Capital Allocation in the Lloyd's Insurance Market

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Market Risk and Reserving Unit

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Contents

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Part 1

Capital allocation in the Lloyd's market

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Risk based capital - Overview

- RBC system applied to corporate members from 1994 and all members from January 1998
- RBC equalises expected loss to the Central Fund per unit of net premium/reserve
- Inputs include:
  - Business mix diversification
  - Profile of reinsurance protection including security
  - Credit for diversification across managing agents
  - Credit for diversification across underwriting years
  - Syndicate specific adjustments

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Lloyd's Chain of Security

corporate members	individual members	end 2001 £m
Premium Trust Funds	**Premium Trust Funds**	**£13,462**
Funds at Lloyd's	**Funds at Lloyd's**	**£7,704**
	Other Personal Wealth	**£327**
Central Fund		**£280***

* an insurance protection as well as an additional callable component is also available

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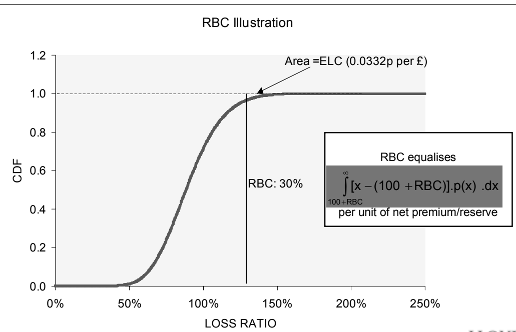
RBC Concept



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Note: RBC calculated using illustrative parameters

Syndicate-Specific Parameters

Previously

- RBC has previously used a market average model
- Average means and variances, imputed reserve exposure
- Differences from different portfolios
- Loadings for catastrophe and management risk
- Discounts for syndicate performance

2003

- 2003 YOA model has syndicate-level adjustments for mean and potentially for variance
- Some Cat loadings in model

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Operating Risk

- Define OR as "Measurable features of a syndicate that can be shown to be associated with better or worse than average performance"
- Add requirement that these pass the reasonableness test

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How to set SSPs : Operating Risk

- Syndicates' actual results not suitable
- Looked instead for Explanatory Variables (EVs)
- 1993 - 2000 years, 50 Risk Groups,
  all syndicates = 11,000 data points
- 40 potential EVs
- Seven were statistically significant
- Reasonableness checks

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Table of EVs

EV	RBC increases with
Size	Smaller syndicates
U/W Experience	Less Experience
U/W Qualification	No ACII/FCII
Syndicate growth	Faster growth
Writing 100% lines	More 100% lines
Relying on one broker	More from largest broker
Reinsurance gearing	More reinsurance spend

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Catastrophes

- Previously potential for loading if certain criteria tripped - based on RDS returns
- Now proposed to use RDS directly in the RBC calculation
- Add 3 specific RDS amounts directly: US Wind,California Earthquake, New Madrid Earthquake
- Old process for others - extend in future years

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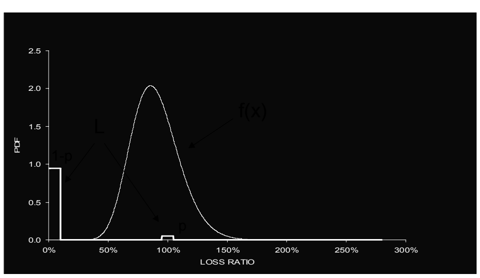
Adding Catastrophe Risk



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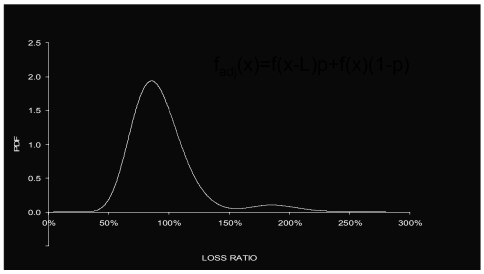
Adding Catastrophe Risk



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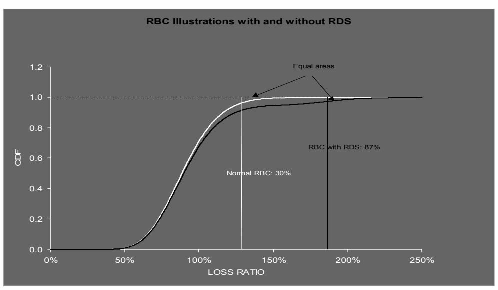
Adding Catastrophe Risk



Note: RBC calculated using illustrative parameters

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Part 2

Allocation of risk capital to pooled liabilities

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Distortion Principles

- Definition of the risk measure (Denneberg (1990), Wang (1996)):

$$\rho(X) = \int_0^\infty g(P_o(X > x))dx$$

$$g' > 0,\ g'' < 0,\ g(0) = 0,\ g(1) = 1$$

- Distortion principles satisfy the axioms of coherent risk measures, plus the requirement for comonotonic additivity

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Allocation of pooled capital

- n portfolios of stochastic liabilities are pooled
- The risk capital that the pool must hold is lower than the aggregate capital requirements would be for the non-pooled liabilities
- Cooperation produces capital savings: how to allocate those to the participants?
- The core of a cooperative game: no disincentives for cooperation

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Example

- 3 Pareto distributed liabilities, α=4, β=3/4.
- Correlation matrix and correlations to the aggregate:

$$r(X) = \begin{pmatrix} 1 & 0.1 & 0.5 \\ 0.1 & 1 & 0.8 \\ 0.5 & 0.8 & 1 \end{pmatrix}, \quad \begin{array}{l} r(X_1, \Sigma_i X_i) = 0.64 \\ r(X_2, \Sigma_i X_i) = 0.75 \\ r(X_3, \Sigma_i X_i) = 0.90 \end{array}$$

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Example (cont'd)

- Aggregate required capital: $\rho(\Sigma_i X_i) = 5.06$
- Allocate proportionally: $\rho(X_i) = 1.69$
- Suppose now that only the first two portfolios co-operate.
- Aggregate required capital: $\rho(X_1 + X_2) = 3.29$
- Allocate proportionally: $\rho(X_1) = \rho(X_2) = 1.64$
- The first two portfolios have an incentive to expel the third one! What went wrong?

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The 'fuzzy core'

- Interested in allocations that add up to the aggregate risk and produce no disincentives for cooperation
- We need to find a vector $d \in R^n$, such that:
  - a $\Sigma_j d_j = \rho(\Sigma_j X_j)$
  - b $\rho(\Sigma_j u_j X_j) \geq \Sigma_j u_j d_j \ \forall u \in [0,1]^n$
- For the distortion principle there is only one such allocation

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A formula for the core allocation

- It turns out that the core allocation is given by:

$$d_i = E[X_i g'(S_{\Sigma X}(\Sigma_j X_j))], \quad S_{\Sigma X}(z) = P_o(\Sigma_j X_j > z)$$

- We can re-write that formula as:

$$d_i = E_Q[X_i], \quad \frac{\partial Q}{\partial P_o} = g'(S_{\Sigma X}(\Sigma_j X_j))$$

- ...and also as:

$$d_i = \int_0^1 \int_0^1 F_{X_i}^{-1}(u) g'(1-v) dC_{X_i, \Sigma_j X_j}(u,v) du dv$$

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Dynamic extension of risk measure and allocation method

- Let $Z = \Sigma_j X_j$. We can write the risk measure as:
  $\rho(Z) = \sup_{P \le g(P_o)} E_P[Z]$
- Assume that the underlying risk processes are Markov on [0,T]. Let B_t be the event:
  $B_t = \{\omega : X_t^1(\omega) = x_t^1, \ldots, X_t^n(\omega) = x_t^n\}$
- Then generalise the risk measure by:
  $\rho(Z_T \mid B_t) = \sup_{P \le g(P_o)} E_P[Z_T \mid B_t]$

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Explicit formulae

- Allocated capital to ith portfolio:
  $d_i = E[D_{t,T} X_T^i \mid B_t]$
- Radon-Nikodym derivative:
  $D_{t,T} = G_t'(P_o(Z_T > z \mid B_t))$
- Updated distortion function:
  $G_t(s) = \frac{g(sP_o(B_t))}{g(sP_o(B_t)) + 1 - g(1 - (1 - s)P_o(B_t))}$

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