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Capital Allocation: Challenges and Options

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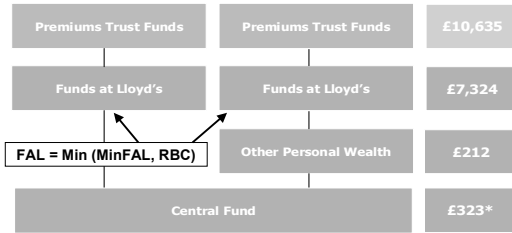
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Lloyd's Market Risk Unit with Imperial College, London

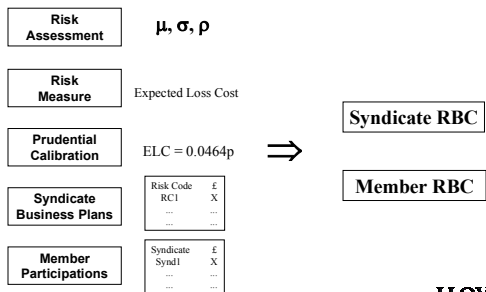
- MRU is a centre of expertise for the Market, with 3 actuaries, 3 actuarial students, 5 technical experts and 2 general analysts
- The Capital Modelling and Systemic Risk teams generate opportunities and requirements for research
- Imperial College, through the Centre for Quantitative Finance, provides tuition on financial mathematics and has seconded a PhD student, Andreas Tsanakas, to work in the MRU for three years
- Although it is expected that the resulting thesis will relate to the MRU's activities, there have also been opportunities to incorporate research within current development plans for RBC

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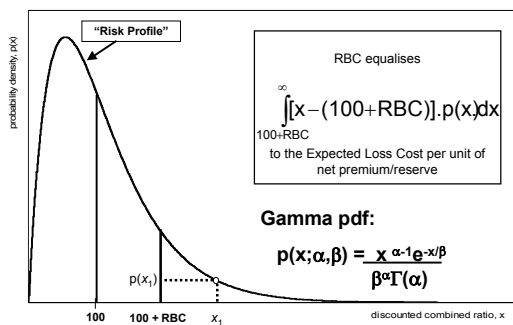
Lloyd's Chain of Security, Funds at Lloyd's and RBC



RBC - Inputs and Outputs



Gamma Distribution and ELC



Properties of RBC

- **Risk Based** \Rightarrow differential capital requirements that reflect the risk posed by each member
- **Equity** \Rightarrow each member poses the same ELC to Central Fund for each £1 of net premium or net reserve
- **Diversification** \Rightarrow recognises benefits from business mix, spread across managing agents and years of account
- **Capital Efficiency** \Rightarrow sub-optimal as diversification within the Central Fund is not reflected in the risk measure

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Rule-based Allocation

- 80:20 solution would suggest that a risk-based approach should focus on larger and more complex entities
- For the remainder, we should seek to achieve a broad reflection of comparative risk, based on some general rules
- Criteria for fixed capital may include:
 - no concentration >20%
 - limited exposure to high risk syndicates

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Coherent Risk Measures

- Risk is defined as the amount of capital required to cover for future liabilities
- A risk measure is a real valued function, defined on the set, G , of all random variables representing risks (losses)
- *Coherent* risk measures satisfy the four properties:
 - Monotonicity: $\mathbf{X}, \mathbf{Y} \in G, \mathbf{X}(\omega) \leq \mathbf{Y}(\omega) \Rightarrow \rho(\mathbf{X}) \leq \rho(\mathbf{Y})$
 - Positive Homogeneity: $\lambda \geq 0, \mathbf{X} \in G \Rightarrow \rho(\lambda \mathbf{X}) = \lambda \cdot \rho(\mathbf{X})$
 - Subadditivity: $\mathbf{X}, \mathbf{Y} \in G \Rightarrow \rho(\mathbf{X} + \mathbf{Y}) \leq \rho(\mathbf{X}) + \rho(\mathbf{Y})$
 - Translation invariance: $\mathbf{X} \in G, a \in \mathbf{R} \Rightarrow \rho(\mathbf{X} + a) = \rho(\mathbf{X}) - a$

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Expected Shortfall

- Expected shortfall:

$$\mathbb{E}[X | X > \text{VaR}_\alpha(X)]$$
- A generalisation of Value at Risk
- “How bad is bad?”
- It is a coherent risk measure and satisfies the properties listed previously
- It is additive under comonotonicity

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Cooperative Games

- Economies of scale: allocating savings from cooperation
- Stability of the grand coalition:
 - > individual rationality
 - > collective rationality
- In our case costs correspond to risk capital
 - The cost function corresponds to a risk measure
- The Shapley value:

$$\varphi_i = \sum_{S: i \in S} \frac{|S|!(n-|S|-1)!}{n!} (\rho(N-S) - \rho(S))$$

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Non-atomic Cooperative games

- Players are (divisible) portfolios: non-atomic games
- The Aumann-Shapley value:

$$\varphi_i = \int_0^1 \frac{\partial \rho(A\gamma)}{\partial a_i} d\gamma = \frac{\partial \rho(A)}{\partial a_i}$$

- Coherent risk measure & AS \Rightarrow Coherent allocation
- For expected shortfall AS is:

$$\mathbb{E} \left[X_i \mid \sum_i X_i > \text{VaR}_\alpha \left(\sum_i X_i \right) \right]$$

- A measure of systemic risk

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Application to Lloyd's

- Lloyd's both accepts excess risk from members and regulates the market
- The two distinct roles suggest different approaches to capital allocation:
 - "Reinsurer": Determine aggregate risk to Central Fund and allocate excess risk to members according to AS - Risk capital is determined indirectly, as a retention.
 - "Regulator": Determine aggregate risk capital and allocate capital directly according to AS

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Equations

- "Reinsurer":

$$R_m = E \left[(X_m - K_m)_+ \mid \sum_j (X_j - K_j)_+ > VaR_\alpha \left(\sum_j (X_j - K_j)_+ \right) \right]$$

- "Regulator":

$$K_m = E \left[X_m \mid \sum_j X_j > VaR_\alpha \left(\sum_j X_j \right) \right]$$

X_m : Claims for member's m portfolio K_m : Total capital for member m
 R_m : Risk contribution for member m (proportional to capacity)

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Capital efficiency

- The capital at Lloyd's is only partially mutualised
- There are several possible allocation methodologies
- Each methodology might result in a different amount of required risk capital
 - Aggregate capital is not fixed!
- We need to investigate which the most capital efficient methodology is

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Dependence Structures

- Modelling dependent risks
- Copulas de-couple marginal behaviour from the dependence structure:

$$P(X \leq x, Y \leq y) = C(P(X \leq x), P(Y \leq y)) \Leftrightarrow$$

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y))$$
- Can model both asymptotically dependent and independent risks
- How does capital efficiency of different methodologies relate to the dependence structure between risks?

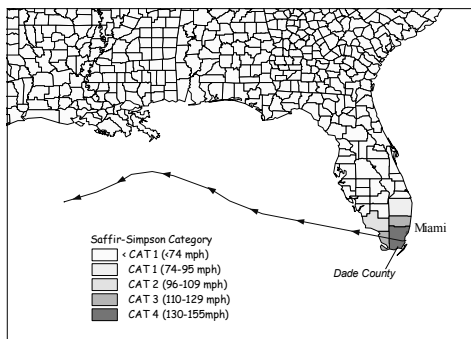
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Key References

- Artzner, P., F. Delbaen, J. Eber, & D. Heath (1999), 'Coherent Measures of Risk', *Mathematical Finance*, 9 (3), 203-228.
- Billera, L. J. and D. C. Heath (1982), 'Allocation of shared costs: a set of axioms yielding a unique procedure', *Mathematics of Operations Research*, 7 (1), 32-39.
- Denault, M. (2001), *Coherent allocation of risk capital*, ETH Preprint, Zurich, <http://www.risklab.ch/Papers.html>.
- Frees, E. W. and E. A. Valdez (1998), 'Understanding Relationships Using Copulas', *North American Actuarial Journal*, 2 (1), 1-25.
- Wirch, J. L. and M. R. Hardy (1999), 'A synthesis of risk measures for capital adequacy', *Insurance: Mathematics and Economics*, 25, 337-347.

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Specific RDS - Florida Windstorm



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Summary

- Lloyd's RBC
- Coherent Risk Measures
- Cooperative Games
- Capital Efficiency
- Dependence Structures
- Systemic Risk

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