The Actuarial Profession
making financial sense of the future

GIRO Conference and Exhibition 2012

The Munich chain ladder: *a personal view*

Anthony Wright

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

Reference:

Quarg, Gerhard, and Thomas Mack, "Munich Chain Ladder: A Reserving Method that Reduces the Gap between IBNR Projections Based on Paid Losses and IBNR Projections Based on Incurred Losses," *Variance* 2:2, 2008, pp. 266-299.

Web link:

http://www.variancejournal.org/issues/?fa=article&abstrID=6514

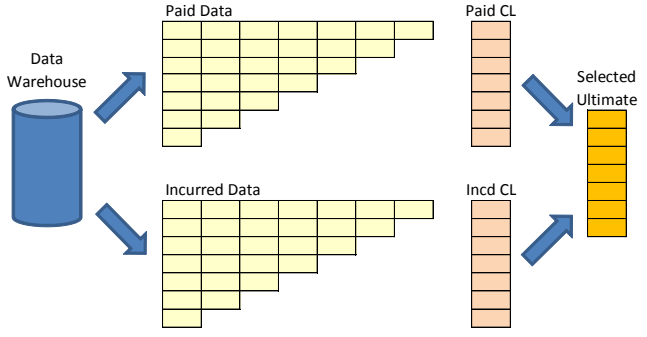
© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

- A problem -
  paid and incurred chain ladders don't agree
- A solution –
  the Munich Chain Ladder
- In practice...

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder
Quarg & Mack's example data

Data Warehouse

Paid Data

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	
1,412	3,758	4,252	4,416	4,494		
2,286	5,292	5,724	5,850			
1,868	3,778	4,648				
1,442	4,010					
2,044						

Incurred Data

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	
2,904	4,354	4,698	4,600	4,644		
3,502	5,958	6,070	6,142			
2,812	4,882	4,852				
2,642	4,406					
5,022						

Paid CL

2,131
2,380
4,652
6,182
5,056
4,934
6,128

Incd CL

2,174
2,445
4,582
6,126
4,839
4,476
8,429

Selected Ultimate

?

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder
P/I ratios

Paid Triangle

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	
1,412	3,758	4,252	4,416	4,494		
2,286	5,292	5,724	5,850			
1,868	3,778	4,648				
1,442	4,010					
2,044						

Incurred Triangle

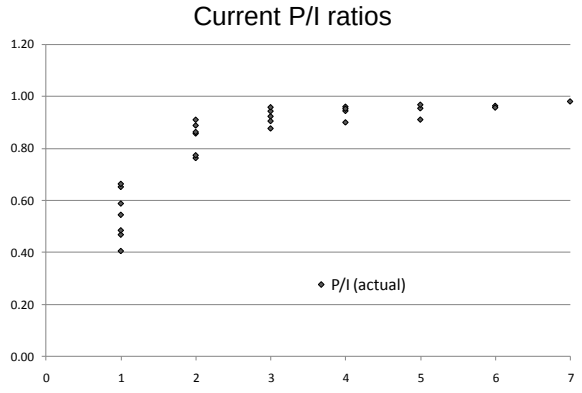
978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	
2,904	4,354	4,698	4,600	4,644		
3,502	5,958	6,070	6,142			
2,812	4,882	4,852				
2,642	4,406					
5,022						

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	
0.49	0.86	0.91	0.96	0.97		
0.65	0.89	0.94	0.95			
0.66	0.77	0.96				
0.55	0.91					
0.41						

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder
P/I ratios

Paid Quadrangle

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	2,380
1,412	3,758	4,252	4,416	4,494	4,589	4,652
2,286	5,292	5,724	5,850	5,971	6,097	6,182
1,868	3,778	4,648	4,784	4,884	4,987	5,056
1,442	4,010	4,536	4,669	4,766	4,867	4,934
2,044	4,981	5,634	5,800	5,920	6,045	6,128

Incurred Quadrangle

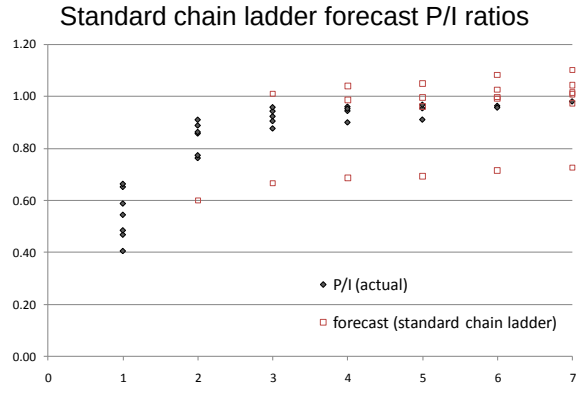
978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	2,445
2,904	4,354	4,698	4,600	4,644	4,598	4,582
3,502	5,958	6,070	6,142	6,210	6,149	6,126
2,812	4,882	4,852	4,851	4,905	4,857	4,839
2,642	4,406	4,488	4,488	4,537	4,493	4,476
5,022	8,297	8,451	8,450	8,544	8,460	8,429

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	0.97
0.49	0.86	0.91	0.96	0.97	1.00	1.02
0.65	0.89	0.94	0.95	0.96	0.99	1.01
0.66	0.77	0.96	0.99	1.00	1.03	1.04
0.55	0.91	1.01	1.04	1.05	1.08	1.10
0.41	0.60	0.67	0.69	0.69	0.71	0.73

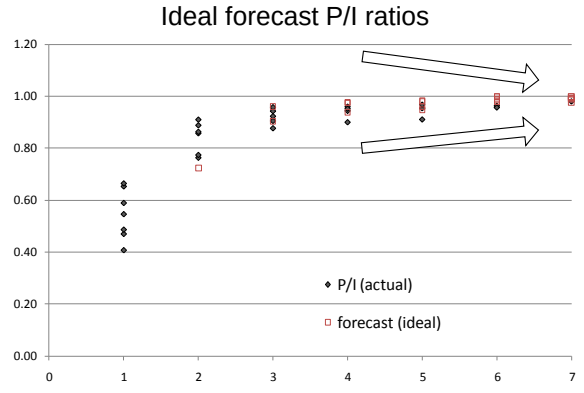
© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder
I/P ratios

Paid Triangle

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	
1,412	3,758	4,252	4,416	4,494		
2,286	5,292	5,724	5,850			
1,868	3,778	4,648				
1,442	4,010					
2,044						

Incurred Triangle

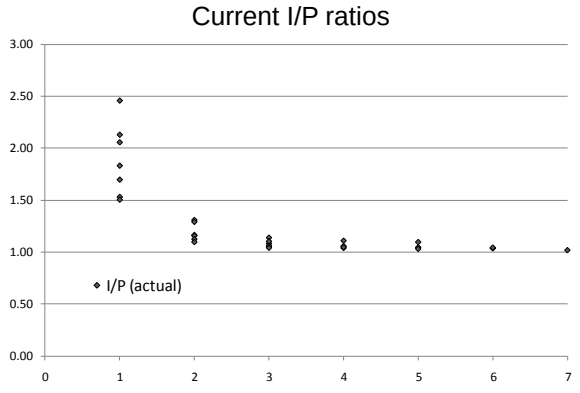
978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	
2,904	4,354	4,698	4,600	4,644		
3,502	5,958	6,070	6,142			
2,812	4,882	4,852				
2,642	4,406					
5,022						

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	
2.06	1.16	1.10	1.04	1.03		
1.53	1.13	1.06	1.05			
1.51	1.29	1.04				
1.83	1.10					
2.46						

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder
I/P ratios

Paid Quadrangle

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	2,380
1,412	3,758	4,252	4,416	4,494	4,589	4,652
2,286	5,292	5,724	5,850	5,971	6,097	6,182
1,868	3,778	4,648	4,784	4,884	4,987	5,056
1,442	4,010	4,536	4,669	4,766	4,867	4,934
2,044	4,981	5,634	5,800	5,920	6,045	6,128

Incurred Quadrangle

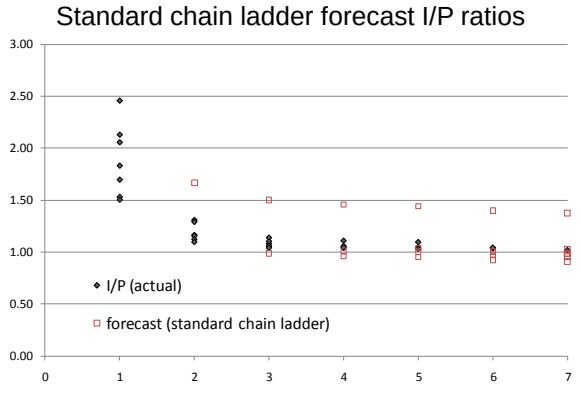
978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	2,445
2,904	4,354	4,698	4,600	4,644	4,598	4,582
3,502	5,958	6,070	6,142	6,210	6,149	6,126
2,812	4,882	4,852	4,851	4,905	4,857	4,839
2,642	4,406	4,488	4,488	4,537	4,493	4,476
5,022	8,297	8,451	8,450	8,544	8,460	8,429

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	1.03
2.06	1.16	1.10	1.04	1.03	1.00	0.98
1.53	1.13	1.06	1.05	1.04	1.01	0.99
1.51	1.29	1.04	1.01	1.00	0.97	0.96
1.83	1.10	0.99	0.96	0.95	0.92	0.91
2.46	1.67	1.50	1.46	1.44	1.40	1.38

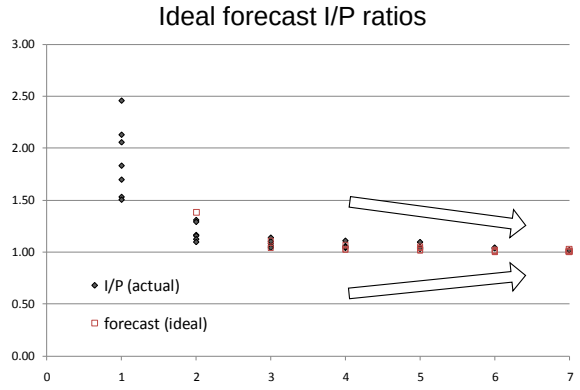
© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder

Paid Data

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	
1,412	3,758	4,252	4,416	4,494		
2,286	5,292	5,724	5,850			
1,868	3,778	4,648				
1,442	4,010					
2,044						
paid df	**2.437**	**1.131**	**1.029**	**1.021**	**1.021**	**1.014**

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	
0.49	0.86	0.91	0.96	0.97		
0.65	0.89	0.94	0.95			
0.66	0.77	0.96				
0.55	0.91					
0.41						
average P/I	**0.53**	**0.85**	**0.93**	**0.95**	**0.95**	**0.96**	**0.98**

Incurred Data

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	
2,904	4,354	4,698	4,600	4,644		
3,502	5,958	6,070	6,142			
2,812	4,882	4,852				
2,642	4,406					
5,022						
incd df	**1.652**	**1.019**	**1.000**	**1.011**	**0.990**	**0.996**

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	
2.06	1.16	1.10	1.04	1.03		
1.53	1.13	1.06	1.05			
1.51	1.29	1.04				
1.83	1.10					
2.46						
average I/P	**1.88**	**1.18**	**1.08**	**1.06**	**1.05**	**1.04**	**1.02**

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder

Paid Data

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	2,380
1,412	3,758	4,252	4,416	4,494	4,589	4,652
2,286	5,292	5,724	5,850	5,971	6,097	6,182
1,868	3,778	4,648	4,784	4,884	4,987	5,056
1,442	4,010	4,536	4,669	4,766	4,867	4,934
2,044	4,981	5,634	5,800	5,920	6,045	6,128
paid df	**2.437**	**1.131**	**1.029**	**1.021**	**1.021**	**1.014**

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	0.97
0.49	0.86	0.91	0.96	0.97	1.00	1.02
0.65	0.89	0.94	0.95	0.96	0.99	1.01
0.66	0.77	0.96	0.99	1.00	1.03	1.04
0.55	0.91	1.01	1.04	1.05	1.08	1.10
0.41	0.60	0.67	0.69	0.69	0.71	0.73
average P/I	**0.53**	**0.85**	**0.93**	**0.95**	**0.95**	**0.96**	**0.98**

Incurred Data

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	2,445
2,904	4,354	4,698	4,600	4,644	4,598	4,582
3,502	5,958	6,070	6,142	6,210	6,149	6,126
2,812	4,882	4,852	4,851	4,905	4,857	4,839
2,642	4,406	4,488	4,488	4,537	4,493	4,476
5,022	8,297	8,451	8,450	8,544	8,460	8,429
incd df	**1.652**	**1.019**	**1.000**	**1.011**	**0.990**	**0.996**

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	1.03
2.06	1.16	1.10	1.04	1.03	1.00	0.98
1.53	1.13	1.06	1.05	1.04	1.01	0.99
1.51	1.29	1.04	1.01	1.00	0.97	0.96
1.83	1.10	0.99	0.96	0.95	0.92	0.91
2.46	1.67	1.50	1.46	1.44	1.40	1.38
average I/P	**1.88**	**1.18**	**1.08**	**1.06**	**1.05**	**1.04**	**1.02**

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder

Paid Data

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	2,380
1,412	3,758	4,252	4,416	4,494	4,589	4,652
2,286	5,292	5,724	5,850	5,971	6,097	6,182
1,868	3,778	4,648	4,784	4,884	4,987	5,056
1,442	4,010	4,536	4,669	4,766	4,867	4,934
2,044	4,981	5,634	5,800	5,920	6,045	6,128
paid df	**2.437**	**1.131**	**1.029**	**1.021**	**1.021**	**1.014**

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	0.97
0.49	0.86	0.91	0.96	0.97	1.00	1.02
0.65	0.89	0.94	0.95	0.96	0.99	1.01
0.66	0.77	0.96	0.99	1.00	1.03	1.04
0.55	0.91	1.01	1.04	1.05	1.08	1.10
0.41	0.60	0.67	0.69	0.69	0.71	0.73
average P/I	**0.53**	**0.85**	**0.93**	**0.95**	**0.95**	**0.96**	**0.98**

low

Incurred Data

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	2,445
2,904	4,354	4,698	4,600	4,644	4,598	4,582
3,502	5,958	6,070	6,142	6,210	6,149	6,126
2,812	4,882	4,852	4,851	4,905	4,857	4,839
2,642	4,406	4,488	4,488	4,537	4,493	4,476
5,022	8,297	8,451	8,450	8,544	8,460	8,429
incd df	**1.652**	**1.019**	**1.000**	**1.011**	**0.990**	**0.996**

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	1.03
2.06	1.16	1.10	1.04	1.03	1.00	0.98
1.53	1.13	1.06	1.05	1.04	1.01	0.99
1.51	1.29	1.04	1.01	1.00	0.97	0.96
1.83	1.10	0.99	0.96	0.95	0.92	0.91
2.46	1.67	1.50	1.46	1.44	1.40	1.38
average I/P	**1.88**	**1.18**	**1.08**	**1.06**	**1.05**	**1.04**	**1.02**

high

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Standard Chain Ladder

Paid Data

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	2,380
1,412	3,758	4,252	4,416	4,494	4,589	4,652
2,286	5,292	5,724	5,850	5,971	6,097	6,182
1,868	3,778	4,648	4,784	4,884	4,987	5,056
1,442	4,010	4,536	4,669	4,766	4,867	4,934
2,044	4,981	5,634	5,800	5,920	6,045	6,128
paid df	**2.437**	**1.131**	**1.029**	**1.021**	**1.021**	**1.014**

P/I ratio

0.59	0.86	0.92	0.94	0.95	0.96	0.98
0.47	0.76	0.88	0.90	0.91	0.96	0.97
0.49	0.86	0.91	0.96	0.97	1.00	1.02
0.65	0.89	0.94	0.95	0.96	0.99	1.01
0.66	0.77	0.96	0.99	1.00	1.03	1.04
0.55	0.91	1.01	1.04	1.05	1.08	1.10
0.41	0.60	0.67	0.69	0.69	0.71	0.73
average P/I	**0.53**	**0.85**	**0.93**	**0.95**	**0.95**	**0.96**	**0.98**

high

Incurred Data

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	2,445
2,904	4,354	4,698	4,600	4,644	4,598	4,582
3,502	5,958	6,070	6,142	6,210	6,149	6,126
2,812	4,882	4,852	4,851	4,905	4,857	4,839
2,642	4,406	4,488	4,488	4,537	4,493	4,476
5,022	8,297	8,451	8,450	8,544	8,460	8,429
incd df	**1.652**	**1.019**	**1.000**	**1.011**	**0.990**	**0.996**

I/P ratio

1.70	1.17	1.08	1.06	1.05	1.04	1.02
2.13	1.31	1.14	1.11	1.10	1.05	1.03
2.06	1.16	1.10	1.04	1.03	1.00	0.98
1.53	1.13	1.06	1.05	1.04	1.01	0.99
1.51	1.29	1.04	1.01	1.00	0.97	0.96
1.83	1.10	0.99	0.96	0.95	0.92	0.91
2.46	1.67	1.50	1.46	1.44	1.40	1.38
average I/P	**1.88**	**1.18**	**1.08**	**1.06**	**1.05**	**1.04**	**1.02**

low

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

Standard chain ladder:

same average development factor applied to all accident years

An improvement:

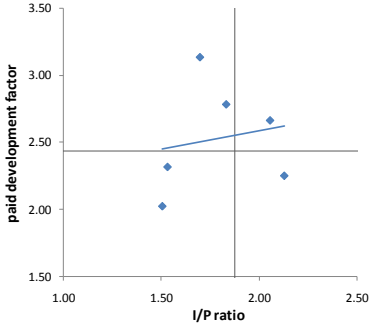
- Paid chain ladder: if the momentary I/P ratio is higher (lower) than average, use a higher (lower) than average paid development factor
- Incurred chain ladder: if the momentary P/I ratio is higher (lower) than average, use a higher (lower) than average incurred development factor

But how much higher or lower?

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

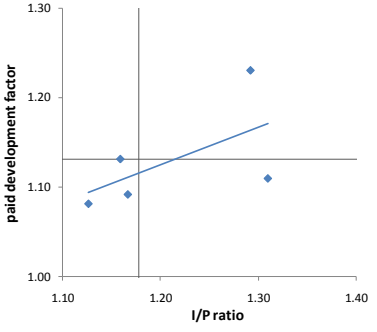
paid development factor (year 1 to year 2)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

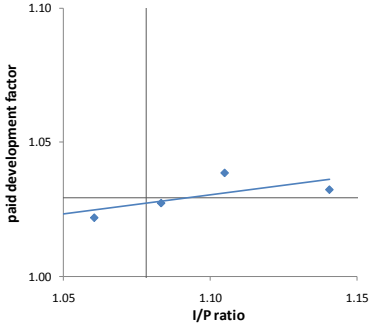
paid development factor (year 2 to year 3)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

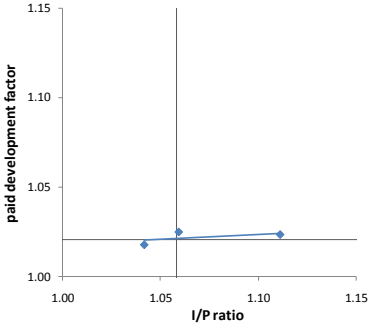
paid development factor (year 3 to year 4)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

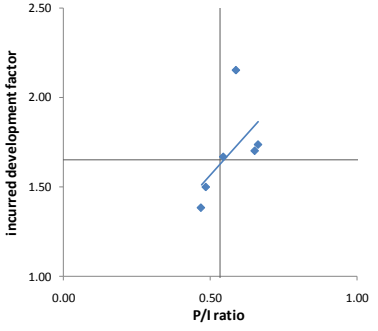
paid development factor (year 4 to year 5)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

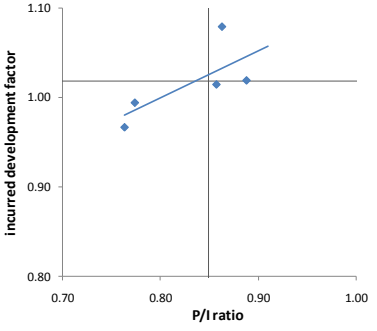
incurred development factor (year 1 to year 2)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

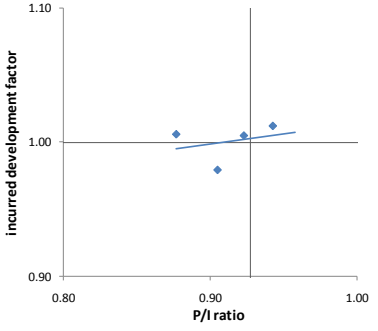
incurred development factor (year 2 to year 3)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

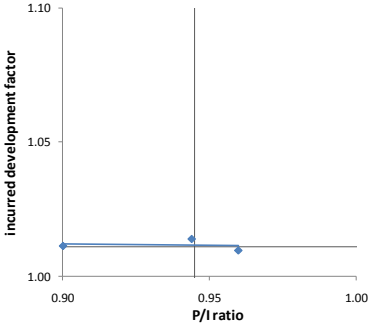
incurred development factor (year 3 to year 4)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

incurred development factor (year 4 to year 5)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

One answer:

linear regression of development factors at each development age

But…

- Sometimes the gradient is steep but the correlation is low
- Sometimes the regression line will slope the “wrong” way
- Where there are few data points the regression parameters are uncertain

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

The Munich chain ladder:

- Uses all the data at once to estimate:
  - a single correlation parameter for adjusting the paid development factors
  - a single correlation parameter for adjusting the incurred development factors
- Takes into account the variance of the paid and incurred development factors
- Takes into account the variance of the P/I and I/P ratios

Key is to use residuals…

© 2011 The Actuarial Profession • www.actuaries.org.uk

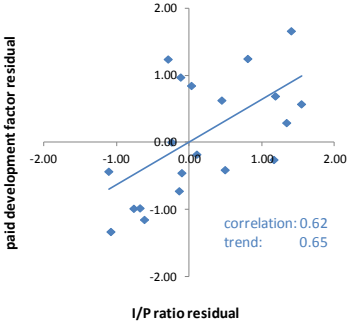
The Munich Chain Ladder

- Plot incremental paid development residual as a function of momentary I/P ratio residual
- Plot incremental incurred development residual as a function of momentary P/I ratio residual
- Look at gradient and correlation

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

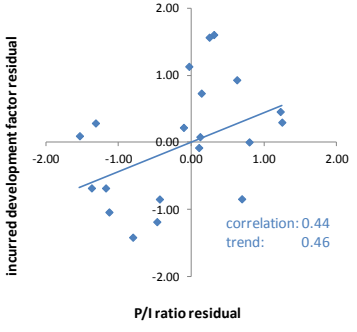
paid development factor residuals (all years)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

incurred development factor residuals (all years)



© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

Standard chain ladder:

- Same paid development factor at each age for all accident years
- Same incurred development factor at each age for all accident years

Munich chain ladder:

- adjust paid development factor for each accident year according to the residual of the momentary I/P ratio
- Adjust incurred development factor each accident year according to the residual of the momentary P/I ratio

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

$$\widehat{f^P_{i,t\to t+1}} = \widehat{f^P_{t\to t+1}} + \widehat{\lambda^P} \cdot \frac{\widehat{\sigma^P_{t\to t+1}}}{\widehat{\rho^P_t}} \left(Q^{-1}_{i,t} - \widehat{q^{-1}_t}\right)$$

i = accident year

t = development year

Q^{-1} = I/P

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

$$\widehat{f^P_{i,t\to t+1}} = \widehat{f^P_{t\to t+1}} + \widehat{\lambda^P} \cdot \frac{\widehat{\sigma^P_{t\to t+1}}}{\widehat{\rho^P_t}} \left(Q^{-1}_{i,t} - \widehat{q^{-1}_t}\right)$$

MCL paid development factor

SCL paid development factor

Correlation parameter

Ratio of standard errors

I/P residual

i = accident year

t = development year

Q^{-1} = I/P

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

$$\widehat{f^I_{i,t\to t+1}} = \widehat{f^I_{t\to t+1}} + \widehat{\lambda^I} \cdot \frac{\widehat{\sigma^I_{t\to t+1}}}{\widehat{\rho^I_t}} \left(Q_{i,t} - \widehat{q_t}\right)$$

- MCL incurred development factor
- SCL incurred development factor
- Correlation parameter
- Ratio of standard errors
- P/I residual

i = accident year

t = development year

Q = P/I

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder
Accident year 7 – low P/I

SCL forecast

Paid d.f.		2.44	1.13	1.03	1.02	1.02	1.01
Paid forecast	2,044	4,981	5,634	5,800	5,920	6,045	6,128
Incurred d.f.		1.65	1.02	1.00	1.01	0.99	1.00
Incurred forecast	5,022	8,297	8,451	8,450	8,544	8,460	8,429
P/I ratio	**0.41**	**0.60**	**0.67**	**0.69**	**0.69**	**0.71**	**0.73**

MCL forecast

Paid d.f.		2.77	1.23	1.03	1.02	1.02	1.01
Paid forecast	2,044	5,659	6,944	7,177	7,330	7,485	7,549
Incurred d.f.		1.56	0.98	0.99	1.01	0.99	1.00
Incurred forecast	5,022	7,828	7,688	7,644	7,727	7,650	7,650
P/I ratio	**0.41**	**0.72**	**0.90**	**0.94**	**0.95**	**0.98**	**0.99**

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder
Accident year 6 – high P/I

SCL forecast

Paid d.f.			1.13	1.03	1.02	1.02	1.01
Paid forecast	1,442	4,010	4,536	4,669	4,766	4,867	4,934
Incurred d.f.			1.02	1.00	1.01	0.99	1.00
Incurred forecast	2,642	4,406	4,488	4,488	4,537	4,493	4,476
P/I ratio	**0.55**	**0.91**	**1.01**	**1.04**	**1.05**	**1.08**	**1.10**

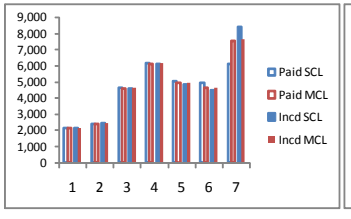
MCL forecast

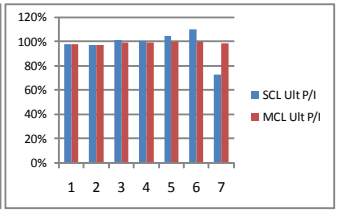
Paid d.f.			1.09	1.02	1.02	1.02	1.00
Paid forecast	1,442	4,010	4,388	4,493	4,574	4,643	4,656
Incurred d.f.			1.04	1.01	1.01	1.00	1.00
Incurred forecast	2,642	4,406	4,567	4,601	4,657	4,646	4,665
P/I ratio	**0.55**	**0.91**	**0.96**	**0.98**	**0.98**	**1.00**	**1.00**

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

Accident Year	Paid SCL	Paid MCL	Paid MCL Adjustment		Incd SCL	Incd MCL	Incd MCL Adjustment		SCL Ult P/I	MCL Ult P/I
1	2,131	2,131	+0	+0.0%	2,174	2,174	+0	+0.0%	98%	98%
2	2,380	2,383	+2	+0.1%	2,445	2,444	-1	-0.1%	97%	98%
3	4,652	4,597	-55	-1.2%	4,582	4,629	+47	+1.0%	102%	99%
4	6,182	6,119	-62	-1.0%	6,126	6,176	+50	+0.8%	101%	99%
5	5,056	4,937	-118	-2.3%	4,839	4,950	+111	+2.3%	104%	100%
6	4,934	4,656	-279	-5.6%	4,476	4,665	+189	+4.2%	110%	100%
7	6,128	7,549	+1,420	+23.2%	8,429	7,650	-779	-9.2%	73%	99%
TOTAL	31,463	32,371	+908	+2.9%	33,071	32,688	-383	-1.2%	95%	99%





© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

In practice…

© 2011 The Actuarial Profession • www.actuaries.org.uk

In practice…

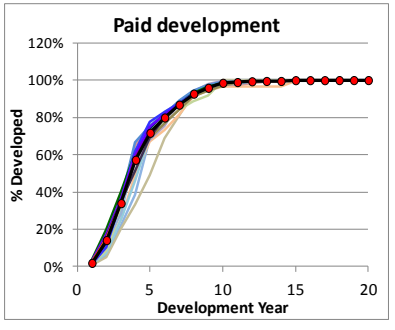
Often works fine without adjustment.

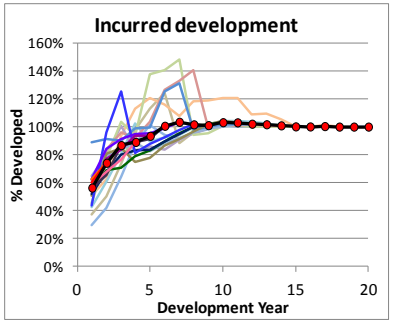
Common challenges:

- Negative incurred development
- Noise
- Zero residuals
- Tail factors

© 2011 The Actuarial Profession • www.actuaries.org.uk

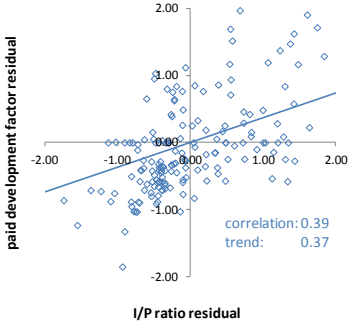
Example 1 – “easy” triangle

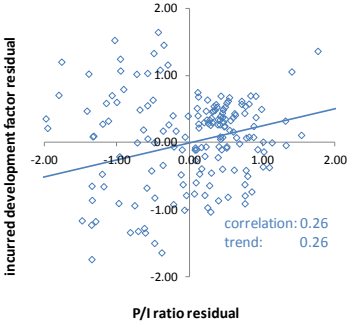




© 2011 The Actuarial Profession • www.actuaries.org.uk

Example 1 - “easy” triangle

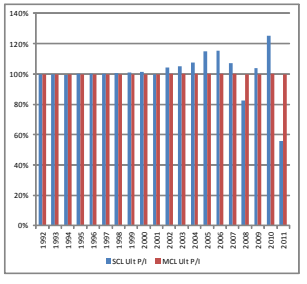




© 2011 The Actuarial Profession • www.actuaries.org.uk

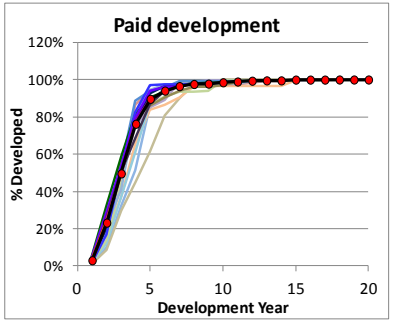
Example 1 - “easy” triangle

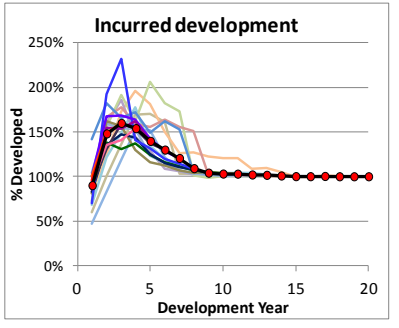
Year	Current Paid	Current Incd	Paid SCL	Incd SCL	Paid MCL	Incd MCL	Paid MCL Adjustment	Paid MCL Adjustment	Incd MCL Adjustment	Incd MCL Adjustment	SCL Ult P/I	MCL Ult P/I
1992	825	829	825	829	825	829	+0	+0.0%	+0	+0.0%	100%	100%
1993	909	911	909	911	909	911	-0	-0.0%	+1	+0.1%	100%	100%
1994	819	823	819	823	819	822	+0	+0.0%	-1	-0.1%	99%	100%
1995	1,116	1,118	1,117	1,114	1,116	1,117	-0	-0.0%	+4	+0.3%	100%	100%
1996	1,100	1,101	1,100	1,100	1,100	1,102	-0	-0.0%	+3	+0.2%	100%	100%
1997	1,399	1,401	1,399	1,399	1,399	1,402	-0	-0.0%	+3	+0.2%	100%	100%
1998	1,250	1,258	1,257	1,247	1,255	1,257	-2	-0.1%	+10	+0.8%	101%	100%
1999	1,624	1,643	1,632	1,616	1,630	1,633	-3	-0.2%	+16	+1.0%	101%	100%
2000	1,569	1,591	1,579	1,559	1,576	1,578	-4	-0.2%	+19	+1.2%	101%	100%
2001	1,925	2,010	1,946	1,951	1,947	1,951	+1	+0.1%	+0	+0.0%	100%	100%
2002	2,941	2,949	2,985	2,859	2,959	2,960	-26	-0.9%	+101	+3.5%	104%	100%
2003	2,574	2,582	2,686	2,555	2,651	2,652	-35	-1.3%	+97	+3.8%	105%	100%
2004	2,228	2,272	2,403	2,236	2,364	2,364	-39	-1.6%	+129	+5.8%	107%	100%
2005	2,391	2,477	2,755	2,395	2,677	2,675	-78	-2.8%	+280	+11.7%	115%	100%
2006	3,098	3,381	3,878	3,361	3,760	3,757	-119	-3.1%	+396	+11.8%	115%	100%
2007	2,409	2,926	3,360	3,133	3,306	3,308	-54	-1.6%	+175	+5.6%	107%	100%
2008	1,859	3,515	3,253	3,949	3,510	3,521	+258	+7.9%	-428	-10.8%	82%	100%
2009	1,509	3,726	4,456	4,295	4,418	4,422	-38	-0.9%	+127	+3.0%	104%	100%
2010	821	3,468	5,857	4,680	5,335	5,336	-522	-8.9%	+656	+14.0%	125%	100%
2011	53	3,036	3,018	5,392	4,507	4,517	+1,489	+49.3%	-876	-16.2%	56%	100%



© 2011 The Actuarial Profession • www.actuaries.org.uk

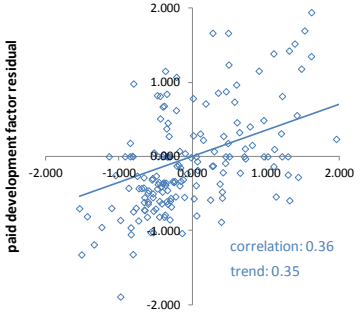
Example 2 – negative incurred development





© 2011 The Actuarial Profession • www.actuaries.org.uk

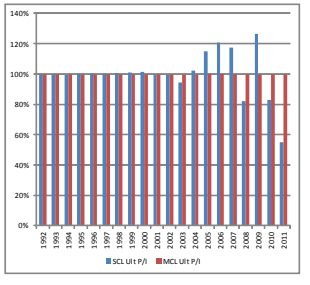
Example 2 – negative incurred development



© 2011 The Actuarial Profession • www.actuaries.org.uk

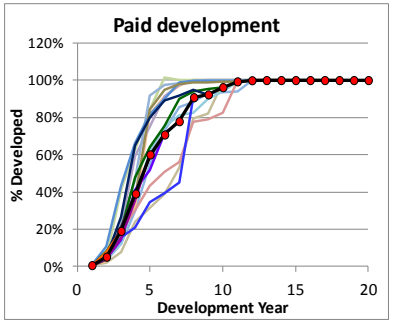
Example 2 – negative incurred development

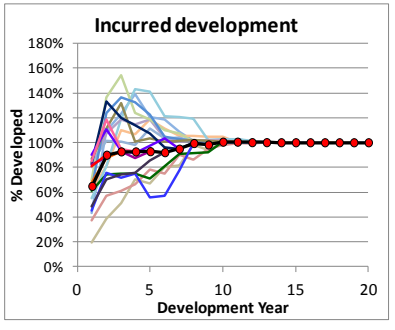
Year	Current Paid	Current Incd	Paid SCL	Incd SCL	Paid MCL	Incd MCL	Paid MCL Adjustment	Paid MCL Adjustment	Incd MCL Adjustment	Incd MCL Adjustment	SCL Ult P/I	MCL Ult P/I
1992	1,980	1,989	1,980	1,989	1,980	1,989	+0	+0.0%	+0	+0.0%	100%	100%
1993	1,999	2,003	1,999	2,003	1,999	2,005	-0	-0.0%	+1	+0.1%	100%	100%
1994	1,719	1,728	1,719	1,728	1,719	1,727	+0	+0.0%	-1	-0.1%	99%	100%
1995	2,233	2,235	2,234	2,228	2,233	2,235	-1	-0.0%	+7	+0.3%	100%	100%
1996	2,199	2,202	2,200	2,200	2,200	2,205	-0	-0.0%	+5	+0.2%	100%	100%
1997	2,937	2,943	2,939	2,939	2,938	2,946	-0	-0.0%	+7	+0.2%	100%	100%
1998	2,876	2,894	2,890	2,869	2,886	2,892	-3	-0.1%	+23	+0.8%	101%	100%
1999	3,736	3,779	3,754	3,719	3,749	3,756	-5	-0.1%	+37	+1.0%	101%	100%
2000	3,608	3,660	3,632	3,588	3,625	3,631	-7	-0.2%	+43	+1.2%	101%	100%
2001	4,427	4,623	4,475	4,492	4,478	4,490	+3	+0.1%	-3	-0.1%	100%	100%
2002	6,427	6,782	6,525	6,583	6,537	6,554	+12	+0.2%	-28	-0.4%	99%	100%
2003	5,437	6,120	5,560	5,882	5,650	5,673	+90	+1.6%	-209	-3.6%	95%	100%
2004	5,124	5,610	5,251	5,142	5,225	5,236	-25	-0.5%	+94	+1.8%	102%	100%
2005	6,111	6,646	6,337	5,517	6,173	6,176	-164	-2.6%	+659	+11.9%	115%	100%
2006	8,803	10,062	9,361	7,748	9,016	9,018	-345	-3.7%	+1,270	+16.4%	121%	100%
2007	7,617	10,095	8,499	7,230	8,194	8,199	-305	-3.6%	+970	+13.4%	118%	100%
2008	5,702	14,012	7,481	9,106	8,052	8,080	+570	+7.6%	-1,027	-11.3%	82%	100%
2009	6,199	15,821	12,511	9,906	11,625	11,637	-886	-7.1%	+1,731	+17.5%	126%	100%
2010	2,059	15,952	8,919	10,776	9,853	9,878	+934	+10.5%	-898	-8.3%	83%	100%
2011	195	11,172	6,806	12,406	9,991	10,022	+3,186	+46.8%	-2,384	-19.2%	55%	100%



© 2011 The Actuarial Profession • www.actuaries.org.uk

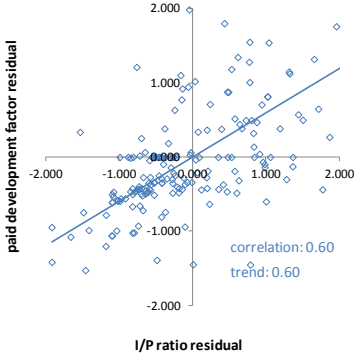
Example 3 – noisy incurred triangle

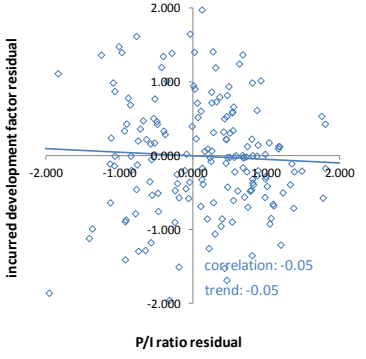




© 2011 The Actuarial Profession • www.actuaries.org.uk

Example 3 – noisy incurred triangle

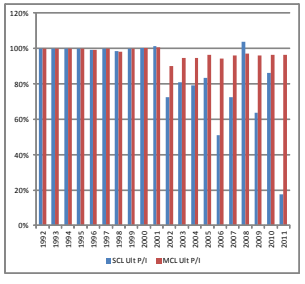




© 2011 The Actuarial Profession • www.actuaries.org.uk

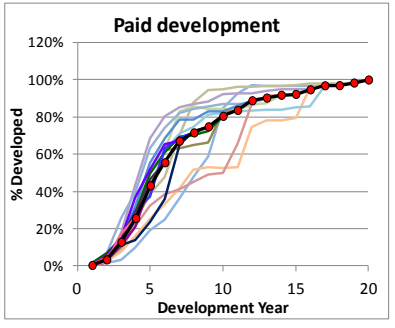
Example 3 – noisy incurred triangle

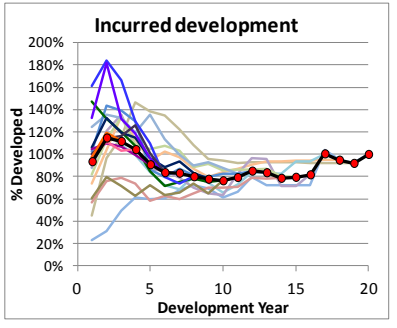
Year	Current Paid	Current Incd	Paid SCL	Incd SCL	Paid MCL	Incd MCL	Paid MCL Adjustment	Paid MCL Adjustment	Incd MCL Adjustment	Incd MCL Adjustment	SCL Ult P/I	MCL Ult P/I
1992	3,559	3,560	3,559	3,560	3,559	3,560	+0	+0.0%	+0	+0.0%	100%	100%
1993	8,726	8,726	8,726	8,726	8,726	8,726	+0	+0.0%	-0	-0.0%	100%	100%
1994	3,134	3,135	3,134	3,135	3,134	3,135	+0	+0.0%	+0	+0.0%	100%	100%
1995	3,661	3,662	3,661	3,662	3,661	3,662	+0	+0.0%	-0	-0.0%	100%	100%
1996	2,463	2,485	2,463	2,485	2,463	2,486	+0	+0.0%	+0	+0.0%	99%	99%
1997	2,292	2,292	2,292	2,292	2,292	2,292	+0	+0.0%	-0	-0.0%	100%	100%
1998	6,069	6,173	6,069	6,171	6,069	6,173	+0	+0.0%	+2	+0.0%	98%	98%
1999	4,238	4,239	4,238	4,235	4,238	4,235	+0	+0.0%	-0	-0.0%	100%	100%
2000	4,395	4,396	4,399	4,392	4,395	4,390	-4	-0.1%	-1	-0.0%	100%	100%
2001	4,744	4,745	4,777	4,722	4,742	4,718	-35	-0.7%	-4	-0.1%	101%	101%
2002	3,826	5,511	3,978	5,485	4,993	5,555	+1,015	+25.5%	+69	+1.3%	73%	90%
2003	3,950	5,189	4,280	5,282	5,047	5,328	+767	+17.9%	+46	+0.9%	81%	95%
2004	6,545	9,033	7,208	9,089	8,707	9,197	+1,499	+20.8%	+108	+1.2%	79%	95%
2005	5,019	7,341	6,444	7,741	7,524	7,806	+1,081	+16.8%	+66	+0.8%	83%	96%
2006	4,051	10,369	5,725	11,269	10,972	11,616	+5,247	+91.6%	+348	+3.1%	51%	94%
2007	4,427	9,422	7,373	10,152	9,907	10,332	+2,534	+34.4%	+180	+1.8%	73%	96%
2008	4,196	9,614	10,756	10,379	10,028	10,325	-728	-6.8%	-55	-0.5%	104%	97%
2009	1,424	10,904	7,503	11,769	11,554	12,010	+4,051	+54.0%	+240	+2.0%	64%	96%
2010	365	7,494	7,181	8,330	8,100	8,388	+919	+12.8%	+59	+0.7%	86%	97%
2011	11	7,582	2,058	11,693	11,525	11,945	+9,467	+460.1%	+251	+2.1%	18%	96%



© 2011 The Actuarial Profession • www.actuaries.org.uk

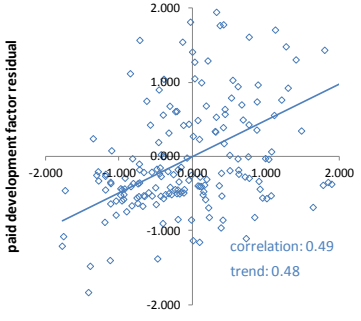
Example 4 – extreme tail

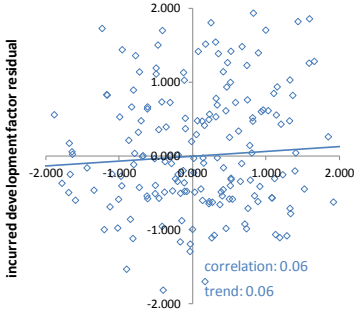




© 2011 The Actuarial Profession • www.actuaries.org.uk

Example 4 – extreme tail

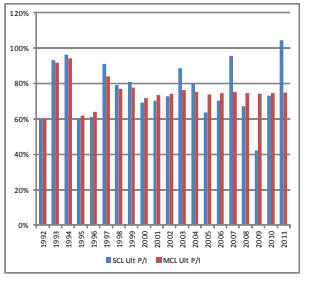




© 2011 The Actuarial Profession • www.actuaries.org.uk

Example 4 – extreme tail

Year	Current Paid	Current Incd	Paid SCL	Incd SCL	Paid MCL	Incd MCL	Paid MCL Adjustment	Paid MCL Adjustment	Incd MCL Adjustment	Incd MCL Adjustment	SCL Ult P/I	MCL Ult P/I
1992	1,525	2,549	1,525	2,549	1,525	2,549	+0	+0.0%	+0	+0.0%	60%	60%
1993	2,087	2,095	2,122	2,277	2,096	2,286	-26	-1.2%	+9	+0.4%	93%	92%
1994	2,123	2,127	2,178	2,263	2,143	2,275	-35	-1.6%	+12	+0.5%	96%	94%
1995	2,278	3,771	2,342	3,892	2,395	3,867	+54	+2.3%	-25	-0.6%	60%	62%
1996	2,679	4,078	2,853	4,672	2,960	4,622	+106	+3.7%	-50	-1.1%	61%	64%
1997	4,304	4,383	4,764	5,223	4,478	5,318	-286	-6.0%	+95	+1.8%	91%	84%
1998	3,820	4,417	4,257	5,375	4,170	5,411	-87	-2.1%	+37	+0.7%	79%	77%
1999	3,831	4,725	4,325	5,348	4,205	5,404	-120	-2.8%	+56	+1.1%	81%	78%
2000	6,634	9,821	7,655	11,055	7,874	10,950	+219	+2.9%	-105	-0.9%	69%	72%
2001	3,585	5,238	4,423	6,277	4,588	6,242	+165	+3.7%	-35	-0.6%	70%	74%
2002	6,798	9,630	8,711	11,951	8,845	11,920	+134	+1.5%	-31	-0.3%	73%	74%
2003	10,575	13,021	14,357	16,223	12,609	16,539	-1,747	-12.2%	+316	+1.9%	88%	76%
2004	4,375	6,416	6,165	7,686	5,831	7,749	-334	-5.4%	+63	+0.8%	80%	75%
2005	3,659	7,441	5,464	8,599	6,225	8,423	+761	+13.9%	-176	-2.0%	64%	74%
2006	5,200	11,709	9,509	13,524	10,030	13,446	+522	+5.5%	-78	-0.6%	70%	75%
2007	5,080	12,339	12,146	12,697	9,824	13,030	-2,321	-19.1%	+333	+2.6%	96%	75%
2008	2,183	14,733	8,617	12,828	9,471	12,687	+854	+9.9%	-141	-1.1%	67%	75%
2009	2,057	44,981	15,432	36,488	26,022	35,022	+10,590	+68.6%	-1,467	-4.0%	42%	74%
2010	365	17,708	9,952	13,580	10,093	13,509	+141	+1.4%	-71	-0.5%	73%	75%
2011	65	17,011	16,346	15,664	11,898	15,904	-4,448	-27.2%	+239	+1.5%	104%	75%



© 2011 The Actuarial Profession • www.actuaries.org.uk

In practice…

Possible extensions:

- Zoning the triangle
  - Rising phase vs. falling phase
  - Noisy tail
  - Missing or obsolete data
- Other correlated triangles
  - Claims count, settled claims
  - Indemnity costs, legal costs

© 2011 The Actuarial Profession • www.actuaries.org.uk

The Munich Chain Ladder

Summary:

- Paid and Incurred chain ladders don't agree – the standard chain ladder ignores any correlation between them
- The Munich chain ladder accommodates correlation between paid and incurred
- The method works well in practice, though sometimes it is best to split the triangle into zones

The Munich Chain Ladder is available in 'R'

If you don't have 'R', ask me for a copy in Excel: wrightar@the-mdu.com

© 2011 The Actuarial Profession • www.actuaries.org.uk