The Actuarial Profession
making financial sense of the future

GIRO conference and exhibition 2010
Richard Shaw (Horgen Capital & Risk)

The Modelling of Reinsurance Credit Risk

12-15 October 2010

Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Challenges

Reinsurance Credit Risk

What is Reinsurance Credit Risk

Definition

"The risk of loss if another party fails to perform its obligations or fails to perform them in a timely manner."

Examples of Risk Factors

- Failure of individual Reinsurers
- Credit Deterioration (of individual reinsurers)
- Bad Debt provision inadequacy
- Correlation in extreme loss scenarios
- Credit Concentration
- Duration of Recoveries
- Willingness to Pay / Dispute Risk

Reinsurance Credit Risk

Why it is important to Understand

Regulatory Capital Requirements

- ICA Capital – VaR (@99.5%) over 12-months
- Solvency II SCR and ORSA Capital

Risk Management Best Practices

- An understanding of risks and issues might translate into better practices e.g. Regular aged debt analysis → highlight issues with reinsurers ('Willingness to Pay')

Capital Markets Solutions

- Securitisation and risk transfer products

Reinsurance Panel Evaluation

- Given a new reinsurance program how should it be placed – 100% with one reinsurer or equal shares with others, what about rating
- Benefits of diversification

The Loss Process

Expected Loss ("EL") and Unexpected Loss ("UL")

Binary Variable

- Let Y_i be a binary variable for obligor i at time 1 year
- Y_i takes values - 1 (Default) or 0 (No Default) given non-default state at t= 0.

Expected Loss

- $EL_i = PD_i \times EAD_i \times LGD_i$

Unexpected Loss

- EAD_i and LGD_i are constant
- $UL_i = [PD_i \times (1 - PD_i)]^{1/2} \times EAD_i \times LGD_i$
  - EAD = Exposure at Default
  - LGD = Loss Given Default (i.e. severity per unit of exposure)
  - PD = Probability of Default
- This further assumes that PD_i, EAD_i and LGD_i are independent

The Loss Process

Expected Loss ("EL") and Unexpected Loss ("UL")

Unexpected Loss

- Otherwise

$$UL_i = [\ PD^2_i \text{ x } EAD^2_i \text{ x } \sigma^2_{LGDi} + EAD^2_i \,.\, LGD^2_i \,.\, \sigma^2_{PDi} + LGD^2_i \,.\, PD^2_i \text{ x } \sigma^2_{EADi} +$$

$$+ PD^2_i \text{ x } \sigma^2_{EADi} \text{ x } \sigma^2_{LGDi} + EAD^2_i \text{ x } \sigma^2_{LGDi} \text{ x } \sigma^2_{PDi} + LGD^2_i \text{ x } \sigma^2_{PDi} \text{ x } \sigma^2_{EADi}$$

$$+ \sigma^2_{PDi} \text{ x } \sigma^2_{EADi} \text{ x } \sigma^2_{LGDi}\ ]^{0.5}$$

The Loss Process

Expected Loss ("EL") and Unexpected Loss ("UL")

Obligor	PD	LGD	EAD	EL	UL
Obligor 1	**2.0%**	**40%**	**2,000**	**16.0**	**131.7**
Obligor 2	**5.0%**	**60%**	**2,000**	**60.0**	**283.5**
Portfolio	**3.80%**	**50%**	**4,000**	**76.0**	**319.8**

Asset Correlation	25%
Joint Default Prob	0.28%
Default Correlation	6.03%

Diversification Benefit	95.5
as % of (UL_1 + UL_2)	23.0%

PD = Probability of Deafult
LGD = Loss Given Default (%)
EAD = Exposure at Default
EL = Expected Loss
UL = Unexpected Loss

$$\mathbf{UL_i = EAD_i \ x \ [LGD^2_i \ x \ PD_i \ x \ (1 - PD_i) + PD_i \ x \ LGD_i \ x \ (1 - LGD_i) / 4]^{0.5}}$$

$$\mathbf{UL_T = (UL^2_1 + UL^2_2 + 2 \ x \ \rho_d \ x \ UL_1 \ x \ UL_2)^2}$$

ρ_d = *Default correlation between obligor 1 and obligor 2*

$\sigma^2_{PDi} = PD_i \ x \ (1 - PD_i)$

$\sigma^2_{LGDi} \sim LGD_i \ x \ (1- LGD_i) / 4$ *(and assuming a Beta Distribution)*

EAD_i = *constant*

The Loss Process

Determining the Probability of Default

Structural Model (“Merton Model”)

- Based on the firm’s capital structure and asset return volatility
- Firm defaults when value of assets < value of liabilities at maturity
- Equity is a call option on the assets of the firm – Black-Scholes framework
- The structural approach uses company-specific information and involves the specification of how a company changes values over time

Reduced Form Model

- The reduced form approach bypasses the company’s financial fundamentals and deals directly with market information.
- Price or spread of a defaultable bond is directly related to a risk-free bond through default and recovery rates that are exogenous.
- The approach is considered mathematically more tractable
- If rating is important then can use can be made of rating agency studies

The Loss Process

Loss Severity

Two ways of modelling loss severity

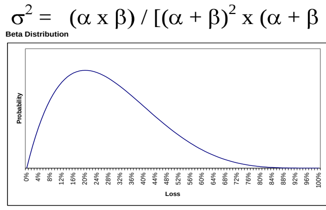
- Recovery % amount is constant
- Recovery % amount is variable
- Beta Distribution is often used to model Loss Severity in this situation

$$f(x) = x^{(\alpha - 1)} \text{ x } (1 - x)^{(\beta - 1)} \text{ x } [\Gamma(\alpha + \beta) / \Gamma(\alpha) \text{ x } \Gamma(\beta)] \text{ .... for } 0 < x < 1$$

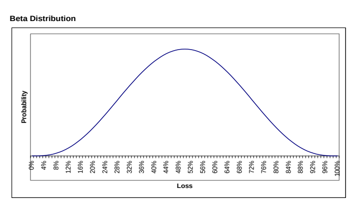
$$0 \text{ .... for } x < 0 \text{ and } x > 1$$

$$\mu = \alpha / (\alpha + \beta)$$

$$\sigma^2 = (\alpha \text{ x } \beta) / [(\alpha + \beta)^2 \text{ x } (\alpha + \beta + 1)]$$



α	2.0	E(X)	28.6%
β	5.0	σ(X)	16.0%



α	4.0	E(X)	50.0%
β	4.0	σ(X)	16.7%

The Loss Process

Reinsurance Credit Exposure

Reinsurance Exposures are Stochastic

- Reinsurance Recoveries – Function of Gross losses and Payment patterns
- Prior year and Current year – different loss dynamics

Reinsurance – Current Year Exposure

- More accurate modelling of Stochastic Gross to Net Losses
- Detailed knowledge of current reinsurance structures
- Gross Loss calibration – Attritional and Large (Frequency / Severity)

Reinsurance – Prior Year Exposure

- Mix of reinsurers different to Current year
- Average credit rating likely to be lower (rating downgrades)
- Gross to Net Process Loss relationship less accuracy unless modelling prior year reinsurance treaties

The Loss Process

Loss Paradigms and Economic Capital

Mark-to-Market Loss Paradigm

- A loss (or gain) also occurs if there is a change in the credit quality
- Values being determined by the discounting of cash flows using credit curve

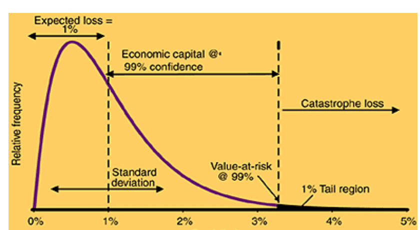
Mark-to-Model Loss Paradigm

- A slight variation on the Mark-to-Market paradigm
- None or limited secondary market – Value estimated by model

Default Loss Paradigm

- A loss is only recognised on default
- e.g. reinsurance default

Economic Capital



The Loss Process
Credit Risk Modelling Challenges (vs Market Risk)

The lack of a liquid market
- Makes it difficult to price products
- Time horizon tends to be longer than for market risk
- Requirement for more refined simulation techniques (evolution of exposures)

"True" default probabilities cannot be observed - need to be estimated
- Historical experience of credit ratings
- Market Prices
- Subjective assessment criteria

Default Correlations are difficult to measure for Risk Aggregation
- Sparse data

Economic Capital calculations
- Tails of asymmetric fat-tailed distributions

Diversification and Correlation

The Aggregation of Risks

Overview

- Default loss is sparse making it difficult to estimate default correlations
- Instead use a model utilising the concept of asset return correlation
- A multivariate distribution is needed.

Copulas

- A way of dealing with this difficulty is to split the problem into two parts:
- Stand alone marginal distribution
- Dependency structure between the risk variables i.e. the copula of the distribution

Single and Multi-Factor Model

- Copula approach involves Monte Carlo simulation and is computationally intensive
- Simplifications can be achieved by imposing more structure on the model by consideration of single or multi-factor models
- A useful starting point is the multivariate normal distribution

Diversification and Correlation

One-Factor Model

$\mathbf{AR_i = [R^2_i]^{0.5} \ x \ X + [1 - R^2_i]^{0.5} \ x \ \varepsilon_i}$

The common economic factor X and the obligor specific component are assumed to be standard normal

Where:

ε_i = Obligor Specific (Non-Systematic) component

Obligors tend to downgrade and default when the economy is in a downturn

X = State of the Economy

R^2_i = Obligor asset return correlation with the Economy

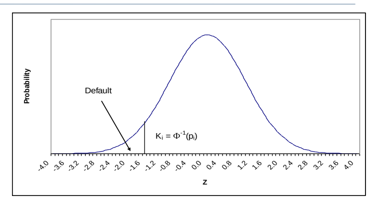
$\rho_A = \text{Corr}(AR_1, AR_2) = [R^2_1]^{0.5} \ x \ [R^2_2]^{0.5}$

Example:

$R^2_1 = 50\%$ and $R^2_2 = 25\%$ then $\rho_A = 35.4\%$

Diversification and Correlation

Asset Return and Default Correlation relationship



$\mathbf{Y_i = 1 \Leftrightarrow X_i \leq D_i \Leftrightarrow AR_i \leq K_i}$

Where:

X_i = Value of the Assets for obligor i at the end of time t.

D_i = Value of the Asset Threshold (or cut-off level) for obligor i at the end of time t.

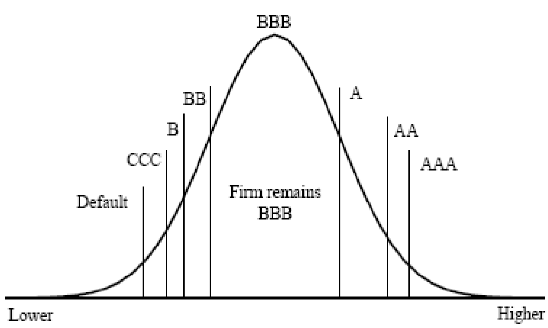
AR_i = Asset Return for obligor i over time t.

K_i = Asset Return threshold for obligor i over time t

Number of defaults within a portfolio of M obligors = $\sum_{i=1}^{M} \mathbf{Y_i}$

Diversification and Correlation

Multi-year modelling – Correlated Credit Migration

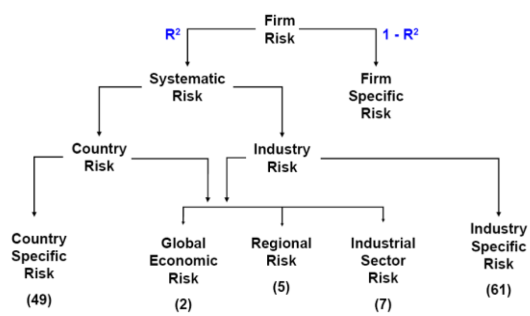


Correlated Credit Migration

- The same logic can be used to determine future rating states
- Consider the case of a counterparty currently rated ‘BBB’. The rating thresholds in one year’s are such that the areas of the standard normal distribution between ratings are equivalent to the credit rating transition probabilities for a bond ‘BBB’

Diversification and Correlation

Multi-Factor Model



Multi Factor Model Example – Moody's KMV

- The systematic risk component is replaced by a linear function of risks factors x_i with coefficients equal to β_i
- These risk factors consisting of primarily (i) country and (ii) industry specific features
- Also relates to states of the economy

Rating Agency Studies

Cumulative Probability of Default

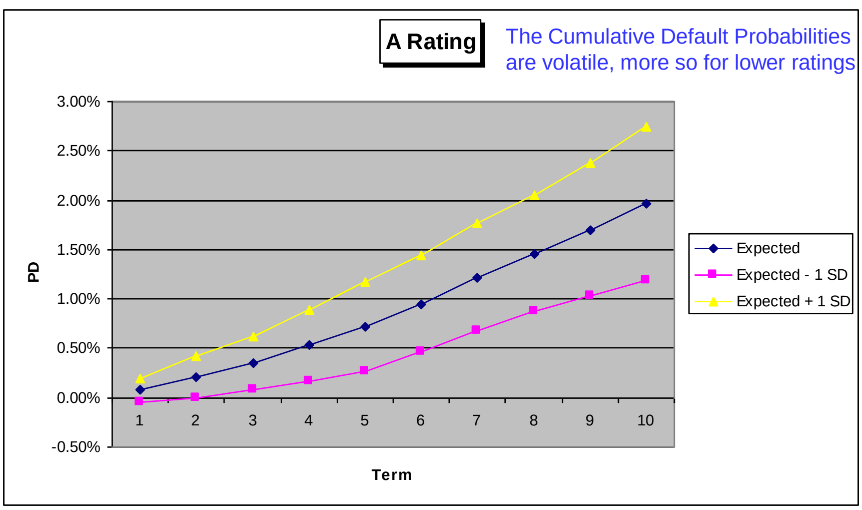
					Time Horizon (years)					
Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00%	0.03%	0.14%	0.26%	0.39%	0.51%	0.58%	0.68%	0.74%	0.82%
AA	0.02%	0.07%	0.14%	0.24%	0.33%	0.43%	0.52%	0.60%	0.67%	0.74%
A	0.08%	0.21%	0.35%	0.53%	0.72%	0.95%	1.22%	1.46%	1.70%	1.97%
BBB	0.26%	0.72%	1.23%	1.86%	2.53%	3.20%	3.80%	4.40%	5.00%	5.60%
BB	0.97%	2.94%	5.27%	7.49%	9.51%	11.48%	13.19%	14.75%	16.21%	17.45%
B	4.93%	10.76%	15.65%	19.46%	22.30%	24.57%	26.47%	28.06%	29.44%	30.82%
CCC/C	27.98%	36.95%	42.40%	45.57%	48.05%	49.19%	50.26%	51.09%	52.44%	53.41%
Investment	0.13%	0.35%	0.60%	0.91%	1.24%	1.58%	1.90%	2.20%	2.50%	2.80%
Speculative	4.44%	8.68%	12.42%	15.46%	17.90%	19.96%	21.72%	23.25%	24.67%	25.96%
All rated	1.63%	3.23%	4.67%	5.89%	6.90%	7.79%	8.55%	9.23%	9.86%	10.45%

Observation

- There are some inconsistencies by rating within term
  - Top-left: Higher rating, shorter time horizon (Rates need to be smoothed)
  - Function of the methodology - Static Pool Methodology
- Corporate Debt statistics – Adaptability for reinsurance default process ?
- Consider use of ‘stressed’ default rates – Impairment, Willingness to Pay

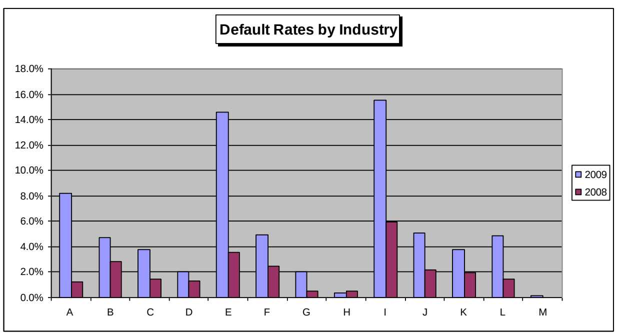
Rating Agency Studies

Volatility of Cumulative Probability of Default



Rating Agency Studies

Annual Corporate Default Rates

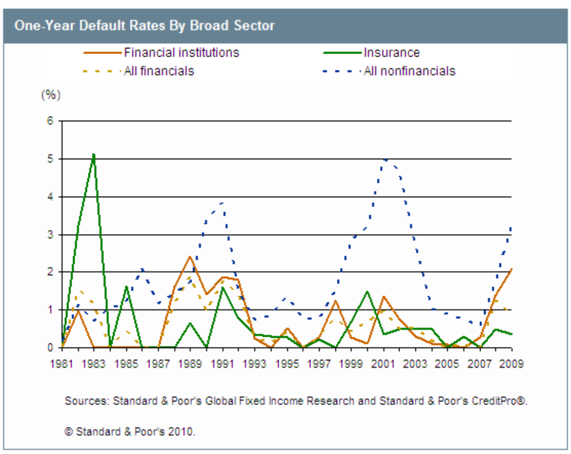


A	Aero / auto / capital goods / metal
B	Consumer / service sector
C	Energy and natural resources
D	Financial Institutions
E	Forest, building prods / homebuilders
F	Health care / chemicals
G	High tech\ / computers / office equip
H	Insurance
I	Leisure time / media
J	Real estate
K	Telecommunications
L	Transportation
M	Utility

- Default Rates vary markedly by:
  - Industry and
  - Calendar Year
- Insurance default rates are low (perhaps higher debt initial ratings)

Rating Agency Studies

One Year Default Rates



- Default Rates are very cyclical

Rating Agency Studies

Transition Matrices

One Year

From / To	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	88.2%	7.7%	0.5%	0.1%	0.1%	0.0%	0.1%	0.0%	3.3%
AA	0.6%	86.6%	8.1%	0.6%	0.1%	0.1%	0.0%	0.0%	4.0%
A	0.0%	2.0%	87.1%	5.5%	0.4%	0.2%	0.0%	0.1%	4.8%
BBB	0.0%	0.1%	3.8%	84.2%	4.1%	0.7%	0.2%	0.3%	6.7%
BB	0.0%	0.1%	0.2%	5.2%	75.5%	7.5%	0.8%	1.0%	9.8%
B	0.0%	0.0%	0.2%	0.2%	5.4%	72.7%	4.7%	4.9%	11.8%
CCC/C	0.0%	0.0%	0.2%	0.3%	0.9%	11.3%	45.0%	28.0%	14.4%

Transition Matrices

- Probability of moving from rating now to one at a future time horizon e.g. one year
- Largest values are along the diagonal
  - Values fall off very quickly moving off the diagonal
- Investment Grade companies tend to exhibit lower ratings volatility
- Transition matrices are based on historical rating changes
  - There is volatility in transition rates from year to year – macroeconomic etc.
- Often used for multi-year modelling of future states - $M_T = (M_1)^T$
  - where M_T = T-year transition matrix
  - assumes Markov Process for transition rates – a convenient modelling approach

Rating Agency Studies

Transition Matrices – Conditional vs Unconditional

Comparison Of Conditional Versus Unconditional Transition Matrices—One Year (1981-2006) (%)

From / To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
AA+	0.30	0.91	1.58	1.15	2.59	0.94	0.00	0.00	0.00	0.00	N.A.	N.A.	N.A.	0.00	N.A.	N.A.	N.A.	N.A.
AA	0.00	0.36	0.94	1.34	2.31	1.49	4.34	0.80	0.00	0.00	0.00	0.00	0.00	N.A.	N.A.	0.00	0.00	N.A.
AA-	0.00	1.30	0.51	0.89	1.63	2.25	1.30	0.75	1.30	0.00	0.00	N.A.	N.A.	0.00	5.22	0.00	N.A.	0.00
A+	N.A.	0.00	0.29	0.37	0.92	1.59	1.74	3.04	1.52	4.45	0.00	0.00	0.00	2.23	0.00	0.00	N.A.	3.71
A	0.00	0.00	0.80	1.34	0.56	0.92	1.84	2.18	0.87	2.15	2.41	0.80	1.21	1.10	0.00	0.00	0.00	2.01
A-	0.00	0.00	0.00	1.61	0.37	0.68	0.93	1.38	1.94	2.15	1.43	2.64	5.36	0.00	4.29	0.00	0.00	2.15
BBB+	N.A.	0.00	0.00	1.25	2.63	0.30	0.38	0.91	1.61	2.61	3.39	3.25	1.25	0.36	1.36	0.00	1.25	1.50
BBB	0.00	0.00	0.00	0.00	0.00	0.89	0.95	0.32	0.95	2.09	1.57	2.44	3.33	1.17	0.67	0.00	5.59	4.14
BBB-	0.00	N.A.	0.00	0.00	0.00	1.17	1.41	0.63	0.45	0.89	2.12	2.59	2.03	2.33	2.35	0.73	1.85	1.55
BB+	1.75	7.02	N.A.	0.00	0.00	0.00	2.81	0.00	0.44	0.48	0.99	1.59	1.72	1.87	1.53	4.21	1.65	3.90
BB	N.A.	N.A.	3.12	N.A.	N.A.	0.00	2.34	1.04	1.20	0.17	0.51	0.93	1.49	1.22	3.74	4.21	2.27	3.21
BB-	N.A.	N.A.	N.A.	22.14	0.00	0.00	0.00	1.23	2.60	0.82	0.74	0.91	0.84	1.53	2.20	1.67	2.82	3.01
B+	N.A.	0.00	N.A.	0.00	N.A.	2.77	1.39	2.22	4.44	0.00	0.85	0.74	0.60	0.91	1.57	1.79	2.06	1.91
B	N.A.	N.A.	0.00	5.26	N.A.	1.32	0.00	0.00	0.00	0.00	1.32	0.29	0.50	0.46	0.90	1.67	2.05	1.92
B-	N.A.	N.A.	N.A.	N.A.	0.00	2.01	N.A.	1.34	2.01	0.00	0.00	3.22	0.31	0.67	0.68	0.81	1.76	2.25
CCC/C	N.A.	N.A.	N.A.	N.A.	2.24	N.A.	2.24	0.00	1.12	0.00	2.24	2.24	0.93	0.71	0.78	0.56	0.79	1.59

Conditional transition matrix generated using entities that experienced downgrade in prior year. N.A.—Not available. *Sources:* Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

Values > 1.0 → Probability of Default greater if downgrade in a prior period.

Markov process for transition matrix assumes only current rating is important

- Conditional – Experienced a ratings downgrade in prior period
  - If Value = 1.0: Transitions conditioned on prior downgrade are no different
  - If Value > 1.0: Future ratings depends on Current AND Prior ratings

Rating Agency Studies

Recovery Rates

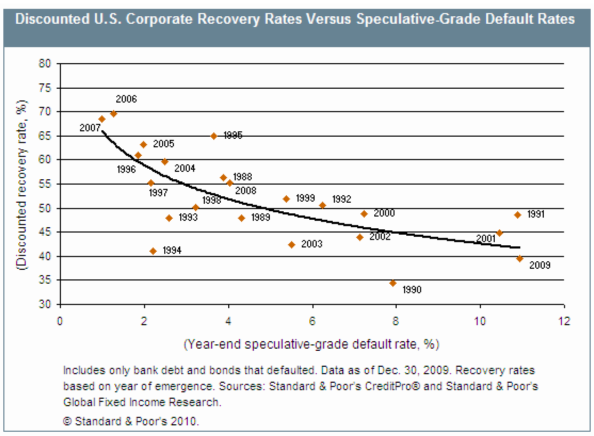
Discounted Recovery Rates By Instrument Type (1987-2009)

Instrument Type	Mean	Median	Std Dev	CV	Count
Term loans	69.4%	80.4%	32.9%	47.4%	616
Revolving credit	78.0%	95.4%	29.5%	37.9%	617
All loans/facilities	73.8%	87.5%	31.3%	42.4%	1,233
Senior secured bonds	57.2%	58.2%	30.9%	54.1%	299
Senior unsecured bonds	43.0%	39.2%	32.8%	76.4%	1,084
Senior subordinated bonds	28.3%	16.6%	32.5%	114.7%	495
All other subordinated bonds	19.4%	8.3%	29.9%	154.0%	425
All bonds	37.4%	29.3%	32.6%	87.3%	2,303
Total defaulted instruments	50.1%	47.9%	36.5%	73.0%	3,536

- Recovery rates are conditional on the level of debt seniority
- Higher security → greater expected recovery
- Standard deviation High
- Measurement does not ‘neutralise’ impact of economic cycle

Rating Agency Studies

Default Rate vs Recovery Rate



- Inverse relationship between Probability of Default and Recovery Rate

Rating Agency Studies

Impairment Rates – A.M. Best Studies

- A.M. Best rated U.S. domiciled insurance companies
- General Corporate Bond Default Rates are inappropriate for insurance:
  - Unique regulatory and accounting environments
- Impairment is a wider category of financial duress than default
  - Impairment often occurs when insurer able to meet policyholder obligations
  - Impairment rates > Default rates for a given rating
- Definition of Impairment
  - Financially Impaired Company (“FIC”) - First official regulatory action taken
    - Ability to conduct normal insurance operations is adversely affected
    - Capital and Surplus inadequate to meet legal requirements
    - General financial condition has triggered regulatory concern
- State Actions – Regulatory Supervision, Rehabilitation, Liquidation, Receivership

Modelling Reinsurance Credit Risk Loss

Assumptions

Loss Process

- Loss only due to default

Time Horizon

- 12-months (as per Solvency II)
- Duration mean-term liabilities (proxy for 12-monthly intervals with rating migration)

Monte Carlo Simulation

- 20,000 Gaussian and t copula simulations using MATLAB
- 20 reinsurers with variable exposure amounts (these assumed to remain constant)
- Variations in
  - Rating
  - Dependency (copulas)
  - LGD – Constant / Variable / Correlated variable

Modelling Reinsurance Credit Risk Loss

Assumptions

Reinsurer	Recoveries
1	10,000
2	15,000
3	20,000
4	25,000
5	30,000
6	35,000
7	40,000
8	45,000
9	50,000
10	55,000
11	60,000
12	65,000
13	70,000
14	75,000
15	80,000
16	85,000
17	90,000
18	95,000
19	100,000
20	105,000

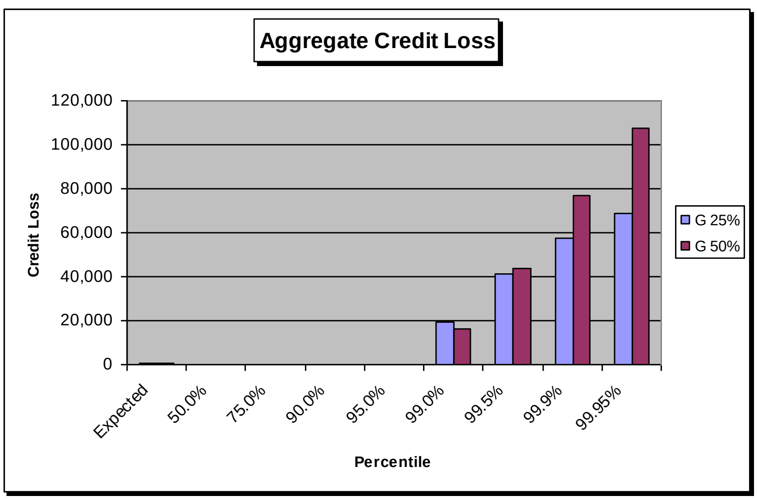
	Time Horizon (Years)				
Rating	1	2	3	4	5
AAA	0.005%	0.030%	0.070%	0.120%	0.160%
AA	0.020%	0.070%	0.140%	0.240%	0.330%
A	0.080%	0.210%	0.350%	0.530%	0.720%
BBB	0.260%	0.720%	1.230%	1.860%	2.530%
BB	0.970%	2.940%	5.270%	7.490%	9.510%
B	4.930%	10.760%	15.650%	19.460%	22.300%
CCC/C	27.980%	36.950%	42.400%	45.570%	48.050%

Rating	PD	E (LGD)	SD (LGD)	Alpha (α)	Beta (β)
AAA	0.005%	40.0%	25.0%	1.14	1.70
AA	0.020%	50.0%	25.0%	1.50	1.50
A	0.080%	55.0%	25.0%	1.63	1.33
BBB	0.260%	58.0%	25.0%	1.68	1.22
BB	0.970%	60.0%	25.0%	1.70	1.14
B	4.930%	65.0%	25.0%	1.72	0.92
CCC	27.980%	80.0%	25.0%	1.25	0.31

Loss Severity is assumed to follow a Beta Distribution

Numerical Examples

A Rating, 1 Yr PD, Gaussian Copula (25% and 50%), Constant LGD

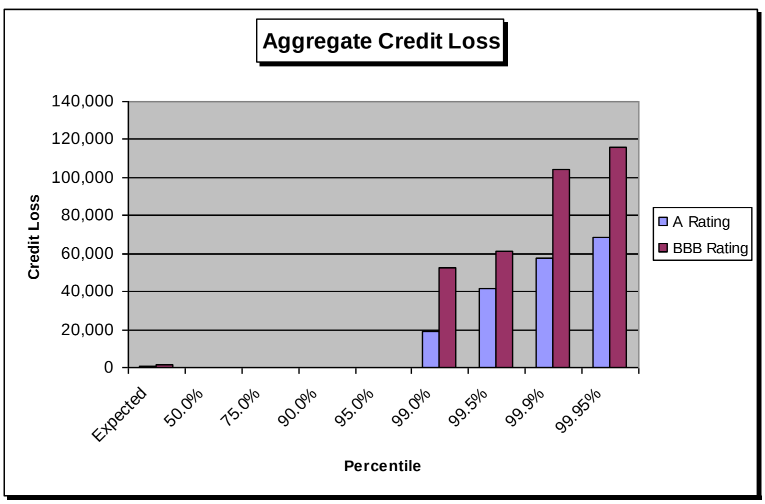


Gaussian 25% zero losses for 98.5% of dist.

VaR (99.5%) = 87x E(Loss)

Numerical Examples

A and BBB Rating, 1 Yr PD, Gaussian Copula 25%, Constant LGD

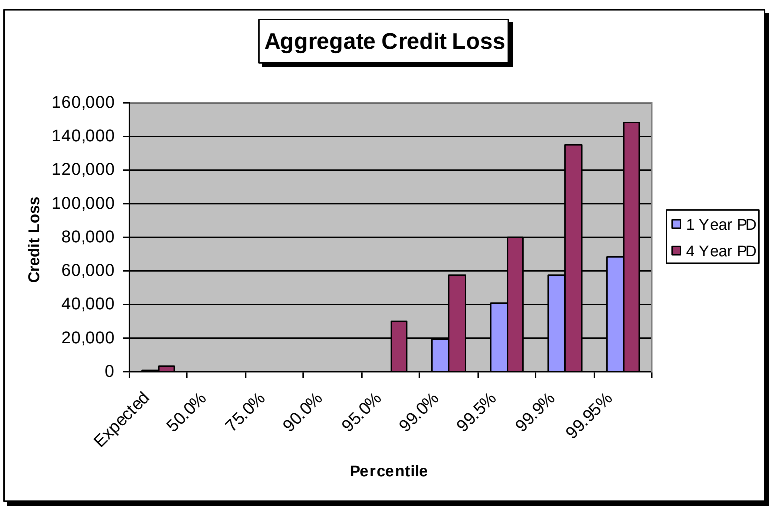


BBB Gaussian 25% zero losses for 95.7% of dist.

VaR (99.5%) = 35x E(Loss)

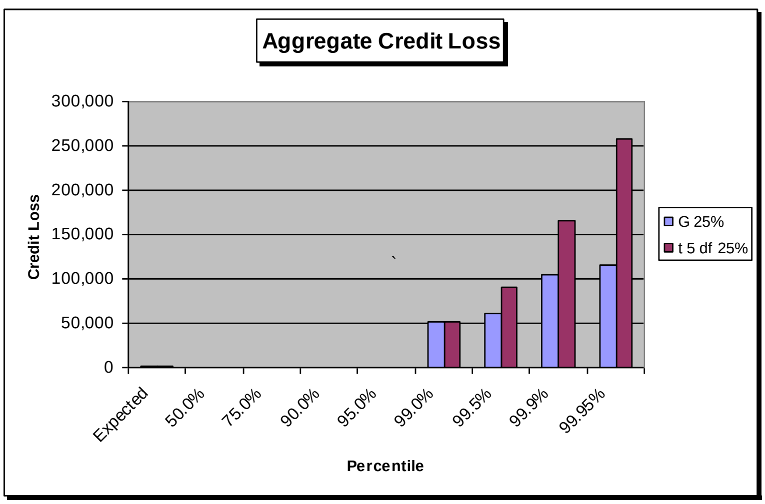
Numerical Examples

A Rating, 1 Yr and 4 Yr PD, Gaussian Copula 25%, Constant LGD



Numerical Examples

BBB Rating, 1 Yr PD, Gaussian 25% and t 5 df 25% Copula, Constant LGD



Modelling Challenges
Assumptions

Setting assumptions for

- Probability of Default (setting “Stressed levels”)
- Loss Given Default
- Asset (or Default Correlation)
- Dependencies
  - Amongst the above e.g. PD and LGD; or Value of Asset Return and LGD
  - Other variables – insurance loss and default rate

Risk Aggregation

- Copulas or Factor Models
- Single vs Multi-Factor Models
- Model Calibration