The Actuarial Profession
making financial sense of the future

GIRO 2011
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Double chain ladder with a touch of Bornhuetter-Ferguson

Agenda

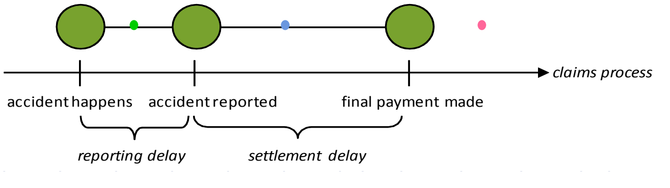
- Introducing the problem: stochastic reserving
  - Current solution: chain ladder methods
- Motivating a model for the problem of stochastic reserving
  - Addressing the limitations of chain ladder methods
- Defining a model for the problem of stochastic reserving
  - Consistency with the chain ladder method

Agenda

- The double chain ladder estimation method
- New insights:
  - Estimating the tail
  - Separation into RBNS and IBNR
  - Introducing prior knowledge
- Simulation methods to obtain statistical distributions
- Conclusions

The individual claims mechanism

- The life of an individual claim in the general claims process:



- Three categories of claim:
  - Reported and settled
  - Reported but not settled, **RBNS**
  - Incurred but not reported, **IBNR**

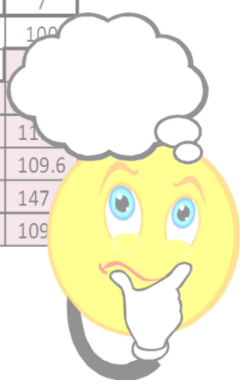
The problem: stochastic reserving

- Outstanding liabilities are impacted by two types of delay during the claims process:
  - Reporting delay
  - Settlement delay

- Objectives:
  - Produce point forecasts for the outstanding reserve and cash flows
  - Produce accompanying distributions

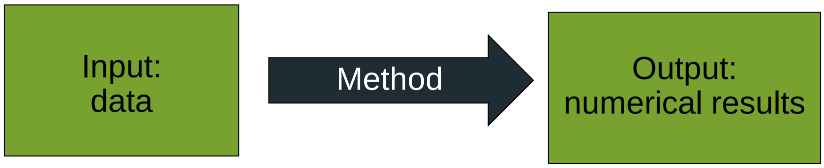
Motivating a model for the chain ladder mean

	1	2	3	4	5	6	7
1	2200	1500	1000	650	300	150	100
2	1900	1400	900	550	250	145	[illegible]
3	2300	1700	1200	750	400	175.9	[illegible]
4	3000	1800	950	500	369.9	183.4	11[illegible]
5	2700	1500	1000	641.8	345.8	171.4	109.6
6	3400	2200	1414.0	865.7	466.4	231.2	147[illegible]
7	2500	1629.0	1042.6	638.3	343.9	170.5	109[illegible]



What is a method?

- A sequence of steps, specifically designed to produce particular results



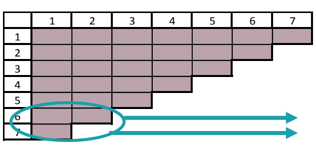
- A method can be inflexible
  – It is hard to adapt it to deal with unsatisfactory results

- An example is the chain ladder method

The chain ladder method

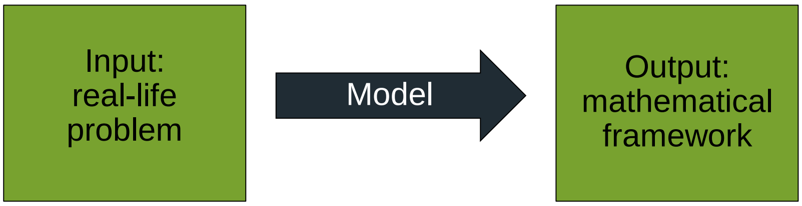
- Current method for calculating loss reserves: chain ladder method (CLM)

- CLM in its most basic form suffers from three main drawbacks:
  - Unstable estimates
  - No information about the tail
  - Unable to separate RBNS and IBNR claims



What is a model?

- A mathematical framework that completely describes a real-life problem



- Translates a real-life problem into a language which we, as mathematicians, can understand and work with
- To apply to a specific data set, we also require an estimation method based on the model

Introducing the model: addressing limitations of CLM

- We will introduce a mathematical model which underlies the CLM
- Using this model we are able to:
  - Reduce the instability of the CLM in a natural way by introducing prior knowledge at a micro level
  - Automatically provide the tail
  - Separate into RBNS and IBNR claims
- With this model, we are creating a vehicle which can incorporate current actuarial techniques in a more natural manner

Summary

- The problem of stochastic reserving includes many dependencies
- These are **implicit** within the chain ladder method
- They will be made **explicit** in our model

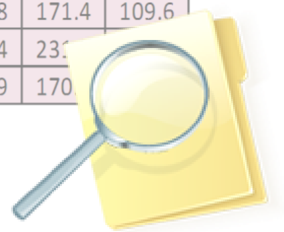
CL predictions for payments

	1	2	3	4	5	6	7
1	2200	1500	1000	650	300	150	100
2	1900	1400	900	550	250	145	88.7
3	2300	1700	1200	750	400	175.9	112.5
4	3000	1800	950	500	369.9	183.4	117.3
5	2700	1500	1000	641.8	345.8	171.4	109.6
6	3400	2200	1414.0	865.7	466.4	231.2	147.9
7	2500	1629.0	1042.6	638.3	343.9	170.5	109.0

Settlement delay
Reporting delay
Inflation
Accident / Development year
Mean of Individual Payment

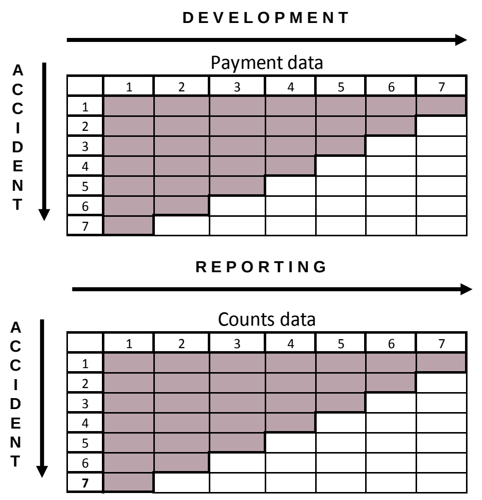
Defining a model for stochastic reserving

	1	2	3	4	5	6	7
1	2200	1500	1000	650	300	150	100
2	1900	1400	900	550	250	145	88.7
3	2300	1700	1200	750	400	175.9	112.5
4	3000	1800	950	500	369.9	183.4	117.3
5	2700	1500	1000	641.8	345.8	171.4	109.6
6	3400	2200	1414.0	865.7	466.4	23[illegible]	[illegible]
7	2500	1629.0	1042.6	638.3	343.9	170[illegible]	[illegible]



The modelled data: two run-off triangles

- We model annual data triangles
  - Incremental aggregated payment data
  - Incremental aggregated counts data, which is assumed to have fully run off



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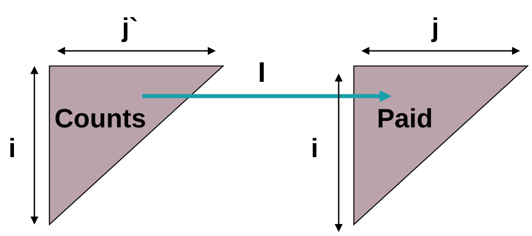
Introducing index notation

- We index the data as follows:
  - Accident year, *i*
  - Reporting delay, *j`*
  - Settlement delay, *l*
  - Development delay, *j*



- Note that j = j` + l

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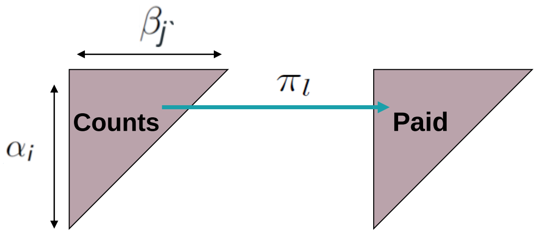
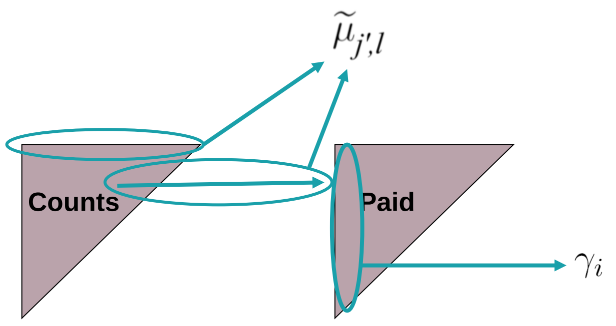
The parameters involved in the model



- Accident year: α_i
  - Represents ultimate claim numbers
- Reporting delay: β_j
  - Represents the proportion of ultimate claims reported with *j* period delay
- Settlement delay: π_l
  - Represents the proportion of claims settled *l* years after being reported

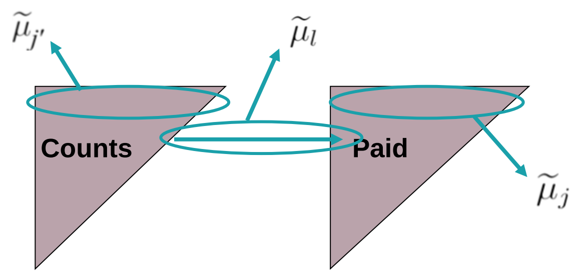
The inflation parameters involved in the model



- Inflation parameters
  - $\tilde{\mu}_{j',l}$ dependency on reporting delay and settlement delay
  - γ_i dependency on accident year
- Individual claim payment mean = $\tilde{\mu}_{j',l}$ x γ_i

The generality of the inflation parameters

- The inflation parameters can account for many dependencies, according to the choice of the practitioner
  - Dependence on the reporting delay: $\widetilde{\mu}_{j',l} = \widetilde{\mu}_{j'}$
  - Dependence on the settlement delay: $\widetilde{\mu}_{j',l} = \widetilde{\mu}_{l}$
  - Dependence on the development delay: $\widetilde{\mu}_{j',l} = \widetilde{\mu}_{j'+l}$



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Deriving an expression for the mean

- Under our model the mean of the total of the incremental payments, for accident year *i* and development delay *j,* is given by:

$$\mathrm{E}[X_{ij}] = \alpha_i \gamma_i \sum_{l=0}^{j} \beta_{j-l} \widetilde{\mu}_{j-l,l} \widetilde{\pi}_l$$

- **Is this consistent with the chain ladder method?**

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The chain ladder mean

- The chain ladder mean of the total of the incremental payments, for accident year *i* and development delay *j,* can be formulated as:

$$\mathrm{E}[X_{ij}] = \widetilde{\alpha}_i \widetilde{\beta}_j$$

  - $\widetilde{\alpha}_i$ represents ultimate payment numbers
  - $\widetilde{\beta}_j$ represents the development delay

- For derivation of this result, see Mack (1991)

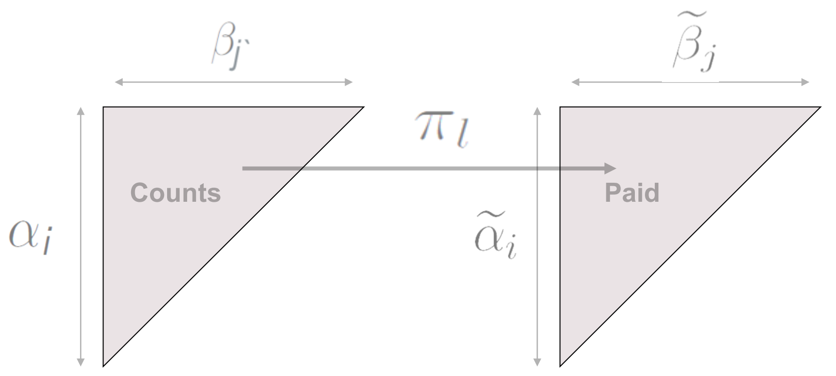
Rediscovering the chain ladder mean

- We impose the following relationships:

$$\boxed{\begin{aligned} \alpha_i \gamma_i &= \widetilde{\alpha}_i \\ \sum_{l=0}^{j} \beta_{j-l} \widetilde{\mu}_{j-l,l} \widetilde{\pi}_l &= \widetilde{\beta}_j \end{aligned}}$$

- This ensures that our model has the same component structure as the one implicitly assumed by CLM

The double chain ladder estimation method



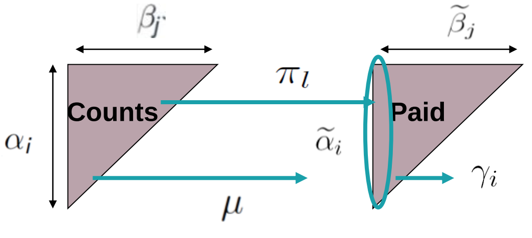
Introducing the double chain ladder method

- DCL is a method like CLM to produce estimations for the total of the incremental payments
- The classical chain ladder algorithm is applied twice to obtain estimates for all of the parameters in the model
- They can give the same value for the point estimates but DCL gives us more information

Over-parameterisation of the chain ladder mean model

- We aim to solve the problem using only two run-off triangles
- Therefore, we have to restrict ourselves to: $\widetilde{\mu}_{j',l} = \widetilde{\mu}_l$
  - Given more data, this restriction may not be necessary
- We rescale to obtain a constant mean: $\mu = \sum_{l=0}^{m-1} \widetilde{\pi}_l \widetilde{\mu}_l$
  - µ represents the mean of individual claim payments in the first accident year
- We can now completely solve the problem

The parameters to estimate by DCL



- Ultimate claim numbers: α_i
- Reporting delay: $\beta_{j'}$
- Settlement delay: π_l
- Development delay: $\widetilde{\beta}_j$
- Ultimate payment numbers: $\widetilde{\alpha}_i$
- Severity inflation: γ_i
- Individual payment mean in first year μ

The DCL method: estimating the parameters

- Apply CLM to count data from a toy example to get the estimates $\widehat{\alpha}_i, \widehat{\beta}_j$.

Count Data

	1	2	3	4	5	6	7
1	230	100	40	10	3	2	1
2	200	110	35	5	2	1	
3	210	85	25	7	2		
4	270	130	50	20			
5	240	100	45				
6	285	135					
7	240						

The DCL method: estimating the parameters

- Apply CLM to count data from a toy example to get the estimates $\widehat{\alpha}_i, \widehat{\beta}_j$.

Estimated Counts

	1	2	3	4	5	6	7
1	227.30	104.53	38.63	10.48	2.52	1.57	1.00
2	208.40	95.84	35.42	9.61	2.31	1.44	0.92
3	195.00	89.69	33.14	8.99	2.16	1.34	0.86
4	280.40	128.98	47.66	12.93	3.11	1.93	1.23
5	236.20	108.64	40.15	10.89	2.62	1.63	1.04
6	287.70	132.32	48.90	13.26	3.19	1.98	1.27
7	240.00	110.38	40.79	11.06	2.66	1.65	1.06

→

$\widehat{\alpha}$
386
353
331
476
401
489
408

$\widehat{\beta}$						
0.589	0.271	0.1	0.027	0.007	0.004	0.003

- Reminder:
  - $\widehat{\alpha}_i$ represents ultimate claim numbers in the i^{th} accident period
  - $\widehat{\beta}_j$ represents the proportion of ultimate claims reported with *j* period delay

The DCL method: estimating the parameters

- Apply CLM to the payment data to obtain the estimates $\widehat{\widehat{\alpha}}_i, \widehat{\widehat{\beta}}_j$

Payment data

	1	2	3	4	5	6	7
1	2200	1500	1000	650	300	150	100
2	1900	1400	900	550	250	145	
3	2300	1700	1200	750	400		
4	3000	1800	950	500			
5	2700	1500	1000				
6	3400	2200					
7	2500						

The DCL method: estimating the parameters

- Apply CLM to the payment data to obtain the estimates $\widehat{\widehat{\alpha}}_i, \widehat{\widehat{\beta}}_j$

Estimated Payments

	1	2	3	4	5	6	7
1	2,292.8	1,494.0	956.2	585.4	315.4	156.3	100.0
2	2,033.8	1,325.3	848.2	519.3	279.8	138.7	88.7
3	2,579.7	1,681.0	1,075.8	658.7	354.9	175.9	112.5
4	2,689.4	1,752.4	1,121.6	686.7	369.9	183.4	117.3
5	2,513.7	1,638.0	1,048.3	641.8	345.8	171.4	109.6
6	3,390.6	2,209.4	1,414.0	865.7	466.4	231.2	147.9
7	2,500.0	1,629.0	1,042.6	638.3	343.9	170.5	109.0

$\widehat{\widehat{\beta}}$

$\widehat{\widehat{\alpha}}$							
5900	*0.389*	0.253	0.162	0.099	0.053	0.026	0.017
5233							
6638							
6920							
6468							
8725							
6433							

- Reminder:
  - $\widehat{\widehat{\alpha}}_i$ represents ultimate payment numbers in the i^{th} accident period
  - $\widehat{\widehat{\beta}}_j$ represents the proportion of ultimate claims that develop in period j

The DCL method: estimating the parameters

- Use the following relationships between the CLM estimates and the parameters to estimate the remaining parameters:

$$\alpha_i \mu \gamma_i = \widetilde{\alpha}_i$$

$$\sum_{l=0}^{j} \beta_{j-l} \pi_l = \widetilde{\beta}_j$$

- Reminder:
  - π_l represents the proportion of claims settled *l* years after reporting
  - γ_i represents the claims inflation in the *i*th accident period
  - µ represents the mean of individual payments in the first accident year

The DCL method: estimating the parameters

- Solving the linear system gives the following values:

$\widehat{\pi}$	*0.66*	0.127	0.105	0.068	0.028	0.01	0.002
$\widehat{\gamma}$	*1*	0.967	1.311	0.951	1.055	1.168	1.033
$\widehat{\mu}$	*15.28*						

- We've now estimated all the parameters, and can apply the formula derived from the model

Estimating the RBNS claims

Count Data

	1	2	3	4	5	6	7
1	230	100	40	10	3	2	1
2	200	110	35	5	2	1	
3	210	85	25	7	2		
4	270	130	50	20			
5	240	100	45				
6	285	135					
7	240						

Payment Data

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2,200	1,500	1,000	650	300	150	100						
2	1,300	1,400	300	550	250	145							
3	2,300	1,700	1,200	750	400								
4	3,000	1,800	350	500									
5	2,700	1,500	1,000										
6	3,400	2,200											
7	2,500												

- RBNS claims contribute to cells to the right of the paid data

Estimating the RBNS claims

Estimated Counts

	1	2	3	4	5	6	7
1	227.3	104.5	38.63	10.48	2.523	1.565	1
2	208.4	95.84	35.42	9.606	2.313	1.435	
3	195	89.69	33.14	8.99	2.164		
4	280.4	129	47.66	12.93			
5	236.2	108.6	40.15				
6	287.7	132.3					
7	240						

$\widehat{\pi}_l \; \widehat{\gamma}_i \; \widehat{\mu}$

RBNS Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1								21.0	7.2	2.5	0.8	0.2	0.0
2							51.6	16.9	5.0	1.3	0.3	0.0	
3						[illegible]	62.1	18.6	4.5	0.8	0.1		
4					340.1	159.1	60.0	16.3	3.4	0.4			
5				525.9	295.7	130.3	44.0	10.3	1.5				
6			837.7	598.9	307.2	116.1	34.6	5.6					
7		479.2	397.3	258.9	107.5	36.2	8.9						

- RBNS claims contribute to cells to the right of the paid data
- We predict RBNS reserve using estimated parameters and estimated count data from the upper triangle
- RBNS point prediction for cell (i,j): $$\widehat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{\min(j,d)} \widehat{N}_{i,j-l}\widehat{\pi}_l\widehat{\mu}\widehat{\gamma}_i$$

Worked example

- For illustration, we focus on payments in cell (1,11)

Estimated Counts

	1	2	3	4	5	6	7
1	227.3	104.5	38.63	10.48	2.523	1.565	1
2	208.4	95.84	35.42	9.606	2.313	1.435	
3	195	89.69	33.14	8.99	2.164		
4	280.4	129	47.66	12.93			
5	236.2	108.6	40.15				
6	287.7	132.3					
7	240						

RBNS Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1													
2													
3													
4													
5													
6													
7													

- RBNS estimation for (1,11) comes from reported counts in the previous six years:
  - We have chosen a maximum delay of six years

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Worked example

Estimated Counts

	1	2	3	4	5	6	7
1	227.3	104.5	38.63	10.48	2.523	1.565	1
2	208.4	95.84	35.42	9.606	2.313	1.435	
3	195	89.69	33.14	8.99	2.164		
4	280.4	129	47.66	12.93			
5	236.2	108.6	40.15				
6	287.7	132.3					
7	240						

$\boxed{2.523} \times \widehat{\pi}_6$

$= 2.523 \times 0.0011$
$= 0.0028$

- Consider the counts from six years ago – cell (1,5)
- Multiply by $\widehat{\pi}_6$ which represents the proportion of claims for which a payment is made after six years
- Gives an estimate for the number of claims reported six years ago that contributes to our cell (1,11)

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Worked example

Estimated Counts

	1	2	3	4	5	6	7
1	227.3	104.5	38.63	10.48	2.523	1.565	1
2	208.4	95.84	35.42	9.606	2.313	1.435	
3	195	89.69	33.14	8.99	2.164		
4	280.4	129	47.66	12.93			
5	236.2	108.6	40.15				
6	287.7	132.3					
7	240						

$$2.523 \text{ x } \widehat{\pi}_6 + 1.565 \text{ x } \widehat{\pi}_5 + 1 \text{ x } \widehat{\pi}_4 = 0.046$$

- Proceed in the same way to find estimates for the number of claims reported four and five years ago that contributes to our cell (1,11)
- Sum to get the total estimate of the number of claims that contribute to (1,11)

Worked example

- We've estimated the total number of claims that contribute to (1,11) as 0.046
- Now we multiply by $\widehat{\mu}$ x $\widehat{\gamma}_1$, which represents the mean of claim payments which occurred in the first accident period
- This gives us our RBNS estimation for cell (1,11):

$$0.046 \text{ x } \widehat{\mu} \text{ x } \widehat{\gamma}_1 = 0.710$$

Estimating the IBNR claims

- Since the accidents are not reported yet, the IBNR reserves are derived from the lower triangle
- This fills in the paid triangle in the purple highlighted section:

Estimated Counts

	1	2	3	4	5	6	7
1							
2							0.9
3						1.3	0.9
4					3.1	1.9	1.2
5				10.9	2.6	1.6	1.0
6			48.9	13.3	3.2	2.0	1.3
7		110.4	40.8	11.1	2.7	1.7	1.1

IBNR Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1													
2							8.9	1.7	1.4	0.9	0.4	0.1	0.0
3						17.8	14.8	5.0	3.6	1.9	0.7	0.2	0.0
4					29.8	24.2	20.1	8.3	5.1	2.5	1.1	0.2	0.0
5				115.9	50.1	41.1	30.8	12.7	6.4	2.7	0.8	0.2	0.0
6			576.3	266.8	159.2	115.1	66.3	25.5	10.7	3.6	1.1	0.3	0.1
7		1,149.8	645.3	379.4	236.4	134.3	65.5	22.9	7.9	2.7	0.8	0.2	0.0

- IBNR point prediction for cell (i,j) : $\widehat{X}_{ij}^{ibnr} = \sum_{l=0}^{\min(i-m+j-1,d)} \widehat{N}_{i,j-l}\widehat{\pi}_l\widehat{\mu}\widehat{\gamma}_i$

Worked example

- For illustration, we focus on payments in cell (3,11)

Estimated Counts

	1	2	3	4	5	6	7
1							
2							0.9
3						1.3	0.9
4					3.1	1.9	1.2
5				10.9	2.6	1.6	1.0
6			48.9	13.3	3.2	2.0	1.3
7		110.4	40.8	11.1	2.7	1.7	1.1

IBNR Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1													
2													
3													
4													
5													
6													
7													

- IBNR estimation for (3,11) comes from incurred but not reported counts in the previous six years:
  - We have chosen a maximum delay of six years

Worked example

Estimated Counts

	1	2	3	4	5	6	7
1							
2							0.9
3						1.3	0.9
4					3.1	1.9	1.2
5				10.9	2.6	1.6	1.0
6			48.9	13.3	3.2	2.0	1.3
7		110.4	40.8	11.1	2.7	1.7	1.1

- Consider the counts from six years ago – cell (3,5)
- Multiply by $\hat{\pi}_6$ which represents the proportion of claims for which a payment is made after six years
- Gives an estimate for the number of claims reported six years ago that contributes to our cell (3,11)

$\boxed{3.1} \times \hat{\pi}_6$

$= 3.1 \times 0.0011$
$= 0.0034$

Worked example

Estimated Counts

	1	2	3	4	5	6	7
1							
2							0.9
3						1.3	0.9
4					3.1	1.9	1.2
5				10.9	2.6	1.6	1.0
6			48.9	13.3	3.2	2.0	1.3
7		110.4	40.8	11.1	2.7	1.7	1.1

$\boxed{3.1} \times \hat{\pi}_6 + \boxed{1.9} \times \hat{\pi}_5 + \boxed{1.2} \times \hat{\pi}_4 = 0.056$

- Proceed in the same way to find estimates for the number of claims reported four and five years ago that contributes to our cell (3,11)
- Sum to get the total estimate of the number of claims that contribute to (3,11)

Worked example

- We've estimated the total number of claims that contribute to (3,11) as 0.056
- Now we multiply by $\widehat{\mu}$ x $\widehat{\gamma}_3$, which represents the mean of claim payments which occurred in the third accident period
- This gives us our IBNR estimation for cell (1,11):

$$0.056 \times \widehat{\mu} \times \widehat{\gamma}_3 = 1.122$$

The predicted reserve: the chain ladder mean

RBNS Estimates

	1	2	3	4	5	6	7
1							
2							51.6
3						158.1	62.1
4					340.1	159.1	60.0
5				525.9	295.7	130.3	44.0
6			837.7	598.9	307.2	116.1	34.6
7		479.2	397.3	258.9	107.5	36.2	8.9

IBNR Estimates

	1	2	3	4	5	6	7
1							
2							8.9
3						17.8	14.8
4					29.8	24.2	20.1
5				115.9	50.1	41.1	30.8
6			576.3	266.8	159.2	115.1	66.3
7		1,149.8	645.3	379.4	236.4	134.3	65.5

Total Estimates

	1	2	3	4	5	6	7
1							
2							60.6
3						175.9	76.8
4					369.9	183.4	80.1
5				641.8	345.8	171.4	74.9
6			1,414.0	865.7	466.4	231.2	101.0
7		1,629.0	1,042.6	638.3	343.9	170.5	74.4

The estimated reserve: the chain ladder mean

RBNS Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1								21.0	7.2	2.5	0.8	0.2	0.0
2							51.6	16.9	5.0	1.3	0.3	0.0	
3						158.1	62.1	18.6	4.5	0.8	0.1		
4					340.1	159.1	60.0	16.3	3.4	0.4			
5				525.9	295.7	130.3	44.0	10.3	1.5				
6			837.7	598.9	307.2	116.1	34.6	5.6					
7		479.2	397.3	258.9	107.5	36.2	8.9						

IBNR Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1													
2							9.9	1.9	1.5	1.0	0.5	0.1	0.2
3						14.4	12.0	4.0	2.9	1.6	0.6	0.3	0.2
4					29.8	24.2	20.1	8.3	5.1	2.5	1.3	0.5	0.2
5				115.9	50.1	41.1	30.8	12.7	6.4	4.3	1.2	0.5	0.2
6			576.3	266.8	159.2	115.1	66.3	25.5	18.7	5.8	1.6	0.6	0.3
7		1,149.8	645.3	379.4	236.4	134.3	65.5	38.8	13.8	4.3	1.2	0.5	0.2

….Or by accident year

Total Estimates

	1	2	3	4	5	6	7	8	9	10	11	12	13
1								21.0	7.2	2.5	0.8	0.2	0.0
2							60.6	18.6	6.4	2.2	0.7	0.2	0.0
3						173.9	76.8	23.6	8.2	2.8	0.8	0.2	0.0
4					369.9	183.4	80.1	24.7	8.5	2.9	0.9	0.2	0.0
5				641.8	345.8	171.4	74.9	23.0	7.9	2.7	0.8	0.2	0.0
6			1,414.0	865.7	466.4	231.2	101.0	31.1	10.7	3.6	1.1	0.3	0.1
7		1,629.0	1,042.6	638.3	343.9	170.5	74.4	22.9	7.9	2.7	0.8	0.2	0.0

Cash flow by calendar year

….Or the total reserve

Using the available information

- Currently, when calculating the RBNS, we use the formula:

$$\widehat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} \widehat{N}_{i,j-l}\widehat{\pi}_l\widehat{\mu}\widehat{\gamma}_i$$

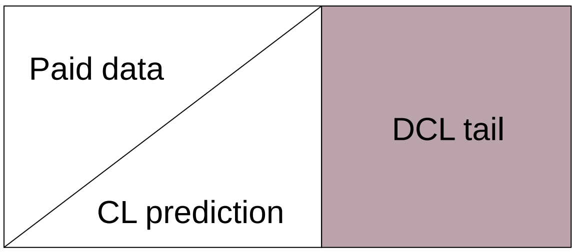
  which involves the estimated counts
  - This produces a result consistent with the CLM
- We could instead use the count data directly in this formula:

$$\widehat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} N_{i,j-l}\widehat{\pi}_l\widehat{\mu}\widehat{\gamma}_i$$

- This leads to greater accuracy, since we are using actual count data rather than estimated counts

Predicting the tail through DCL

- With CLM, when a triangle has not run-off one needs to fit a tail
- DCL provides the tail prediction as an intrinsic part of the model



DCL and introducing prior knowledge

- CLM (and therefore DCL) provides a prediction for the reserve which is heavily dependent on the figures in the bottom left of the triangle

	1	2	3	4	5	6	7
1	227.3	104.5	38.63	10.48	2.523	1.565	1
2	208.4	95.84	35.42	9.606	2.313	1.435	
3	195	89.69	33.14	8.99	2.164		
4	280.4	129	47.66	12.93			
5	236.2	108.6	40.15				
6	287.7	132.3					
7	240						

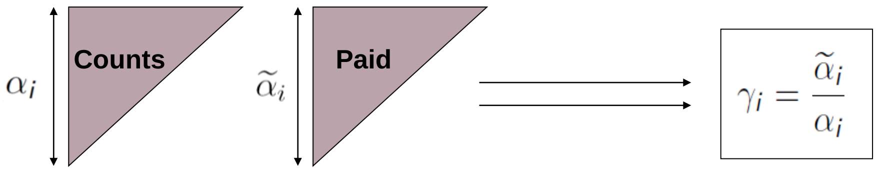
- The estimators from CLM seem to be unstable
- Methods such as the Bornhuetter-Ferguson method propose to improve the estimates for recent accident periods by incorporating prior knowledge

Locating the source of the instability

- The model breaks down the chain ladder estimates into their individual components

$$\tilde{\alpha}_i = \alpha_i \gamma_i$$

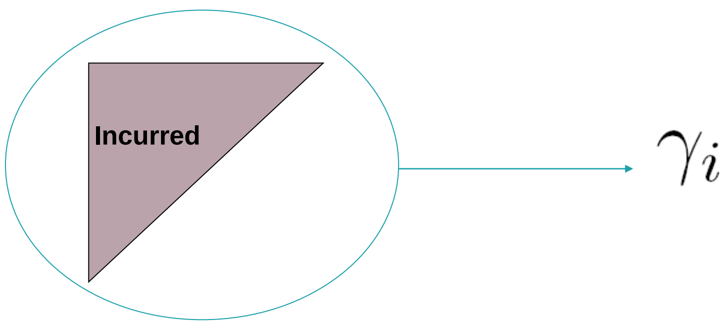
- The instability comes from the estimation of the severity inflation



Looking for information in the incurred data

- **The proposed solution:**

  Take a more realistic estimation of the inflation from the incurred triangle using BDCL (Bayesian Double Chain Ladder)

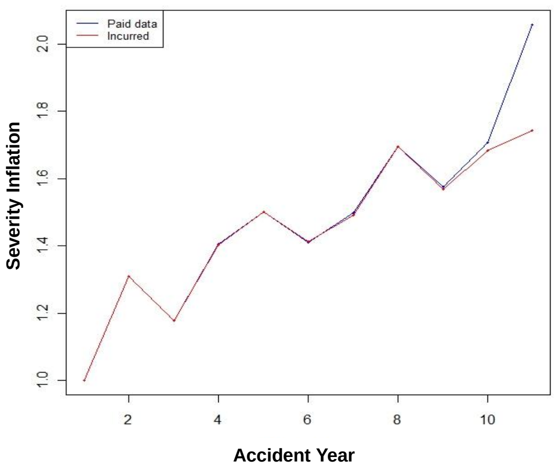


An example with real data

- We consider a liability dataset consisting of three triangles: payment, counts and incurred data
- Apply DCL estimation method to obtain point forecasts for future calendar years
- Total reserve estimated at approximately **£14 million**

	DCL		
Future	RBNS	IBNR	Total
1	11,302,982	975,297	12,278,280
2	781,910	712,483	1,494,393
3	329,991	81,801	411,792
4	171,565	31,225	202,790
5	0	18,199	18,199
6	0	3,002	3,002
7	0	1,123	1,123
8	0	359	359
9	0	42	42
10	0	11	11
11	0	4	4
12	0	0	0
13	0	0	0
Total	12,586,449	1,823,547	14,409,995

Comparison of inflation estimates



- The instability within the paid data can be seen in the estimates for the inflation in the last 2 accident years
- The estimates from the incurred data are more stable in the final accident periods

Using BDCL to obtain a more realistic reserve

- DCL reserve using estimates for inflation from the paid data
- BDCL reserve using estimates for inflation from the incurred data
- The total reserve is **13% lower** using the incurred data to estimate the inflation

	BDCL			DCL		
Future	RBNS	IBNR	Total	RBNS	IBNR	Total
1	9,741,548	832,613	10,574,161	11,302,982	975,297	12,278,280
2	705,311	610,024	1,315,334	781,910	712,483	1,494,393
3	300,007	71,688	371,695	329,991	81,801	411,792
4	146,369	27,373	173,743	171,565	31,225	202,790
5	0	15,791	15,791	0	18,199	18,199
6	0	2,675	2,675	0	3,002	3,002
7	0	993	993	0	1,123	1,123
8	0	309	309	0	359	359
9	0	37	37	0	42	42
10	0	10	10	0	11	11
11	0	3	3	0	4	4
12	0	0	0	0	0	0
13	0	0	0	0	0	0
Total	10,893,235	1,561,517	12,454,751	12,586,449	1,823,547	14,409,995

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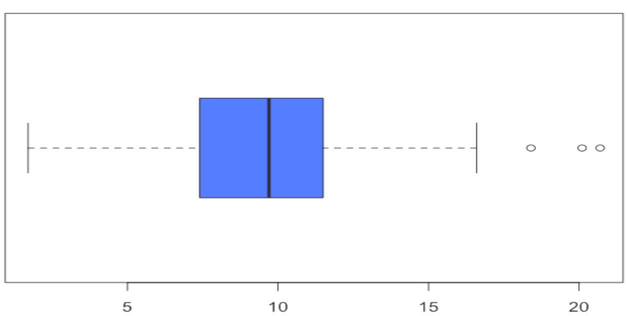
The full statistical model



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Obtaining a distribution

- So far we have only discussed point estimates of the individual payments
- We have at no point mentioned anything about the variance or the distribution of the reserve estimations
- Now we will discuss how the introduction of a model allows us to obtain full distributions based on our model assumptions

Parameters and distributions

- We will only introduce a single new parameter: the variance of the individual payments
- The following statistical distributions are assumed for each of the components in the model:

Component	Distribution
Count data	Poisson
Settlement delay	Multinomial
Individual payments	Gamma

Estimates for simulation

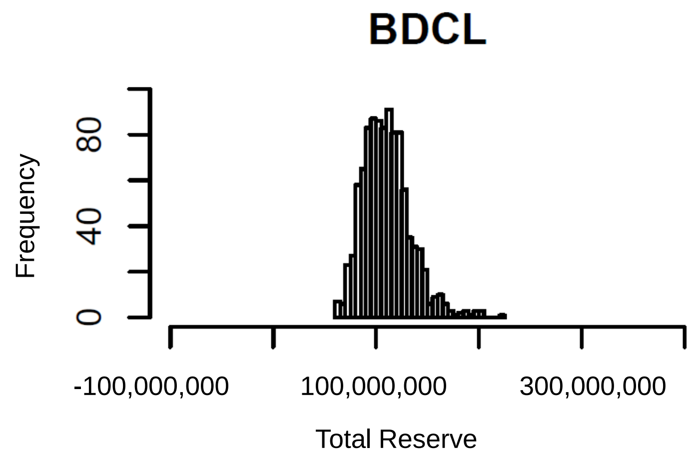
- We already have estimates for many of the parameters
  - Only need to estimate $\widehat{\sigma}^2$ via the method of least squares
- Now we have all the information we need to simulate the data
- We derive empirical distributions of:
  - The cash flows
  - The total reserve

Empirical illustration

- Consider the following results produced from a motor dataset

	Simulated predictive distribution from BDCL		
	RBNS ('000s)	IBNR ('000s)	Total ('000s)
Mean	97,508	9,127	106,635
SD	18,776	5,429	21,804
0.50%	61,165	1,221	65,882
1%	62,110	1,943	69,645
5%	70,856	2,908	76,602
10%	76,141	3,700	81,728
25%	85,040	5,401	91,913
50%	95,383	7,886	103,781
75%	107,979	11,661	119,122
90%	120,950	15,603	134,064
95%	130,938	19,248	146,686
99%	152,070	26,404	171,998
99.50%	165,542	32,460	183,404

Distribution histogram



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Conclusions

- The chain ladder model is a solid framework for loss reserving
- Provides a natural method for introducing prior knowledge
- Intrinsic tail estimation
- Separates RBNS and IBNR reserves
- Gives distribution forecasts as required by Solvency II
- Does **not** rely on proprietary software

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- Martinez-Miranda, M.D., Nielsen, J.P. and Verrall, R. (2011) Double Chain Ladder. Revised and resubmitted to *ASTIN Bulletin*

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