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How long is forever?

Using annual capital re-allocation to calculate a rate of return on capital

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ALLIED WORLD**



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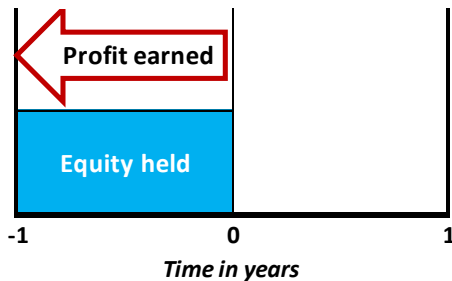
I. What are we aiming for?

Defining the ROC

The hindsight view: Return on Equity (ROE)

$$ROE = \frac{\text{Net Profit}}{\text{Shareholder's Equity}}$$

- The **ROE** compares the profits generated in the year to the equity held
- Calculated from the Financials
- A widely used measure of investment performance



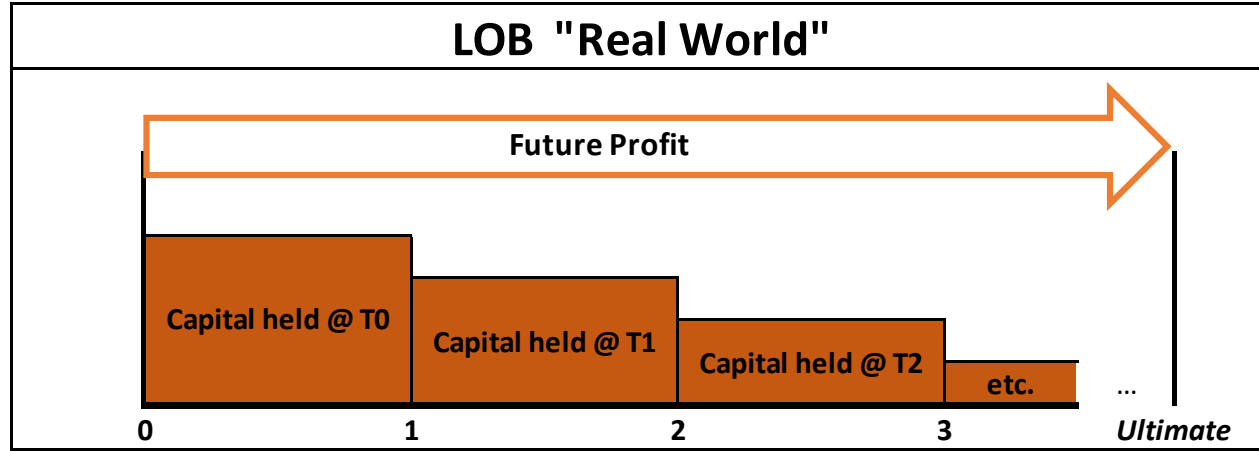
Key figures of non-life insurers (in CHF thousands)

	2017	2016
Gross premiums written	49,241,703	47,967,106
Claims paid out	25,961,281	25,811,840
Cost for the change in technical liabilities	597,371	12,584
Cost for the change in other actuarial liabilities	798,173	645,157
Costs for underwriting	10,097,966	9,548,200
Taxes	739,231	757,187
Gains/losses from investments	6,060,406	6,517,964
Annual profits	5,339,225	7,018,905
Balance sheet total	169,054,218	167,036,747
Investments	150,572,335	148,589,158
Technical liabilities	84,669,696	83,445,422
Equity (before profit allocation)	38,987,401	39,385,332
Return on investments (in %)	4.06%	4.44%
Return on equity (in %)	13.69%	17.82%
Loss ratio (in %)	66.7%	61.6%

The prospective view (I)

- Planning objective: to maximize the ***reward*** (profit earned) vs. the ***risk*** to the balance sheet from writing the business
- Return on capital (ROC) is a risk/reward metric
 - *Risk* is measured using economic capital
 - *Reward* is the expected (ultimate) profit
- How should we define a ROC for an insurance LOB? Remember that in the real world:
 - The profit may be earned over many years
 - Capital is released as claims are paid
 - Capital must be held as long claims are open

The prospective view (II)



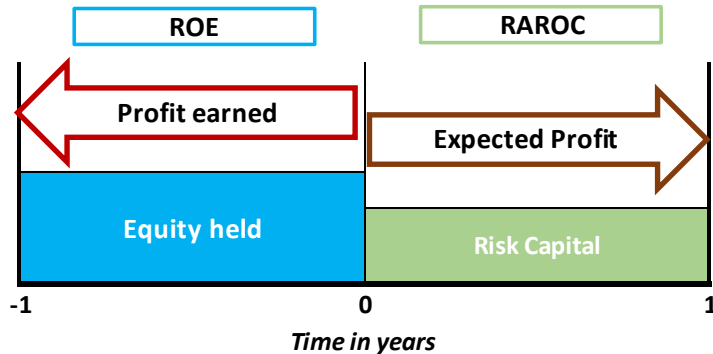
- Let's look at two potential analogs from finance
 - Risk Adjusted Return on Capital (RAROC)
 - A mortgage APR

RAROC: From banking...

- Concept developed at Banker's Trust in the 1970s

$$RAROC = \frac{\text{Revenue} - \text{Costs} - \text{Expected losses}}{\text{Risk capital}}$$

- Like a prospective ROE, using economic capital



...to insurance

- Now widely used in insurance, e.g., *Risk-Adjusted Performance Measurement for P&C Insurers* by Goldfarb (on CAS syllabus)

Table 23: Comparison of RAROC - Using Co-CTE Allocation

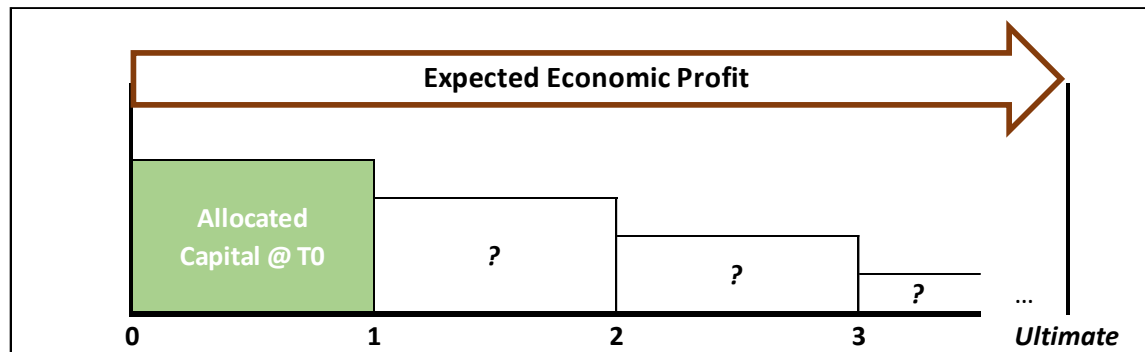
Earned over the lifetime of the claims "to ultimate"

Allocated ultimate capital @T0

	Economic Profit	Co-CTE (99%) Allocated Capital	RAROC
Line A	496,000	2,117,082	23.4%
Line B	880,000	4,225,340	20.8%

RAROC assessment

- But does it “work” for insurance?

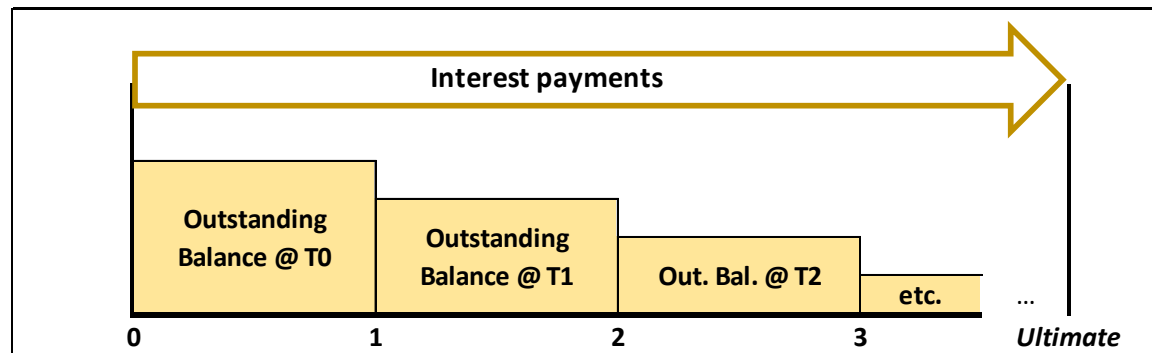


- In general – “No”*
 - The *reward* is the ultimate profit
 - The *risk* is the capital held in the first year only
 - A mismatch: There can be no cost of capital without the capital

* Goldfarb (2010) presents alternative approaches that account for this; see p. 41

Mortgage APR

- Now for something more familiar...
- In a typical mortgage, for the bank/lender:
 - The *reward* is the interest payments received
 - The (debt) capital at *risk* is the outstanding loan amount
 - The principal repaid is capital returned



Mortgage APR assessment

- How does it compare to insurance? In general – quite well

	Mortgage	Insurance
Capital type	Debt (loan)	Economic, GAAP, etc.
Capital recipient	Home owner	Underwriter
Investment	Houses	Insurance contracts
Profit	Interest on loan	UW profit
Investment horizon	Term of mortgage	Time claims are open
Maximum loss	Outstanding balance	Capital held at start of year
Annual rate of return	APR	Annual <i>rate</i> of ROC



Mortgage APR

- Mortgage analogy for RAROC:
 - The initial loan amount and total interest are known, but not the repayment schedule
 - (A bad idea)
- We can build on these comparisons to define an annual rate of ROC for an LOB...

*So many
different
investment
opportunities...
how can I
compare them?*



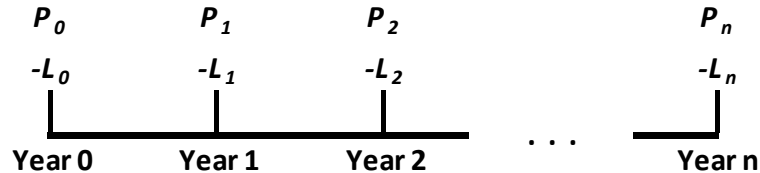
Four principles for R , the rate of ROC

1. **Capital must be held as long as claims are open** (*investment horizon*)
2. The total capital requirement varies by year according to the downside risk of open claims (*amount of total capital*)
3. The amount of capital allocated to a LOB depends on its contribution to the total capital requirement (*amount of allocated capital*)
4. **R is the effective annual rate of return paid to the capital providers over the period that their capital is invested** (*normalizes for the amount of capital and the investment horizon*)

The UW account

Underwriting Account

Receives **premium** and pays **loss and expense**



Earns the risk free rate r



A cheque is written for the balance at n years

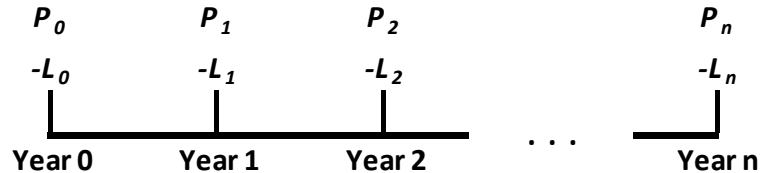
$$\sum_{t=0}^n (P_t - L_t) * (1 + r)^{n-t}$$



Two accounts, one value

Underwriting Account

Receives **premium** and pays **loss and expense**



Earn the risk free rate r



Cheques of equal amount are written for the balances at n years

$$\sum_{t=0}^n (P_t - L_t) * (1 + r)^{n-t}$$

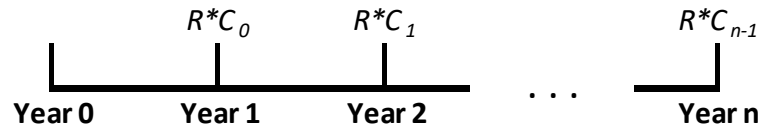


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Dividend Account

Receives $R * (\text{Capital held @ BOY})$ at year-end



$$\sum_{t=0}^{n-1} R * C_t * (1 + r)^{n-t-1}$$

Solving for R

- R is the value that makes the two cheques equal
- Dividing by $(1+r)^n$ to get present values:

$$R * \sum_{t=0}^{n-1} C_t * (1+r)^{-t-1} = NPV(UW Profit)$$

- Simple! Looks just like the S2 risk margin formula
 - $R \leftrightarrow$ CoC (6%)
 - $NPV(UW Profit) \leftrightarrow$ Risk Margin

Solving for C_t is the central challenge

- Except now:
 - We are solving for R
 - C_t is for a LOB
- How do we solve for C_t for each LOB?
- We can't just run-off C_0 as we might for the SCR
 - The size of the total portfolio capital pie changes, AND
 - The relative share of each LOB changes

Running off C_0 doesn't always work

- A hypothetical two-line portfolio example:
 - Large Property LOB, paid out after 5 years
 - Small Casualty LOB, paid out after 10 years
 - The Casualty LOB could have small/nil/negative allocated capital @ $T=0$
 - Not so for years 6+
- Allocation applies *at a point in time*



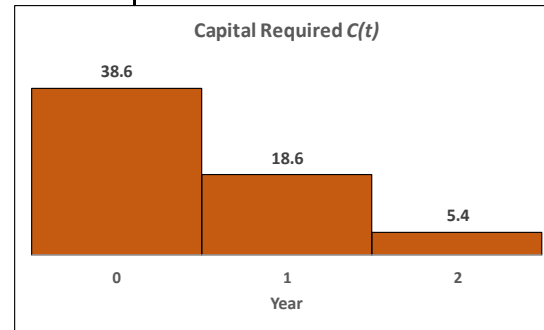
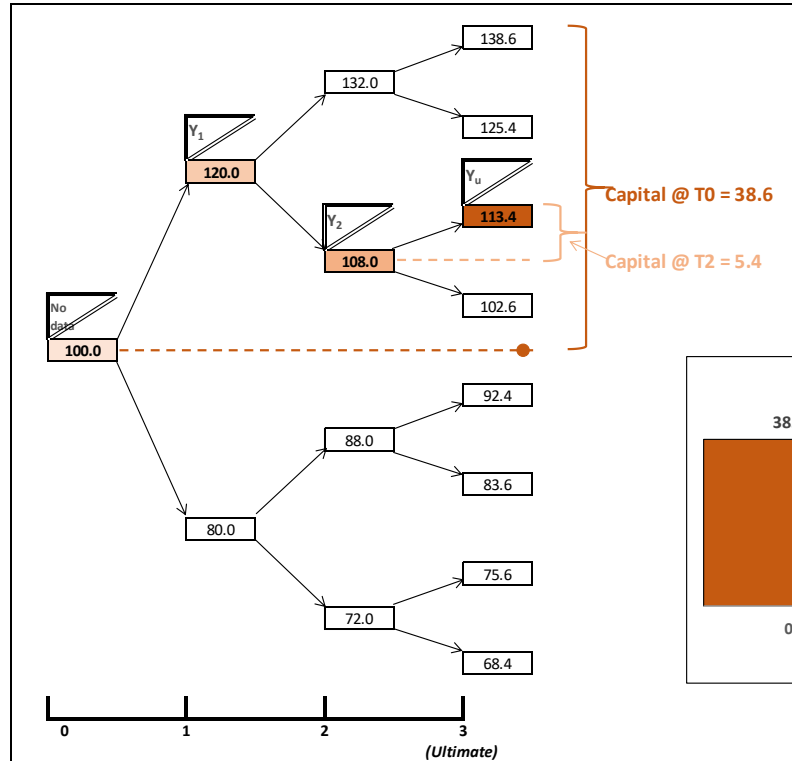
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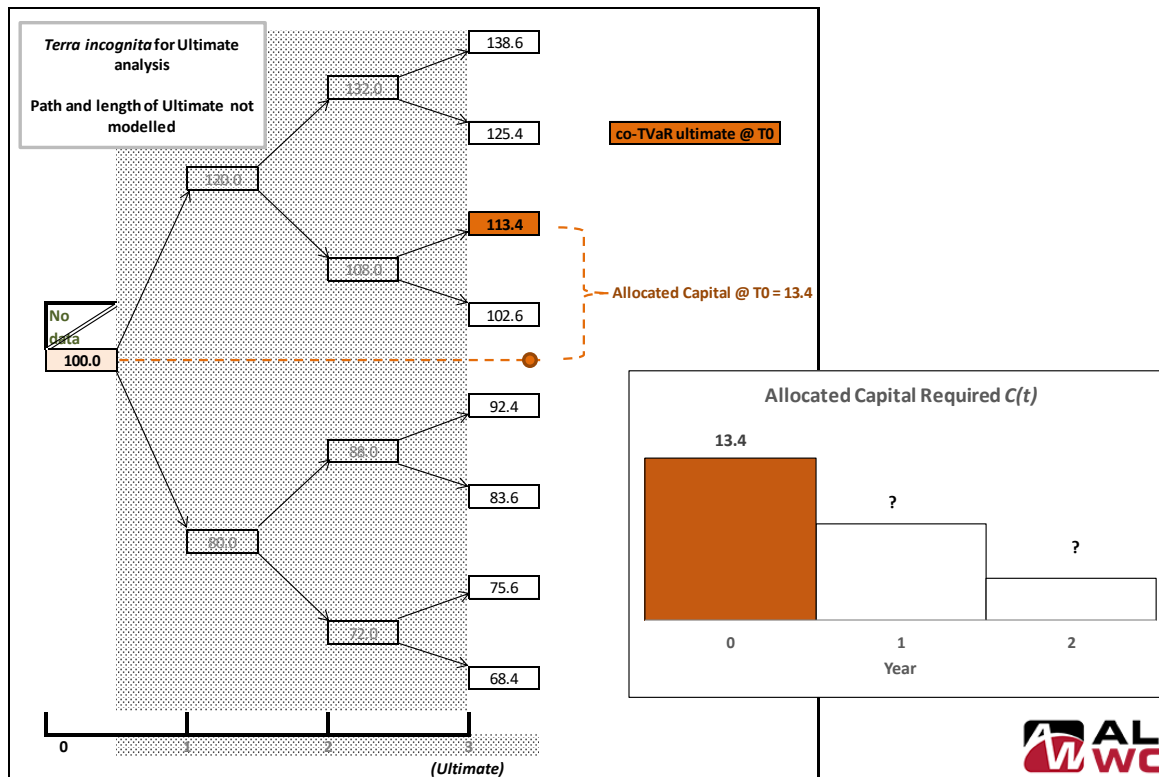
II. How do we get there?

A practical methodology

Path of the projected ultimate



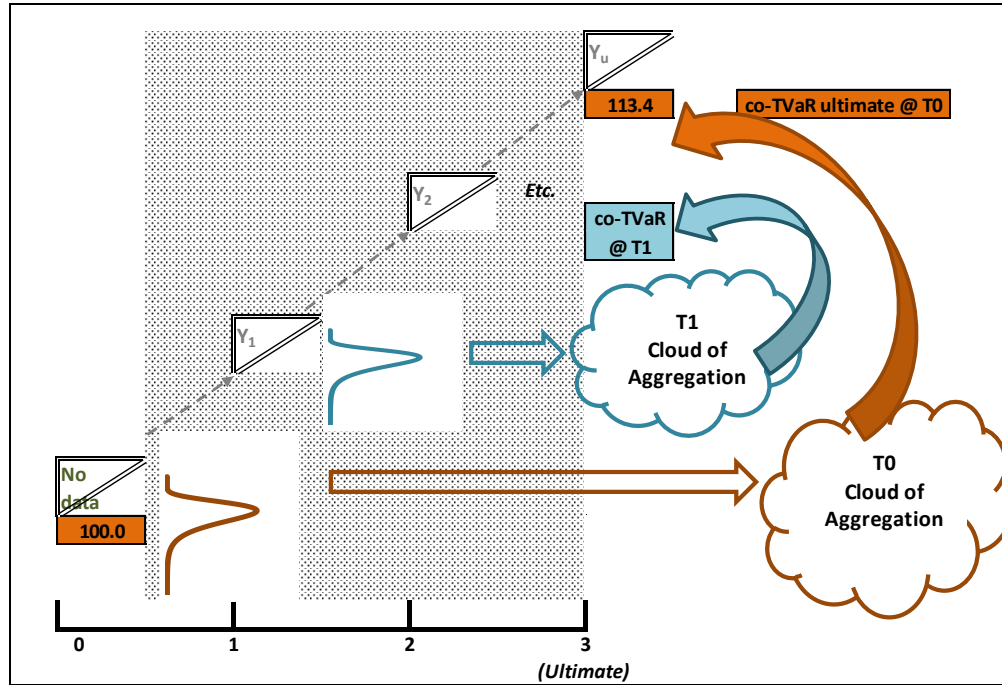
LOB modelled ultimates @ T=0



The algorithm

- **Question:** What capital would we expect to allocate at the start of each year if the true final ultimate is the co-TVaR ultimate @ $T=0$?
- **Answer:** At $T = 1, 2, \dots$, for each LOB:
 1. Recognize a share of the deterioration from best estimate to co-TVaR
 2. Parameterize a lognormal using a CoV and the updated best estimate from step 1
 3. Simulate an ultimate for each LOB and in total
 4. Re-allocate capital to each LOB

Proposal for re-allocating LOB capital



- Claims experience converges to co-TVaR ultimate modelled @ $T=0$
- At each time step:
 1. A distribution is parameterized
 2. Claims are simulated by LOB and aggregated
 3. Capital is re-allocated

A simplified LOB example

Model outputs @ T0

Premium	110.0		Risk-free rate 0.0%
Expected Loss & ALAE	100.0	(A)	
Expected UW Profit	10.0	(B)	
TVaR 99p	180.0		
Co-TVaR 99p	125.0	(C)	
Co-TVaR 99p Stress	25.0	(D) = (C) - (A)	

The Cloud of Aggregation
(all LOB)

Dependency Matrix				
Sim #	LOB1	LOB2	...	LOB n
1	0.763	0.020		0.171
2	0.726	0.827		0.124
3	0.969	0.550		0.467
⋮				
10000	0.128	0.579		0.845

The Thunder of Allocation

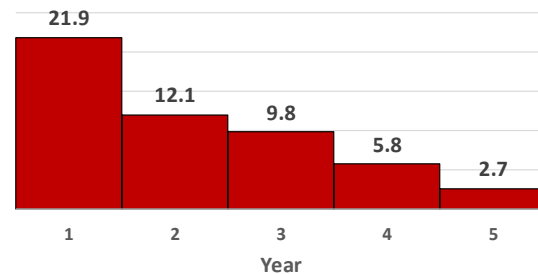
Year	Paid as % of Ultimate	% Recognition of co-TVaR Stress	Opening Liability (BE)	Stress Recognition = (2)*(D)	Paid in Year	Closing Liability (BE) = (3)+(4)-(5)	COV (to Ultimate)	co-TVaR (of Opening Liability)	RiM @ 6%	Fair Value Premium	C _i Allocated Ultimate Capital
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10) = (3)+(9)	(11) = (8)-(10)
1	10%	0%	100.0	0.0	10.0	90.0		125.0	3.1	103.1	21.9
2	25%	35%	90.0	8.8	28.1	70.7	15%	103.9	1.8	91.8	12.1
3	30%	30%	70.7	7.5	37.5	40.7	15%	81.6	1.1	71.8	9.8
4	20%	25%	40.7	6.3	28.6	18.4	15%	47.0	0.5	41.2	5.8
5	15%	10%	18.4	2.5	20.9	0.0	15%	21.2	0.2	18.5	2.7
TOTAL	100%	100%		25.0	125.0						

$$R * \sum_{t=0}^{n-1} C_t * (1 + r)^{-t-1} = NPV (UW Profit)$$

$$R * (21.9 + 12.1 + \dots + 2.7) = 10$$

$$R = 19.2\%$$

Allocated Capital C_i by Year





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III. How will we know when we've arrived?

Validating and interpreting results

“Validation” of R

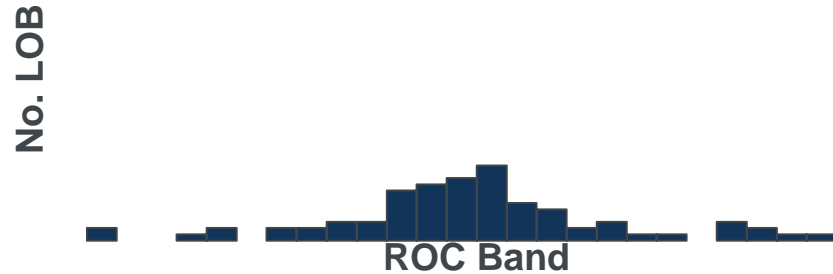
- R for individual LOB can't be validated
 - R depends on the portfolio as a whole and the methodology
- But we can have expectations for the portfolio:
 - Extreme outliers (high or low) not realistic
 - Reason: *Competition/UW discipline place upper/lower bounds*
 - Similar LOB should have a similar ROC
 - Reason: *Comparable risk vs. reward*
 - No trend in ROC vs. premium
 - *But more variability expected for smaller LOB*
- A good methodology will satisfy these criteria



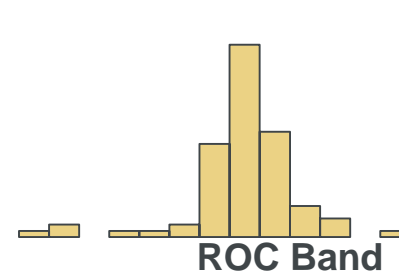
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Test 1: Fewer extreme outliers

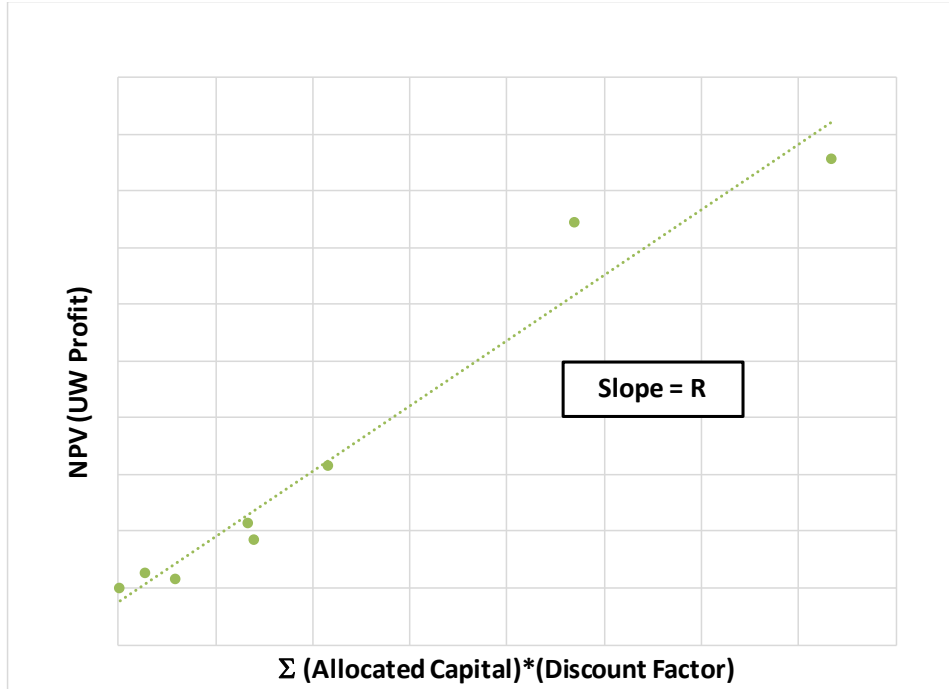
ROC Distribution
No Re-allocation



ROC Distribution
With Re-allocation

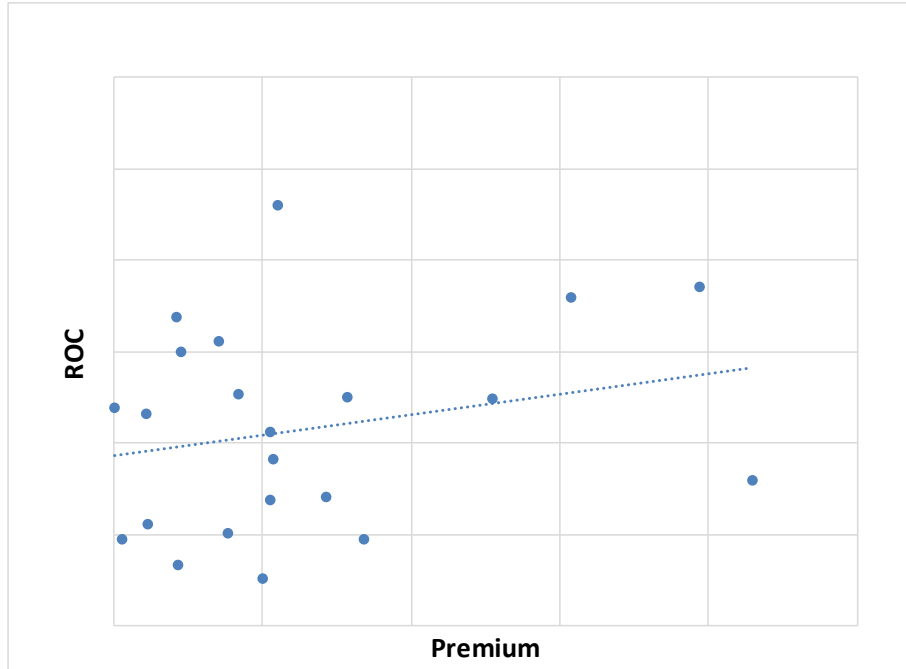


Test 2: Efficient frontier



- A fitted line should start at the origin (no risk, no reward)
- Result depends on portfolio *and* ROC methodology

Test 3: No trend in R vs. premium



- Small (or large) LOB shouldn't have inherently higher/lower ROC
- But it is reasonable to expect more volatility for smaller LOB



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IV. What are the limitations of the approach?

Hazards, pitfalls and considerations

ROC pitfalls & considerations

- ...of which there are many
- General
 - Capital is not legally divisible and the allocation method is a choice
 - Many assumptions are arbitrary and can yield very different ROCs
- Specific to this method (to name a few)
 - How to align reserve risk volatilities (low granularity) with UW LOB (high granularity)
 - The capital release pattern requires many simulations
 - Explaining to key stakeholders

Take away

- The first step is always the most important: ***choose the right metric***
 - The ROC should be a ***rate*** of return
- Not all methodologies are equal – develop ***a validation toolkit*** for separating the good, the bad, and the ugly
- ***Communication*** to stakeholders is key – buy-in to the results depends on buy-in to the method

Questions



How long is
FOREVER?

Sometimes,
just 1 second!



Now I know my
invested capital for
each year, from now
until ultimate...*until*
FOREVER!

Questions

Comments

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