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34TH ANNUAL GIRO CONVENTION

CELTIC MANOR RESORT, NEWPORT, WALES

Claims reserving including count data

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Outline

- Motivation - why use count data
- Data - count data
- A claim - incurred, reported, paid
- IBNR and RBNS
- Stochastic model
- Data study - Auto TPL
- Conclusion

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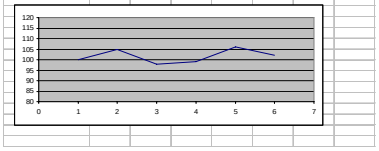
Motivation

An introductory example

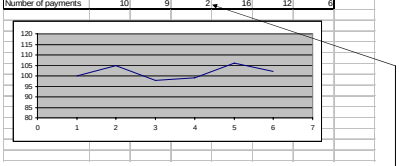
Payment stream						
Time	1	2	3	4	5	6
Aggregated Payments	100	105	98	99	106	102



This 'business' is obviously not very volatile, or?

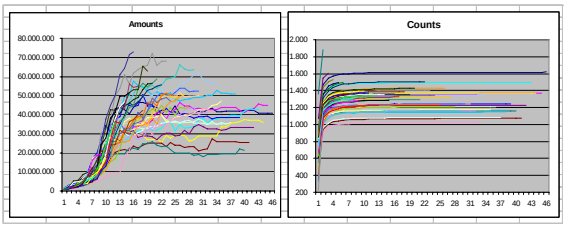
An introductory example

Payment stream						
Time	1	2	3	4	5	6
Aggregated Payments	100	105	98	99	106	102
Number of payments	10	9	2	16	12	6



Maybe it is!?

Auto TPL data and Prediction



Finish all graphs up to time 46...

Example with coin flips

- Monday: 49% tail
- Tuesday: 41% tail
- Wednesday: 53% tail
- Thursday: 59% tail
- Friday: 51% tail

Example with coin flips

- Monday: 49% tail, 2500 flips
- Tuesday: 41% tail, 250 flips
- Wednesday: 53% tail, 500 flips
- Thursday: 59% tail, 300 flips
- Friday: 51% tail, 2100 flips

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Data discussion

Which data do we use?

- The number of reported claims
- The aggregate paid amounts
- Both in triangular arrays

Why use this data format?

- Availability
- Relation to existing method
- Both incurred and paid data

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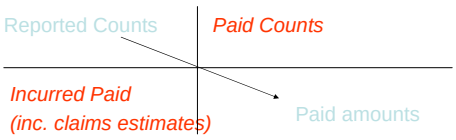
Why use this data format?



- Incurred Paid is not 'real data' - claim estimates. Cashflow.
- Paid Counts means what? - more payments per claim
- Reported Counts - one claim, one count
- Paid Amounts is 'real data' - no claim estimates

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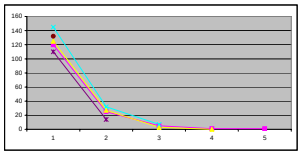
A thought up data example

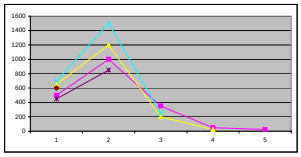
Repoted Counts

	1	2	3	4	5
1	120	25	5	1	1
2	125	27	3	0	
3	145	32	7		
4	110	14			
5	132				



Paid amounts

	1	2	3	4	5
1	500	1000	350	50	25
2	650	1200	200	25	
3	700	1500	250		
4	450	850			
5	600				



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A claim - IBNR and RBNS

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The two delays...

- The claim incur
- The claim is reported
- The claim is (fully) paid

Incur —IBNR→ Reported —RBNS→ Paid

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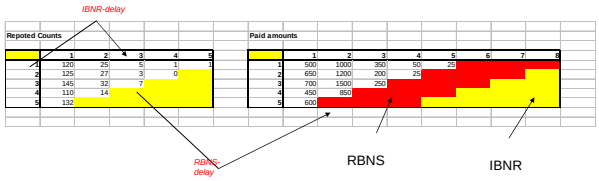
A thought up example; Conditional information on future payments

We know 'something' about the future.



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Stochastic model

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The Reported Counts

Given Θ_i

$$N_{ij} \sim \text{Pois}(\Theta_i E_i \overline{p}_j)$$

where

$$\sum_{j=1}^{n} \overline{p}_j = 1$$

and E_i is the expected exposure (e.g. the net premium) the ith period.

CHAIN LADDER for the REPORTED COUNTS (given Θ).

Given (Θ_i) then (N_{ij}) are mutually independent.
(Θ_i) are iid.
One can also choose value structure to $\Theta_i E_i \overline{p}_j \delta_{i+j-1}$ given Θ_i to correct for claims inflation.

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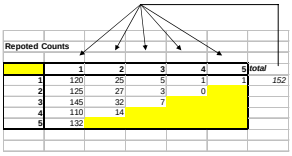
The Reported Counts - a remark

Given $\sum_{j=1}^{n} N_{ij}$ we have

$$(N_{i1},....,N_{in}) \sim Multi\left(\sum_{j=1}^{n} N_{ij}\ ;\ \overline{p}_1,....,\overline{p}_n\right)$$



Repoted Counts

	1	2	3	4	5	total
1	120	25	5	1	1	152
2	125	27	3	0		
3	145	32	7			
4	110	14				
5	132					

Payment of Reported Claims

$$\begin{array}{lllllll} N_{i1} & \rightarrow & N_{i11} & N_{i12} & N_{i13} & \cdots & N_{i1k} \\ N_{i2} & \rightarrow & & N_{i21} & N_{i22} & N_{i23} & \cdots & N_{i2k} \\ \vdots & & & & \ddots & & \ddots & \\ N_{in} & \rightarrow & & & N_{in1} & N_{in2} & N_{in3} & \cdots & N_{ink} \\ \hline & & N_{i1}^{Paid} & N_{i2}^{Paid} & N_{i3}^{Paid} & & & & N_{ik}^{Paid} \end{array}$$

Given (N_{ij}) we have

$$(N_{ij1},....,N_{ijk},N_{ij}^{Zero}) \sim Multi\left(N_{ij}\ ;\ p_1,....,p_k,p^{Zero}\right)$$

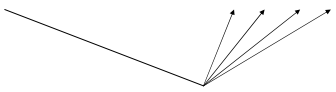
A thought up example - Payment of Reported Claims

Repoted Counts

	1	2	3	4	5
1	120	25	5	1	1
2	125	27	3	0	
3	145	32	7		
4	110	14			
5	132				

Paid amounts

	1	2	3	4	5	6	7	8
1	500	1000	350	50	25			
2	650	1200	200	25				
3	700	1500	250					
4	450	850						
5	600							

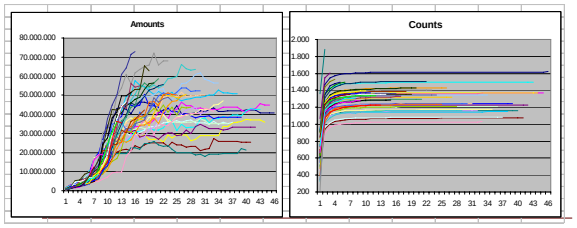


- RED AREA - NOT observed
- RED AREA - depend on GREEN AREA

Payment of Reported Claims

(Y_{ijk}) are iid claims and

$$X_{ij} = \sum_{k=1}^{N_{ij}^{Paid}} Y_{ijk}$$



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Data study TPL

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Inflation corrected data

Counts	1	2	3	4	5	6	7	8	9	10
1	6238	831	49	7	1	1	2	1	2	3
2	7773	1381	23	4	1	3	1	1	3	
3	10306	1093	17	5	2		2	2		
4	9639	995	17	6	1	5	4			
5	9511	1386	39	4	6	5				
6	10023	1342	31	16	9					
7	9834	1424	59	24						
8	10899	1503	84							
9	11954	1704								
10	10989									

Paid/100	1	2	3	4	5	6	7	8	9	10
1	451289	339519	333371	144988	93243	45511	25217	20406	31482	1729
2	449828	512882	168468	130674	56045	33397	56071	26523	14346	
3	693574	497737	202272	120754	125046	37155	27609	17864		
4	652043	546406	244475	200897	106802	106753	63689			
5	566082	503971	217839	145182	165519	91313				
6	606607	562544	227374	153552	132743					
7	536976	472525	154205	150565						
8	554833	590880	300964							
9	537238	701112								
10	684944									

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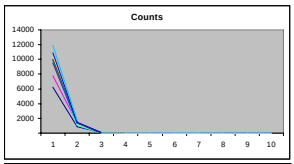
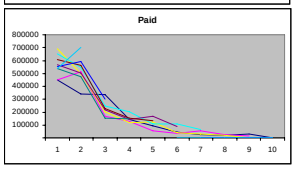
Counts and Paid





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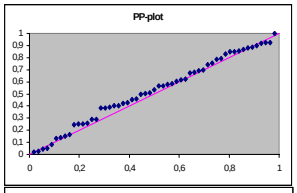
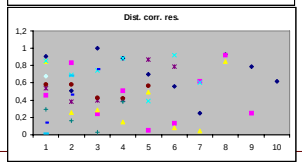
Model validation





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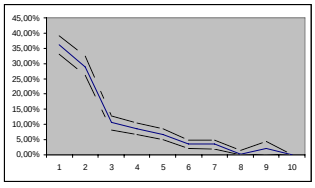
The delay function

Delay	1	2	3	4	5	6	7	8	9	10
Mean	36,18%	28,79%	10,66%	8,52%	6,71%	3,51%	3,46%	0,15%	2,03%	0,00%
Sd	1,57%	1,56%	1,16%	0,94%	0,93%	0,66%	0,72%	0,40%	0,92%	0,00%
2,5%-Quant.	33,06%	26,16%	8,12%	6,67%	5,07%	2,10%	1,88%	0,00%	0,31%	0,00%
97,5%-Quant.	39,24%	32,34%	12,80%	10,40%	8,63%	4,77%	4,89%	1,50%	4,33%	0,00%



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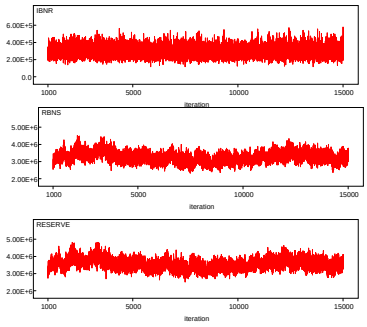
Outcome of MCMC simulation



Results

- Reserve: 3,546 * 10^8
- Conf. Bands: (3,001 * 10^8 ; 4,187 * 10^8)

- IBNR: 2,965 * 10^7
- Conf. Bands: (2,931 * 10^7 ; 4,225 * 10^7)

- RBNS: 3,25 * 10^8
- Conf. Bands: (2,729* 10^8 ; 3,865 * 10^8)

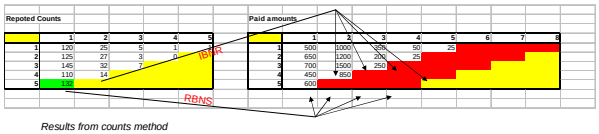
- Mean claim: 16.346,96 DKK
- Single claim distribution:

$$\Gamma(0.01568,\ 9.592*10^{-5})$$

Comparison to the Bootstrap method on Paid

- Reserve: 3,311 * 10^8 (7% less then counts method)
- Conf. Bands: (3,116 * 10^8 ; 3,924 * 10^8)

Notice: Chain ladder reserve contains both IBNR and RBNS...



Results from counts method

- Reserve: 3,546 * 10^8
- IBNR: 2,965 * 10^7
- RBNS: 3,25 * 10^8

Conclusion

- Data is available
- Gain understanding of the volatility of IBNR, RBNS and the related delays
- Use both paid and incurred data
- Method closely related to chain ladder
- Cashflows can be modelled effectively

Future work...

- Use claims estimates in counts model (delay)
- Generalise to other distributions for the single claims (Gamma)
- Develop model with more payments per claim
- Make models operational