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Game Theory in General Insurance
How to outdo your adversaries while they are trying to outdo you

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- **Abstract:**

  This paper explores the application of some of the existing game theory to the problem of strategically managing non-life insurance products where current methods of setting price do not typically consider reactions of competitors. We explore the standard Bertrand and Hotelling models and based on these develop a model applicable for the insurance market. Using these models, we show what the effect of the impending EU Gender Directive will have on the prices set by insurers and also the likely impact of potential regulation of the way in which ancillary products are sold. Through some examples we show how insurers can use commitments, promises and threats as strategic moves to manipulate the rules to their own individual advantage. This shows that sometimes a firm is better off as being one of the small guys.
- **Keywords**. Game theory, strategic modeling, general insurance, pricing, Bertrand model, Hotelling model, competition, EU Gender directive, Chinese drywall,

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By

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1. INTRODUCTION

"Strategic thinking is the art of outdoing an adversary knowing that the adversary is trying to outdo you."
- Dixit & Nalebuff: Thinking Strategically, 1991

How firms have gone about setting the price to charge for underwriting non-life insurance risk has evolved significantly since the dawn of the industry. A world of "big data", where there is more and more information both of customers being insured and the perils being underwritten, together with greater computational capabilities and bright actuaries and statisticians chasing the application of increasingly-complex statistical models, has led to smaller and smaller insurance risk groups and ever-increasing degrees of differentiation of price. However, driven by the belief that better estimates of the marginal cost of a policy will enable a firm to attract currently overcharged policies and avoid undercharged ones, the focus of these models has been to understand the expected cost of providing the cover (product) rather than what is the strategically optimal price to quote. Admittedly, some modern price optimization methods do attempt to determine the price which mathematically optimizes the profit of the portfolio given models of costs and customers' price elasticity, but these methods mostly assume a static market where competitors keep their prices unchanged.

This behavior of not considering what options or choices competitors are making is arguably naïve. While in some cases such an approach may work out, it could lead to dismal failure. As actuaries we recognize the value of information, so by throwing away valuable strategic information in this way, it seems likely that the decision will be suboptimal and poorer outcomes will often be generated.

Game theory can help insurers with including this valuable information when making decisions. The Oxford English Dictionary defines game theory as:

> *[mass noun]*the branch of mathematics concerned with the analysis of strategies for dealing with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants. Game theory has been applied to contexts in war, business, and biology.

What is important is the notion of strategic interdependence. That is, the payoff to a firm of a choice it makes will depend on which choices are open to its competitors and how they respond. It then becomes impossible to know for sure what is the payoff of any action that is taken, even if the firm has full information about its customers and its costs.

1.1 Research Context

Von Neumann and Morgenstern (1944) is the classic work in game theory. Luce and Raiffa (1957) and Schelling (1960) are early textbooks covering some basic concepts and main ideas. Dixit, Skeath, and Reiley (2009) provides an introduction to the field of game thery with a variety of illustrative cases. Osborne and Rubinstein (1994) presents the main ideas of game theory from a mathematical perspective with full proofs of results.

A 2002 GIRO paper by Andrew Smith (2002) sets out an introduction to the concepts of game theory for general insurance actuaries and hints at where the theory might have some application. Among other examples, the paper introduces the model developed by Antoine Cournot (1838) to address the issue of competition between two suppliers to a market who need to make decisions on how much to produce.

A 2009 GIRO working party (Rothwell, et al., 2009) applied the **winner's curse** concept of auction theory to setting the price of an insurance policy. This suggests that firms bid to underwrite a customer's risk by quoting a premium. However, the true cost of that policy is unknown as it is not possible to predict precisely what will be the claims cost. In this event, insurers will determine the price they will bid by considering their expectation of what is the cost of claims. The winner's curse theory suggests that an insurer who bids the lowest price and wins the business is likely to have underestimated the cost and therefore is likely to be cursed by less profit than expected. Auction theory suggests that bidders should "shade" their bids to allow for the impact of winner's curse.

1.2 Objective

The winner's curse model assumes that insurers are identical, independent and compete for the same risks at the same level of profitability. Therefore the consumer has no preference for an insurer and will always pick the organization that provides the cheapest price. The price submitted by the insurer is based on their perception of the risk driven from their own data. This brings in uncertainty in that the premium will be based on certain distributional assumptions. The winner's curse and the related auction theory provides an interesting perspective for an insurance company, but is rather limited in practice. Firstly, it does not explicitly allow for any differentiation between products and therefore does not deal with the observation that some consumers will buy a product from one insurer while others will buy a product from another insurer in spite of marginal differences in the price of the two products. Also, the approach does not provide a robust framework under which an insurer can identify actions which would change the rules of the game to its own individual advantage.

Similarly, the Cournot competition model (Cournot, 1838) might provide a framework for exploring how different insurance firms may interact. However, in reality, most insurance companies execute their strategy by deciding what price to set, rather than deciding what quantity to produce, as assumed under the Cournot model.

This paper aims to provide an introduction to some elements of game theory relevant to a general insurance firm and to develop an initial framework which insurers can use to strategically manage their business in a competitive and non-cooperative environment. The paper will rely on the work undertaken by Bertrand (1883) and Hotelling (1929), which address the issue of price competition and differentiation between products.

What our model hopes to do is allow for the fact that insurance companies are not necessarily identical. Even though technology has changed the way that insurance is purchased, for example price comparison websites have led to commoditization and increased price transparency of motor insurance, there is still a difference in consumers' preferences for different products driven by brand, benefits and other factors. This creates opportunities for insurers to win profitable business in a highly competitive market. We look at examples, including under the impending change in gender discrimination law, of how an insurer can do just this and so outdo its adversaries.

1.3 Disclaimer

While this paper is the product of a GIRO working party, its findings do not represent the official view of the Institute and Faculty of Actuaries, or that of the employers of the working party's members. Moreover, while we believe the approaches we describe are very good examples of how to address the issue of applying game theory to general insurance business, we do not claim they are the only acceptable ones.

1.4 Outline

The remainder of the paper proceeds as follows. Section 2 will provide a recap of the economic basics of determining price. Section 3 will set out a brief background to some basic game theory concepts, including the Bertrand and Hotelling models, which are required for understanding the rest of the paper. Section 4 will explore the impending EU Gender Directive legislation and its impact on the insurance industry. We will take advantage of this example to show some actions players can take to change the game and gain some strategic advantage over competitors through strategic moves. Section 5 develops our own model, based on expanding the Bertrand and Hotelling models within the context of general insurance business. Section 6 applies the model to explore the interactions between brokers, insurers and direct insurance firms. Finally, section 7 draws out some of the key conclusions.

2. CLASSICAL ECONOMIC PRICING

This section is meant as an aide memoir of the basic economic theory underlying the determination of price.

2.1 Market Price and Natural Price, and Supply and Demand

Adam Smith, in *The Wealth of Nations*, described the fundamental difference between natural price and market price. **Natural price** is used in classical economics to refer to the value of a commodity affected by the various costs and incentives of producing the commodity. In other words, it is the costs of producing the commodity plus an allowance for the ordinary rate of profit. **Market price**, on the other hand, is the actual price at which the commodity is commonly sold. It may be either above, or below, or exactly the same with its natural price. (Smith A. , 1776)

Economic theory asserts that in a free market economy the market price reflects interaction between supply and demand. Price is interrelated with both of these measures. The relationship between price and demand is generally negative (the **law of demand**), meaning that the higher the price climbs, the lower amount of the supply is demanded. Conversely, the lower the price, the greater the supply is demanded. A **demand curve** is typically used to indicate the total quantity consumers want to buy at any given price, everything else being equal (ceteris paribus). Market price is just the price at which goods and services are sold. Adam Smith argued that the market price of a commodity would gravitate towards its natural price, since market prices above the natural price would attract additional suppliers to the market thus driving prices down, while market prices below the natural price would lead to some suppliers exiting the market. (Smith A. , 1776) A **supply curve** indicates the total quantity firms want to supply at any given price ceteris paribus. Therefore, the market price will settle at the natural price or **economic equilibrium price and quantity** at a point where the quantity demanded by consumers (at current price) will equal the quantity supplied by producers (at current price). In reality, the price may be distorted by other factors, such as tax and other government regulations.

2.2 How Firms Determine Price

A firm's pricing strategy will be set by considering the relation between supply and demand in the market. The firm will also need to understand how its price and quantity produced translate into costs and revenues, and hence profits.

Marginal revenue (MR) is the additional revenue from selling one additional unit of the product. It is related to **total revenue** (TR), **price** (p) and **quantity** (q) through the following formula:

$$MR = \frac{\partial TR}{\partial q} = \frac{\partial pq}{\partial q} = p \tag{2.1}$$

Total cost (C) is a function of quantity and is the sum of **fixed costs** (FC) and **variable costs** (VC) defined as:

$$C(q) = FC + VC(q) \tag{2.2}$$

Marginal cost (MC) is defined as the increase in costs from selling one additional unit of product and can be expressed as:

$$MC(q) = \frac{\partial C(q)}{\partial q} \tag{2.3}$$

Average cost (AC) is defined as:

$$AC(q) = \frac{C(q)}{q} \tag{2.4}$$

The quantities of supply and demand discussed earlier are determined by the marginal utility of the product to different buyers and to different sellers. Basic economic theory makes an argument for setting price at the point when marginal revenue equals marginal cost. The reasoning is as follows. Firstly, for each unit sold, **marginal profit** ($M\Pi$) equals marginal revenue minus marginal cost.

$$M\Pi = MR - MC \tag{2.5}$$

If marginal revenue is greater than marginal cost, marginal profit is positive, and if marginal revenue is less than marginal cost, marginal profit is negative. When marginal revenue equals marginal cost, marginal profit is zero. Since total profit increases when marginal profit is positive and total profit decreases when marginal profit is negative, it must reach a maximum where marginal profit is zero or where marginal cost equals marginal revenue. If there are two points where this occurs, maximum profit is achieved where the seller has collected positive profit up until the intersection of MR and MC but would make negative profit thereafter.

This means profit maximization occurs when

$$MR = MC \quad \text{and} \quad \frac{\partial MR}{\partial q} < \frac{\partial MC}{\partial q} \tag{2.6}$$

2.3 Market Structures

The structure of the market will determine the extent to which individual firms will be able to influence the price and their share of the market and total value produced.

Perfectly competitive markets are those where: (1) there are many, small and independent buyers and sellers; (2) there is a homogeneous product of known quality; (3) there is perfect information about the price; and (4) buyers and sellers are price takers. **Monopoly** is the case of a single seller that can adjust the supply and price at will. **Monopsony** is a market with a single buyer and many sellers. **Oligopoly** is a market with so few sellers that they must take account of their actions on the market price or each other. Game theory may be used to analyze such a market.

Most (deregulated) insurance markets can be considered as an oligopoly as there are relatively small numbers of firms who collectively influence the price and supply of insurance policies. In some cases, e.g. motor third party liability, insurance can be a compulsory purchase for specified consumers.

2.4 Business Decisions

Fixed costs can be ignored if they are sunk, which they are usually in the short term, but in longer term they may not be sunk. If a firm is not covering its fixed costs in the long term, i.e. $P < AC$, then it should shut down. If in the short term a firm is not covering its variable costs, i.e. $P < AVC$, then it should shut down.

2.5 Definition of Profit

A distinction should be made between accounting and economic profits. **Accounting profit** is the difference between the revenue received from the output and the costs of the inputs and producing the output. **Economic profit** is the difference between the revenue received from the sale of an output and the opportunity cost of the inputs used. This can be used as another name for "economic value added" (EVA).

Classic economic theory suggests that a firm can make positive economic profits in the short run if it can read the market. However, zero economic profits will be made in the long run, i.e. a firm will make as much profit as it would have made with other use of capital. If firms are more or less efficient due to different technologies, resources, or abilities, however, and these cannot be reproduced, some firms will earn positive economic profits.

3. GAME THEORY INTRODUCTION

This section is intended to set out some basics of game theory to a level sufficient for providing context for the remainder of the paper. Interested readers can find more detailed explorations of these and other topics in the various texts listed in the bibliography.

Game theory is the study of strategic decision making. More formally, Roger Myerson defined it as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another's welfare." (Myerson, 1991) Robert J. Aumman described it as "the interactive behavior of Homo Rationalis – rational man... [An] important function of game theory is the classification of interactive decision situations." (Aumann, 1987)

3.1 Ingredients of a Game Theory Problem

Players are those participating, including possibly nature. Players can make **moves** at certain times. There is certain **information** available to a player when a move is made. The **payoff** for each player is a function of all of the players' moves.

How a player uses any information at his disposal to select his moves determines his strategy for playing the game. A **strategy** is a complete plan of action for a player - it specifies a move for every feasible circumstance for that player or it can be an expectation (by others) of his behavior. A **strategy profile** collects together strategies for every player.

3.2 Rational Behavior

The models assume that each player is **rational** in the sense that the player is aware of their alternatives, forms expectations about any unknowns, has clear preferences and chooses their actions deliberately after some process of optimization. A model of rational choice contains the following:

i. A set of strategies from which the player makes a choice

ii. A set of consequences of these strategies

iii. A consequence function which associates a consequence with each strategy

iv. A payoff function or utility function on the set of consequences

Almost all game theory modeling decisions under uncertainty uses the theories of von Neumann & Morgenstern (1944) and of Savage (1972). That is, if the payoff function is stochastic then the player will behave as if he maximizes the expected value of a function (**von Neumann-Morgenstern utility**) that attaches a value to each outcome.

In practice, there may well be differences between individual players in their abilities or information at their disposal. For example, some players may have a greater ability to analyze a situation. Dealing with these are more advanced game theory, including models of **bounded rationality**, and are not covered in this paper.

3.3 Simultaneous-Move Games

Simultaneous-move games are those in which all players are required to make their moves at the same time and therefore no one will know what moves other players have made until all players have made their moves.

A simultaneous-move game consists of:

i. a finite set N of players

ii. for each player $i \in N$ a nonempty set S_i of strategic actions available to player i

iii. for each player $i \in N$ a consequence function $g_i : S \to C$

iv. for each player $i \in N$ a payoff function $h_i : C \to \mathbb{R}$

For example, we might want to model an oligopoly insurance market. In this case we would take the set of players to be the set of firms and the set of strategies to be the set of prices. Under the assumption that each firm cares about maximizing its profit, we would set the consequence function to determine the profit for each price. However, the profit is uncertain given the stochastic nature of the claims cost. We can then introduce a probability space Ω and adjust the consequence function to be $g_i : S \times \Omega \to C$ with the interpretation that $g_i(s, \omega)$ is the consequence when the strategic action is $s \in S$ and the realization of the random variable is $\omega \in \Omega$.

Games which are limited to two or three players can be conveniently represented by a **payoff matrix**. Figure (3.1) is an example of a two-player simultaneous-move game in which both players have two moves. One player's moves are identified by the rows and the other player's moves are identified by the columns. The two numbers in each cell show the payoffs of the row player and column player respectively. In figure (3.1), the strategies of the row player are $\{T, B\}$ and the strategies of the column player are $\{L, R\}$, and the row player's payoff from the outcome $\{T, L\}$ is w_1 and the column player's payoff is w_2.

	L	R
T	$w_1; w_2$	$x_1; x_2$
B	$y_1; y_2$	$z_1; z_2$

Figure 3.1 A payoff matrix of a two-player simultaneous-move game

One of the most well-known examples of such games is the **Prisoners' Dilemma** (Tucker, 1950), which is a case where each player has a personal incentive to do something that ultimately leads to a result that is bad for all players when everyone ultimately does what their personal interests dictate. The classic story involves two suspects who are arrested for a crime and interviewed separately. If they both keep quiet (i.e. they cooperate with each other) then they both go to prison for a year. If one suspect supplies incriminating evidence (i.e. defects) then that one is freed and the other one is imprisoned for nine years. If both defect then they are imprisoned for six years. Their preferences are solely contingent on any jail term they individually serve. The payoff matrix is set out in figure (3.2).

Another example is the **Chicken** or **Hawk-Dove** game. In this case two combatants engage in a contest. A combatant can either fight (play "Hawk") or can chicken out and defer to his opponent (play "Dove"). A Hawk beats a Dove, wins the prize, and there is no conflict. Two Doves share the prize equally. Two Hawks destroy the prize and are damaged fighting. The payoff matrix is set out in figure (3.3).

The **Battle of the sexes** game is one whose name derives from the sexist 1950's (Dixit, Skeath, & Reiley, Jnr, 2009). A husband and wife choose between going to a rugby match or a ballet. The husband prefers the rugby while the wife prefers the ballet, but both would prefer being with the other than going alone. The payoffs are shown in figure (3.4).

	Cooperate (don't confess)	*Defect (confess)*
Cooperate	-1; -1	-9; 0
Defect	0; -9	-6; -6

Figure 3.2 Prisoners' Dilemma

	Hawk	*Dove*
Hawk	-1; -1	9; 1
Dove	1; 9	3; 3

Figure 3.3 Chicken or Hawk-Dove

		Sally	
		Rugby	*Ballet*
Harry	*Rugby*	2; 1	0; 0
	Ballet	0; 0	1; 2

Figure 3.4 Battle of the sexes

	Rugby	*Pub*
Rugby	2; 2	0; 0
Pub	0; 0	1; 1

Figure 3.5 Assurance

Finally, an **assurance** game is where both players have similar preferred outcomes but this is only reached if each has enough certainty or assurance that the other is choosing the appropriate action. For example, in figure (3.5) there are two friends who would like to meet each other but have no means of communicating their actions – maybe while trying to agree to meet up, one of their phone's battery went flat mid-way during the call. Each would prefer to meet at the rugby match rather than the pub, and each knows that they share the same preferences. They were unable to decide definitely on the meeting place before communication was cut.

3.4 Solving Problems - Best Response and Nash Equilibrium

We can define a **best response function** of a player to be a profile of all strategies of that player that give the best payoff given the other players' strategic actions. If we denote the set of strategies for all players except i as S_{-i} then for any $s_{-i} \in S_{-i}$ the best response function $B_i(s_{-i})$ is the set of player i's best strategies given s_{-i}.

$$B_i(s_{-i}) = \{s_i \in S_i : h_i(s_{-i}, s_i) \geq h_i(s_{-i}, s_i') \text{ for all } s_i' \in S_i\} \tag{3.1}$$

A **dominant strategy** is a strategy that offers a higher payoff than any other strategy regardless of the choices made by the other players. A rational player will always play a dominant strategy if he has one. A **dominated strategy** is a strategy which yields a lower payoff than another strategy regardless of the choices made by the other players. An optimizing player would never play a dominated strategy. Hence an opponent anticipates it will not be played and so deletes it from consideration. Notice that this utilizes two assumptions: (1) the first player plays optimally, avoiding dominated strategies; and (2) the second player believes that the first is playing optimally. Further iterations require players to believe that others believe that they believe that others will play optimally and so on! However, we might expect dominated strategies, which are actions that can never be optimal to take, to be weeded out over time. Nevertheless, this does not always give a unique answer.

The most commonly used solution concept is that of a **Nash equilibrium**. A Nash equilibrium strategy profile consists only of best replies. Therefore, a Nash equilibrium strategy profile is a profile s^* of moves for which

$$s_i^* \in B_i(s_{-i}^*) \text{ for all } i \in N \tag{3.2}$$

For s^* to be a Nash equilibrium it must be that no player i has a strategy which would yield an outcome that he would prefer to that generated when he chooses s_i^* given that every other player j chooses their equilibrium action s_j^*. In other words, no player can deviate profitably, given the strategies of other players.

As an example, figure (3.2) shows the prisoner's dilemma. Each player has a dominant strategy of 'defect' because the player's payoff will be higher if they defect regardless of what the other player chooses. The Nast equilibrium is therefore the pair of choices {defect, defect}. In figure (3.3), neither player has a dominant strategy. If one player picks 'hawk' then the other player will be best to pick 'dove' and similarly if they had picked 'dove' then the other player will do best to pick 'hawk'. There are therefore two Nash equilibria at {hawk, dove} and {dove, hawk}. Similarly, there are two Nash equilibria in figure (3.5) at {rugby, rugby} and {pub, pub}.

3.5 Bertrand Price Competition

The Bertrand model is named after Joseph Louis François Bertrand (1822-1900). It describes interactions among firms (sellers) that set prices and their customers (buyers) that choose quantities at that price. Firms simultaneously choose prices. A firm's demand is decreasing in its own price, but increasing in the prices of its competitors.

The model rests on the following assumptions:

i. There are at least two firms producing homogeneous (undifferentiated) products (perfect substitutes);

ii. Firms do not cooperate;

iii. Firms compete by setting prices simultaneously;

iv. Consumers buy everything from a firm with a lower price. If all firms charge the same price, consumers randomly select among them.

v. No capacity constraints

vi. Consumers learn about prices instantly

vii. Same constant marginal cost (denoted c) and no fixed costs.

Bertrand's model considers a duopoly market of two participants, firm A and firm B. Firm A's optimum price depends on where it believes firm B will set its price. Pricing just below the other firm will obtain full market demand, though this is not optimal if the other firm is pricing below marginal cost as that would entail negative profits.

Quantity (demand) for firm A would be:

$$Q_A(p_A, p_B) = \begin{cases} D & \text{for all } p_A < p_B \\ \dfrac{D}{2} & \text{for all } p_A = p_B \\ 0 & \text{for all } p_A > p_B \end{cases} \tag{3.3}$$

Firms choose prices simultaneously and non-cooperatively. The set of strategies S is therefore the set of prices p. Firms are concerned about maximizing their profits. The consequence function for firm A is therefore defined as the profit:

$$\Pi_A(p_A, p_B) = (p_A - c) \times Q_A(p_A, p_B) \tag{3.4}$$

Given the definition of the demand function in equation (3.3), the best response function of firm A will be to charge just less than the price p_B charged by firm B with a minimum of the marginal cost c. Firm B's best response is determined in a similar manner.

The Nash Equilibrium is a pair of prices p_A^*, p_B^* such that:

$$\Pi_A(p_A^*, p_B^*) \geq \Pi_A(p_A, p_B^*) \text{ for all } p_A \in S_A$$
$$\Pi_B(p_A^*, p_B^*) \geq \Pi_B(p_A^*, p_B) \text{ for all } p_B \in S_B \tag{3.5}$$

This can be shown to occur at $p_A^* = p_B^* = c$. This is also shown graphically in figure (3.6). The result of this model is referred to as the **Bertand Paradox** – it takes only two firms to obtain (perfect) competition.

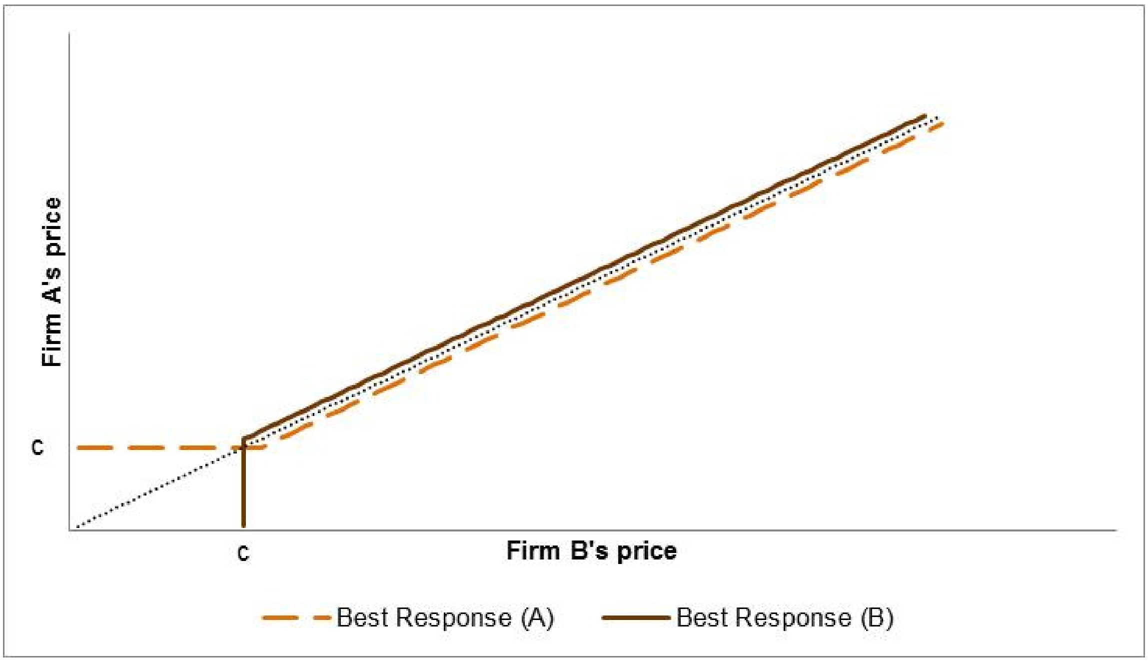

Figure 3.6 Best response functions of two firms in simple Bertrand competition

3.6 Hotelling's Model for Differentiated Products

The simple Bertrand competition model dealt with the condition where products were undifferentiated. Harold Hotelling noted that what was being overlooked was the fact that of all the purchasers of a commodity, some buy from one seller, while others buy from another, in spite of moderate differences in price. Hotelling (1929) expanded the Bertrand price competition model to allow for differentiated products in a duopoly. Hotelling introduced the concept that the buyers of a commodity will be supposedly uniformly distributed along a line and that distances between points on the line will reflect consumer preferences impacted by such things as product design, customer service, branding, etc. Consmers will buy from the firm which is nearest to their location on this line.

Assume that consumers have unit demand and that consumer preferences are uniformly distributed on the interval $[0,1]$. Hotelling likened the interval to a street in which firms represent shops. Consumers decide from which firm to purchase on consideration of price and transportation costs. In our case, the street is a spatial representation of consumer preference and their purchase decision is driven by price and the utility that they derive from each product. The products are homogenous with gross utility v. A consumer with a preference location of x incurs a utility cost of kx to purchase from firm A and a utility cost of $k(1-x)$ to purchase from firm B, assuming that the firms are located at the extremes of the interval and $k>0$. If we rather assumed that firm i is located on the interval at position l_i, $i \in \{A,B\}$, then we can denote the net utility the consumer derives from each firm's product by means of the following functions:

$$u_A(p_A,x) = v - p_A - k\left|x-l_A\right|^{\gamma} \text{ where } \gamma>0 \tag{3.6}$$

$$u_B(p_B,x) = v - p_B - k\left|x-l_B\right|^{\gamma} \text{ where } \gamma>0 \tag{3.7}$$

The above net utility functions imply that when $\gamma=0$ then the model approaches the Bertrand model and when $\gamma=1$ the utility cost is linear. When $\gamma=2$ the utility cost is quadratic, meaning that a customer's utility derived from a product reduces more strongly as the distance between their preference and the product increases. In this paper, for simplicity we assume that $\gamma=1$.

We can consider v being a reserve price. If u_i for some individual consumers becomes negative for the price-location combination of both firms, then these consumers will not buy any products. Under the assumption that prices are not too high relative to v the market is fully covered. In such a case, and assuming that $0 \le l_A \le x \le l_B \le 1$, we can find the marginal consumer x who is indifferent between the two products and then we have:

$$\begin{aligned} u_A(p_A,x) &= u_B(p_B,x) \\ v-p_A-k(x-l_A) &= v-p_B-k(l_B-x) \\ x &= \frac{(p_B-p_A+k(l_A+l_B))}{2k} \end{aligned} \tag{3.8}$$

We can then define the demand functions for the two firms as follows:

$$Q_A(p_A,p_B) = Dx = D\frac{(p_B-p_A+k(l_A+l_B))}{2k} \tag{3.9}$$

where $p_A \le p_B + k(l_A+l_B)$ and $p_A \le v$. If $p_A > v$ or $p_A > p_B + k(l_A+l_B)$ then $Q_A = 0$.

$$Q_B(p_A,p_B) = D(1-x) = D\frac{(p_A-p_B+k(2-l_A-l_B))}{2k} \tag{3.10}$$

where $p_B < p_B + k(l_A+l_B)$ and $p_B \le v$. If $p_B > v$ or $p_B > p_A + k(l_A+l_B)$ then $Q_B = 0$.

As before, the set of strategies S is the set of prices p and firms are concerned about maximizing profit. Assuming that the firms have unit costs c_A and c_B respectively, then the consequence functions for each firm relates to profit Π and can be defined as the following:

$$\begin{aligned} \Pi_A(p_A,p_B) &= (p_A-c_A)Q_A(p_A,p_B) \\ &= (p_A-c_B)(p_B-p_A+k(l_A+l_B))\frac{1}{2k} \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} \Pi_B(p_A,p_B) &= (p_B-c_B)Q_B(p_A,p_B) \\ &= (p_B-c_B)(p_A-p_B+k(2-l_A-l_B))\frac{1}{2k} \end{aligned} \tag{3.12}$$

We can determine the best response functions by maximizing each of equations (3.11) and (3.12), which can be done by setting the partial differential of each to zero.

This gives:

$$\frac{\partial \Pi_A(p_A, p_B)}{\partial p_A} = -(p_A - c_A) + \left(p_B - p_A + k\left(l_A + l_B\right)\right) = 0$$
$$p_A = \frac{\left(p_B + c_A + k\left(l_A + l_B\right)\right)}{2} \tag{3.13}$$

and

$$\frac{\partial \Pi_B(p_A, p_B)}{\partial p_B} = -(p_B - c_B) + \left(p_A - p_B + k\left(2 - l_A - l_B\right)\right) = 0$$
$$p_B = \frac{\left(p_A + c_B + k\left(2 - l_A - l_B\right)\right)}{2} \tag{3.14}$$

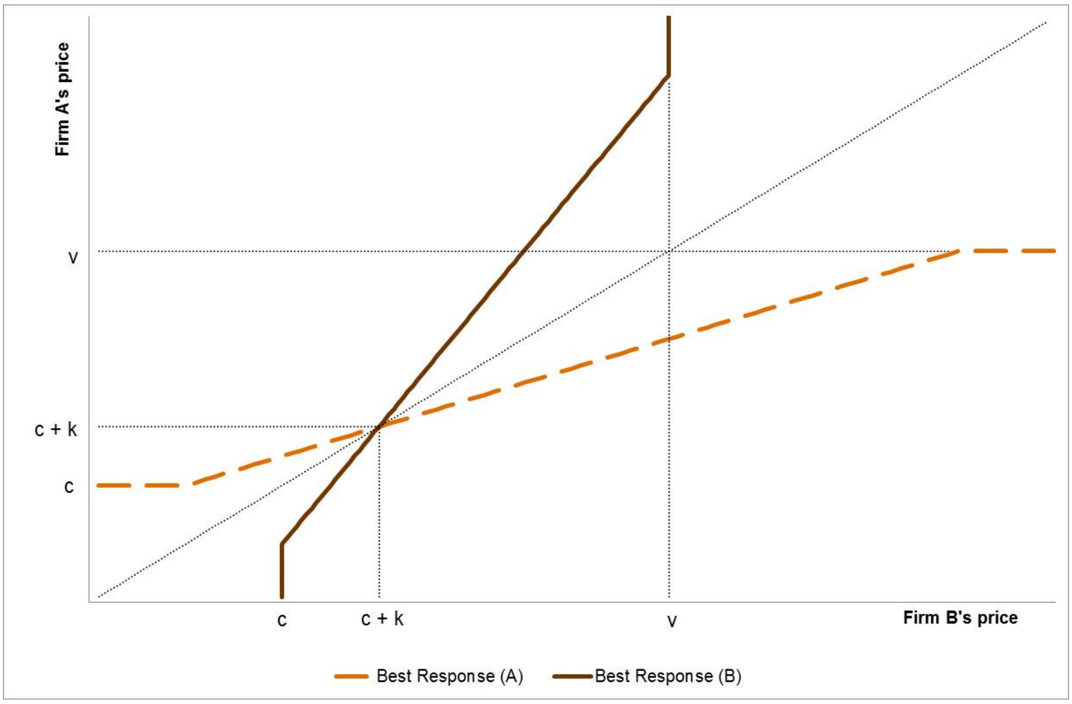

Figure 3.7 Best response functions of two firms in Hotelling competition model

We can then calculate the Nash equilibrium to be the intersection of the two equations, which occurs at the following prices of the two firms

$$p_A^* = \frac{2c_A + c_B}{3} + \frac{k\left(2 + l_A + l_B\right)}{3} \tag{3.15}$$

and

$$p_B^* = \frac{2c_B + c_A}{3} + \frac{k\left(4 - l_A - l_B\right)}{3} \tag{3.16}$$

If the marginal costs of the two firms are equal and the firms are located on the extremes of the interval, i.e. $c_A = c_B$ and $l_A = 0$ and $l_B = 1$, then the Nash equilibrium occurs at $p_A^* = p_B^* = c + k$. Figure (3.7) illustrates the case for two firms with equal costs. This model shows that there is no Bertrand Paradox with differentiated products. When the products are more differentiated (i.e. larger k) then prices are higher. When $k = 0$ then the model approaches Bertrand competition with homogeneous products.

4. CHANGING THE GAME

This section explores some of the actions that a firm could take to change the game in a way which benefits it. We use the impending EU Gender Directive as a backdrop for some examples applicable to the insurance industry.

4.1 Modeling the EU Gender Directive

The European Court of Justice ruled in March 2011 to remove the ability of insurers to use gender as a factor in pricing and benefits from 21 December 2012. We will use the impending EU Gender Directive as a setting to explore how the models developed earlier are applied to a hypothetical duopoly of two firms competing in the insurance market. We will also explore some actions that a firm could take to change the game in some way to its own advantage.

The model rests on the following assumptions:

i. Two firms produce a homogeneous product and sell one unit to each consumer;

ii. Firms do not cooperate;

iii. There are no capacity constraints;

iv. For each firm $i \in \{A, B\}$ a nonempty set S_i of strategic actions available to firm i which is the price it quotes for the policy. Firms compete by setting prices simultaneously;

v. For each mutually exclusive group of consumers $g \in \{m, f\}$ where m denotes male and f denotes female, consumers are uniformly distributed on the unit interval $[0,1]$. This interval represents consumer preferences. Consumers learn about prices instantly;

vi. Firms are able to position themselves on each interval through advertising, product design, customer service or some other means of altering customer preference. Firm i is located at point l_{gi};

vii. Each firm $i \in \{A, B\}$ must charge the same price for consumers even if they are in different groups;

viii. For each consumer group $g \in \{m, f\}$ the two firms have constant marginal costs c_{gi} and no fixed costs;

ix. For each firm $i \in \{A, B\}$ a consequence function $g_i : S \to C$ and a payoff function $h_i : C \to \mathbb{R}$

Consumers are subject to the following net utility function:

$$u_{gi}(p_i, x) = v - p_A - k_g \left|x - l_{gi}\right|^{\gamma} \text{ where } \gamma > 0 \tag{4.1}$$

For the sake of simplicity, we will assume that $\gamma = 1$. This means that the disutility cost for a consumer is linear in the distance of the product from the consumer's preferences.

The demand curves for each of the two firms are therefore:

$$\begin{aligned} Q_A(p_A, p_B) &= D_m x_m + D_f x_f \\ &= D_m \frac{\left(p_B - p_A + k_m \left(l_{mA} + l_{mB}\right)\right)}{2k_m} + D_f \frac{\left(p_B - p_A + k_f \left(l_{fA} + l_{fB}\right)\right)}{2k_f} \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} Q_B(p_A, p_B) &= D_m \left(1 - x_m\right) + D_f \left(1 - x_f\right) \\ &= D_m \frac{\left(p_A - p_B + k_m \left(2 - l_{mA} - l_{mB}\right)\right)}{2k_m} + D_f \frac{\left(p_A - p_B + k_f \left(2 - l_{fA} - l_{fB}\right)\right)}{2k_f} \end{aligned} \tag{4.3}$$

The consequence function is the profit set out as follows:

$$\begin{aligned} \Pi_A(p_A, p_B) = &\left(p_A - c_m\right) \frac{\left(p_B - p_A + k_m \left(l_{mA} + l_{mB}\right)\right)}{2k_m} \\ &+ \left(p_A - c_f\right) \frac{\left(p_B - p_A + k_f \left(l_{fA} + l_{fB}\right)\right)}{2k_f} \end{aligned} \tag{4.4}$$

and

$$\begin{aligned}\Pi_B(p_A, p_B) = \left(p_B - c_m\right)&\frac{\left(p_A - p_B + k_m\left(2 - l_{mA} - l_{mB}\right)\right)}{2k_m}\\ +\left(p_B - c_f\right)&\frac{\left(p_A - p_B + k_f\left(2 - l_{fA} - l_{fB}\right)\right)}{2k_f}\end{aligned} \tag{4.5}$$

Each firm's best response function is calculated by setting the partial derivatives to zero. This results in the following:

$$p_A = \frac{p_B\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)}{2\left(k_m + k_f\right)} \tag{4.6}$$

and

$$p_B = \frac{p_A\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)}{2\left(k_m + k_f\right)} \tag{4.7}$$

We can then calculate the Nash equilibrium to be the intersection of the two equations, which occurs at the following prices of the two firms

$$p_A^* = \frac{3k_m c_f + 3k_f c_m + k_m k_f\left(4 + l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)}{3\left(k_m + k_f\right)} \tag{4.8}$$

and

$$p_B^* = \frac{3k_m c_f + 3k_f c_m + k_m k_f\left(8 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)}{3\left(k_m + k_f\right)} \tag{4.9}$$

Appendix A contains a detailed derivation of the above two functions.

Having constructed a general model, we now use some examples to explore some further ideas around the impact of the directive.

Example 4.1.a. *There are two firms, EastSure and WestCover, competing for a share of a particular market segment, which is made up of 12,500 male and 12,500 female consumers. Both firms sell a motor insurance policy on an aggregator website, ConfuseTheAardvark.com; prices are loaded on the website simultaneously. The* *insurance policies have the same benefits, terms and excesses. Research undertaken by the two firms showed that some consumers were willing to pay £80 more for one firm's policy than the other, and vice versa. Men attach a value of £500 to having the policy, while women attach a value of £400 to the policy. The expected marginal cost for each firm is the same at £195 per policy for men and £95 per policy for women. Consumers are similar in all other aspects of risk and behavior. Consider what price each firm is likely to set prior to 21 December 2012.*

Solution: Prior to 21 December 2012, each firm is able to charge a different price for men and women. We can therefore deal with each group of men and women separately.

Applying equations (3.15) and (3.16), where for men $v = 500$, $k = 80$, $c_E = c_W = 195$, $l_W = 0$ and $l_E = 1$ and for women $v = 400$, $k = 80$, $c_E = c_W = 95$, $l_W = 0$ and $l_E = 1$, we get the equilibrium prices $p^*_{mW} = p^*_{mE} = 275$ for men and $p^*_{fW} = p^*_{fE} = 175$ for women.

At these prices, equations (3.9) and (3.10) suggest that each firm will have a market share of 50% for both men and women, i.e. $Q^*_{mW} = Q^*_{mE} = 6,250$ and $Q^*_{fW} = Q^*_{fE} = 6,250$. Similarly, equations (3.11) and (3.12) suggest that the firms will make profits of $\Pi^*_{mW} = \Pi^*_{mE} = 500,000$ and $\Pi^*_{mW} = \Pi^*_{mE} = 500,000$. Thus both firms make a total profit of £1,000,000 each.

This suggests that prior to the EU Gender Directive, competition applies independently to the two gender groups. ■

Example 4.1.b. *Consider what price each firm is likely to set post 21 December 2012 from when the EU Gender Directive prevents the firms from charging different prices for men and women.*

Solution: Post 21 December 2012, each firm must charge the same price for men and women. We therefore deal with each group of men and women as being dependent.

Applying equations (4.8) and (4.9), where for men $v_m = 500$, $k_m = 80$, $c_{mE} = c_{mW} = 195$, $l_{mW} = 0$ and $l_{mE} = 1$ and for women $v_f = 400$, $k_f = 80$, $c_{fE} = c_{fW} = 95$, $l_{fW} = 0$ and $l_{fE} = 1$, we get the equilibrium prices $p^*_W = p^*_E = 225$ for both men and women.

At these prices, equations (4.2) and (4.3) suggest that each firm will have a market share of 50% for both men and women, i.e. $Q^*_{mW} = Q^*_{mE} = 6,250$ and $Q^*_{fW} = Q^*_{fE} = 6,250$. Similarly, equations (4.4) and (4.5) suggest that the firms will make profits of $\Pi^*_{mW} = \Pi^*_{mE} = 187,500$ and $\Pi^*_{mW} = \Pi^*_{mE} = 812,500$ for men and women respectively. Thus both firms make a total profit of £1,000,000 each.

Our conclusion is that, post the EU Gender Directive, if firms are unable to apply any intervention that provides them with some advantage over others, then the equilibrium price will be the average of what the male and female prices were prior to the directive. Also, the overall profits of the market and individual firms will be unchanged. ■

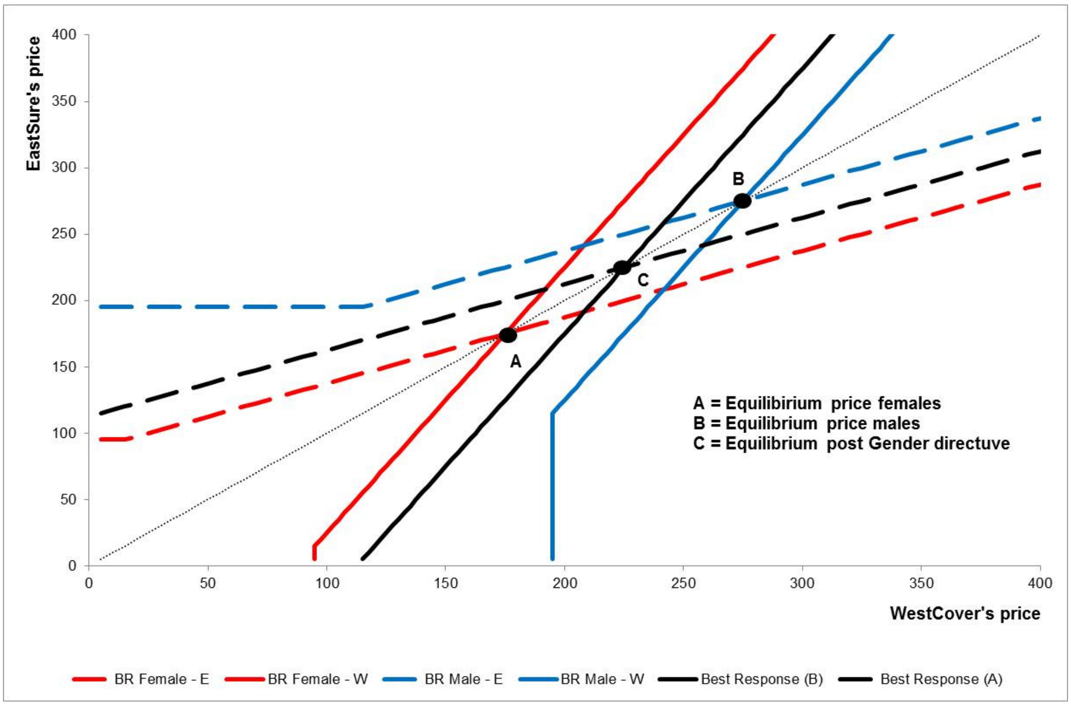

Figure 4.1 Nash equilibria for two insurers pre and post EU Gender directive

4.2 Innovation

A firm has an incentive to introduce an innovation which would give it some advantage over the other players. **Innovation** is the creation of better or more effective products, processes, services, technologies, or ideas that are readily available to markets. Innovation refers to the use of a better and, as a result, novel idea or method. Innovation differs from improvement in that it refers to the notion of doing something different rather than doing the same thing better. A firm introducing an innovation should find that it can either produce the same product at lower marginal costs than its competitors, or enhance the attractiveness of its product over those of its competitors.

Example 4.1.c. *The marketing team of WestCover formulates a new plan to target women through offering free nail treatments and pedicures. This dramatically increases the quality of the experience of WestCover's customers. The new plan has a once-off cost of £600,000. WestCover's consultant estimates that the quality increase of the product offering will increase the willingness to pay of ladies by 15%, but that there is unlikely to be any change to that of men.*

Solution: The innovation introduced by WestCover increases the ladies' willingness to pay by $0.15v = 60$. This has the effect of moving the location of WestCover on the unit interval to $60/k = 0.75$.

Applying equations (4.8) and (4.9), where for men $v_m = 500$, $k_m = 80$, $c_{mE} = c_{mW} = 195$, $l_{mW} = 0$ and $l_{mE} = 1$ and for women $v_f = 400$, $k_f = 80$, $c_{fE} = c_{fW} = 95$, $l_{fW} = 0.75$ and $l_{fE} = 1$, we get the equilibrium prices $p_W^* = 235$ and $p_E^* = 215$.

At these prices, equations (4.2) and (4.3) show that WestCover will be able to command a larger market share. WestCover will have a 38% share of the male consumers and 75% of the females, i.e. $Q_{mW}^* = 4,688$, $Q_{fW}^* = 9,375$, $Q_{mE}^* = 7,813$ and $Q_{fE}^* = 3,125$. Therefore, WestCover will have a 56% share of the overall market, $Q_W^* = 14,063$ and $Q_E^* = 10,937$. Similarly, equations (4.4) and (4.5) suggest that WestCover will make larger profits than EastSure, with $\Pi_W^* = £1,500,000$ and $\Pi_E^* = £531,250$. WestCover is therefore much better off from being attractive to women. ■

An interesting observation can be made from this example. WestCover had written 6,250 women and 6,250 men, and made profits of £1,000,000. The quality increase raised the willingness to pay of the ladies by £60. Other things being equal, WestCover should benefit by $6,250 \times 60 = £375,000$. However, reduction in demand for EastSure prompts it to reduce its price, resulting in WestCover reducing its price in response. The strategic effect of this is still favorable however, as WestCover then makes profits of £1,500,000, i.e. a net gain of £500,000. This outcome results from the fact that the directive requires a single price for men and women, and that women are subsidizing the men. EastSure sees a reduction in demand from the profitable ladies and an increase in demand from the loss-making men. It is therefore unable to reduce the price too far as the losses made from the extra men it attracts will more than offset the profits from the ladies.

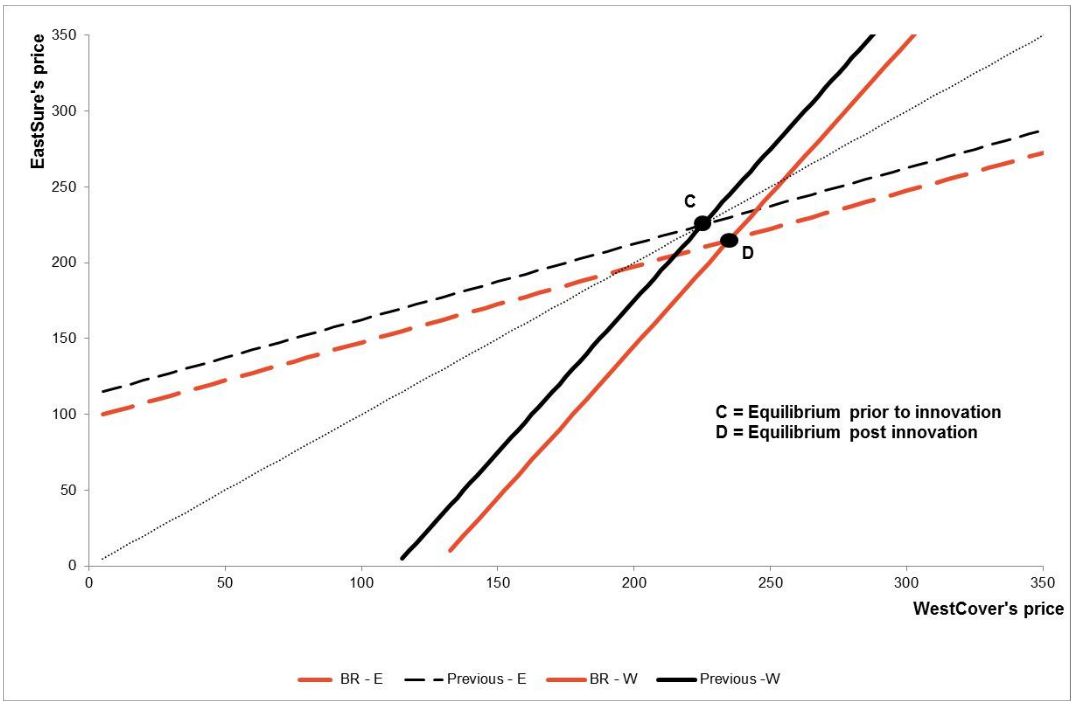

Figure 4.2 Nash equilibria post implementation of an innovation by one of the firms

4.3 Strategic Moves – Commitments, Threats and Promises

Unless the rules of a game are fixed by an outside authority, each player has an incentive to manipulate the rules to produce an outcome that is more to their own advantage. Devices to manipulate a game in this way are called **strategic moves**. Strategic moves and how to apply them are discussed in more detail by Dixit, Skeath, and Reiley (2009), and Dixit and Barry (2008).

A strategic move creates a new two-stage game. The first stage of a game with strategic moves now specifies how a player will act in the second stage, which aims to alter the outcome of the game to his advantage. Importantly, the success of the strategic move is dependent on the credibility of the move, in other words whether the other players believe that at the second stage the player in question will indeed do what he declared at the first stage. To make strategic moves credible, some ancillary actions will be needed in the first stage. These actions need to be (1) observable by other players; and (2) irreversible. (Dixit & Barry, 2008)

Strategic moves can be grouped into **commitments**, **threats** or **promises**. A commitment is a declaration in stage one of an unconditional move in stage two, which is irrespective of the other player' moves. If credible, this is tantamount to changing the order of the game at stage two (or converting a simultaneous-move game into a sequential-move game in which the player making the commitment makes the first move). A threat is a conditional move where one player declares that unless the other player's action conforms to his wish, then he will respond in a way which hurts the other player. Finally, a promise is a conditional move where one player declares that if the other player's action conforms to his wish, then he will respond in a way which rewards the other player.

Example 4.2. *"Chinese drywall" refers to an environmental health issue involving defective drywall (or plasterboard) manufactured in China and imported to the US starting from 2001, particularly during the construction boom between 2004 and 2007. Importation was further spurred by a shortage of US-made drywall due to rebuilding demand from nine hurricanes that hit Florida in 2004 and 2005. Toxic boards emitted sulfur gases and homeowners reported a variety of symptoms including respiratory problems, chronic headaches and sinus issues. Also numerous incidents of corroding copper and other metals in the home were reported. Class action lawsuits have been filed against home builders, drywall suppliers and a Chinese drywall manufacturer. Legal claims have been made against the firms, but the international aspect of the litigation complicates matters. China does not enforce US judgments. Since Chinese manufacturers of this drywall presumably are no longer doing business in the US, there is likely to be no way to get at those companies' assets in the US. The insurance industry received many related claims.*

Discussion: This example sets out a strategic interaction between the drywall manufacturers and the homebuilders. In the first stage of the game, homebuilders decide to purchase the drywall from the manufacturers at the agreed price. In return, the manufacturers commit to deliver the drywall product of stated quantity and quality. But how should homebuilders be sure that the manufacturer's commitment is credible?

Manufacturers create credibility by giving the homebuilders the right to sue them if they do not deliver on the contract. This is a credible threat, since at the second stage of the game whether or not the manufacturer is sued is in the control of the homebuilder, who will be sure to carry out this threat if the manufacturer has reneged (the move is observable and irreversible). The likely costly outcome of being sued at the second stage limits the options for the manufacturer; they would maximize their payoff by delivering on the contract. However, in this case homebuilders did not fully appreciate that while this threat was credible for US manufacturers, because it is not possible in practice to sue Chinese companies in the US, the threat is not credible when dealing with Chinese manufacturers. Therefore, Chinese manufacturers' acted rationally by supplying substandard products to maximize their payoff, given that there was no threat of a costly lawsuit, and homebuilders, if aware of the full legal facts, should not have been surprised by the outcome. ■

Figure 4.3 Photos of defective Chinese drywall and corrosion it's caused to pipework

There are a number of different examples of strategic moves and it is impossible to set out an exhaustive list. Instead we explore one such example of a common strategic move which is price-matching or "beat the competition".

Example 4.1.d. *Having recently introduced the quality increase, WestCover recognizes that EastSure may respond by aggressively reducing its price to maintain market share. As a result, it introduces a "match the competition" clause by publicly guaranteeing that if it is underpriced by EastSure, then customers have the right to buy from them at the lower price.*

Solution: By WestCover introducing the guarantee that it will match EastSure's price if it is lower, it has made a credible threat to EastSure. If EastSure quotes a price less than WestCover's then WestCover will match the lower price. This guarantee has been committed by WestCover in advance and is irreversible. In this event, customers will make the choice between the two products at the same price, but women will still prefer to buy from WestCover because of the quality difference. The best response function for EastSure should therefore be amended to be the maximum of its previous best response based on equation (4.7) and WestCover's price. This is shown in figure (4.4).

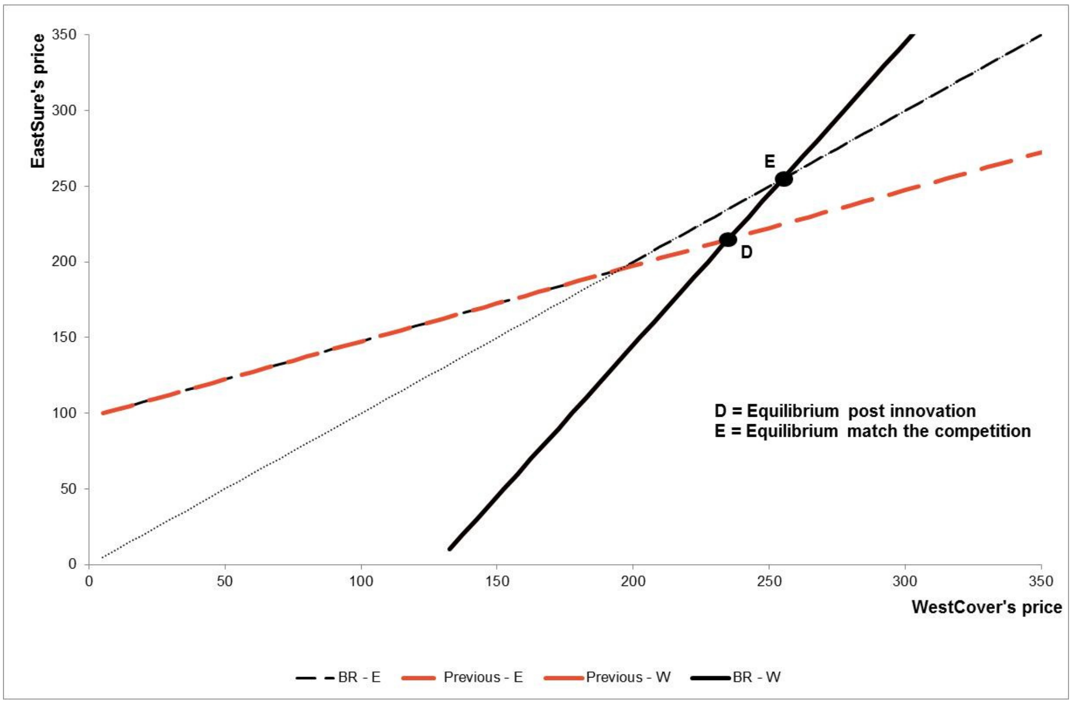

Figure 4.4 Nash equilibrium with a strategic move of matching competitor's price

The new equilibrium prices are then $p_E^* = p_W^* = 255$ and the resulting market shares are $Q_{mE}^* = Q_{mW}^* = 6,250$ (50% each) and $Q_{fW}^* = 10,938$ (88%) and $Q_{fE}^* = 1,562$ (12%). WestCover is able to write 69% of the total market, leaving 31% for EastSure. WestCover is also able to generate total profits of £2,125,000, while EastSure will make profits of £625,000. The fact that the equilibrium price is higher than before means that the profits made in the market are higher, but WestCover is able to command a larger share of this. Both firms are actually better off than they were before.

Note that EastSure is not in a position to credibly make such a price matching guarantee. If it were to institute such a guarantee, then WestCover would not be threatened as at the same price female customers have a preference for WestCover's product over Eastsure's. ■

The degree of success of a strategic move will depend on a number of factors, including the temptation to cheat, the ability or chance of other players observing a defection, the chances of other players being able to effectively punish a defector, and whether or not the game is repeated. (Dixit, Skeath, & Reiley, Jnr, 2009)

5. A MODEL FOR INSURANCE

We now develop a price competition model for the insurance industry.

5.1 Model Setting

The model rests on the following assumptions:

i. A finite set N of n insurance firms;

ii. Each insurer $i \in N$ produces a homogeneous product and sells one unit to each consumer;

iii. For each insurer $i \in N$ a nonempty set S_i of strategic actions available to insurer i. These actions are usually taken to be price p_i;

iv. For each insurer $i \in N$ a consequence function $g_i : S \rightarrow C$

v. For each insurer $i \in N$ a payoff function $h_i : C \rightarrow \mathbb{R}$

vi. Insurers do not cooperate;

vii. There are no capacity constraints;

viii. Insurers compete by setting prices simultaneously;

ix. For each insurer $i \in N$ marginal costs of c_i and no fixed costs;

5.1.1 Demand

The demand for each insurer is a function of its price as well as the price of all other firms. A firm's demand is driven by the conversion rate, being the proportion of consumers purchasing the firm's product given that a quote is made, so we have:

$$Q_i(p_1, p_2, ..., p_n) = D \times z_i(p_1, p_2, ..., p_n) \tag{5.1}$$

where D is the market demand, z_i is the conversion rate of insurer i which is a function of premiums charged by all firms in the market, and p_i is the premium charged by insurer i.

An important function that relates demand and price is the point-price elasticity e.

The elasticity for insurer i is defined as:

$$e_i(p_1, p_2, ..., p_n) = \frac{\partial Q_i(p_1, p_2, ..., p_n)}{\partial p_i} \times \frac{p_i}{Q_i(p_1, p_2, ..., p_n)} \tag{5.2}$$

The above formula usually yields a negative value, due to the inverse nature of the relationship between price and quantity demanded, as described by the law of demand. However, economists often refer to price elasticity as a positive value, i.e. in absolute value terms.

A decrease in the price of a good normally results in an increase in the quantity demanded by consumers because of the law of demand, and conversely, quantity demanded decreases when price rises. The price elasticity for a good or service is referred to by different descriptive terms depending on whether the elasticity coefficient is greater than, equal to, or less than -1. That is, the demand for a good is called:

i. Perfectly inelastic when $e = 0$, i.e. there is no change in the quantity demanded for a change in price;

ii. Relatively inelastic when $-1 < e < 0$, i.e. the percentage change in quantity demanded is less than the percentage change in price;

iii. Unit elastic when $e = -1$, i.e. the percentage change in quantity demanded is equal to the percentage change in price;

iv. Relatively elastic when $-\infty < e < -1$, i.e. the percentage change in quantity demanded is greater than the percentage change in price; and

v. Perfectly elastic when $e = -\infty$.

Various research methods are used to calculate price elasticities in practice, including analysis of historic sales data, both public and private, and use of surveys of customers' preferences to build up test markets capable of modeling such changes. Alternatively, conjoint analysis (a ranking of users' preferences which can then be statistically analyzed) may be used.

5.1.2 Simplification of Conversion Model

Current price optimization techniques used in the insurance market to set price tend to adopt a simplified conversion model. There may well be too many insurers in the market to develop a model where the conversion model is a function of each and every competitor. In such an event, many insurers will look at an aggregated market price $p_{m,-i}$ which is dependent on all insurers except insurer i. The precise definition selected will need to consider the nature of the market and how consumers fundamentally select products. One option, for example, would be to take the minimum premium of all other competitors, which will give

$$p_{m,-i} = \min_{j \neq i} \{ p_j \} \tag{5.3}$$

This would be suitable when consumers will tend to always select the cheapest price only. Another option might be to take the average premium of all other competitors, which will give:

$$p_{m,-i} = \frac{1}{n-1} \sum_{j=1,\neq i}^{n} p_j \tag{5.4}$$

or, alternatively, the average of the five lowest premiums excluding insurer i. This latter approach may be suitable where consumers are driven by both price and brand or some other product differentiation.

Yet a further simplification that could be made is to the functional form of the conversion rate z such that it only depends on a single combined measure of p_i and $p_{m,-i}$. Options include using the ratio $p_i / p_{m,-i}$ or the difference $p_i - p_{m,-i}$. The conversion rate then becomes a function of one parameter. This is not necessarily true, but could simplify the calculation and gives interesting results. The conversion rate is then

$$z_i(p_1, p_2, ..., p_n) = f(p_i - p_{m,-i}) \text{ or } z_i(p_1, p_2, ..., p_n) = f(p_i / p_{m,-i}) \tag{5.5}$$

5.1.3 Consequences and Payoffs

For each insurer $i \in N$ there is a consequence function $g_i : S \rightarrow C$ and a payoff function $h_i : C \rightarrow \mathbb{R}$. Each insurer will seek to maximize its payoff h_i. In the Bertrand and Hotelling models discussed earlier, we have assumed that the consequences and payoffs are profit. However, these could be profit, market share, a mixture of the two, or anything else that reflects the objectives of the insurer for that matter.

Profit

If profit is used, the measure is

$$g_i = h_i = \Pi_i = D \times z_i(p_i, p_{m,-i}) \times (p_i - c_i) - FC_i \tag{5.6}$$

where c_i is the variable costs and FC_i is the fixed costs. In this analysis, c_i is considered as known and fixed. Interestingly, the inclusion of fixed costs is mathematically redundant as it will fall out during the differentiation when maximizing the function.

More Complex Profit Calculation

As discussed in section 2.5, a distinction is usually made between accounting and economic profit. However, there may be two further adjustments that could be made to the profit measure which would allow for a more accurate assessment of the profits.

The first is to allow for the cost of capital required to support the business and the other profit and loss items not in equation (5.6), including for example investment income. In such a case, rather than looking at absolute profits, the measure may be return on capital, or even some risk-adjusted variant.

The second enhancement that might be made is to allow for the stochastic nature of the estimated marginal costs. How to allow for stochastic payoffs was discussed briefly in section 3.3 and so equation (5.6) can be easily adjusted to allow for the random nature of the claims and other costs. However, where the payoff function is stochastic then the player will behave as if he is maximizing the expected value of the outcome. (von Neumann & Morgenstern, 1944). We have therefore chosen to ignore this adjustment in this paper. That said, such an adjustment can also allow for the differing levels of information and abilities of market participants to estimate the expected costs (Dixit, Skeath, & Reiley, Jnr, 2009) and for the winner's curse (Rothwell, et al., 2009).

Market Share

If a firm is concerned solely about maximizing the volume of business it writes, then it would charge a price equal to marginal costs and make no profit. Companies with such a strategy include charitable or mutual companies, where the objective is to only cover the costs of the business so that it is sustainable, while delivering the promised services to its customers (or members).

Another situation where an insurer may be concerned purely with volume is where an outside authority has set the price. Examples of insurance markets where the price is fixed by the local regulators include compulsory earthquake cover in Turkey (DASK) and compulsory motor third party liability insurance in India. A profit-maximizing insurer in such a market will need to decide how much of the tariffed business to write, together with other nontariff classes, which will maximize its profits.

Customer Lifetime Value

Our discussion in the paper so far has assumed a single occurrence of the game. However, the game is repeated on a regular basis, for example as policies come up for renewal. Insurers should consider how it and its competitors might behave in future games, and how actions taken in the first period can limit or create options in future games. We consider here the concept of customer lifetime value, but discuss some considerations about the implications it has in repetitive situations at the end of this section.

Customer Lifetime Value (CLV) suggests that a firm should consider the net profit of a customer attributable to its entire future relationship with that customer. Lifetime value is typically used to assess the appropriateness of the acquisition costs of a customer. So one way to define the payoff measure could be

$$g_i = \Pi_i = D \times z_i(p_i, p_{m,-i}) \times (p_i - c_i + LV_i) - FC_i \tag{5.7}$$

where LV_i is a constant that reflects the discounted future average net profit from that customer segment and the marginal costs c_i include all marginal and acquisition costs related to the policy. A special case of equation (5.7) is when $LV_i = 0$, which is then the same as the profit function in equation (5.6). For this reason, we will focus our subsequent analysis on this form of consequence function.

There are a number of critical factors needed for such an approach to be successful. Firstly, it must be possible for an insurer to charge more than other insurers for a customer in the future without that customer changing loyalty. Secondly, the insurer may discounted the value of net profits to allow for the time value of money. Finally, acquiring business at a loss in the hope of generating profits in the future is a dangerous strategy, particularly when there are potential outcomes outside your control. For example, competitors could take actions which result in your losing customers and therefore foregoing the future profits you were relying on. We explore the reasoning about whether or not allowing for future profits is sensible or not in section 5.6.

5.2 Best Responses

The best response for insurer i can be calculated by setting to zero the partial derivative of equation (5.7) with regards to p_i. That is, the price at which insurer i will maximize its profit given the prices charged by its competitors. This gives:

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{\partial z_i}{\partial p_i} D\left(p_i - c_i + LV_i\right) + Dz_i = 0 \qquad (5.8)$$

The definition of elasticity in equation (5.2) can be reworked as follows:

$$\frac{\partial z_i D}{\partial p_i} = e_i \times \frac{z_i D}{p_i} \qquad (5.9)$$

Substituting this into equation (5.8), it gives

$$\frac{z_i}{p_i} e_i D\left(p_i - c_i + LV_i\right) + Dz_i = 0 \qquad (5.10)$$

and finally

$$-e_i(p_i, p_{m,-i}) = \frac{p_i}{p_i - c_i + LV_i} \qquad (5.11)$$

The right side of equation (5.11) only depends on p_i but the left side depends on both p_i and $p_{m,-i}$. Both sides of the function can be plotted on a chart to determine at what p_i there is an intersection, if any, which will then be the best response premium (although we should check whether this is a maximum or minimum value!).

This is illustrated in figure (5.8) for a specific case assuming that the elasticity coefficient α_i is constant, that the conversion rate is modeled using a logit link function so that $z_i = 1\big/\left(1+\exp(\alpha_i \, {}^{p_i}\!/_{p_{m,-i}} - \alpha_i)\right)$ and that $p_{m,-i} = 1$, $c_i = 1$ and $LV_i = 0.3$.

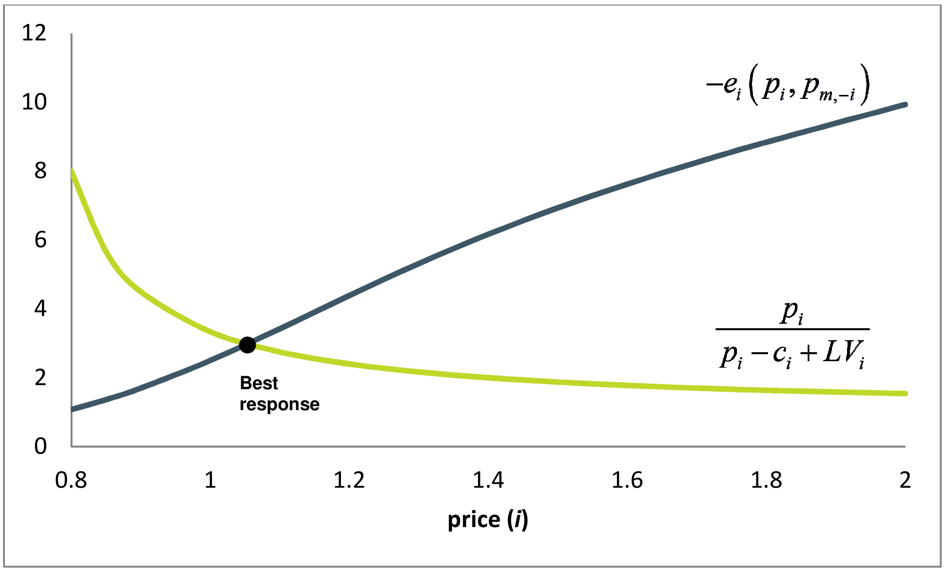

Figure 5.8 Example calculation of best response price of insurer *i* for $c_i = 1$, $p_m = 1$ and $LV_i = 0.3$

The solution to equation (5.11) depends on the shape of the assumed conversion rate z and elasticity function $-e$, and there is no general analytical solution. However, we can make several useful observations as follows:

5.2.1 Existence of solution

The existence of a solution to equation (5.11) is not guaranteed. The function $p_i/\left(p_i - c_i + LV_i\right)$ takes any value between 1 and ∞ for all $p_i > c_i - LV_i$. So if $-e$ is a monotonic increasing function and takes a value higher than 1 (for any p_i), then there will be a solution. In practice, we often find that this is the case for conversion models of new business, where elasticity is greater than 1 and a logit link function is used.

Another important case is when $-e$ is always less than 1 and, therefore, there is no solution. The implication is that the best response premium is ∞, i.e. the market is so inelastic that the firm can charge any price. In practice, it is more likely that the modeling of elasticity needs to be reviewed as most insurance markets tend to have a price at which demand will reduce significantly.

5.2.2 Uniqueness of solution

As $p_i/\left(p_i - c_i + LV_i\right)$ is monotonically decreasing for $p_i > c_i - LV_i$, if $-e$ is a monotonic increasing function then the solution is unique if it exists. If there is more than one solution, the second-order differentiation needs to be calculated to check which solution is the optimal one. This also potentially leads to two or more equilibrium points.

5.2.3 Monotonicity with regard to market premium

How p_i changes when market premium $p_{m,-i}$ changes is critically important when determining the equilibrium price. Generalized mathematical derivation is difficult; however, intuitively the elasticity should decrease when market price increases, as shown in figure (5.8). When the market premium increases, there is a greater difference between the firm's price and what is available from competitors and so the firm should see an increase in demand. This causes the elasticity curve $-e$ to shift southeast. Because the function $p_i/\left(p_i - c_i + LV_i\right)$ is fixed in terms of the market premium, the intersection point will move towards the right. So the best response premium of insurer i will increase given an increase in market premium. This is consistent with the Bertrand and Hotelling models discussed earlier.

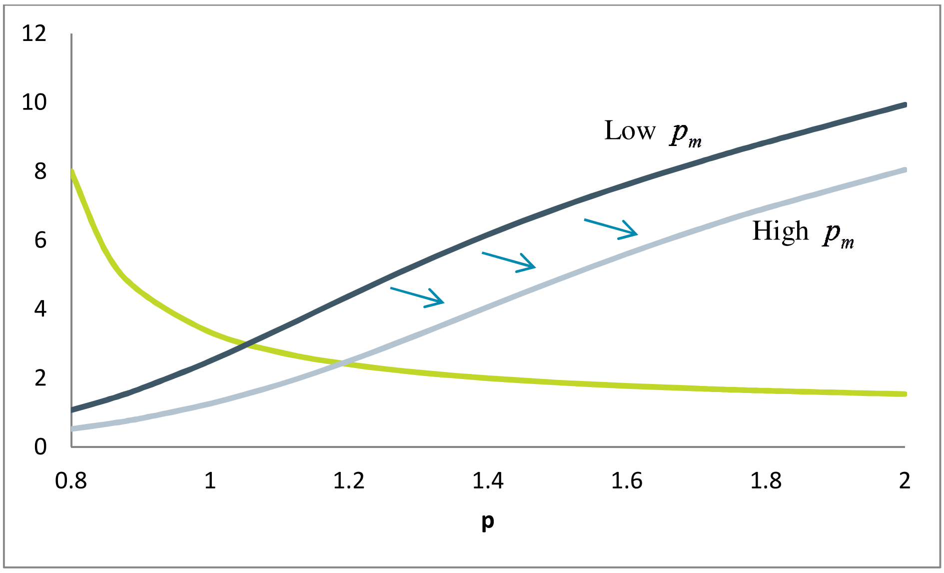

Figure 5.8 Example best response of insurer i for $c_i = 1$, $p_m = 1$ (low) or $p_m = 1.2$ (high) and $LV_i = 0.3$

For insurer i, the best response premium can be calculated for every given market premium $p_{m,-i}$. The relation between the best response premium and the market premium can be defined as a function $B_i\left(p_{m,-i}\right)$.

Example 5.1.a. *It is not obvious that a closed-form solution to deriving the best response function exists but the result can be easily calculated in Excel by using its solver functionality. Assume an insurance company has marginal costs of 1 and does not allow for any future lifetime value. The company has modeled its conversion rates using a logit link function and has estimated a constant elasticity coefficient* α *of 5.*

Solution: The example has $c_i = 1$, $LV_i = 0$ and $z_i = 1 \big/ \left(1 + \exp(5\, {}^{p_i}\!/_{p_{m,-i}} - 5\right)$. Using Excel's solver function, the best response can be calculated for each possible market premium. This is shown by the blue line in figure (5.9). For interest, the best response function from the Bertrand model is plotted in red on the same graph.

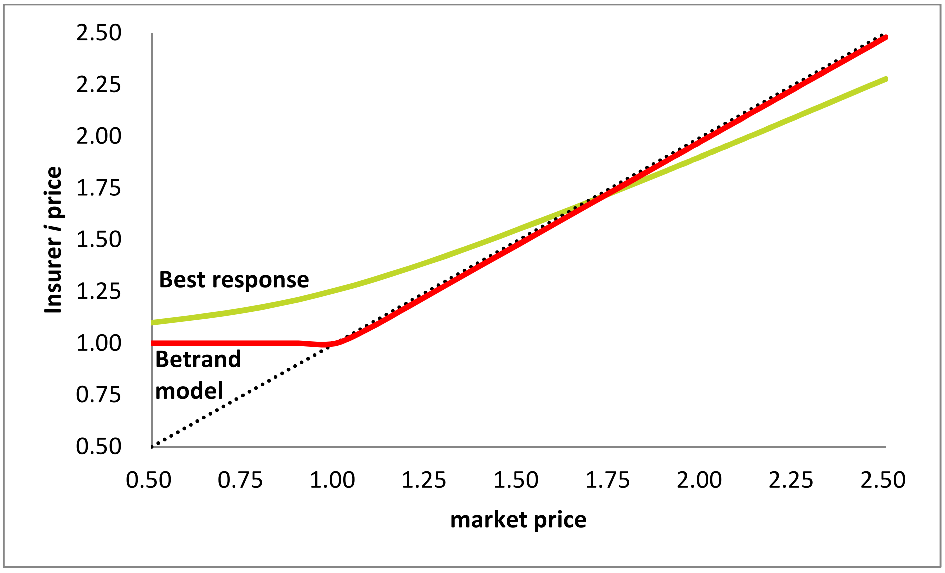

Figure 5.9 Best response function of insurer i for $c_i = 1$ and $LV_i = 0$ for all market premiums p_m

The best response curve cuts the $y = x$ line at around a price of 1.7, which means that if the market price p_m is lower than 1.7, the insurer will do best by charging a price higher than the market premium, and vice versa. The best response premium is also never lower than the expected claim cost c_i, which is consistent with the Bertrand and Hotelling models discussed earlier. ■

Example 5.1.b. *Now assume the insurance company still has marginal costs of 1 but now decides to allow for future lifetime value by calculating the net present value of future net profits to be 0.3. The company has modeled its conversion rates using a logit link function and has estimated a constant elasticity coefficient of 5.*

Solution: The example has $c_i = 1$, $LV_i = 0.3$ and $z_i = 1 \big/ \left(1 + \exp(5\, {}^{p_i}\!/\!_{p_{m,-i}} - 5\right)$. Again, using Excel's solver function, the best response can be calculated for each possible market premium. This is shown by the blue line in figure (5.10).

By allowing for the future lifetime value of a customer, the best response for the insurer can indicate a value less than the marginal cost, which for an insurer is largely the expected claims cost. ■

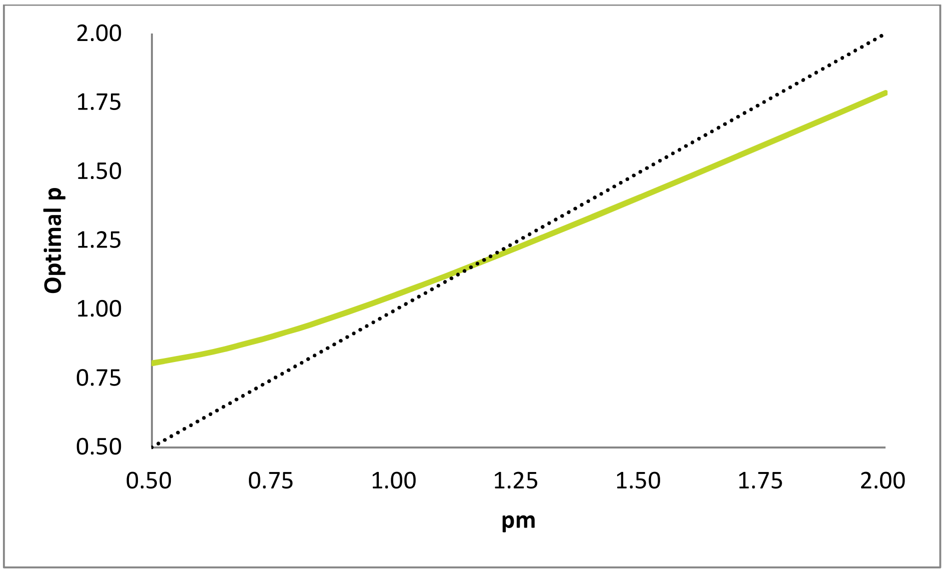

Figure 5.10 Best response function of insurer i for $c_i = 1$ and $LV_i = 0.3$ for all market premiums p_m

5.3 Equilibrium Price

Every insurer has the freedom to set its premium to the perceived best response level given the prices charged by other insurers. Any change in an insurer's premium will then affect the market premium and therefore the best response premium that other insurers will choose. This should continue until equilibrium is reached, when there is no insurer that will be better off by changing its premium. As in earlier sections, we can calculate the equilibria by finding the points of intersection of the various insurers' best response functions.

In the case that all insurers have the same marginal costs and elasticity, the best response premium functions will be the same, i.e. $B_i(\cdot) = B(\cdot)$. Similarly to the Bertrand and Hotelling models, it can be shown that the equilibrium premiums will be the same for all insurers, i.e. $p_i^* = p_{j\neq i}^* = p^*$ for all $i \in N$ and $j \in N$. It follows that $p_{m,-i}^* = p_i^* = p^*$ for all $i \in N$. This means that $p^* = B(p^*)$, the solution of which is the intersection of curve $B(\cdot)$ with the $y = x$ line.

The best response premium function is generated from the solution to equation (5.11), which means that the equilibrium price p^* will be the solution p' from the following equation:

$$-e(p', p') = \frac{p'}{p' - c + LV} \tag{5.12}$$

which gives

$$p' = \frac{(c - LV) \cdot e(p', p')}{e(p', p') + 1} \tag{5.13}$$

This usually has to be calculated numerically, as described in section 5.1.4. However, if further simplification is made to $e(\cdot,\cdot)$ as described in section 5.1.2, then we can derive a useful result. If $e(\cdot,\cdot)$ only depends on p/p_m, then equation (5.13) becomes

$$p' = \frac{(c - LV)e_1(1)}{e_1(1) + 1} \tag{5.14}$$

in which $e_1(\cdot)$ is used to reflect that it is special case for elasticity function. If $e(\cdot,\cdot)$ only depends on $p - p_m$, then equation (5.13) becomes

$$p' = \frac{(c - LV)e_2(0)}{e_2(0) + 1} \tag{5.15}$$

where $e_2(\cdot)$ is used to reflect that it is the second special case.

Example 5.1.c. *Let us revisit our insurance company with marginal costs of 1 and the two scenarios of whether or not it allows for the future lifetime value. In the first scenario it does not allow for future lifetime value, and in the second it allows for future lifetime value by calculating the net present value of future net profits to be 0.3. The company has modeled its conversion rates using a logit link function and has estimated a constant elasticity coefficient of 5.*

Solution In this example we are using the first special form of $e_1(\cdot)$. It can be calculated that $e_1(1) = -2.5$, so that the solutions for the two scenarios follow:

$$p^* = \frac{(1-0)\times 2.5}{2.5-1} \approx 1.7 \text{ for } LV = 0$$

and

$$p^* = \frac{(1-0.3)\times 2.5}{2.5-1} \approx 1.2 \text{ for } LV = 0.3$$

These results are consistent with those derived earlier. ■

An important conclusion is that the equilibrium premium is higher than the expected claim (marginal cost), which is different to the Bertrand model, where the equilibrium premium is equal to the marginal cost, but is similar to the Hotelling model, where differentiation causes some consumers to buy from one firm rather than another despite differences in price. This suggests that there is a direct relationship between the consumer preference location concept used by Hotelling and the price elasticity used in this model. The level of equilibrium premium depends on the elasticity of the market, for example the lower the elasticity then the higher the equilibrium premium. No cooperation between insurers is needed to achieve that. Finally, the Bertrand model is a special case where elasticity is equal to ∞ so that $p^* = c - LV$.

5.3.1 Duopoly Market

For a duopoly market where there are only two insurers $i \in \{1, 2\}$ in the market, the market premium simplifies to the other firm's price. The equilibrium premiums will occur at the intersection of the two best response functions $B_1(\cdot)$ and $B_2(\cdot)$.

Example 5.2. *Assume that there are two insurance companies, North Re and South Re. Both insurers have marginal costs of 1. North Re is concerned only about maximizing its profit. South Re, however, would like to maximize profit subject to writing a minimum volume of business, which it does by giving policies a future lifetime value of 0.3. Both companies have modeled conversion rates using a logit link function with a constant elasticity coefficient of 5.*

Solution In this case North Re will set $LV_N = 0$ and $c_N = 1$ while South Re will set $LV_S = 0.3$ and $c_S = 1$. The best response premium curves, which were derived by using Excel's Solver, are plotted in figure (5.11). The equilibrium price is calculated from the intersection at approximately $p_N = 1.5$ and $p_S = 1.4$. Because of South Re's strategy to balance volume with profit by taking into account the future profitability of the business when setting the initial price, it chooses to charge a lower premium than North Re. ■

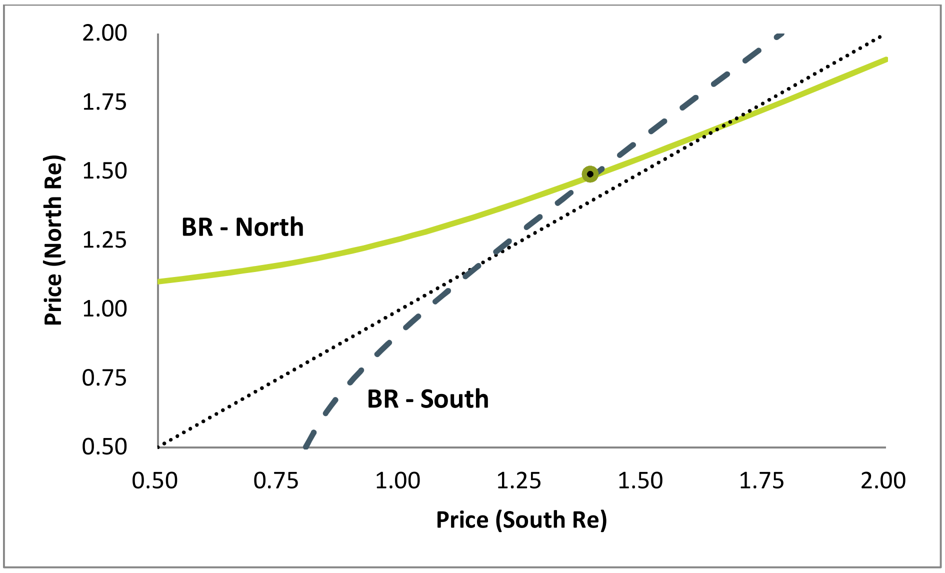

Figure 5.11 Best response functions and equilibrium price for North Re and South Re

5.3.2 Stability of equilibrium

As discussed in section 5.1.4, $B_i(\cdot)$ is monotonically increasing under certain normal market situations and if all best response functions are monotonically increasing, it can be shown that the equilibrium point is stable. So the higher-than-marginal-cost equilibrium point is stable.

5.4 Viability of Lifetime Value

By reducing the price charged today for the expected lifetime value means that the insurer is heavily dependent on actually realizing the future profits underlying the calculation of the expected lifetime value amount. The majority of insurance policies are annual or other fixed duration, and policyholders can usually move between insurance companies relatively easily at renewal. We can explore the viability of the concept of lifetime value by reasoning through the following example.

Assume that there are two insurers, *A* and *B*, who are identical. Both insurers have received an application to insure a person, who we can call Bob for the sake of identity. Both insurers know for certain that Bob will have a claim during the year and both know that their total costs for administering the policy and settling the claim will be £10. Finally, both insurers know that Bob will no longer need insurance from the end of the year (we will leave you to think of an appropriate reason). The insurers have to provide their quotes simultaneously, without knowing what the other has quoted. So what price does each insurer quote?

Following the logic used throughout this paper, *A* will want to quote a value just below *B*, but a quote of less than £10 will make a loss. If *A* thinks that *B* will charge £12 then it would charge £11. *B* will go through a similar thought process, so if *A* quotes £11 then *B* would want to charge £10.50, say. This means that a Nash equilibrium exists when both insurers' charge £10 and the winning insurer will make £0 profit.

Now let's assume that Bob approaches the insurers a year earlier. Both insurers know that Bob will only require two years of insurance. The policy is an annually renewable policy and Bob will have a claim in each year. Both insurers know that to administer and settle claims will cost the winning insurer £10 in each year. What price will each quote at the start of the first year? If *A* were to win the business in the first year, then in the second it will have to quote £10 or less because charging more than this would mean losing the business to *B*, who is likely to quote the equilibrium price of £10. This means that it should not discount the price in the first year, since it will not be able to recoup any losses in the second year. Also, if it quotes more than £10 in the first year, it is likely that *B* could quote less and win the business. This means that both insurers will quote £10 in the first year and then £10 again in the second year when the policy is up for renewal.

Repeating this process means that regardless of the number of years, as long as they are finite, each year has the same result. Each insurer, knowing that it will have to charge the equilibrium price in the final year, is forced to charge the equilibrium price in each preceding year or it will make a loss which will be impossible to recover in the future.

Under this scenario, there are no profits which arise in later years that can be used to discount the price in the earlier years. That is, however, unless there is a difference between the price that the incumbent insurer can charge relative to outside insurers.

Let us now assume that there are two years to go and that Bob prefers to stay with his current insurer (regardless of who that is) on renewal and is willing to pay up to £1 more in that case. Therefore, if *A* wins the business in the first year then it can quote £11 in the final year to make a profit of £1. *B* will still quote £10 in the final year because charging more than this will not lead to it winning the business and any less would make a loss. This means that in the first year, *A* can charge £9 knowing that it can charge £11 in the final year. Both insurers therefore quote £9 in the first year and then the winning insurer quotes £11 in the second year and the other quotes £10.

Assuming a year earlier, i.e. there are three years of cover, then a similar thought process will lead to both insurers quoting £9 in the first year, the winning insurer will then quote £10 in the second year and £11 in the third year. The losing insurer will quote £9 in the second year and then, if it still hadn't won the business, £10 in the final year.

This example is set out graphically in figure (5.12). If the incumbent insurer charges more than £10 in an interim period, then it is at risk of another insurer quoting £9 and winning the business. If the insurer charges less than £10 in an interim year, then it will have discounted too much and will be unlikely to recoup the full discount given over the lifetime of the policy

In conclusion, allowing for the future lifetime value when setting the price can be a complex consideration. The insurer will need to have an accurate assessment of the price elasticity of its customers at renewal at future dates, and how this might differ from that of a new customer. It will then need to remember the discounts given to a policy in the past and be sure not to repeat these discounts needlessly in the future – for example, in figure (5.12) any discount in the second or subsequent years would not be recovered in the future. Getting this wrong can be an expensive mistake.

Year	1	2	3	4	...	n-2	n-1	n
Costs	*£10*	*£10*	*£10*	*£10*		*£10*	*£10*	*£10*
Equilibrium premium for an insurer quoting to win business with x number of years left of policy lifetim								
1 year left								£10
2 years left							£9	£11
3 years left						£9	£10	£11
n-4 years left				£9	...	£10	£10	£11
n-3 years left			£9	£10	...	£10	£10	£11
n-2 years left		£9	£10	£10	...	£10	£10	£11
n-1 years left	£9	£10	£10	£10	...	£10	£10	£11

Figure 5.12 Best responses for an insurer where a policy renewals each year and costs are known by all firms in the market and policyholder is willing to pay £1 more to their current insurer on renewal

6. PRICE OPTIMIZATION IN THE BROKERAGE MARKET

This section considers the price optimization case for an insurer writing its business through a broker distribution channel, where price elasticity is assumed to be constant. Further research is needed to understand whether the conclusion is extendable to general cases.

6.1 Model Setting

The model rests on the following assumptions:

i. A finite set N_B of n_B brokerage firms;

ii. For each broker $b \in N_B$, a finite set $N_{I,b}$ of $n_{I,b}$ insurance firms writing business through that broker;

iii. A finite set N_D of n_D insurance firms writing business directly to the consumer;

iv. Each firm $i \in N_B \cup N_D$ produces a homogeneous product and sells one unit to each consumer;

v. For each broker or direct insurer $i \in N_B \cup N_D$ a nonempty set S_i of strategic actions available to firm i. These actions are taken to be the gross price gp_i charged to the consumer;

vi. For each insurer $i \in N_{I,b}$ for all $b \in N_B$ a nonempty set S_i of strategic actions available to insurer i. These actions are taken to be the net price $np_{i,b}$ charged to broker *b*;

vii. For each firm, a consequence function $g_i : S \to C$

viii. For each firm, a payoff function $h_i : C \to \mathbb{R}$

ix. Firms do not cooperate;

x. There are no capacity constraints;

xi. For each firm, marginal costs of c_i and no fixed costs;

xii. For each firm, additional profit a_i is derived from selling ancillary products to customers.

6.1.1 Demand and conversion

The demand for each broker or direct insurer is a function of its price as well as the price of all other brokers and direct insurance firms. As before the firm's demand is driven by the conversion rate, being the proportion of consumers purchasing the firm's product given that a quote is made, so we have:

$$Q_i(gp_1, gp_2, ..., gp_{n_B+n_D}) = D \times z_i(gp_1, gp_2, ..., gp_{n_B+n_D}) \tag{6.1}$$

where D is the market demand, z_i is the conversion rate of firm i, and gp_i is the premium charged by firm i, for all $i \in N_B \cup N_D$.

The point-price elasticity e is defined as before and is assumed to be constant.

The demand for an insurer writing business through a broker is different, however. In this case, the broker will select the insurer on its panel which quotes the lowest net premium. Therefore, this is more like the Bertrand competition in nature. This would indicate that the demand function for an insurer i on the panel of broker b would be as follows:

$$Q_{i,b}(np, gp) = \begin{cases} 0 & \text{if } np_{i,b} > \min\limits_{j \neq i}\{np_{j,b}\} \\ \dfrac{D \cdot z_b(gp_1, gp_2, ..., gp_{n_B+n_D})}{n_{\min}} & \text{if } np_{i,b} = \min\limits_{j \neq i}\{np_{j,b}\} \\ D \cdot z_b(gp_1, gp_2, ..., gp_{n_B+n_D}) & \text{if } np_{i,b} < \min\limits_{j \neq i}\{np_{j,b}\} \end{cases} \tag{6.2}$$

where we assume that the broker will share business equally between those insurers who equally quote the lowest net price.

6.1.2 Payoff and Consequences

Each broker $b \in N_B$ will seek to maximize its payoff h_b, which is also taken to be profit:

$$\Pi_b = D \times z_b(gp_1, gp_2, ..., gp_{n_D+n_B}) \times \left(gp_b - np_b' - c_b + a_b\right) \tag{6.3}$$

In equation (6.3) the net premium a broker receives from its panel of insurers may well have already been subject to a competitive tender on the panel and therefore is likely to be at its own equilibrium position.

Each direct insurer $d \in N_D$ will have the following payoff function:

$$\Pi_d = D \times z_d(gp_1, gp_2, ..., gp_{n_D+n_B}) \times (gp_d - c_d + a_d) \quad (6.4)$$

Each insurer $i \in N_{I,b}$ writing business through a broker $b \in N_B$ will seek to maximize its payoff h_i. Assuming that insurers are profit maximizing, then we will have

$$\Pi_{i,b} = Q_{i,b}(np_{1,b}, np_{2,b}, ..., np_{n_b,b}, gp_1, gp_2, ..., gp_{n_B+n_D}) \times (np_{i,b} - c_i + a_i) \quad (6.5)$$

where $D_b = D \times z_b(gp_1, gp_2, ..., gp_{n_D+n_B})$ is the demand generated by the broker.

An insurer writing business through a broker wants to set the net premium np_i in order to maximize its own profit, as set out in equation (6.5). However, the broker's conversion rate z_b is affected by the gross premium set by the broker, which is outside the control of the insurer. It therefore competes with other insurers on the panel to take part in the provision of the policy but then is at the mercy of the broker's actions which determine whether or not the policy is actually sold.

6.2 Analysis

6.2.1 Best Response of Broker

We can derive a solution to the problem by working backwards. The first step is to calculate the best response gross premium for broker *b*, given $np'_{i,b}$ and a_b. This is determined by calculating the partial derivative of equation (6.3) with respect to gp_b. This gives:

$$\frac{\partial \Pi_b}{\partial gp_b} = Dz_b(gp_1, gp_2, ..., gp_{n_D+n_B}) + (gp_b - np'_b - c_b + a_b)\frac{\partial z_b(gp_1, gp_2, ..., gp_{n_D+n_B})}{\partial gp_b} = 0 \quad (6.6)$$

Rearranging equation (5.2) and substituting it into equation (6.6) gives the following:

$$D \cdot z_b + (gp_b - np'_b - c_b + a_b) \cdot \frac{D \cdot e_b \cdot z_b}{gp_b} = 0 \quad (6.7)$$

which gives the following best response gross premium function:

$$gp_b = \frac{(np'_{i,b} + c_b - a_b)e_b}{e_b + 1} \quad (6.8)$$

Depending on the broker's additional profit a_b, the best response gross premium could be higher or lower than the net premium. The function is similar in form to equation (5.15) derived earlier.

Example 6.1. *In the UK motor market, the current market conditions suggest an average net premium of the order of £400 per policy and elasticity is approximately 10. If gross premium is to be less than net premium then* $gp_b = \left(np'_{i,b} - a_b\right)e_b \big/ \left(e_b + 1\right) < np'_{i,b}$. *This leads to the condition that* $a_b > np'_{i,b} \big/ \left(-e_b\right)$. *Given the current market conditions, we would expect that brokers would be willing on average to charge less than the net premium as long as they are able to generate at least £40 in additional profits from ancillary products.* ■

6.2.2 Best Response of Insurer

Having determined the broker's best response, we can now look at the insurer's best response. As with the Bertrand model in Section 3, an insurer's best response will be to charge just less than the minimum price quoted by other insurers on the panel with a minimum of the marginal cost less ancillary profits. This derives from the demand function in equation (6.2). If the insurer does not quote the lowest price then it will not write any business. The Nash equilibrium for insurer i is therefore the price $np^*_{i,b}$ such that

$$\Pi_{i,b}(np^*_{i,b}, \underset{j \neq i}{\left\{np^*_{j,b}\right\}}) \geq \Pi_{i,b}(np_{i,b}, \underset{j \neq i}{\left\{np^*_{j,b}\right\}}) \text{ for all } np_{i,b} \in S_{i,b} \tag{6.9}$$

If all insurers have the same expected marginal costs and ancillary product profits, then this can be shown to occur where $np^*_{i,b} = \underset{j \neq i}{\left\{np^*_{j,b}\right\}} = c_{i,b} - a_{i,b}$.

Note that in this case we are assuming that the gross premium is set independently of the net premium.

6.2.3 Special Case of Insurer with an Exclusive Arrangement with the Broker

There are cases in practice where a broker will have an exclusive arrangement with an insurer such that there is only a single insurer on the panel. We will call such a relationship a 'solus' broker arrangement. In practice, many such arrangements exist, for example, in bancassurance markets where the bank has an agreement with a single insurer for the provision of products to its banking customers. In this case, the insurer is not in competition with other insurers on the panel. In this event the Bertrand model does not apply.

To maximize the profit of the solus insurer, we calculate the partial derivative of equation (6.5) with respect to $np_{i,b}$. This gives:

$$\frac{\partial \Pi_{i,b}}{\partial np_{i,b}} = D \cdot z_b(gp_1, gp_2, ..., gp_{n_B+n_D}) + \left(np_{i,b} - c_{i,b} + a_{i,b}\right) \cdot D \cdot \frac{\partial z_b(gp_1, gp_2, ..., gp_{n_B+n_D})}{\partial np_{i,b}} = 0 \quad (6.10)$$

in which, because of equations (5.2) and (6.8), we have:

$$\begin{aligned}
\frac{\partial z_b(gp_1, gp_2, ..., gp_{n_B+n_D})}{\partial np_{i,b}} &= \frac{\partial z_b(gp_1, gp_2, ..., gp_{n_B+n_D})}{\partial gp_b} \cdot \frac{\partial gp_b}{\partial np_{i,b}} \\
&= \frac{e_b \cdot z_b}{gp_b} \cdot \frac{e_b}{(e_b+1)} \\
&= \frac{e_b^{\ 2} z_b}{\frac{\left(np_{i,b} + c_b - a_b\right) e_b}{e_b+1} \cdot (e+1)} \\
&= \frac{e_b z_b}{\left(np_{i,b} + c_b - a_b\right)}
\end{aligned} \quad (6.11)$$

As a result, we are able to derive the following:

$$\frac{\partial \Pi_{i,b}}{\partial np_{i,b}} = D \cdot z_b + \left(np_{i,b} - c_{i,b} + a_{i,b}\right) D \cdot \frac{e_b z_b}{\left(np_{i,b} + c_b - a_b\right)} = 0 \quad (6.12)$$

which gives the following function:

$$np_{i,b} = \frac{a_b - c_b + e_b\left(c_{i,b} - a_{i,b}\right)}{e_b + 1} \quad (6.13)$$

Substituting equation (6.13) into equation (6.8) gives:

$$gp_b = \frac{\left(c_{i,b} + c_b - a_{i,b} - a_b\right) e_b^{\ 2}}{\left(e_b + 1\right)^2} \quad (6.14)$$

6.2.4 Best Response of Direct Insurer

For a direct insurer, which sells business directly to consumers, the best response function can be determined in a similar manner, by differentiating equation (6.4) and setting to nil, which can be shown to give the following:

$$gp_d = \frac{\left(c_d - a_d\right) e_d}{e_d + 1} \quad (6.15)$$

6.3 Observations and Insights

We use the models above to consider some interesting observations of the relationships between brokers, insurers and direct insurers.

We can make the following immediate observations:

i. The best response of the direct insurer is to charge a price which is equal to expected marginal costs less expected ancillary profits, adjusted for elasticity, which is a function of other prices quoted in the market. The best response of the broker is similar, with the exception that the expected marginal costs are largely the net premium charged by its panel. Using the findings of the Hotelling model in section 3 and appendix B to interpret the results, the equilibrium price is likely to occur at the marginal costs less expected ancillary profits (or some weighted average if these are not equal between firms) plus a loading for differentiation (which is modeled within the elasticity). The outcome is therefore very similar to the situation of a market with only direct insurers.

ii. A firm (broker or direct insurer) that is able to generate greater ancillary product profits, all else being equal, will have a strategic advantage and be able to underprice its competitors to demand a higher market share and larger profits.

iii. The best response of an insurer on a broker's panel is to charge a net premium that is just less than those charged by others on the panel. This leads to an equilibrium price equal to the expected marginal costs less expected ancillary profits. This means that insurers on the panel have little market power and will be unable to demand a share of any excess profits that the broker is able to create, for example through differentiation. The Bertrand paradox applies here and perfect competition is possible with only two insurers on the panel.

iv. In a solus broker arrangement, the best response of the solus insurer is also dependent on the broker's ancillary profit and elasticity. The solus insurer is in a more powerful position being the only insurer on the panel and, therefore, demands a share of both the broker's ancillary profits and any excess profits from differentiation.

We now explore a few further hypotheses based on the interaction between the brokers, their insurers, and direct insurance companies.

Hypothesis 6.1. *Solus-insurer panel broker arrangements do not achieve profits which are superior to that of a direct insurer without some source of competitive advantage or differentiation. In fact, such arrangements place the broker at a disadvantage to other firms in the market.*

Discussion: The brokers and direct insurers compete in the market for the business from the same consumers. If there is no differentiation between these firms and marginal costs and ancillary profits are equal, then the equilibrium price will be equal to the marginal costs less ancillary profits, as per the Bertrand model. Where there is a difference in the marginal costs or ancillary profits, or some degree of differentiation between products, then the equilibrium price will settle at an average of marginal costs less ancillary profits plus a differentiation loading (as per the Hotelling model or our model developed in section 5). The firm with the lowest marginal costs, highest ancillary profits and/or highest differentiation loading will be able to set its price lower than other participants and, therefore, command a higher market share and larger profits.

Where there is sufficient competition on the broker's panel, there is no difference in the determination of the equilibrium price and resulting profits between a broker and a direct insurer. This is evident from a comparison of equations (5.13), (6.8) and (6.15). Where a broker has a panel with at least two insurers (from the Bertrand paradox), and the lowest net price is selected, then the Betrand model applies and the equilibrium net premium will result in no profit for the insurers.

However, when a broker implements a solus insurer arrangement, it increases its marginal costs because the solus insurer is able to demand a portion of the brokers' ancillary profits and any differentiation loading. This puts the broker at a disadvantage to other firms in the market; it needs to increase its price and therefore achieves a lower market share and smaller profits. Other firms in the market are able to set price slightly below those of the solus brokerage group and command a greater share of the market and superior profits at the disadvantage of the solus arrangement. Compared to a direct insurer or a competitive broker arrangement, the solus broker arrangement will always result in an equilibrium price *higher* than the other firms and correspond to a smaller market share. There is therefore a strategic cost to the broker were it to enter a solus insurance arrangement. Appendix B sets out a simple example highlighting this effect.

The direct insurance firm is able to optimize its profits across the distribution and manufacturing operations, while the solus brokerage operation optimizes profits for these separately. This overarching view puts it at an advantage over the solus brokerage arrangement.

This effect is exacerbated in a non-elastic market. So, in a period of hardening rates, for example, when the market is generally less elastic, the direct insurer will be able to increase its profits further by offering a higher premium while still maintaining the same market share. Figure (6.1) shows the equilibrium prices for a solus broker and direct insurer for different values of elasticity, all other items being constant.

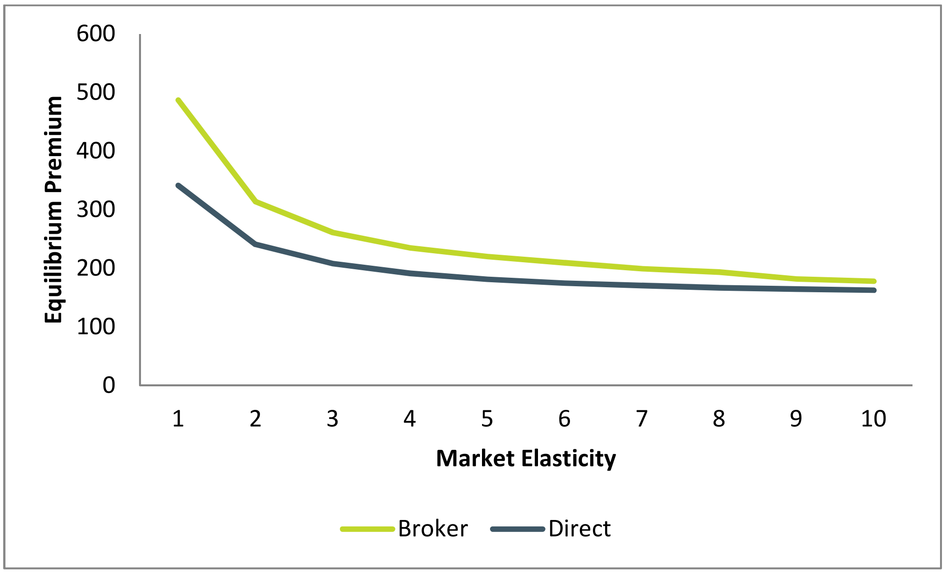

Figure 6.1 Equilibrium prices for varying elasticity coefficient values for each of a solus broker and a direct insurance firm

In a less elastic market conversion rates are higher, given the same price. Consumers may be less likely to shop around and for a given change in premium there is much lower impact on conversion in a low elasticity environment. In these times, the solus insurer will still command the larger proportion of the increased profits arising from increasing prices, which it does by charging a higher net premium to the broker. This results in the broker increasing its price by a greater proportionate amount than the direct insurer in a less elastic market. The increase in price by the broker allows the direct insurer to increase its own price in response.

As the elasticity of the market increases and approaches a perfectly competitive market, the solus broker and direct insurer premiums tend to converge.

A broker and the insurers on its panel all act non-cooperatively, i.e. they each will take the strategic action which is to their own individual best interests (usually the action which maximizes profits) even if this is at the detriment to the whole group.

i. Assuming that there are no side agreements and products/services offered by the panel insurers are homogeneous, then the broker will maximize its profits by selecting the lowest net price offered on the panel and charging a gross price which considers the potential rating actions of other firms and its price-demand elasticity.

ii. The broker will also set its gross price to not be less than its marginal costs, including the net price charged by its panel, offset by expected ancillary product profits.

iii. If the panel is not competitive, then the single panel insurer is able to charge a higher net premium and still win the business from the panel. The insurer will want to increase its price to increase its own profits, but will be conscious that increasing the net price will start reducing the demand of consumers buying from the broker. So, the insurer will increase its net price, which results in the broker raising its gross price in response. This in turn leads to other participants in the market increasing their gross prices. (In example B.2. in appendix B, if the solus insurer increases its net price from £70 to £82, then the best response of the broker will be to increase its gross price from £90 to £96 to make up for the lost profit. The direct insurer is then able to increase its own price from £90 to £93 while still gaining some additional demand. The broker in turn increases its price to respond to reduced profits, and so on, until equilibrium is reached where the broker charges £98 and the direct insurer charges £94.)

iv. Where the broker has a single panel insurer, the resulting higher net price results in the broker being at a strategic disadvantage to other market participants who are able to set their gross price below that of the broker. The broker will, therefore, need to settle for a smaller share of the market and pass the greater proportion of the profits to its panel insurer. This is evident from example (B.2).

So the single panel insurer has the strategic advantage over the broker. The insurer acts non-cooperatively in order to maximize its own profits, which in turn leads to reduced profits from the broker. This strategic interaction between the broker and its panel insurer, however, benefits the other participants in the market who are able to charge more for their own product as a result.

Finally, it is worth noting that the consumer also loses out as the product becomes more expensive to buy. ■

Hypothesis 6.2. *A broker with a solus insurer arrangement could take some strategic moves to change the game and avoid the strategic costs of the standard solus arrangement. Sometimes is pays to be the little guy.*

Discussion: As discussed earlier, the broker and its single panel insurer act non-cooperatively. The broker could take some strategic moves in order to change the game between it and the panel insurer to its own advantage. Strategic moves were discussed in section 4.3.

Firstly, the broker could seek to employ a mechanism which ensures that the panel insurer charges the minimum possible net price to enable the broker to compete with the rest of the market on at least equal terms. However, other moves are possible which may give the broker an even better outcome. We discuss below some possible strategic moves which could result in an improved position for the broker. Note that we ignore any transactional costs in this example, as we have done elsewhere in the paper. Other moves may also exist and the below is not intended to be exhaustive.

i. Where the cost of the product can be estimated in advance, the broker could put the appointment of the solus insurer to an auction in advance. The winning insurer will be the firm that commits to offering the lowest price given the stipulated minimum service levels. A legally-enforceable contract with suitable penalties will need to be put in place to ensure that the insurer sticks to its commitment. This ensures sufficient competition at the first stage of the new game to ensure that the broker is able to derive the cheapest net price possible in the second stage. In example (B.2), such a mechanism would result in the broker being able to achieve the maximum profits of £50,000. However, this mechanism would not deal efficiently with a very dynamic or competitive market where there are regular movements in the market price or where it is difficult for the insurer to commit to a price in advance.

ii. The broker could implement a profit share arrangement, whereby the profits made by the broker and the panel insurer are combined and shared in an agreed manner.

   a. Such an arrangement may not necessarily change the behavior of the insurer, however. For instance, in example (B.2) the combined profits of the broker and panel insurer was £18,000 + £36,000 = £54,000, which was larger than the £50,000 profit from a fully competitive panel. Since the insurer would prefer a portion of the larger total rather than the smaller, it is likely to behave in the same way as if there was no profit share arrangement.

   b. The broker in example (B.2.) could contract with the insurer in advance by making the following offer: all profits made by the insurer will be shared with 35/36 going to the broker and 1/36 to the insurer; otherwise, if the insurer does not accept, then the broker will implement a fully competitive panel. The insurer would accept this offer because it can expect to make total profits of £36,000 of which it will retain £1,000 and give £35,000 to the broker. The broker can therefore expect to make profits of £18,000 + £35,000 = £53,000, which is better than if had a fully competitive panel (although with a smaller market share). If the insurer does not accept the offer, then it can expect the broker to implement the fully competitive panel, in which event the insurer will make profits of £0 and the broker will make £50,000. In this event, the broker is smaller than its direct insurance competitor but is able to achieve greater profits than if it tried to command a larger market share; i.e. it is better off being the small guy.

c. Theoretically, the broker could engineer an even better outcome by putting the profit share arrangement to auction. In this event, with a sufficient number of bidders, the broker should expect to get all of the expected profit.

There are therefore a number of strategic moves that the broker can use to manipulate the rules of the game and generate a superior return for itself. Interestingly, the broker may want to limit its pricing options by allowing the panel insurer to charge a high net premium, but then demand the largest share of the resulting profit. This would give a more lucrative outcome than if it had entered into full competition on the lowest net price. This outcome will also benefit its competitors, but cost consumers. ■

Hypothesis 6.4. *A broker should ensure that it has a minimum of two insurers quoting non-cooperatively on its panel to avoid the strategic costs. Should a Broker always have a comprehensive panel versus a limited panel or solus arrangement?*

Discussion: If a broker has a competitive panel it effectively forces the panel insurers to price at cost and make zero profit (as per the Bertrand model). A panel arrangement of at least two insurers does conform to the simple Bertrand model. If any particular insurer is more expensive than the other then demand for that insurer is zero. Products and service are homogenous and since the product is sold through the broker, the insurer's brand and other product differentiators are irrelevant. The broker therefore selects the insurer on the panel with the lowest net premium.

While we have limited the discussion so far to the consideration of a homogenous target market segment, in reality insurers on the panel will be quoting for policies for consumers in a variety of target segments. The broker will therefore need to ensure that there are a sufficient number of insurers quoting in each segment. Also, the broker will need to ensure that insurers on the panel do not collude, for example by agreeing not to compete against each other in certain segments.

Insurers on the panel will also need to be considerate of the “winner’s curse” (Rothwell, et al., 2009) and in some cases may have more information about the risks for which they are quoting, enabling them to derive a more accurate estimate of the marginal costs. While not explored further here, there are some other concepts from auction theory discussed in Dixit, Skeath, & Reiley (2009) which could help insurers and the broker think through the design and optimal responses in such a competitive panel.

Figure (6.2) sets out a representation of the market coverage of a broker’s panel which is perfectly complementary. In this case, the broker will have at least one insurer returning a net premium for each application it receives. While this setup is better than the case in which the broker has an incomplete panel and is missing out on quoting for some applications, as shown in figure (6.3), having only one insurer quoting for some applications for insurance is not necessarily optimal for the broker since it is essentially acting as a solus broker arrangement for those particular segments.

Rather a more optimal set up for a broker would be where at least two insurers on the panel are competing for each application of insurance. This is shown in figure (6.4). In this setup the Bertrand paradox will take effect and the broker can be assured of the best net price on each application.

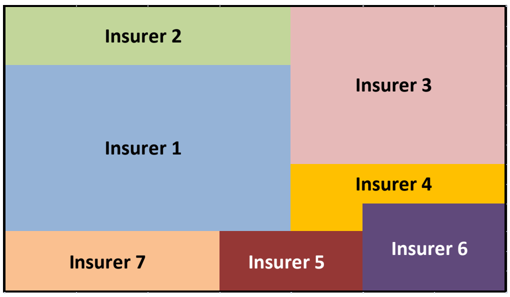

Figure 6.2 Representation of a perfectly complementary broker panel where there is at least one insurer quoting for each application for insurance

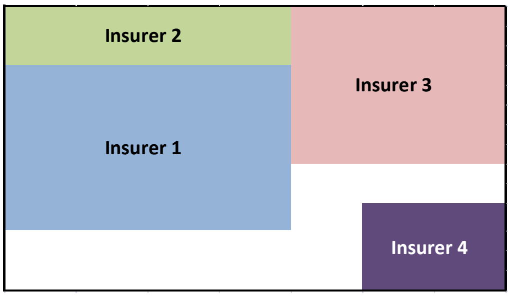

Figure 6.3 Representation of an incomplete broker panel where there are some applications for insurance where there are no quotes being offered from the panel insurers

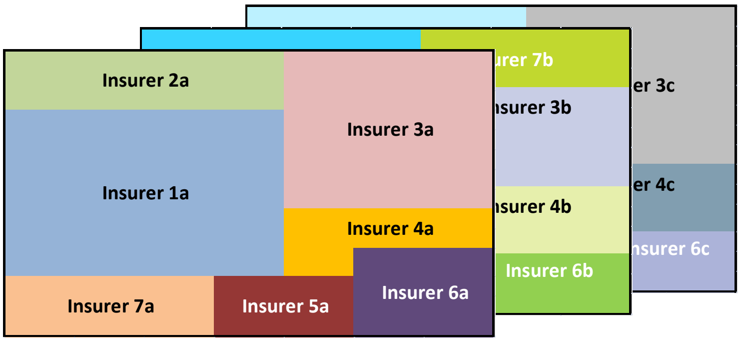

Figure 6.4 Representation of a perfectly complementary broker panel where there is at least two insurers quoting for each application for insurance

Hypothesis 6.5. *In the event that firms are prevented from generating excessive margins from ancillary products, the price of the base product will increase and those firms who were able to generate larger ancillary profits will lose their competitive advantage. Consumers would not necessarily be better off.*

Discussion: In the UK personal lines market, it is common for insurers and brokers to make large profit margins on the ancillary products, which are usually sold to consumers as add-on products after the base product has been sold. This means that consumers will often select the product before knowing about what additional products may be offered or what they may cost. This lack of competition when purchasing these ancillary products means that the margins can be much higher.

In the event that an outside authority, such as the OFT in the UK for example, should regulate the margins on ancillary products or the manner in which these are sold, then this would change the manner in which brokers and insurers have been setting price. The models developed indicate that the ancillary profits have the effect of reducing the marginal costs for each firm. Firms therefore will use this expected profit from the ancillary products to reduce the price charged for the base policy. This suggests that a reduction in the ancillary profit will lead to an increase in the price of the base policy. This means that consumers would not necessarily be better off with such a change.

However, such a change may change which participants in the market have strategic advantages over others. For example, if a firm is better at selling on ancillary products and therefore has higher ancillary profits than its peers, it will have lower overall net marginal costs and therefore be able to undercut the others. Removing the ability of the firm to capitalize on its capability of selling ancillary products better than others will handicap the firm strategically.

7. CONCLUSIONS

This section summarizes some of the key results and conclusions arising from the paper.

Strategic decisions, including the setting of price, that do not consider what options competitors have or choices competitors are making and how competitors may react to the choices you make will be suboptimal at best. Firms should not discard this valuable data. Game theory provides useful tools and frameworks to do this.

The basic principle of game theory is to determine the expected payoffs for each player under each option and for each choice made by the other players. Rational players will be aware of their alternatives, form expectations of any unknowns, and will choose their actions deliberately so as to maximize their expected payoff. We can therefore determine each player's best response strategies which give the best payoff given the other players' actions. Ideally, one or more Nash equilibrium is present, which is a solution to the game, where each player selects their best response action and no player will be able to improve their payoff by changing their response, given the strategies of the other players.

Most insurance markets can be considered as oligopolistic. Such markets are particularly susceptible to analysis through game theory. Insurance companies also usually execute strategy by setting price and then customers decide whether to buy. The application of game theory does not replace the basic principles of economics, which should still apply. Economic theory suggests that firms will set the price at the point where marginal revenue equals marginal costs because that will maximize profits. In the short run, a firm will shut down if price is below average variable costs, and in the long run if price is below average fixed costs.

The Bertrand price competition model (Bertrand, 1883) shows that in a competitive market where consumers select products purely on the lowest price, the equilibrium will occur where price is equal to the marginal cost, i.e. $p_A^* = p_B^* = c$ (where firms have the same costs). The model results in the Bertrand Paradox – it takes only two firms to obtain (perfect) competition.

Hotelling's model (Hotelling, 1929) allows for differentiated products and the fact that some purchasers of a product will buy from one seller and others from another seller in spite of differences in price. The model shows that when firms have the same costs the Nash equilibrium occurs at $p_A^* = p_B^* = c + k$. There is no Bertrand Paradox with differentiated products. When the products are more differentiated (i.e. larger k) then prices are higher. When $k = 0$ then the model approaches Bertrand competition with homogeneous products. Where costs differ between firms, then the equilibrium price will be some weighted average of the costs and *k*. An intuitive interpretation of *k* is that it is the amount in excess of the price charged by other sellers that a firm's most ardent supporter is willing to pay for that firm's product.

We developed an application of these models for insurance companies under the impending changes in gender discrimination law in Europe. This showed that if firms were prevented from charging different prices to men and women, all else being equal, and if there is a difference in the costs underlying each gender, then the new equilibrium price will simply be an average of the previous individual prices for men and women. Also, there will be no change in the market shares or profitability of the participants in the market. However, if a firm is able to introduce some innovation to make itself more attractive to women, then it will be able to outdo its adversaries by commanding a greater market share and larger profits. Such a firm will also have the opportunity to use various other strategic moves, being commitments, threats and promises, to manipulate the rules of the game. In our example we showed a firm using competitor price matching to increase the equilibrium price, its market share and its profits, even further. To be credible, these strategic moves need to be observable and irreversible.

We developed a model for insurance business that uses elasticity and demand modeling rather than the customer utility interval of Hotelling's model. We also incorporated lifetime value to allow for profits on future years. However, the existence of profit in future years is dependent on a policyholder willing to pay more for the incumbent insurer's policy on renewal than moving to a new insurer, i.e. having a greater utility cost arising from moving insurer. Also, assuming that consumers are aware of what other insurers are quoting when their policy is up for renewal, the insurer should only discount the price in the first year of the policy's life (as new business) – competition with other insurers will mean that charging more than the marginal costs plus any normal differentiation (or utility) loading in interim years will result in the business being lost (i.e. the insurer ends up charging the equilibrium amount the insurer would charge in the normal single period game). The firm will only be able to charge more than the marginal costs and differentiation loading in the final year. Where consumers do not revisit their purchase decision at renewal, then the insurer can get away with charging more (although that interaction itself could be considered as a game). Discounting for future profits is a complex process and can be costly to an insurer if it gets it wrong.

Insurance companies that are able to generate additional profits from ancillary products, which are sold to customers after they have purchased the main policy and in a non-competitive environment, will use this to discount the cost of the base product. This reduces the equilibrium price of the base policy. Should an outside authority prevent insurers from selling these ancillary products in such a manner, the equilibrium price of the base policy will increase by a corresponding amount; consumers will not necessarily be better off in this situation. However, firms who are able to sell a higher amount of ancillary products than others will lose their strategic advantage.

Our model shows that the equilibrium price occurs at some average of the marginal costs less expected ancillary product profits less expected future lifetime profits plus a differentiation loading based on elasticity. The complexity of the elasticity models means that the solution needs to be derived numerically.

We applied the model to a market in which there is a broker, its insurers, and direct insurance firms. This shows that where the broker panel comprises more than one insurer, then the broker and direct insurers will compete for business given their individual marginal costs and any differentiation advantages, which will determine the profits that each makes. On the other hand, the insurers on each broker panel will be subject to the Bertrand paradox and therefore make nil profits. A broker who chooses to have a single insurer on its panel will incur a strategic cost, however; the solus insurer will increase its net price, the broker's marginal costs will be higher and it will have to increase its costs in response. This allows others in the market to increase their price but to a lesser degree. The net effect is that the broker will lose market share and profits, while the insurer and the direct insurer will benefit. Consumers will lose out as the equilibrium price increases. We show however that such a broker can actually engineer an outcome which is even better than it would have achieved under a perfectly competitive panel situation by the appropriate use of strategic moves (commitments, promises and threats). In our example, the broker contracts only with its panel insurer, but by limiting its options it is able to increase the market's equilibrium price. While the broker is only able to achieve a smaller market share than before, it is actually able to achieve greater profits than if it had competed for a larger market share with other market participants on the same terms – sometimes it pays to be the little guy.

APPENDIX A

Proof of Gender Directive Game Nash Equilibrium. The calculations below show how the equilibrium prices are derived in equations (4.8) and (4.9)

The two best response functions are:

$$2\left(k_m + k_f\right) p_A = p_B\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)$$

and

$$2\left(k_m + k_f\right) p_B = p_A\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)$$

$$p_A\left(k_m + k_f\right) = 2\left(k_m + k_f\right) p_B - k_m c_f - k_f c_m - k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)$$

$$p_A = \frac{2\left(k_m + k_f\right) p_B - k_m c_f - k_f c_m - k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)}{\left(k_m + k_f\right)}$$

Equating these two equations gives:

$$\begin{aligned} p_B\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right) &= 4\left(k_m + k_f\right) p_B - 2k_m c_f - 2k_f c_m \\ &\quad - 2k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right) \end{aligned}$$

$$\begin{aligned} 3\left(k_m + k_f\right) p_B &= k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right) + 2k_m c_f + 2k_f c_m \\ &\quad + 2k_m k_f\left(4 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right) \end{aligned}$$

$$p_B^* = \frac{3k_m c_f + 3k_f c_m + k_m k_f\left(8 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)}{3\left(k_m + k_f\right)}$$

and

$$2\left(k_m + k_f\right) p_A = p_B\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)$$

$$= \frac{3k_m c_f + 3k_f c_m + k_m k_f\left(8 - l_{mA} - l_{mB} - l_{fA} - l_{fB}\right)}{3\left(k_m + k_f\right)}\left(k_m + k_f\right) + k_m c_f + k_f c_m + k_m k_f\left(l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)$$

$$p_A^* = \frac{3k_m c_f + 3k_f c_m + k_m k_f\left(4 + l_{mA} + l_{mB} + l_{fA} + l_{fB}\right)}{3\left(k_m + k_f\right)}$$

■

APPENDIX B

Application of Hotelling Model to Solus Brokerage Arrangement. The calculations below show how the equilibrium prices are derived for a solus brokerage arrangement under the Hotelling model assuming that the market has three firms, being an insurer *I*, which writes business through a broker *B*, and a direct insurer *D*, which writes business directly to the consumers.

The profit functions for each firm are as follows:

$$\Pi_D = \left(p_D - c_D + a_D\right) \cdot \left(\frac{p_B - p_D + k}{2k}\right) \tag{8.1}$$

$$\Pi_B = \left(p_B - c_B - np_I + a_B\right) \cdot \left(\frac{p_D - p_B + k}{2k}\right) \tag{8.2}$$

$$\Pi_I = \left(np_I - c_I + a_I\right) \cdot \left(\frac{p_B - p_D + k}{2k}\right) \tag{8.3}$$

where the broker and direct insurer are located on the extremes of the unit interval $[0,1]$ which spatially represents consumer preferences, k is the consumer's preference utility cost, p is the gross premium, c is the marginal costs, a is the profits from ancillary products, and np is the net premium charged by the brokerage insurer.

Partial derivatives of each of these profit functions give, for the direct insurer:

$$\begin{aligned} \frac{\partial \Pi_D}{\partial p_D} &= 0 \\ \left(\frac{p_B - p_D + k}{2k}\right) + \left(p_D - c_D + a_D\right) \cdot \left(\frac{-1}{2k}\right) &= 0 \\ \left(p_B - p_D + k\right) - \left(p_D - c_D + a_D\right) &= 0 \\ p_D &= \frac{p_B + c_D - a_D + k}{2} \end{aligned} \tag{8.4}$$

for the broker:

$$\begin{aligned}
\frac{\partial \Pi_B}{\partial p_B} &= 0 \\
\left(\frac{p_D - p_B + k}{2k}\right) + \left(p_B - c_B - np_I + a_B\right)\cdot\left(\frac{-1}{2k}\right) &= 0 \\
\left(p_D - p_B + k\right) - \left(p_B - c_B - np_I + a_B\right) &= 0 \\
p_B &= \frac{p_D + c_B + np_I - a_B + k}{2}
\end{aligned} \tag{8.5}$$

and for the brokerage insurer:

$$\begin{aligned}
\frac{\partial \Pi_I}{\partial np_I} &= 0 \\
\left(\frac{p_D - p_B + k}{2k}\right) + \left(np_I - c_I + a_I\right)\cdot\left(\frac{-1}{2k}\right)\cdot\frac{\partial p_B}{\partial np_I} &= 0 \\
\left(\frac{p_D - p_B + k}{2k}\right) + \left(np_I - c_I + a_I\right)\cdot\left(\frac{-1}{2k}\right)\cdot\frac{1}{2} &= 0 \\
\left(p_D - p_B + k\right) - \left(\frac{np_I - c_I + a_I}{2}\right) &= 0 \\
np_I - c_I + a_I &= 2\left(p_D - p_B + k\right) \\
np_I &= c_I - a_I + 2\left(p_D - p_B + k\right)
\end{aligned} \tag{8.6}$$

The Nash equilibrium will be at the intersection of equations (8.4), (8.5) and (8.6), which is done as follows.

Substituting equation (8.5) into (8.4) gives

$$\begin{aligned}
p_D &= \frac{p_B + c_D - a_D + k}{2} \\
2p_D &= \frac{p_D + c_B + np_I - a_B + k}{2} + c_D - a_D + k \\
4p_D &= p_D + c_B + np_I - a_B + k + 2c_D - 2a_D + 2k \\
p_D &= \frac{\left(c_B + np_I\right) + 2c_D}{3} - \frac{a_B + 2a_D}{3} + k
\end{aligned} \tag{8.7}$$

Substituting equation (8.6) into (8.7) gives

$$
\begin{aligned}
p_D &= \frac{(c_B + np_I) + 2c_D}{3} - \frac{a_B + 2a_D}{3} + k \\
3p_D &= c_B + \left(c_I - a_I + 2(p_D - p_B + k)\right) + 2c_D - (a_B + 2a_D) + 3k \qquad (8.8) \\
p_D &= c_B + c_I + 2c_D - a_I - a_B - 2a_D - 2p_B + 5k
\end{aligned}
$$

Substituting equation (8.8) into (8.4) gives

$$
\begin{aligned}
p_D &= \frac{p_B + c_D - a_D + k}{2} \\
\frac{p_B + c_D - a_D + k}{2} &= c_B + c_I + 2c_D - a_I - a_B - 2a_D - 2p_B + 5k \qquad (8.9) \\
p_B + c_D - a_D + k &= 2c_B + 2c_I + 4c_D - 2a_I - 2a_B - 4a_D - 4p_B + 10k \\
p_B^* &= \frac{2c_B + 2c_I + 3c_D - 2a_I - 2a_B - 3a_D + 9k}{5}
\end{aligned}
$$

Substituting equation (8.9) into (8.4) gives

$$
\begin{aligned}
p_D &= \frac{p_B + c_D - a_D + k}{2} \\
2p_D &= \frac{2c_B + 2c_I + 3c_D - 2a_I - 2a_B - 3a_D + 9k}{5} + c_D - a_D + k \qquad (8.10) \\
10p_D &= 2c_B + 2c_I + 3c_D - 2a_I - 2a_B - 3a_D + 9k + 5c_D - 5a_D + 5k \\
p_D^* &= \frac{c_B + c_I + 4c_D - a_I - a_B - 4a_D + 7k}{5}
\end{aligned}
$$

Substituting equations (8.10) and (8.9) into (8.6) gives

$$
\begin{aligned}
np_I &= c_I - a_I + 2(p_D - p_B + k) \\
np_I &= c_I - a_I + 2\Bigg(\frac{c_B + c_I + 4c_D - a_I - a_B - 4a_D + 7k}{5} \\
&\qquad - \frac{2c_B + 2c_I + 3c_D - 2a_I - 2a_B - 3a_D + 9k}{5} + k\Bigg) \qquad (8.11) \\
np_I^* &= \frac{3c_I - 2c_B + 2c_D}{5} - \frac{3a_I - 2a_B + 2a_D}{5} + \frac{6k}{5}
\end{aligned}
$$

■

Example B.1. *There are two firms, Stark Insurance and Nakatomi Trading Corp, participating in a market providing insurance to a segment of consumers who are homogenous in terms of risk. The market is made up of 10,000 consumers. Prices are set simultaneously. The insurance policies have the same benefits, terms and excesses. Research undertaken by the two firms showed that some consumers were willing to pay £10 more for one firm's policy than the other, and vice versa. The expected marginal cost for each firm is the same at £100 per policy and both firms expect to sell ancillary products to each customer which will give rise to £20 additional profit per policy.*

Solution: Adjusting equations (3.15) and (3.16) to allow for the expected ancillary product profits, where $k=10$, $c_S = c_N = 100$, $a_S = a_N = 20$, $l_S = 0$ and $l_N = 1$, we get the equilibrium prices $p_S^* = p_N^* = 100 - 20 + 10 = 90$.

At these prices, equations (3.9) and (3.10) suggest that each firm will have a market share of 50%, i.e. $Q_S^* = Q_N^* = 5{,}000$. Similarly, equations (3.11) and (3.12) suggest that the firms will make profits of $\Pi_S^* = \Pi_N^* = 5{,}000(90 - 100 + 20) = 50{,}000$. ■

Example B.2. *Nakatomi Trading Corp, decides to outsource the underwriting of the insurance policies to Globex Insurance so that it can focus on the distribution. The expected marginal cost for Globex is £75 per policy and Nakatomi's marginal cost reduces to £25 per policy. Globex expects to sell ancillary products to each customer which will give rise to £5 additional profit per policy while Nakatomi expects to make £15 per policy on ancillary products.*

Solution: Applying equations (8.9), (8.10) and (8.11), where $k=10$, $c_S = 100$, $a_S = 20$, $c_N = 25$, $a_N = 15$, $c_G = 75$ and $a_G = 5$, we get the equilibrium prices

$$p_S^* = \frac{75 + 25 + 4(100) - 5 - 15 - 4(20) + 7(10)}{5} = 94$$

$$p_N^* = \frac{2(75) + 2(25) + 3(100) - 2(5) - 2(15) - 3(20) + 9(10)}{5} = 98$$

$$np_G^* = \frac{3(75) - 2(25) + 2(100)}{5} - \frac{3(5) - 2(15) + 2(20)}{5} + \frac{6(10)}{5} = 82$$

At these prices, we can use equations (3.9) and (3.10) to calculate that Stark Insurance will have a 70% market share and Nakatomi Trading Corp will have a 30% market share, i.e. $Q_S^* = 5,000 \times (98 - 94 + 10)/2(10) = 3,500$ and $Q_N^* = 1,500$. Similarly, equations (8.1) , (8.2) and (8.3) suggest that the firms will make the following profits:

$$\Pi_S^* = (94 - 100 + 20) \cdot (7,000) = 98,000$$

$$\Pi_N^* = (98 - 82 - 25 + 15) \cdot (3,000) = 18,000$$

$$\Pi_G^* = (82 - 75 + 5) \cdot (3,000) = 36,000$$

In this case, because Nakatomi decides to outsource its underwriting to a single insurer, Globex, it increases its marginal costs from £100 to £107 and reduces its ancillary profits from £20 to £15 (a net effect of £12 per policy). Nakatomi increases its price as a result, which in turn leads Stark Insurance to increase its price.

There is therefore a strategic cost to Nakatomi of implementing a solus arrangement with Globex. The total profit from the combined business is shared two thirds to Globex and one third to Nakatomi. It is important to note that this is purely a strategic effect relating to the position of the parties in the market structure, rather than any compensation for the amount of risk taken by either party.

Globex is in a powerful position and demands the larger share of profits from the brokerage arrangement. If Nakatomi had put the underwriting of its policies out to tender by having, say, two insurers on its panel, then the Bertrand model would suggest that the two insurers would charge a net price of £70 $= 75 - 5$. As a result, Nakatomi would see no change in its marginal costs net of ancillary profits. In this event, the equilibrium prices would remain at £90 each, Nakatomi and Stark would both make profits of £50,000 each and the insurers on the panel would make £0 profit. ■

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Abbreviations and notations

Collect here in alphabetical order all abbreviations and notations used in the paper

AC, average costs

AFC, average fixed costs

AVC, average variable costs

FC, fixed costs

GLM, generalized linear models

MC, marginal costs

Π , denotes profit

p , price

q , quantity

p^* , denotes equilibrium price

$\mathbb{R}$, denotes the set of real numbers