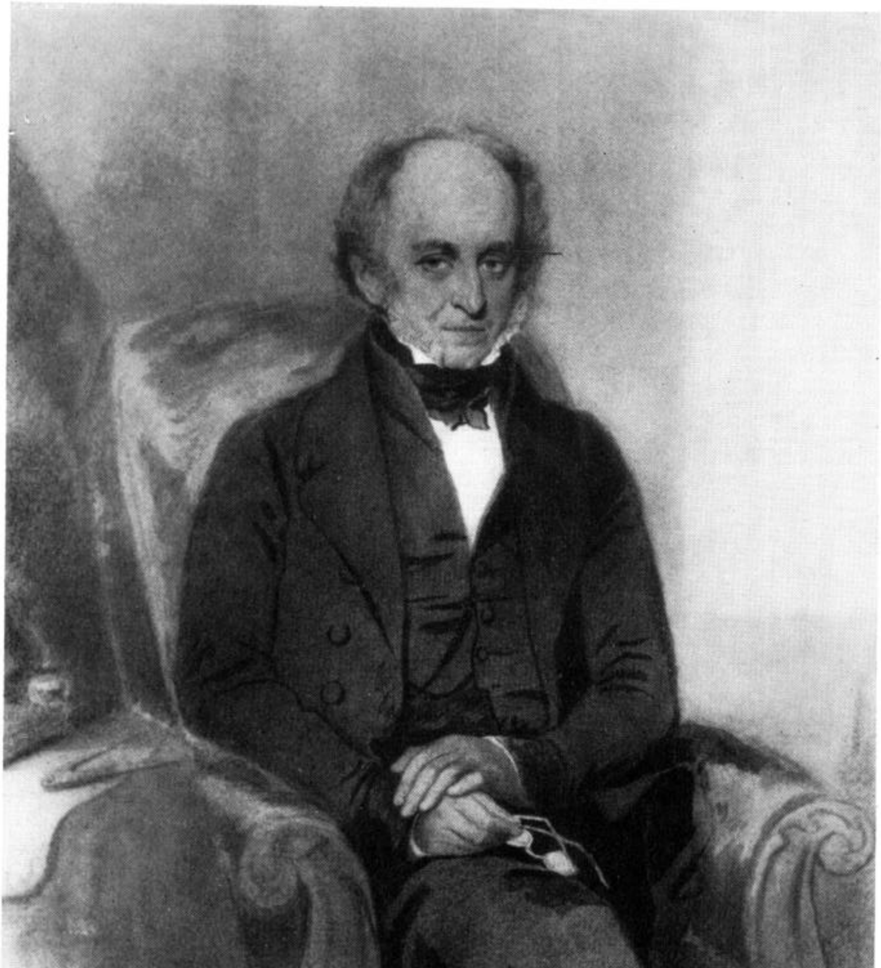

BENJAMIN GOMPERTZ, 1779–1865

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BENJAMIN GOMPERTZ

5 March 1779–14 July 1865

To mathematicians, the death of Benjamin Gompertz 100 years ago represented the passing of an era. Gompertz has been described [24] as 'the last of the learned Newtonians'; out of respect for the memory of Newton he continued to use the old language of fluxions until his death, by which time it had been obsolete in the English mathematical world for nearly half a century. To the actuarial profession, however, Gompertz's paper of 1825, in which he propounded his well-known 'law' of mortality, marked the beginning of a new era, not merely because his formula was, for several reasons, an enormous improvement on others which had been suggested previously (see *Assce. Mag. and J.I.A.* **13**, 14) but because it opened up a new approach to the life table. Previously, the table had been regarded as little more than a record of the number of persons surviving to successive integral ages out of a given number alive at an earlier age; Gompertz introduced the idea that l_x was a function connected by a mathematical relationship with a continuously operating force of mortality.

A memoir of Gompertz was written shortly after his death by his friend M. N. Adler.[23] The author of the present note owes much to Adler's memoir and to the other sources in the appended list.

Benjamin Gompertz, a member of a distinguished Jewish family, was born on 5 March 1779 in London, where his father and grandfather had been successful diamond merchants. His mother was Dutch by birth; her grandfather was the famous Jonas Cohen of Amersfoort, who befriended William, Prince of Orange, during a revolution in the eighteenth century.

Benjamin and his four brothers showed little taste for a commercial life, but each of them excelled in his own way at literary or academic pursuits. Barnet, the eldest brother, was an eminent amateur musician, Isaac was a poet, and Ephraim a mathematician. The youngest brother, Lewis, was the main founder of the Society for the Prevention of Cruelty to Animals (now the R.S.P.C.A.), and maintained that it was wrong to turn any animal to a use that was not beneficial to the animal itself; he was also an inventor and the author of several contributions to mechanical science.

Benjamin himself displayed brilliance from his boyhood. Being debarred by his religion from a college education, he studied without guidance and at an early age was familiar with the writings of the English and French mathematicians of the eighteenth century; Newton, Maclaurin and Emerson were his favourite authors. It is said that so great was his thirst for knowledge while he was a boy that frequently, when his parents had removed all candles to prevent him from injuring his health by studying too late at night, he stole out into the garden and pursued his investigations by moonlight.

As early as 1798, Gompertz took a prominent part among the mathematicians who proposed and answered the ingenious questions contained in the *Gentleman's Mathematical Companion*, a publication to which eminent men of science contributed. From 1812 to 1822 he distinguished himself by winning every year, without exception, the annual prize offered by that magazine for the best solution to a prize problem.

His first paper of importance was submitted to the Royal Society in 1806. It described the application of a method of differences to a species of series whose sums had been obtained 'by Mr Landen by the help of impossible quantities'.

He next turned his attention to the theory of imaginary quantities. He would have liked the Royal Society to publish the results of his work on this subject but his paper was rejected by the Society, apparently on the basis that it was too profound and that no one would understand it. Undeterred by this, Gompertz arranged for two tracts to be privately printed in 1817 and 1818. (A sequel was printed in 1850.)

These two tracts appear to have established his reputation as a brilliant mathematician and in 1819 he was elected a Fellow of the Royal Society. Later, in 1832, he became a member of the Council.

In 1820 the formation of the Astronomical Society (now the Royal Astronomical Society) opened to Gompertz a fresh field of activity. He was not strictly a founder of the Society but was one of its warmest and most active supporters from its foundation. He was elected a member of the Council in 1821 and for about 10 years he actively participated in the work of the Society, contributing valuable papers on the theory of astronomical instruments, the aberration of light, the differential sextant (a device invented by himself) and the convertible pendulum. He also supplied explanatory notes to papers submitted to the Society by other authors.

It is characteristic of Gompertz that, although he enriched the *Memoirs* of the Astronomical Society, he never became a practical astronomer and did not habitually use the instruments with whose construction and limitations he was so familiar. It must not be assumed from this that his work was entirely of a theoretical rather than a practical nature. On the contrary, many of his papers were intensely practical and some of them were supported by a great deal of arithmetical work. However, he does seem to have derived more pleasure from the study of the methods which he advocated than from their practical application. The following extract from one of his papers(11) may perhaps help to explain his attitude:

> In the contemplation of the sciences there is, besides the pleasure arising from the acquirement of knowledge of practical utility, a peculiar charm bestowed by the reasoning faculty in a well-directed pursuit of facts; and though the results shown by the arguments are frequently considered to be the only objects of value by the unlearned, the man of absolute scientific ardour will often, whilst he is enraptured with the argument, have not the least interest for the object for which his argument was instituted.

However, in thus professing indifference towards the objects of his research, he was hardly doing himself justice. A more pleasing explanation is that Gompertz, being a very modest and kindly man who was always ready to acknowledge the help which he had received from studying the writings of his predecessors, was also keen in his turn to facilitate the work of those who were to follow him.

A very practical investigation which Gompertz commenced in 1822 was undertaken jointly with Francis Baily; it was the reduction of apparent to mean places of the fixed stars. To quote the words of Sir John F. W. Herschel, in his memoir of Baily in the *Monthly Notices of the Royal Astronomical Society* for 8 November, 1844,

> It seems almost astonishing that these computations, which lie at the root of all astronomy, and without which no result can be arrived at, and no practical observer can advance a single step, should have remained up to so late a period as the twentieth year of the nineteenth century in the loose, irregular and troublesome state which was actually the case.... Each of the uranographical corrections had to be separately computed by its own peculiar tables, and with coefficients on whose magnitude no two astronomers agreed ... the calculations were formidable and onerous in the extreme to private astronomers....

Realizing the need, Baily and Gompertz proceeded to meet it. They investigated the subject generally and devised a method of making the corrections for aberration and solar and lunar nutations. They had already completed some of the tables when they learned that Bessel had been working on similar lines but had allowed for corrections which they had not included, so they willingly gave way. However, their work was not by any means wasted. The complete catalogue of stars of the Royal Astronomical Society is partly the fruit of the labours of these two men.

Although the last published paper by Gompertz on astronomy was produced in 1829, he maintained an interest in the subject until his death, studying other people's papers and investigating meteors, shooting stars, comets, etc.

It was as an actuary, however, that Gompertz's most lasting work was performed. His two famous papers on the subject of life contingencies were submitted to the Royal Society in 1820 and 1825.

In the 1820 paper he set out a notation which appears rather clumsy to those of us who are familiar with the modern International Actuarial Notation, but which no doubt served Gompertz's purpose very well. He then applied the method of fluxions to investigate various contingencies involving two or more lives and suggested that it could be assumed that l_x decreases either in arithmetical or in geometric progression over successive periods of years. He was careful to explain that he was not suggesting that either of these assumptions was accurate, but he pointed out that by making the periods sufficiently short any desired degree of accuracy in the arithmetical result could be obtained.

One problem which he investigated in this paper may be expressed as follows: 'If two lives who were aged a and b n years ago are both dead, to what formula must l_x conform in order to justify the statement that it is equally likely that (a) or (b) was the first to die?' Gompertz produced three answers to this problem, the second and third being special cases of the first:

(i) $1-l_{a+t}/l_a = k(1-l_{b+t}/l_b)$
(ii) l_x decreases in arithmetical progression (not necessarily the same progression for the two lives)
(iii) $l_x - l_{x+1}$ proceeds in geometric progression.

His reason for investigating the problem was that certain other writers, in their efforts to simplify complicated survivorship problems, had assumed that both orders of death were equally likely. He pointed out, as W. Morgan and Baily had done before him (see *The Doctrine of Life Annuities and Assurances*, F. Baily, 1813, **1**, 123), that this assumption might lead to error if n was large.

Any modern student of life contingencies could easily reproduce Gompertz's three solutions to this problem by means of the relationship

$$_nq^2_{xy} = \int_0^n (1-{}_tp_y)\,{}_tp_x\,\mu_{x+t}\,dt.$$

That Gompertz succeeded in obtaining the solutions by the awkward method of fluxions, not only without using any symbol for the force of mortality but without even introducing the concept of a force of mortality into his reasoning, is greatly to his credit.

In the 1825 paper, Gompertz introduced his now famous 'law' of mortality with the following words: 'It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction.' He went on to point out that, if only the first of these two causes operated, the force (or, as he called it, the intensity) of mortality would be constant and the number of lives surviving from a certain number living at a given earlier age would decrease in geometric progression. If only the second cause operated, and if the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then the force (intensity) of mortality at age x might be denoted by Bc^x (in modern notation).

Gompertz then proceeded to test several mortality tables which were in use at the time and to show that they followed his 'law' approximately over a limited range of ages such as 10 to 50 or 15 to 55. He apparently did not wish to claim more for his 'law' than this. In fact he seems to have attached little importance to it, for the remainder of his paper consists of an exposition (with extensive tables appended) of a method of approximating to the values of functions based on the assumption that, over sufficiently short periods, l_x decreases in geometric progression, i.e.

$$\mu_x = A$$

To anyone reading Gompertz's paper now, it seems strange that his idea of two 'generally co-existing' causes of death did not lead him immediately to Makeham's modification

$$\mu_x = A + Bc^x$$

As Makeham himself points out (*J.I.A.* **28**, 154) Gompertz's train of thought here is not easy to follow. However, it should be borne in mind that in 1825 actuarial science was very much in its infancy and that Gompertz was groping his way towards entirely new concepts. Ideas which seem elementary and obvious to us represented, in his time, a distinct advance in thought. The methods set out in his 1820 paper were a considerable improvement on those of W. Morgan (in the *Philosophical Transactions*) and Baily (in *The Doctrine of Life Annuities and Assurances*) and in his 1825 paper he carried his ideas still further.

It is unfortunate that Gompertz, like many other brilliant men, does not seem to have possessed the gift of being able to explain his ideas in such a way as to enable lesser minds to comprehend them easily. The failure of his 1820 and 1825 papers to receive immediately the wide recognition which they deserved is probably due partly to this fact, partly to his use of the method of fluxions and partly to the numerous errata which occurred in the two papers. However, it is pleasant to be able to record that several eminent mathematicians and actuaries were quick to appreciate Gompertz's genius and very ready to speak up on his behalf when the opportunity offered. For example, Professor A. De Morgan, who was a staunch supporter of Gompertz on more than one occasion, said (37) of his 1825 paper that 'this ingenious paper . . . must always be considered a very remarkable page in the history of the enquiry before us', and Sir John F. W. Herschel, in an unpublished letter* dated 1 June 1823 said 'In your memoir on the value of life contingencies in the *Philosophical Transactions* for 1820 it is easy to trace the hand of a perfect master of his subject, and one familiar with the most difficult parts of its theory'. Others who set a high value on his work were T. B. Sprague and W. S. B. Woolhouse (see below).

Gompertz was a man of such varied activities that it is not practicable to adhere to a strict chronological order in dealing with all his achievements. However, perhaps we might now refer to his business life. At the age of 30, he became a member of the Stock Exchange, without relinquishing his mathematical pursuits.

* The letter from which the quotation is taken is in the archives of the Royal Society and is reproduced with the Society's kind permission.

In 1824, as a result of the initiative of Nathan M. Rothschild and Sir Moses Montefiore, the Alliance British and Foreign Life and Fire Assurance Company was founded. Gompertz, who had married Montefiore's sister Abigail in 1810, was appointed the first actuary and head officer under the deed of settlement. According to one colourful story,[39] Nathan Rothschild, learning with indignation that an application by Gompertz for the vacant actuaryship of a large insurance company had been rejected because of his religion, decided on the spur of the moment to form a new insurance company with a larger share capital and a more influential board of directors than any that had yet been established, so that Gompertz could be its first actuary. A more prosaic explanation of the circumstances leading to the foundation of the Alliance is given by Sir William Schooling,[41] who suggests that the founders may have been influenced in their decision to form a new company by the consideration that there were available, for their guidance in all matters connected with life assurance, the services of Gompertz, who was one of the best mathematicians of the day and a recognized authority on questions of mortality and probability.

Immediately after the formation of the Alliance as a life and fire company, the directors applied to Parliament for a repeal of the Act of 1720, whereby only two companies (the London Assurance and the Royal Exchange Assurance) were permitted to transact marine insurance. The necessary Act was passed on 24 June 1824 and, as the original prospectus of the Alliance did not permit it to transact marine insurance, a separate company, the Alliance Marine Insurance Company, was formed; Gompertz was appointed chief manager.

The Alliance progressed steadily under the guidance of Gompertz, who was more concerned to use a sound basis for calculating premiums than to secure a large volume of new business. He was quick to realize that the Carlisle Table was a suitable one for calculating life assurance premiums, since it appeared to over-estimate slightly the mortality that was likely to be experienced.

As early as 1820 (in the preface to his paper to the Royal Society) Gompertz had criticized the prevalent practice among life assurance societies of using mortality tables that were known to be unsuitable in the hope that they were favourable to the society. He recommended that the actuaries of the different societies should combine to collect their mortality experience for the good of all. It was not until 1838 that this wish was fulfilled and a committee, of which he was a member, was formed to collect the data of seventeen offices 'to afford the means of determining the law of mortality which prevails among assured lives'.

Until 1847, when age and ill-health made it necessary for him to ask the directors of the Alliance to allow him to retire from the position of actuary, he led a very active life. He was frequently consulted by Friendly Societies and other institutions and by the Army Medical Board. He was also an authority on reversions and was actuary of the National Reversionary Investment Company from its formation in 1837 until his retirement.

He was a member of several learned societies in addition to the Royal Society and the Royal Astronomical Society, already mentioned. The first one which he joined was the Society of Mathematicians of Spitalfields, which had been formed in 1717; he was an active member of this society and served as its President for a time. It was absorbed by the Royal Astronomical Society in 1846. He took part in the formation of the Royal Statistical Society and was one of the promoters of the Society for the Diffusion of Useful Knowledge, on the committee of which he served from 1828. On several occasions he was a steward of the Royal Literary Fund. He was an original member of the London Mathematical Society, which was formed in January 1865, and he was actually preparing a paper for this Society at the time of his death on 14 July 1865.

He was a prominent member of some of the leading Jewish charities, and took an active part in their work, in addition to contributing to their funds on a very generous scale.

He was an honorary member of the Institute of Actuaries; his name is preserved in the records(42) as a result of a characteristic action of his in 1857, when he sent a cheque for £10 for the purpose of increasing the Library.

Although his health continued to deteriorate after his retirement in 1847, his mind remained active and in 1860 he contributed a paper to the International Statistical Congress. In this paper he suggested modifications to his 'law' of mortality which would make it applicable over the whole period of life from birth to old age. Another paper on the same subject was presented by him to the Royal Society in 1862, in the form of a supplement to his previous papers of 1820 and 1825. He later prepared a second supplement, which was placed before the Royal Society on 12 May 1864. This supplement contained some suggestions on Barrett's commutation tables and professed to give a method of controlling competition among Assurance Offices. Owing to his failing health, he was unable to complete the paper and it now consists only of a summary of about fifteen lines in the *Proceedings of the Royal Society* and some manuscript pages in the Institute Library.

This account of Gompertz's life would unfortunately not be complete without mention of the rather abusive correspondence in *Assce. Mag. and J.I.A.* **9** and **10**. In 1832, T. R. Edmonds had published a work in which he claimed not only that he had discovered a law of mortality but that his discovery was 'independent of the imperfect one of Mr Gompertz'. This claim to originality was disputed by Prof. De Morgan in his article on Mortality in the *Penny Cyclopaedia* and subsequently in the *Journal*. De Morgan went so far as to accuse Edmonds of having unfairly adopted Gompertz's ideas without anything approaching a sufficient acknowledgment. Edmonds replied by an article in which he again sought to establish his claims. This was followed by another communication from De Morgan, and by one from T. B. Sprague in which he said that De Morgan's charge was completely substantiated and accused Edmonds of bad taste. Edmonds promptly returned to the attack and, instead of naming Sprague, referred to him as 'the new advocate of Mr Gompertz'. Sprague retorted with a twelve-page letter in which he refrained from naming Edmonds and repeatedly called him 'the plagiarist of Mr Gompertz'. This provoked another long letter from Edmonds prolonging the argument. The controversy was closed by W. S. B. Woolhouse, who described Gompertz as 'one of the greatest mathematicians of Europe' and said that he possessed 'the conscientiousness of a great mind'. Gompertz's own contribution to this discussion consists of one short letter which is notable for its restraint and is in marked contrast to the lengthy and acrimonious writings of Edmonds.

If one looks for a moral to this unhappy episode, it is surely to be found in the fact that, whereas Edmonds is now remembered only for his disparagement of the work of a man of genius and for his persistent efforts to convince others that his own achievement was more praiseworthy, Gompertz's name will be known by future generations of actuaries not only because it cannot be omitted from any text-book on life contingencies but because his outstanding brilliance as a mathematician was equalled by his modesty and generosity. It is a pity that Edmonds did not take to heart some earlier words of Gompertz:[2]

> To a true philosopher, it will ever be much more pleasing to grant even more praise than is actually due than to pluck the laurel from the deserving brow.

P. F. HOOKER

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(2) 1820, **110**, 214. A sketch of an analysis and notation applicable to the value of life contingencies.

(3) 1825, **115**, 513. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies.

(4) 1862, **152**, 511. A supplement to the two papers of 1820 and 1825.

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(5) 1864, **13**, 228. Extract from a second supplement to the two papers of 1820 and 1825.

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(8) 1850. Hints on porisms, in a letter to T. S. Davies, F.R.S., F.S.A., with a scholium not contained in the letter, being a sequel to the two tracts on imaginary quantities.

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(10) 1822, **1**, 275. Note on Prof. Littrow's paper on the correction of the transit instrument.

(11) 1822, **1**, 283. On the aberration of light.

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(19) 1853, **4**, 245. Letter on the interest question.

(20) 1861, **9**, 296. Letter on his papers.

(21) **16**, 329. Reprint of his paper to the *International Statistical Congress* (see next item).

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(22) On one uniform law of human mortality from birth to extreme old age, and on the law of sickness.

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