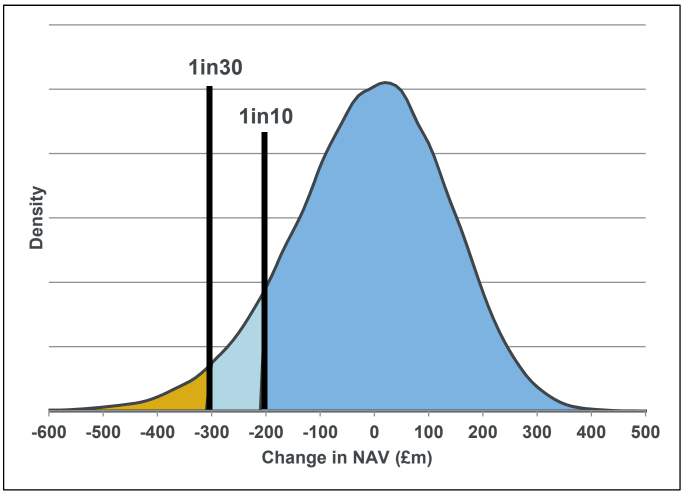
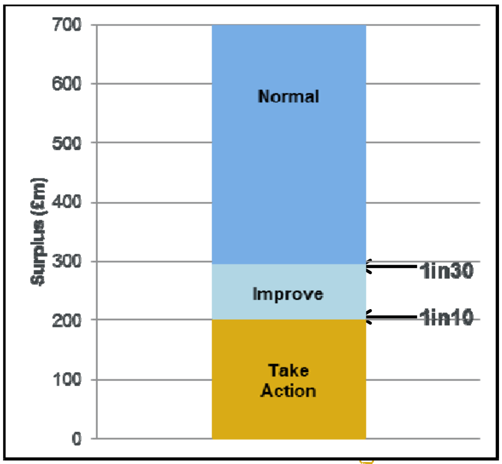
Risk appetite





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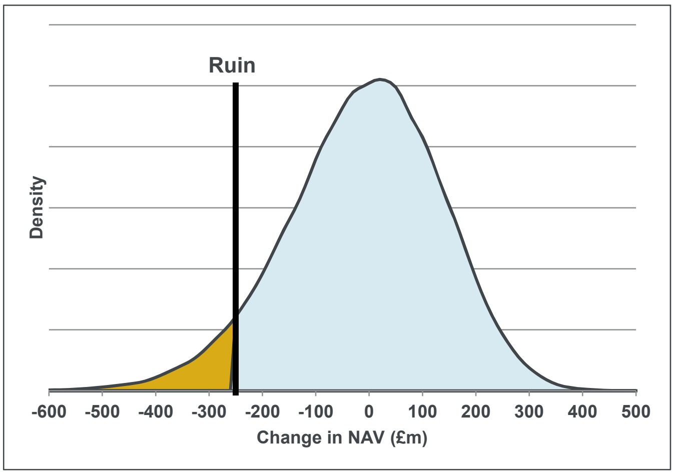
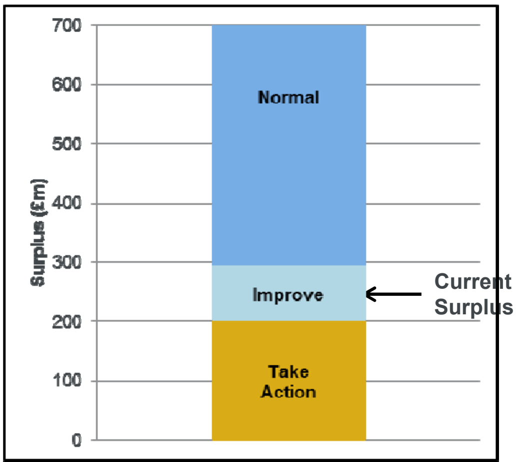
Ruin probabilities





Ruin Prob = 5.9%

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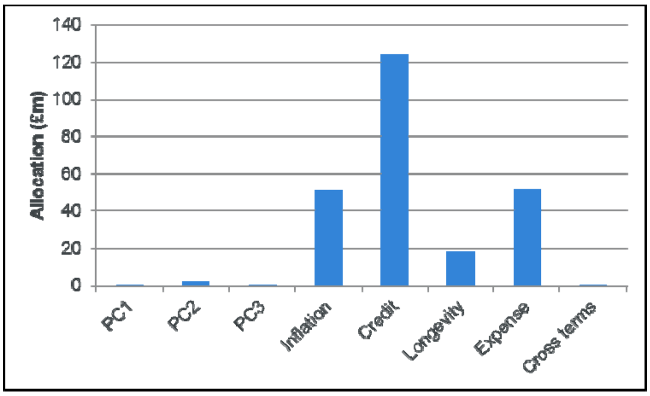
Euler allocation by risk – ruin event of £250m loss



Interest	inflation	credit	longevity	expense	cross
4	72	262	-23	-25	-39
-10	65	-13	74	117	18
28	54	263	-47	-34	-13
16	136	100	5	13	-20
-16	255	76	-78	53	-40
-12	-64	223	41	71	-9

- Euler Allocation of 250m loss

Risk A allocation $= -E[\, X_A | X_{total} = -250m\}$

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Ruin Events

Euler example

- Risks A and B, multivariate standard normal
- Correlation -99.9%

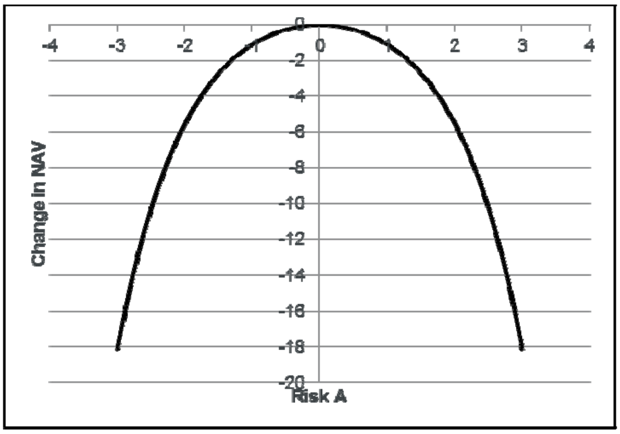
$$Change\ in\ NAV = 2 - (e^{A} + e^{B})$$

1in200 capital = 14.8m

Euler allocations:

Risk A = 7.4m

Risk B = 7.4m



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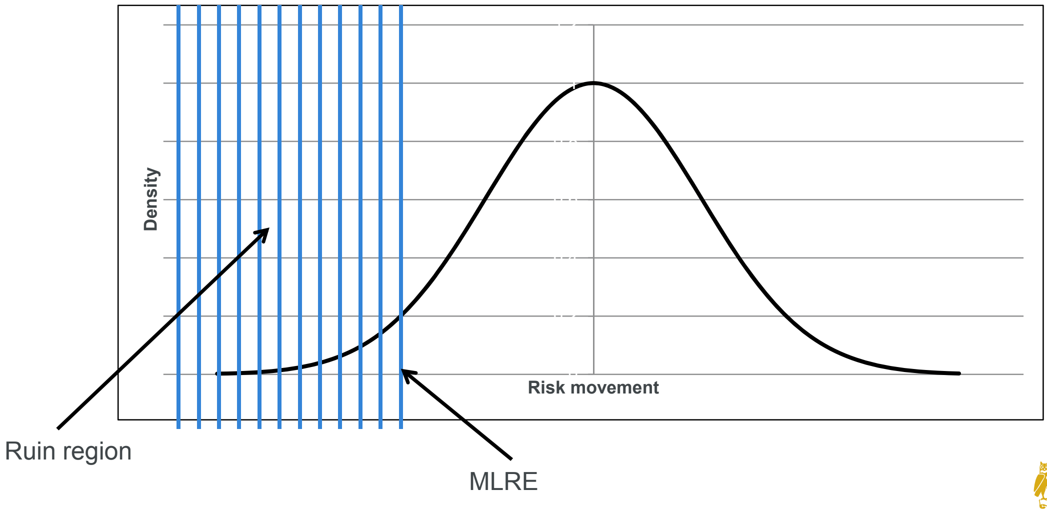
Ruin events – Most Likely Ruin Event



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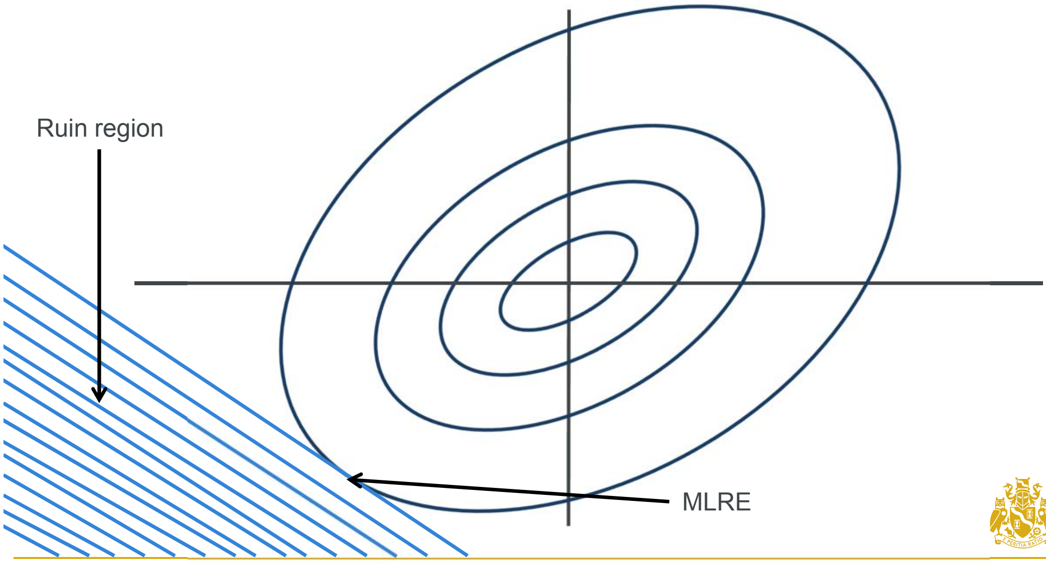
Ruin events – Most Likely Ruin Event



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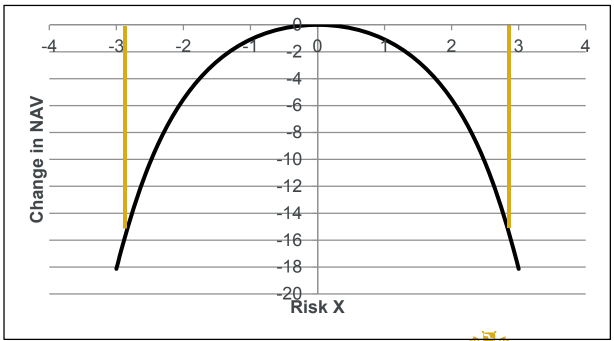
Ruin events – Euler example

- Risk distribution is multivariate normal (-99.9% correlation)
- Density function of A,B is well known f(A,B)
- Can solve for the maximum of f(A,B) subject to constraint $\mathbf{2-(e^{A}+e^{B}) = -14.8}$

Max at (A,B) = (-2.8,2.8) and (2.8,-2.8)



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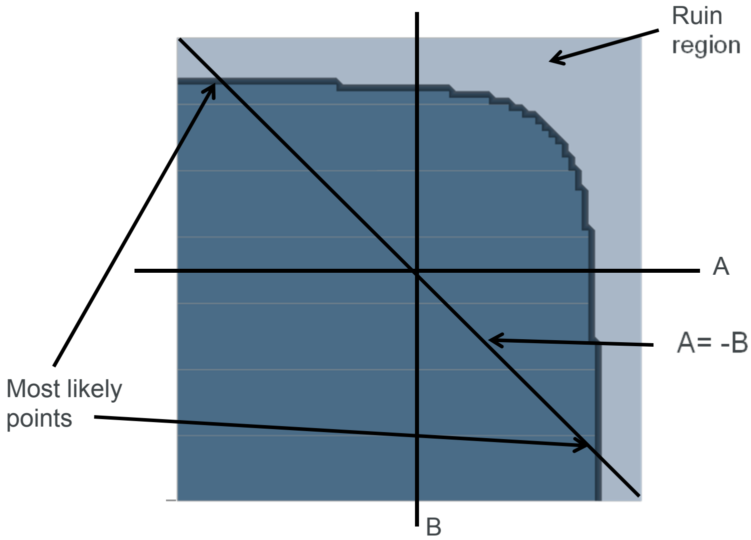
Ruin events – Euler example



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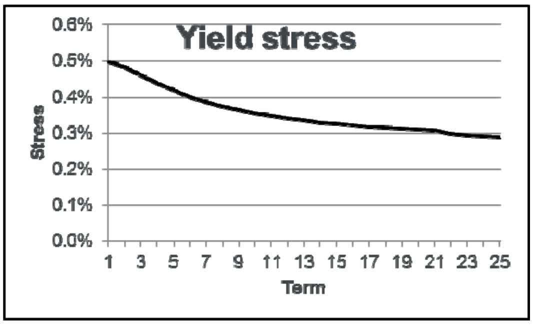
Ruin events – annuity model

- Find MLRE by maximising probability density subject to change in NAV < -£250m
- Risk distribution ~ Multivariate normal so density is well defined
- Change in NAV estimated using proxy functions.

Risk	Movement	Percentile	1 in X
PC1	0.437	0.67	3.0
PC2	- 0.209	0.42	2.4
PC3	- 0.128	0.45	2.2
Inflation	0.6%	0.78	4.6
Credit (spreads)	- 1.8%	0.12	8.1
Longevity (q_x)	- 3.8%	0.31	3.2
Expense (level)	14.1%	0.77	3.0



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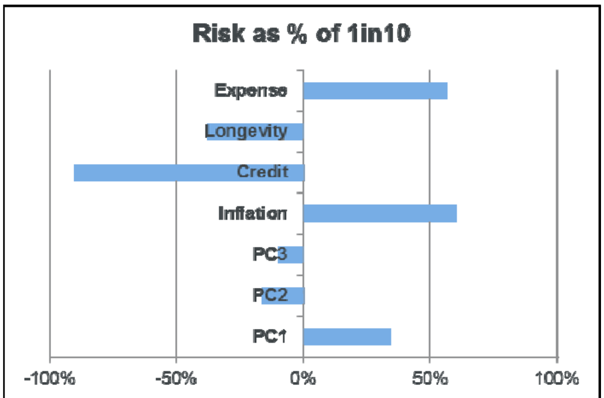
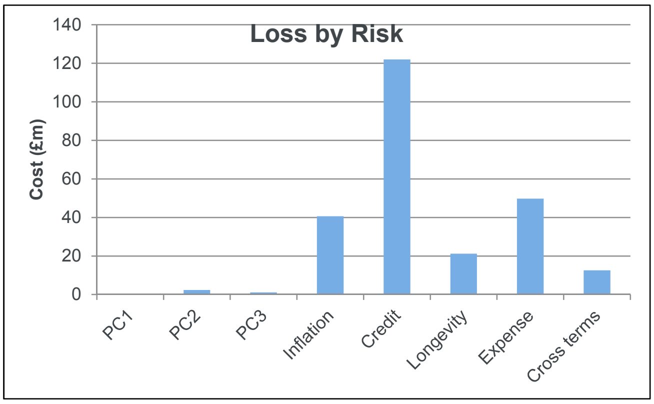
Ruin events – annuity model





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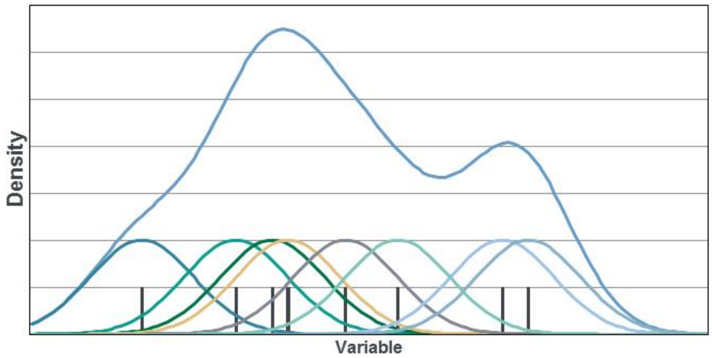
Ruin events – Kernel Density Estimation

- We can use Kernel Density Estimation (KDE) to estimate the density function of the joint risk distribution from the simulations



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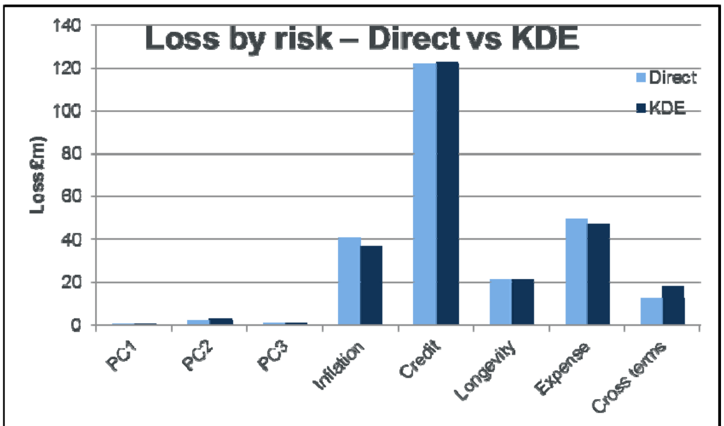
Ruin events – annuity model



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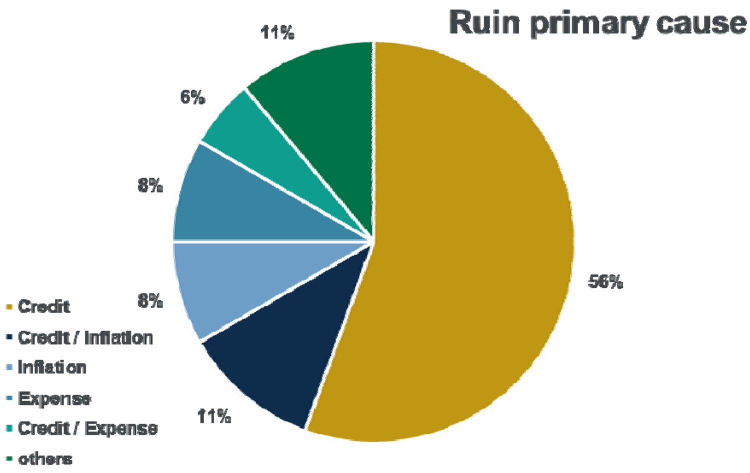
Ruin cause



No.	Event	%
1	Credit	56
2	Credit / Inflation	11
3	Inflation	8
4	Expense	8
5	Credit / Expense	6
	Others	11

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Ruin cause

No.	Event	Credit (change in spreads)	Inflation (change in RPI)	Expense (change in level)
1	Credit	-3.5%		
2	Credit / Inflation	-2.6%	0.9%	
3	Inflation		2.8%	
4	Expense			68.7%
5	Credit / Expense	-2.5%		19.6%

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Ruin events summary

- Euler is about capital allocation not events
- MLREs give insight into actual events
- KDE can be used to get the density from the simulations
- We can plan what we would do under the ruin events

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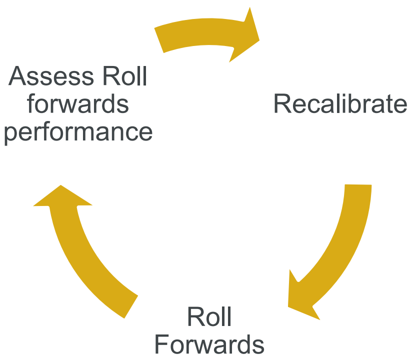
Uses of the model

The roll forward cycle



Roll Forwards for:

- Run off
- New Business
- Economics
- Basis Changes
- Model changes
- Risk calibration changes

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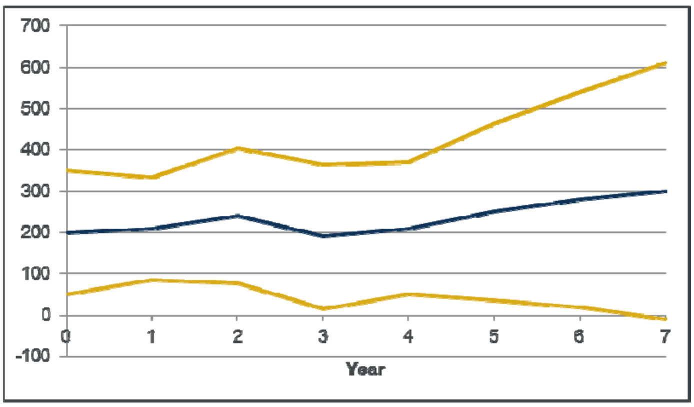
Projections

- Is our balance sheet getting more or less stable over time?
- Is our ruin probability getting better or worse?



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Projections

- Proxy functions normally express changes in NAV as a function of risk movements

Proxy change in NAV = f(L,I,E,PC1,PC2,PC3)

- For projections (e.g. of risk appetite), we need to calibrate as a function of risks and time

Proxy change in NAV = f(L,I,E,PC1,PC2,PC3,Time)

- We can use run off drivers by risk and product to scale the proxy functions over time

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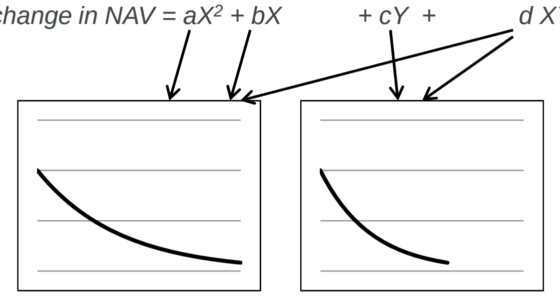
Projections - example

- For Risks X and Y

Proxy change in NAV = $aX^2 + bX \quad + cY \; + \quad d\,XY$



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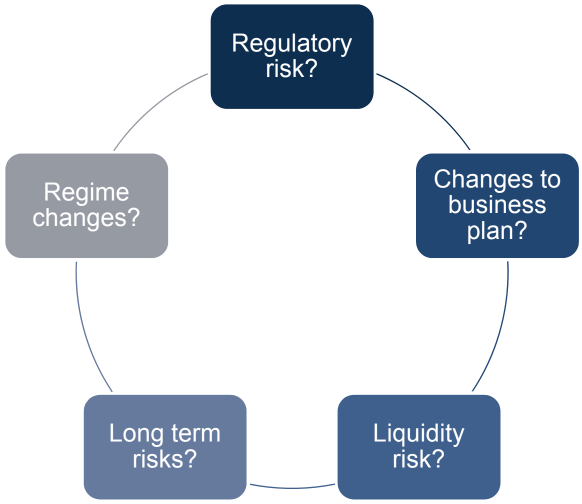
What isn't in your model?



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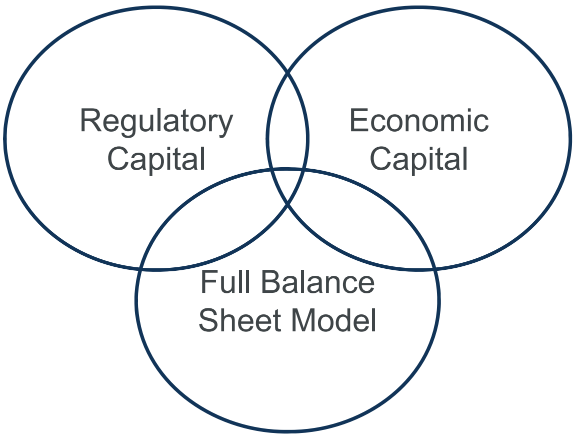
Types of group model



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Questions

Comments

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