The Actuarial Profession
making financial sense of the future

Open Forum – 15th May 2012

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Coherent asset allocation in the presence of stress events

Plan for this session

- Introduction to Bayesian Nets
- Case Study
- Extensions
- Q&A

Why is a new approach needed?

"The financial crisis has shown that estimating ex-ante the probabilities of stress events is problematic

The statistical relationships used to derive the probability tend to break down in stressed conditions

In this respect, the crisis has underscored the importance of giving appropriate weight to expert judgment in defining relevant scenarios with a forward-looking perspective"

The Basel Committee - Principles for sound stress-testing - 2009

How are fat tails traditionally modelled?

- Fat-tailed distributions (t Student, Cauchy etc.)
- Extreme Value Theory
- Econophysics

Why is a new methodology required?

- **Modelling fat tails has become more important**
  - Violent gyrations and “black-swan” events appear to occur more frequently than expected even with fat-tailed distributions
  - Past historical datasets do not always contain information about current weaknesses (e.g. Euro break-up)
  - World is subject to continuous structural changes

Introduction to Bayesian Nets

Outline of the New Methodology

1) Identify the "Body" and "Tails" of the asset price distributions

- (a) Analyse historical data into "Body" and "Outlier" parts. Model the "Body" with well-known statistical techniques
- (b) Build new "Tails" using Bayesian Net and specified stress levels, and replace the "Outliers" in (a)

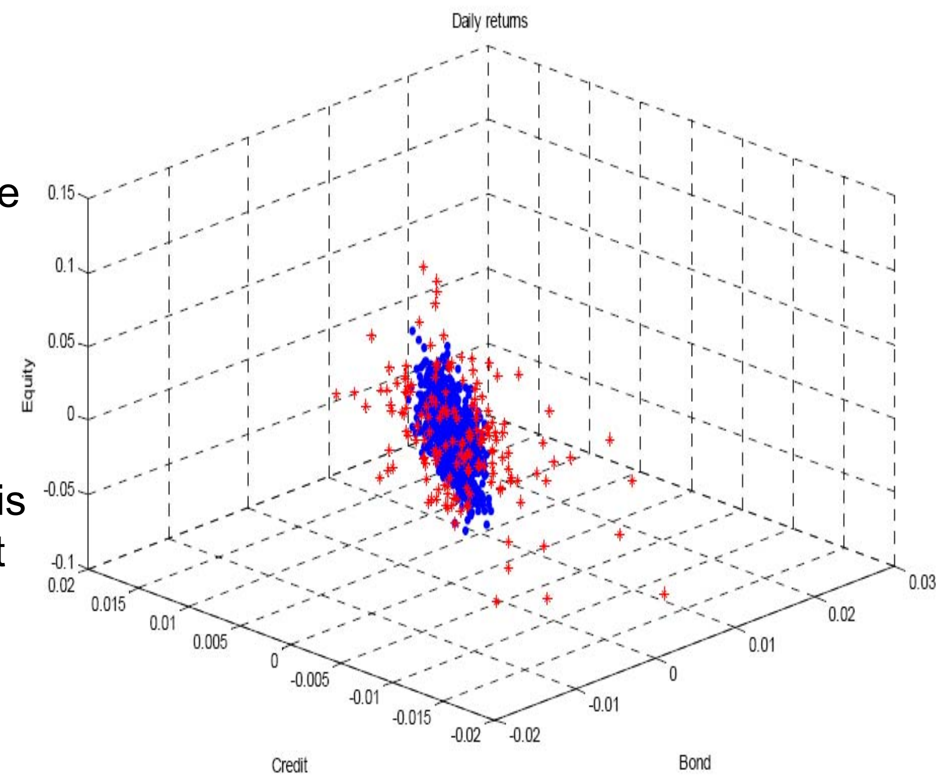
2) Combine Body and Tail to obtain a complete joint probability distribution

1a) Determining the Body of the distribution

"Body" and "Outliers"

Several techniques are available to identify such `body' of 'normal' returns of a distribution e.g.

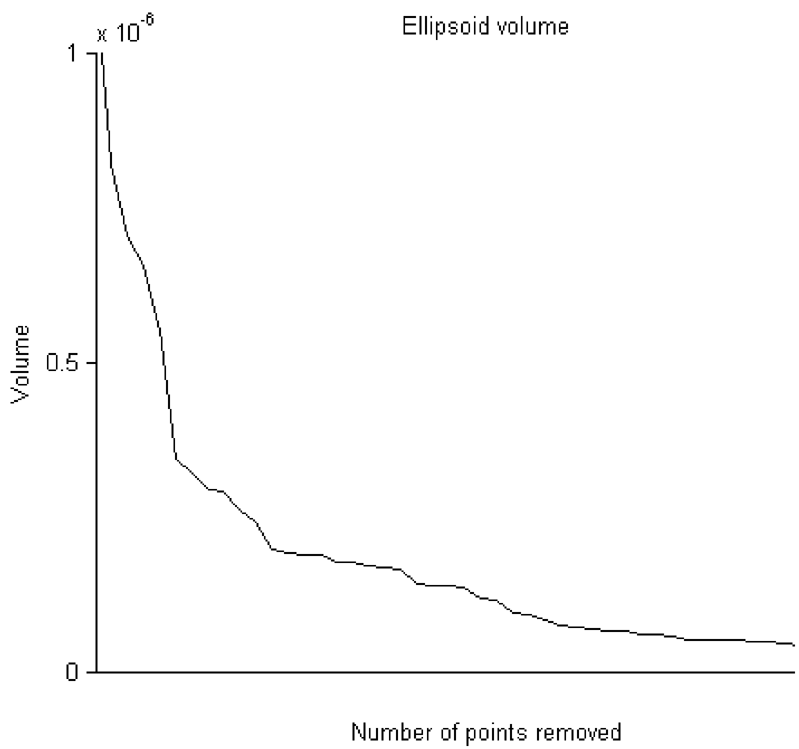
- Truncate all the data points over a threshold in the univariate distribution of each asset class (e.g., a given number of standard deviations) – this approach neglects the joint behaviour of asset classes
- Truncate all the point above a certain Mahalanobis distance – meaningful in the multivariate case but could be contaminated by the outliers
- Minimum Volume Ellipsoid (MVE) and Minimum Covariance Determinant (MCD) techniques – multivariate and robust to outliers. **The approach we adopted**

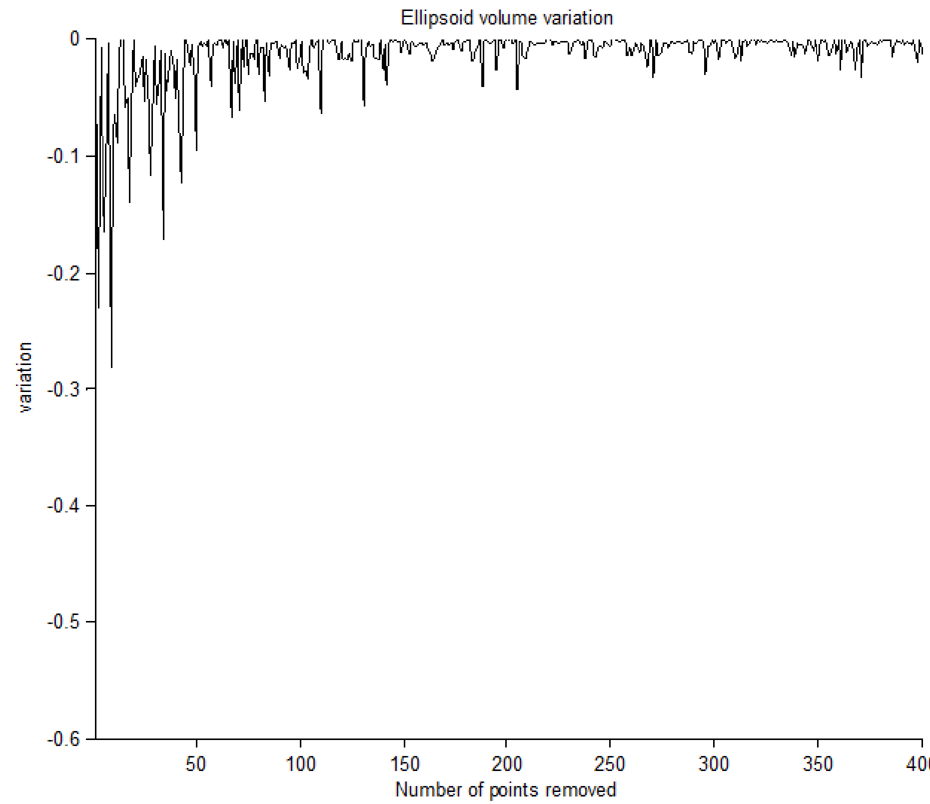


1a) Determining the Body of the distribution

Body and Outliers

We monitor the change in some key quantities (volume of the MVE, determinant of the covariance matrix, eigenvalues of the correlation matrix etc.) as the most distant outliers are removed one by one until these quantities show a certain stabilization. This is taken a borderline between ‘Body’ and ‘Outliers’.

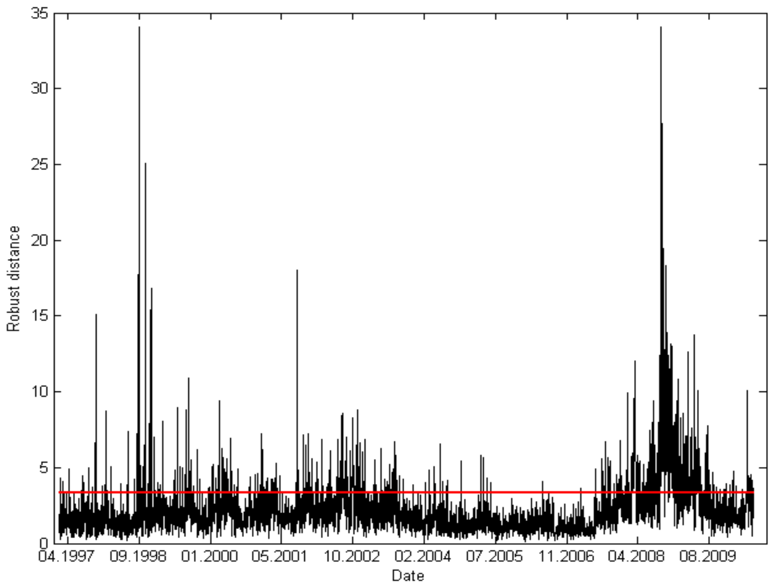




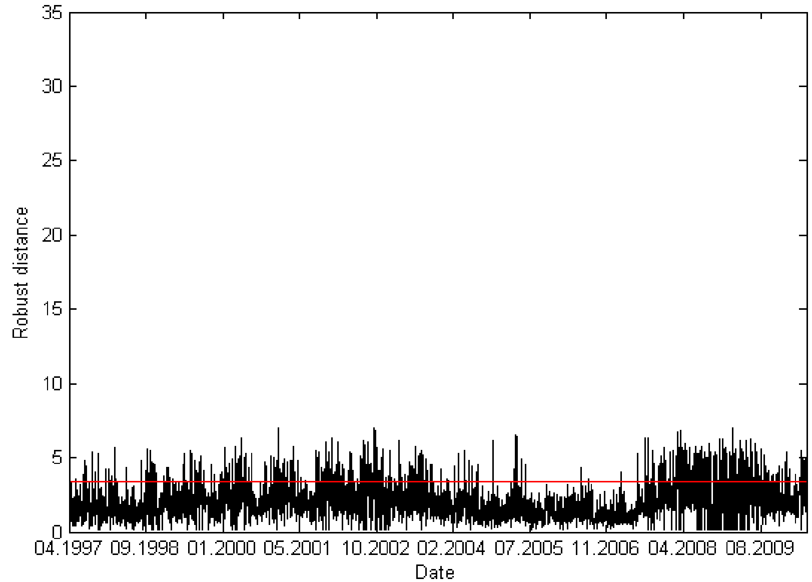
1a) Determining the Body of the distribution

The distribution of Robust Mahalanobis distances before/after the removal of the Outliers shows significant differences

Before



After

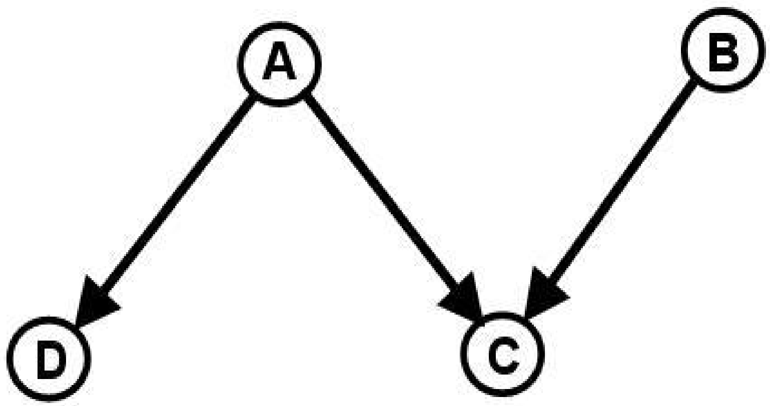


1b) “Tail” events

- **A Bayesian network is used to model tail events:**
  - It has a graphical component that describes causality among tail events
  - Nodes in the Bayesian network denote tail events
  - Probabilities are assigned to certain tail events, which then determine the joint probabilities of interest through the structure of the network
  - One is required to provide (subjective) assessments of the likelihood of tail events or at least relative likelihood of one tail event versus another

1b) What is a Bayesian Network? An Example

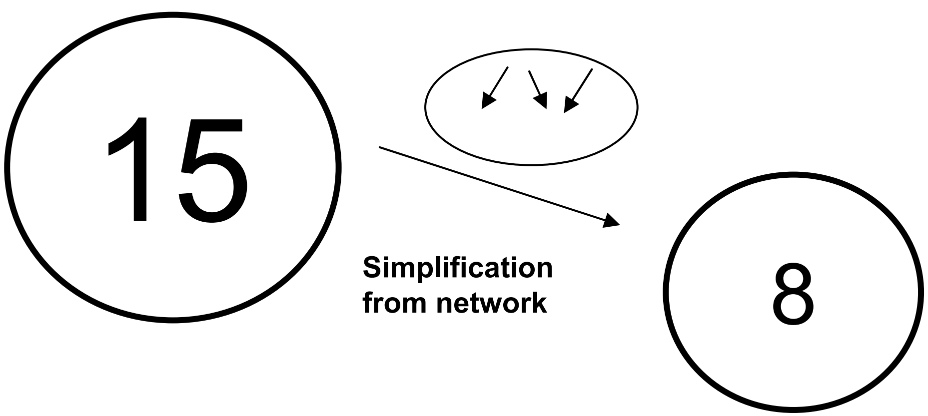
- **A Bayesian Network is a graphical model that represents conditional (in)dependencies in a set of events**



- **Arrows indicate "causal" links**
- **Nodes have "parents" and "children"**
  - For example, node C has nodes A and B as parents, but it has no children.
  - In this example, a double dip recession or UK downgrade (or both) can lead to a UK equity market crash
- **Conditional independence given parents**
  - Given the events of a double dip recession and a UK downgrade, UK equity market crash is independent of a Greek default

1b) Why Use a Bayesian Network?

- **Investment professionals have intuition and experience regarding potential tail events and how they could influence one another**
- **Formulating these links with a Bayesian network simplifies the process of specifying input probabilities**
  - If there are n tail events, then there are 2^n (-1) joint probabilities: Most would find it onerous to specify all the required joint probabilities, particularly when n is large.
  - For the Bayesian network considered earlier, 8 (relatively simple) probabilities need to be specified, instead of 15



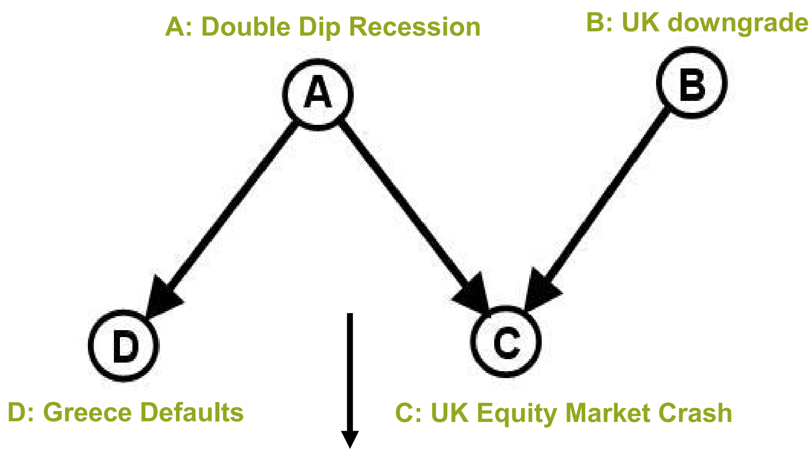
- **Conditional probabilities that need to be specified are the conditional probabilities of child events given their parents**
- **The joint probability factorizes as**

$$p(x) = \prod_{v \in V} p\left(x_v \,\middle|\, x_{\mathrm{pa}(v)}\right)$$

Where the product is over all the nodes and the conditional probability given their parents **only**

1b) Determining Joint Probabilities: An Illustration

- **If the network has a relatively simple structure, marginal and "low-order" conditional probabilities describe the joint probabilities**



Assign marginal probabilities

P(double dip occurs) = 2.37%

P(UK downgrade) = 1.79%

Assign conditional probabilities **(given parents)**

P(Greece defaults | double dip) = 48.1%

Determine joint probabilities →

A	B	C	D	Joint Probability
1	1	1	0	0.23%
0	1	1	1	0.00%
1	1	1	1	0.21%
0	1	1	0	0.01%
1	1	0	0	0.03%
0	1	0	1	0.02%
1	1	0	1	0.03%
0	1	0	0	1.26%
1	0	1	0	0.24%
0	0	1	1	0.00%
1	0	1	1	0.22%
0	0	1	0	0.01%
1	0	0	0	0.73%
0	0	0	1	0.33%
1	0	0	1	0.68%
0	0	0	0	96.00%

Ensuring Input Probabilities Are Valid

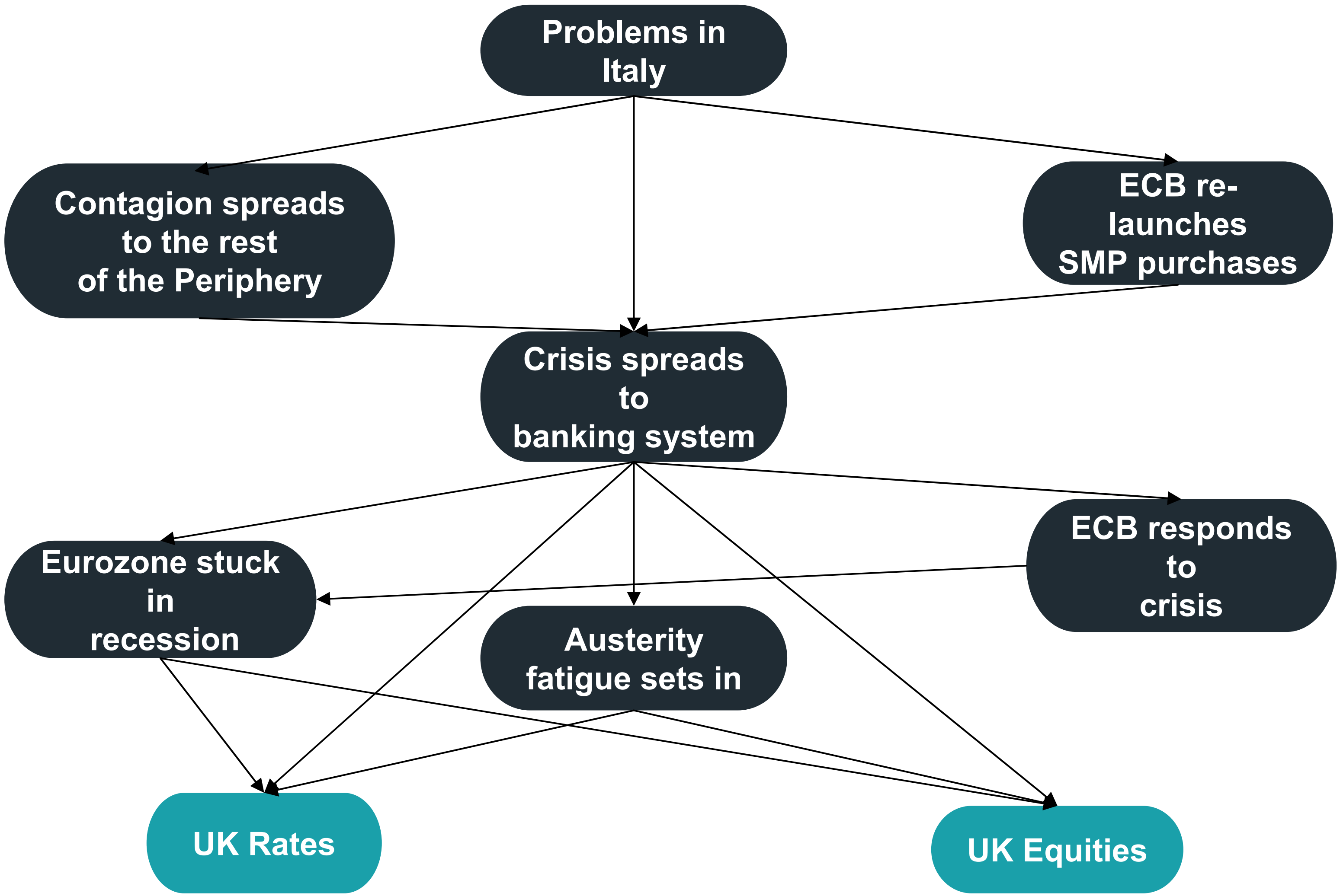
- **Input probabilities are underspecified**
  - We have not specified enough probabilities to determine all the joint probabilities we need
  - We need to pin down the additional degrees of freedom by resorting to other techniques, e.g. **maximum entropy**

- **Input probabilities could be inconsistent**
  - Example: The joint probability P(A and B) can be factored in two ways: P(A and B) = P(A|B) P(B) = P(B|A) P(A)
  - We need to find bona-fide probabilities that are closest to our inputs in some sense: **Linear programming** provides one possible solution

Mapping to losses

- Once the events have been identified and the CPTs assigned, the next important step is the mapping to stress losses for the nodes which express PnL. In the previous example:
  - **Non-PnL nodes** - Double Dip Recession, UK downgrade, Greece Defaults
  - **PnL nodes** - UK Equity Market Crash
- The assignment of the stressed returns is subjective and might again rely on frequentist information. A certain level of uncertainty should be embedded in the determination of stress returns e.g. the UK Stock Market will crash 20% +/- 5%
- The uncertainty can be included in the optimisation process and be taken into account in the asset allocation

European Crisis - Scenario example



European Crisis - Scenario example - explanation

Trigger:
Indicators of Italian GDP in 2012 H1 suggest it will be much weaker than expected

Likely implication:
Italian finances look less sustainable
Periphery sells off, especially BTPs

First key policy decision:
Does the ECB start buying BTPs via SMP or not

Likely implication of ECB inaction:
Crisis spreads straight to banks who have bought even more peripheral sovereign debt
Bank equity falls, CDS and funding costs rise, deposit flight accelerates
Concerns around Spain rise
Expectations of Portuguese PSI harden

Second key policy decision:
ECB has to choose whether to launch third 3yr LTRO + relax collateral requirements
AND/OR cut the refinance rate (say by 50 bps)

Likely implication of ECB inaction:
Feedback into real economy - hence sharp contraction in 2012 H2
Increased chance of political fatigue in periphery
Fears of euro break-up rise

Likely implication for Rates
Yield curve flattening in the far end – both real and nominal as a consequence of a flight-to-quality effect

Likely implication for Equity
Stock market down by 20%

Portfolio Optimisation: Setting Up the Problem

- **Traditional asset allocation methods have limitations**
  - Stress tests and scenario analysis are frequently performed in an "ad hoc" fashion …
  - … without regard to the probabilities of the tail events under consideration
  - Optimal portfolios are typically determined first, and protection is bought *ex-post*

- **Objective**
  - Integrate tail events coherently with asset allocation decisions in a unified manner

Portfolio Optimisation: Setting Up the Problem

- **Example of asset allocation over five asset classes**
  - Treasuries, Linkers, Corporate bonds, Equity, Property / Alternatives
- **Specify**
  - **Bayesian network** for stress events that involve these five asset classes
  - **Stress losses** when tail events occur
    - E.g. a large widening of credit spreads leads to a loss of -20%
- **Returns: Tail-events, Non-tail events**
  - **Enough conditional probabilities** that determine all required joint probabilities
    - E.g. a collapse in equity markets occurs with probability of 7%
    - Given that bonds sell off, then the probability for an equity sell off is 50%
  - What happens to returns **if tail events do not occur**?
    - Link marginal t-distributions with Gaussian copulas, say
    - Other routes are possible

Coherent asset allocation in the presence of stress events

Case Study

Case Study: UK Pension Scheme

- Liabilities of £100m, all inflation-linked
- Assets of £80m split as:

Funding Ratio = 80%

Fixed Income*	UK Gilts	£5m	12.5%	50%
	UK Linkers	£5m		
	Corporate bonds	£30m	37.5%	
Non-Fixed Income	Equity	£28m	35%	50%
	Property / Alternatives	£12m	12%	
		£80m	**100%**	**100%**

* of which 2/3rds has swap overlay to match liability PV01, EI01

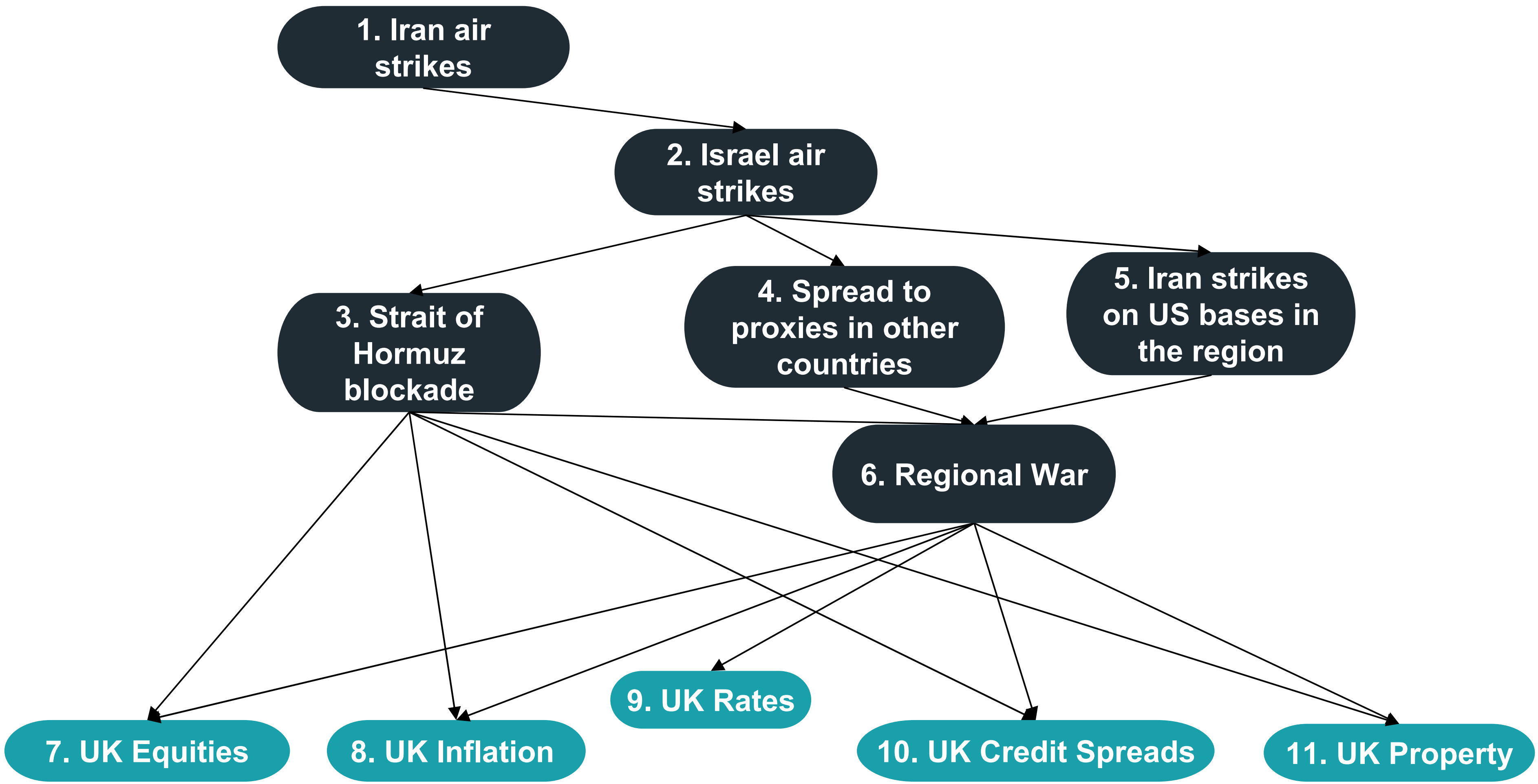
Case Study: Return drivers

Modelled Drivers

Assets

	UK Nominal Rates	UK Inflation	UK Credit Spreads	Equity	Property
UK Gilts	X				
UK Linkers	X	X			
UK Corporates	X		X		
UK Equity				X	
UK Alternatives					X

Case Study: Scenario example



Scenario example - explanation

Trigger:

Due to mounting tensions, the recent threats for pre-emptive strikes by Iran materialise

Likely implication:

Israel strikes Iran (N.B. by itself this event has a high background probability)

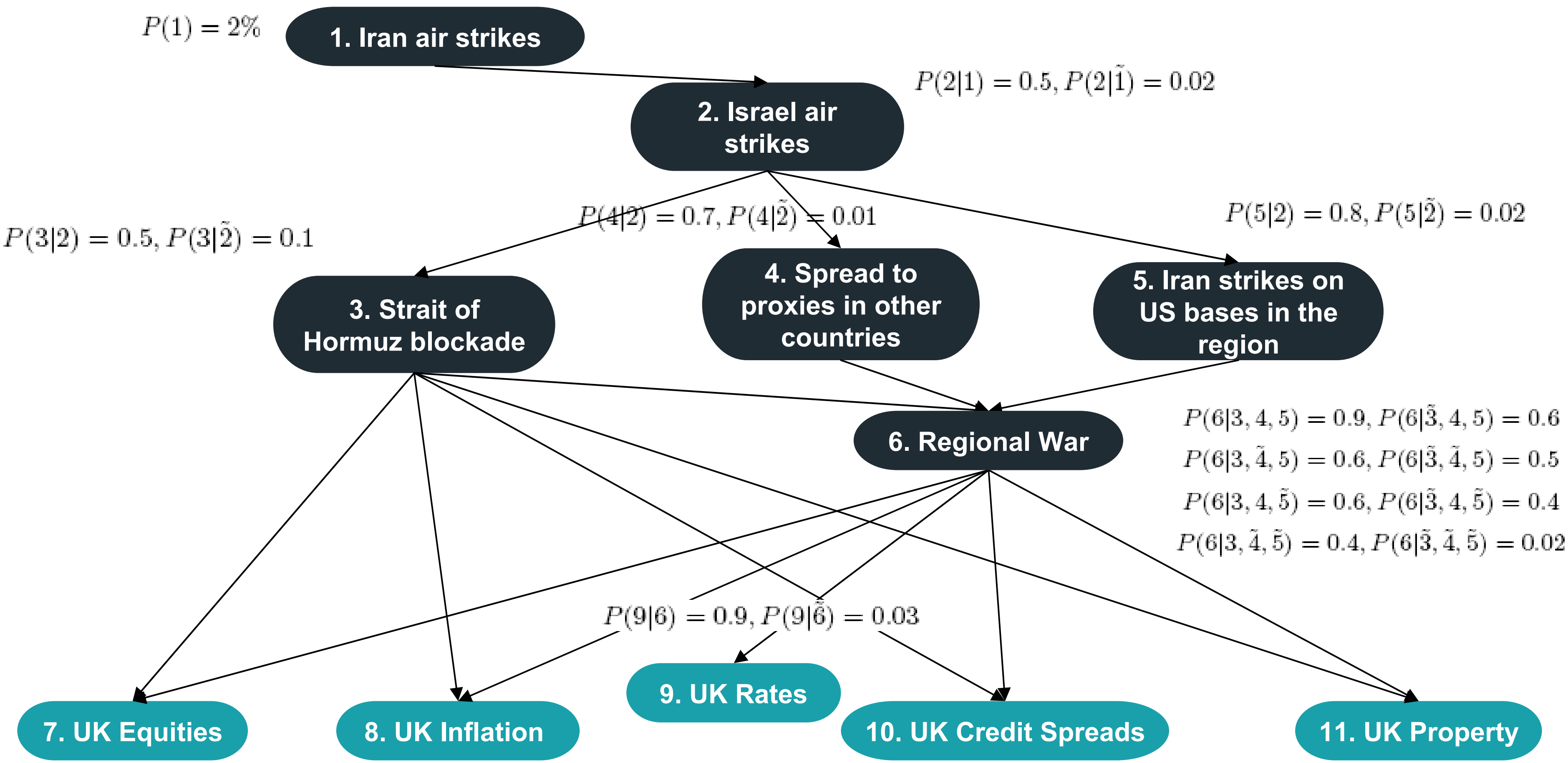
Consequences:

1) Iran tries to block the strait of Hormuz (ships, sea-mines etc.)
2) Growing unrest in other countries (Lebanon, Palestine, Saudi Arabia etc.)
3) Iran strikes US military bases in the Middle East (Qatar, Bahrain, Afghanistan etc.)

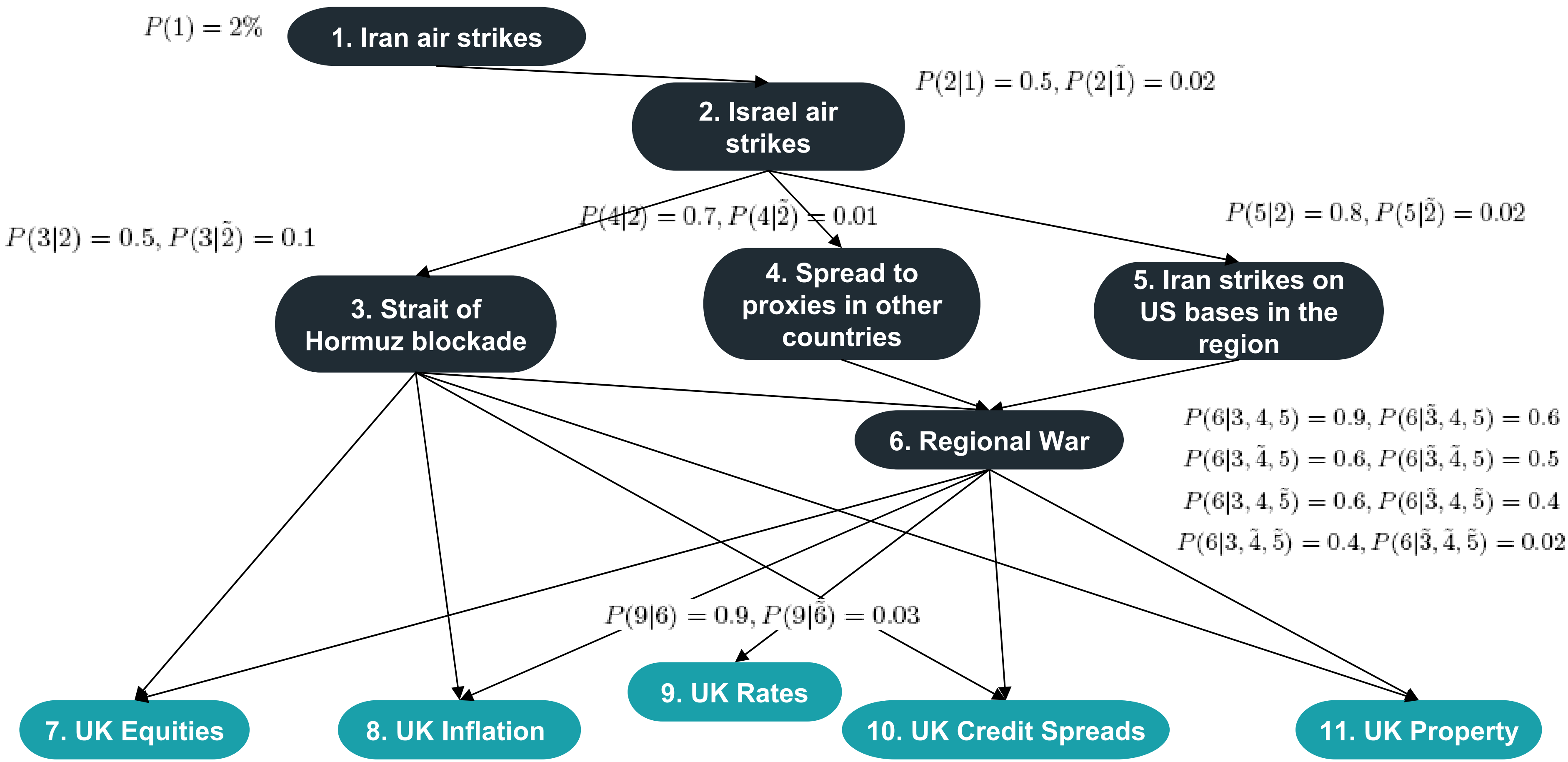
Escalation:

The conflict escalates to a regional war with air strikes, missile launches, crude missiles (‘dirty’ bomb)

Case Study: Scenario example



Case Study: Scenario example



$P(7|3,6) = 0.9, P(7|\tilde{3},6) = 0.8$

$P(7|3,\tilde{6}) = 0.4, P(7|\tilde{3},\tilde{6}) = 0.02$

$P(8|3,6) = 0.9, P(8|\tilde{3},6) = 0.8$

$P(8|\tilde{3},6) = 0.7, P(8|\tilde{3},\tilde{6}) = 0.005$

$P(10|3,6) = 0.99, P(10|\tilde{3},6) = 0.9$

$P(10|\tilde{3},6) = 0.5, P(10|\tilde{3},\tilde{6}) = 0.02$

$P(11|3,6) = 0.8, P(11|\tilde{3},6) = 0.7$

$P(11|\tilde{3},6) = 0.4, P(11|\tilde{3},\tilde{6}) = 0.02$

Case Study: Joint probability table (JPT)

Equity	*Inflation*	*Rates*	*Credit*	*Property*	*Joint Probability*
0	0	0	0	0	77.7%
0	0	0	0	1	1.8%
0	0	0	1	0	1.9%
0	0	1	0	0	2.4%
0	1	0	0	0	1.1%
1	0	0	0	0	1.8%
0	0	0	1	1	0.3%
0	0	1	0	1	0.1%
0	0	1	1	0	0.1%
0	1	0	0	1	0.5%
0	1	0	1	0	0.8%
0	1	1	0	0	0.1%
1	0	0	0	1	0.2%
1	0	0	1	0	0.3%
1	0	1	0	0	0.1%
1	1	0	0	0	0.5%

Equity	*Inflation*	*Rates*	*Credit*	*Property*	*Joint Probability*
0	0	1	1	1	0.1%
0	1	0	1	1	0.6%
0	1	1	0	1	0.1%
0	1	1	1	0	0.3%
1	0	0	1	1	0.2%
1	0	1	0	1	0.1%
1	0	1	1	0	0.2%
1	1	0	0	1	0.4%
1	1	0	1	0	0.6%
1	1	1	0	0	0.1%
0	1	1	1	1	0.7%
1	0	1	1	1	0.7%
1	1	0	1	1	0.8%
1	1	1	0	1	0.2%
1	1	1	1	0	1.3%
1	1	1	1	1	4.3%

Scenario example – specify the tail stresses

- **Effects on asset drivers:**

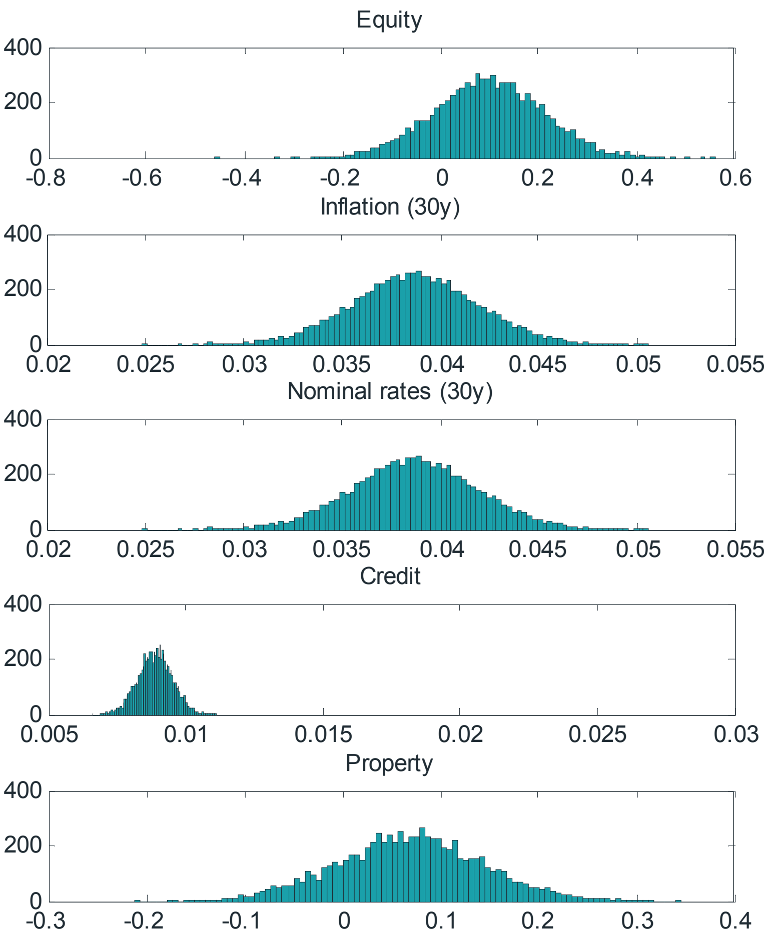
		Comment	Assumed stress
UK Equities	↓	Due to fears and risk aversion	**(25%)**
UK Inflation (L-term)	↑	Oil prices supply shock	**0.3%**
UK Nominal Rates (L-term)	↓	Safe-haven effect	**(0.8%)**
UK Credit Spreads	↑	Company profits fall	**1.6%**
UK Alternatives	↓	Property prices fall	**(15%)**

Case Study: Completeness considerations

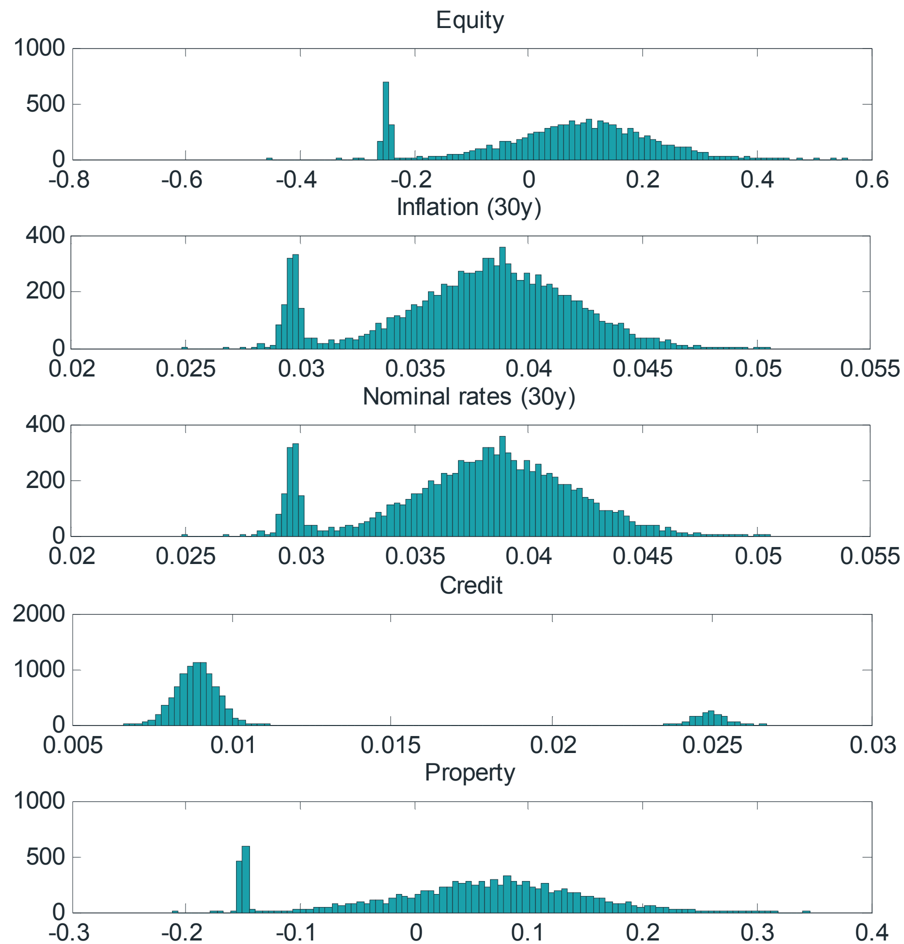
- Must ensure complete set of stress nodes …
- … such that no materially detrimental cases are missed …
- … for the intended application
- For N drivers, need evaluate 2^N possible extremities

Combining the Body and Tail: impact on asset drivers

Without Bayesian Net



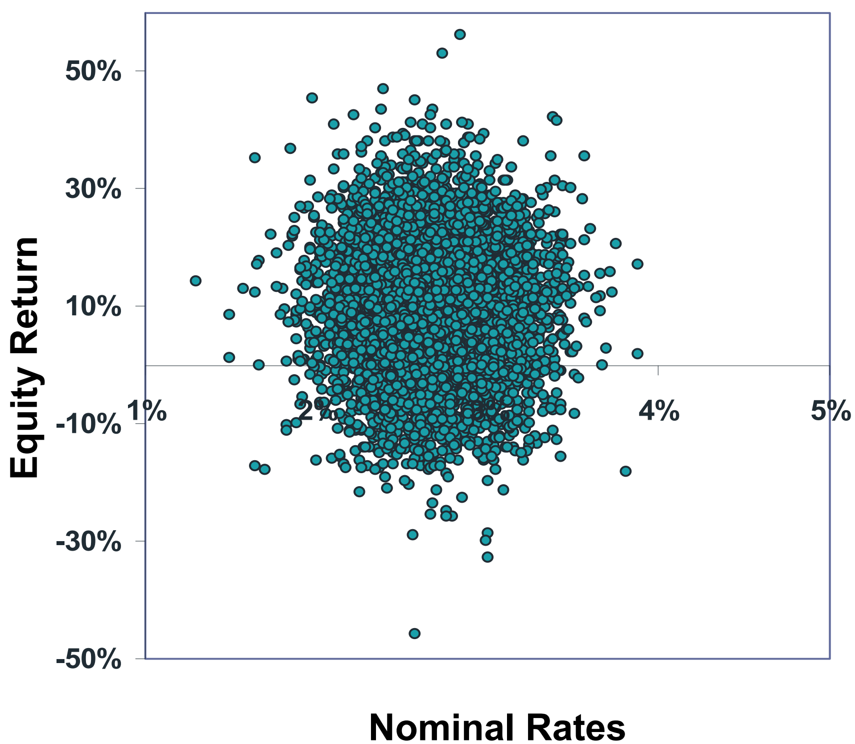
With Bayesian Net



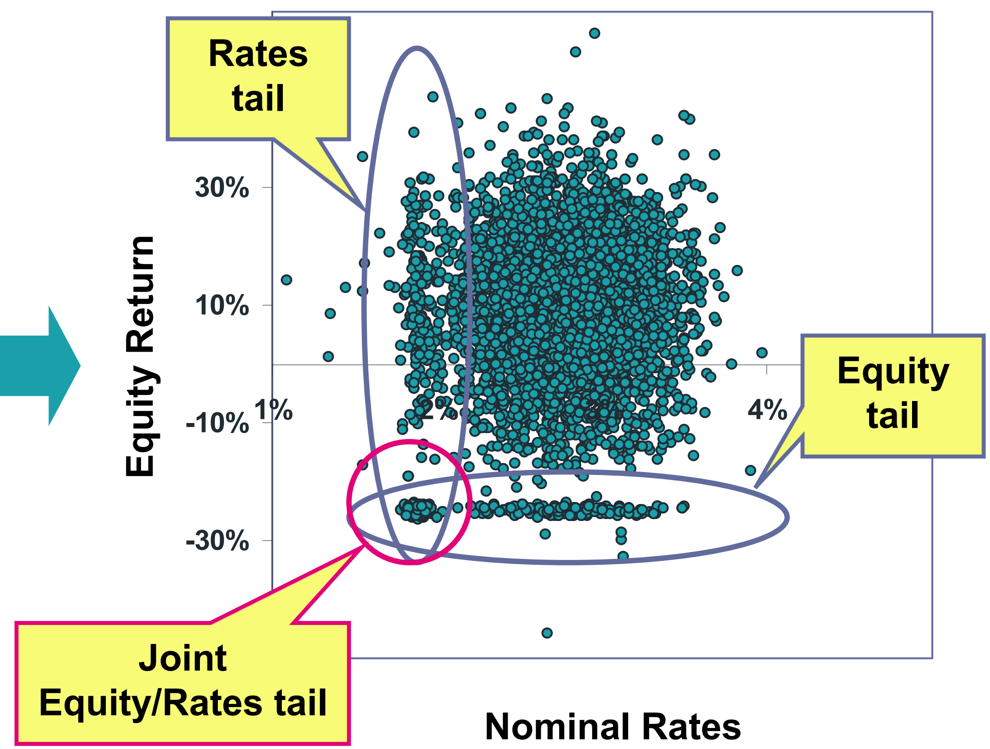
Combining the Body and Tail: impact on returns

- **Effects on returns** - illustrated by Equity and Rates:

Without Bayesian Net

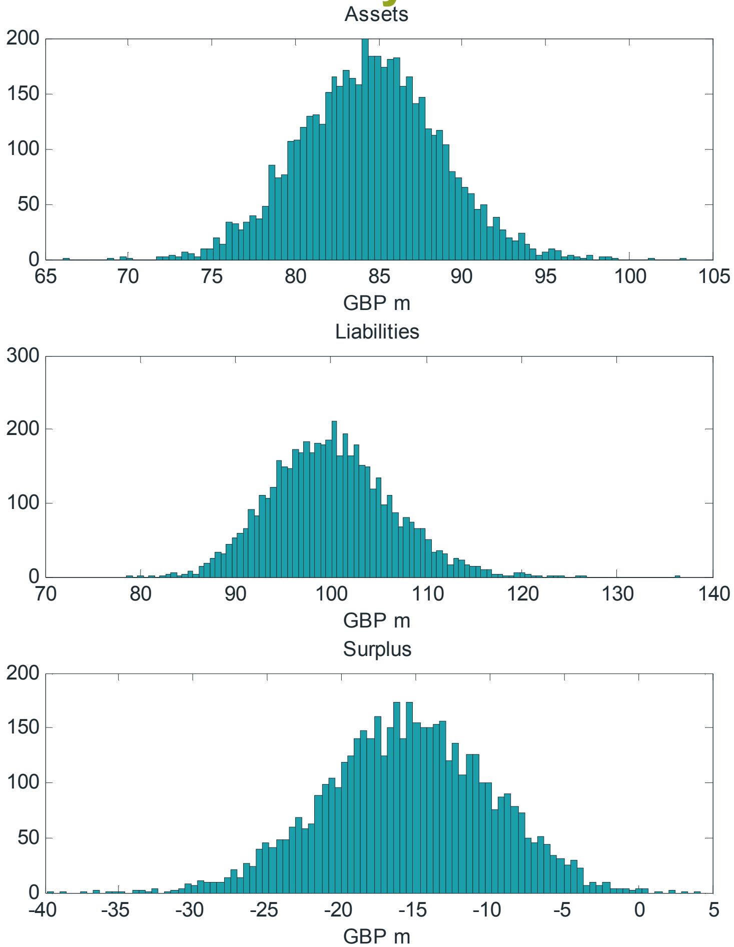


With Bayesian Net

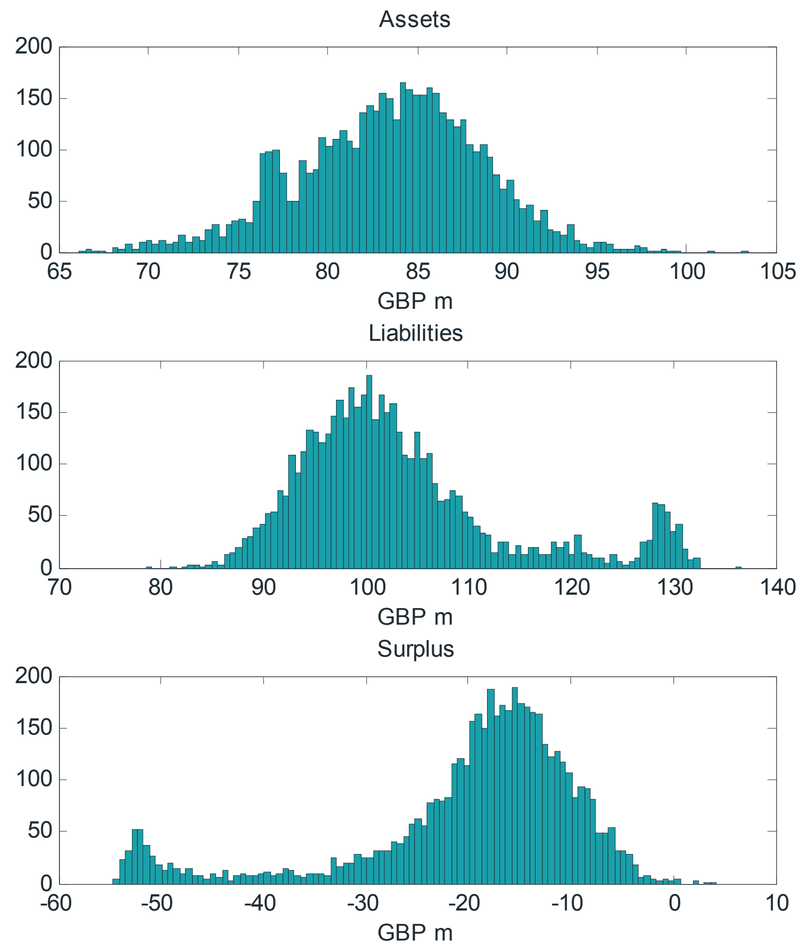


Current portfolio: Projected 1-year

Without Bayesian Net

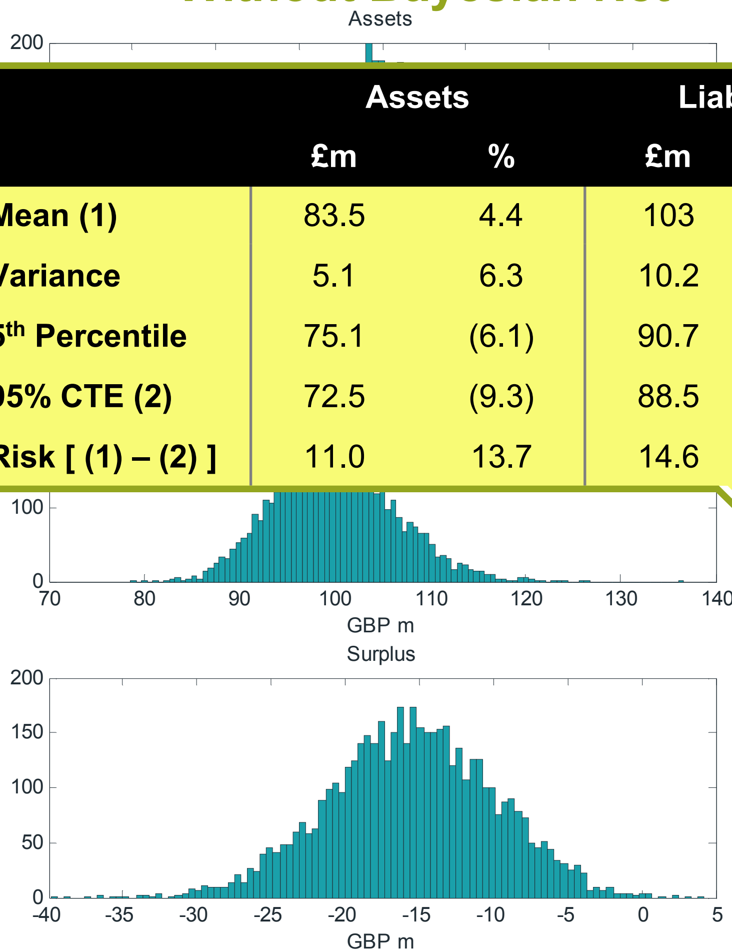


With Bayesian Net

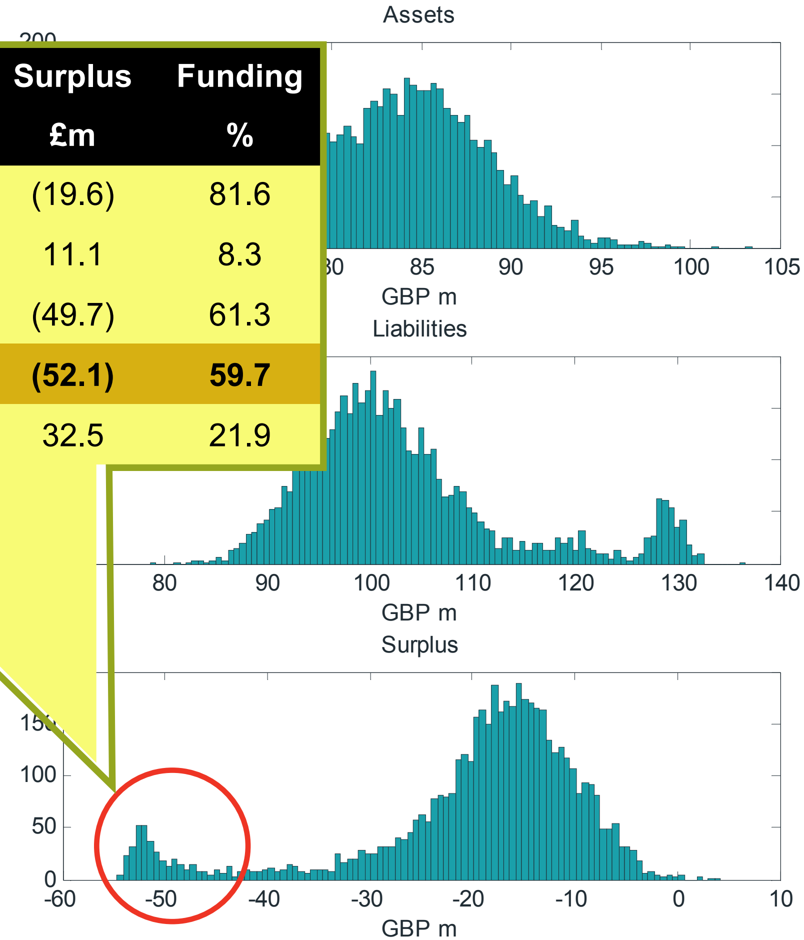


Current portfolio: Projected 1-year

Without Bayesian Net



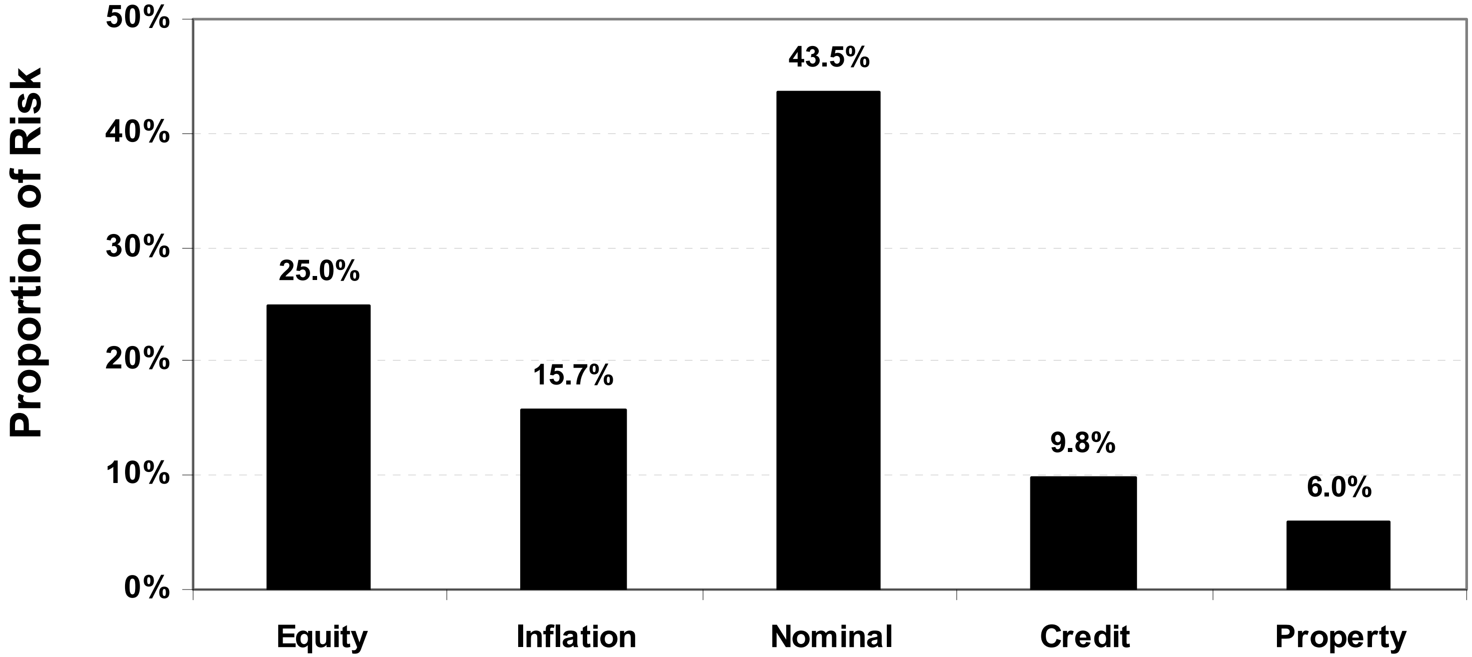
With Bayesian Net



	Assets		Liabilities		Surplus	Funding
	£m	%	£m	%	£m	%
Mean (1)	83.5	4.4	103	3.1	(19.6)	81.6
Variance	5.1	6.3	10.2	10.2	11.1	8.3
5th Percentile	75.1	(6.1)	90.7	(9.3)	(49.7)	61.3
95% CTE (2)	72.5	(9.3)	88.5	(11.5)	**(52.1)**	**59.7**
Risk [(1) – (2)]	11.0	13.7	14.6	14.6	32.5	21.9

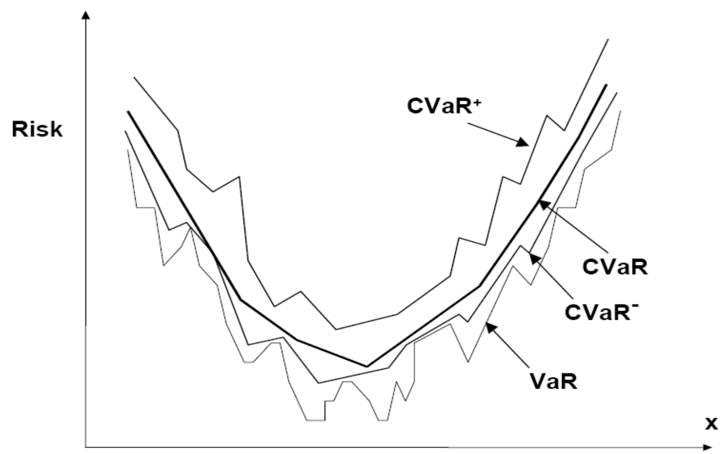
Current portfolio: Risk decomposition

- **Component CVaR at 95%**



Portfolio Optimisation: Methodology

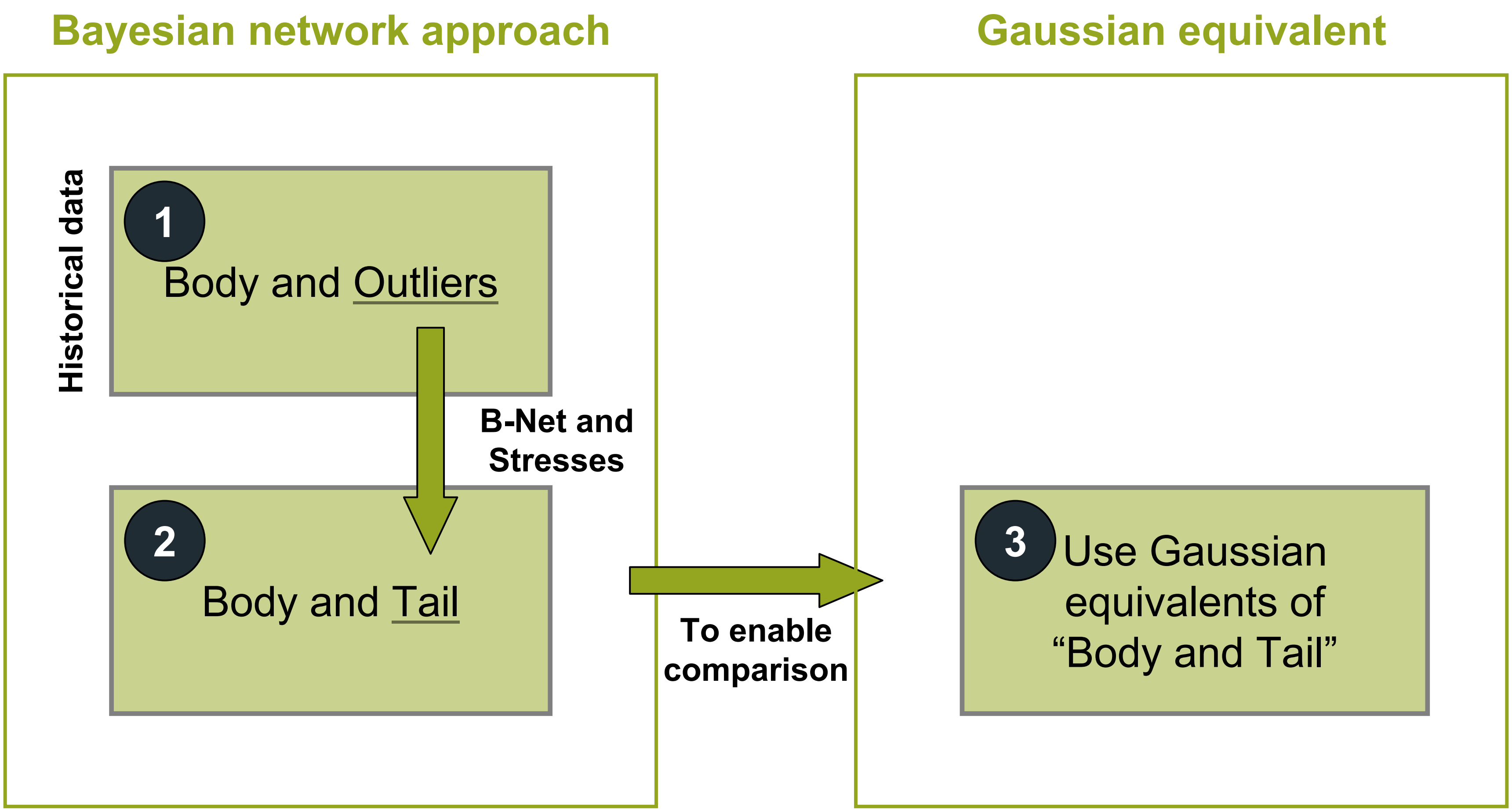
- **Method**
  - CVaR as introduced by Rockafellar and Uryasev. Threshold 95%
  - The CVaR measure can consistently be extended to the case of discrete probability lumps in the tails as might sometimes be wanted or can happen in the application of Bayesian Networks*



CVaR is convex, but VaR, $CVaR^-$, $CVaR^+$ may be non-convex, inequalities are valid: $VaR \le CVaR^- \le CVaR \le CVaR^+$

*** Rockafellar and Uryasev, Conditional value-at-risk for general loss distributions (2002), Journal of Banking and Finance, 26, 1443-1471**

Portfolio Optimisation: Gaussian benchmark



Portfolio Optimisation: Parameters

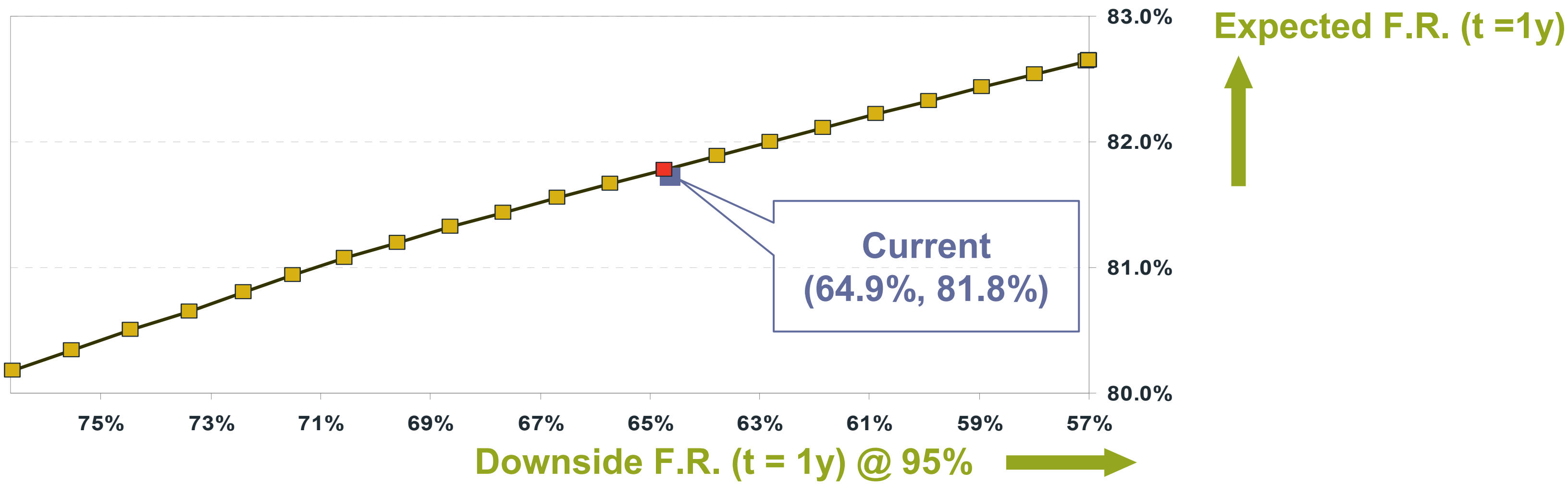
- **Constraints**
  - Equity + Alternatives <= 80%
  - Government bonds >= 20% Total Fixed income
  - Maximum 75% swap overlay for each of UK Gilts, Linkers, Corporates
  - Alternatives weight floored at 5%

- **Risk Budget**
  - On the Gaussian-equivalent returns:
    - 5^{th}-percentile CTE = 64.9% Funding Ratio, £42.4m deficit
  - **We will use the 64.9% Funding Ratio as risk budget**
  - Other alternatives possible – e.g. lock the deficit

Portfolio Optimisation: Without Bayesian Net

Slightly increased allocation to Equity at 64.9% downside Funding Ratio

Efficient Frontier

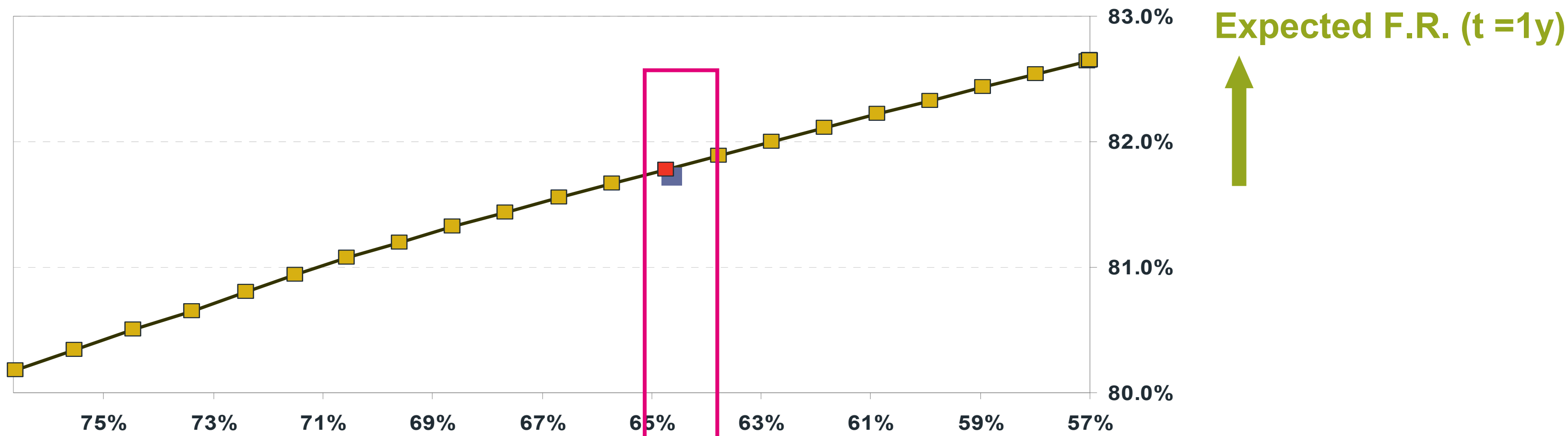


Source: RBS

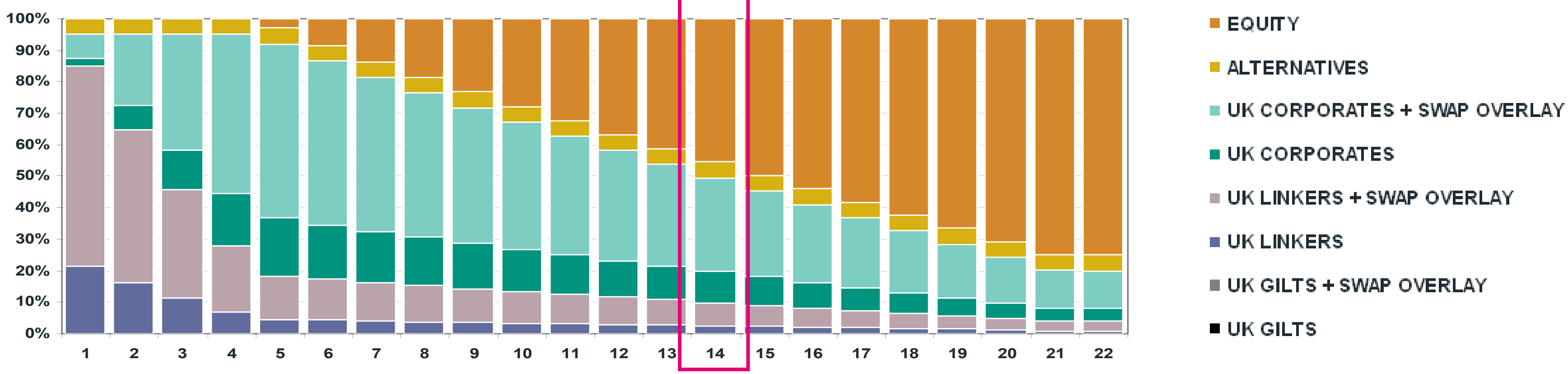
Portfolio Optimisation: Without Bayesian Net

Slightly increased allocation to Equity at 64.9% downside Funding Ratio

Efficient Frontier



Asset Allocation



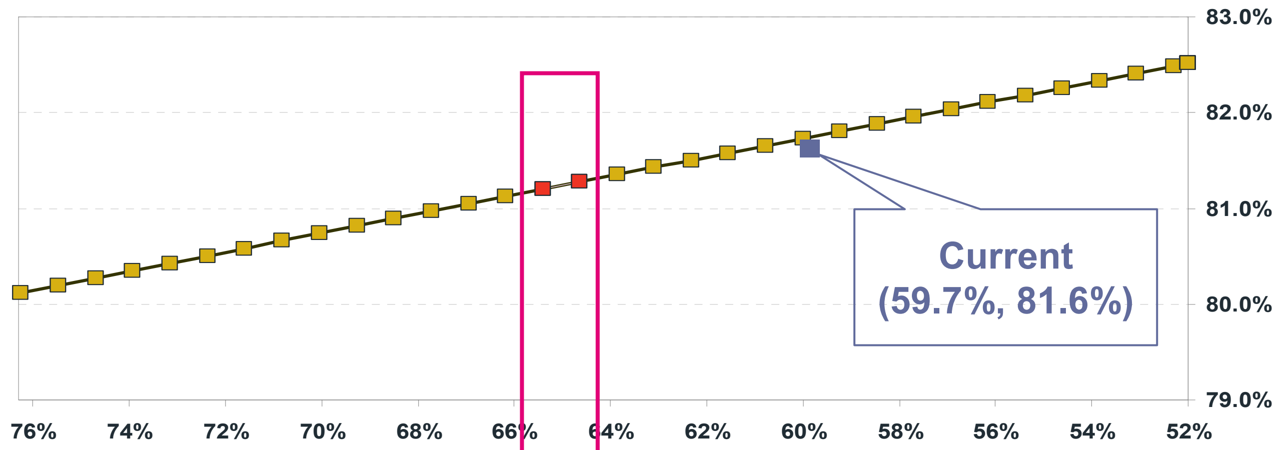
Equity = 41% (c.f. Current = 35%)

Source: RBS

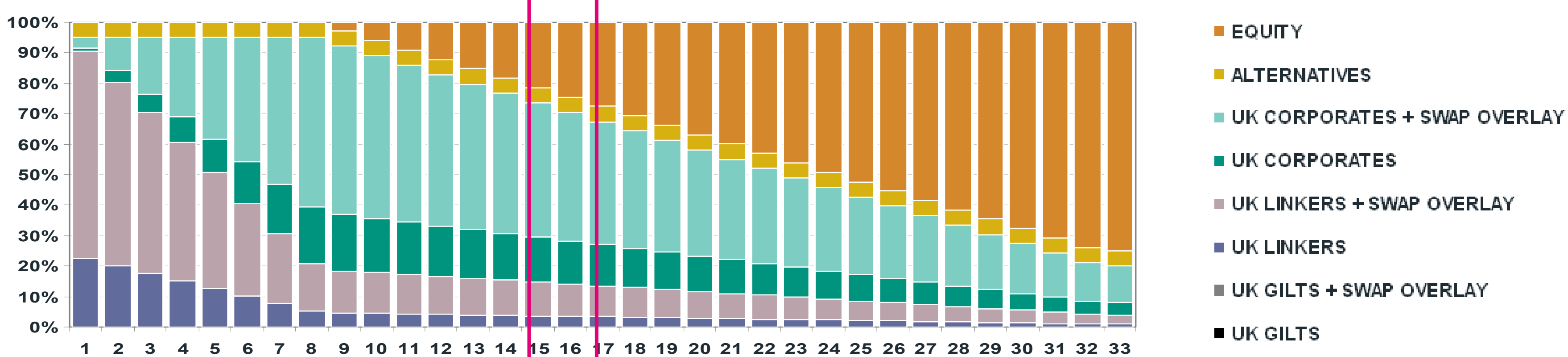
Portfolio Optimisation: With Bayesian Net

Reduced allocation to Equity at 64.9% downside Funding Ratio

Efficient Frontier



Asset Allocation



Equity = 22% (c.f. Current = 35%)

Source: RBS

Optimisation results compared:
Compare Current and Optimised portfolios

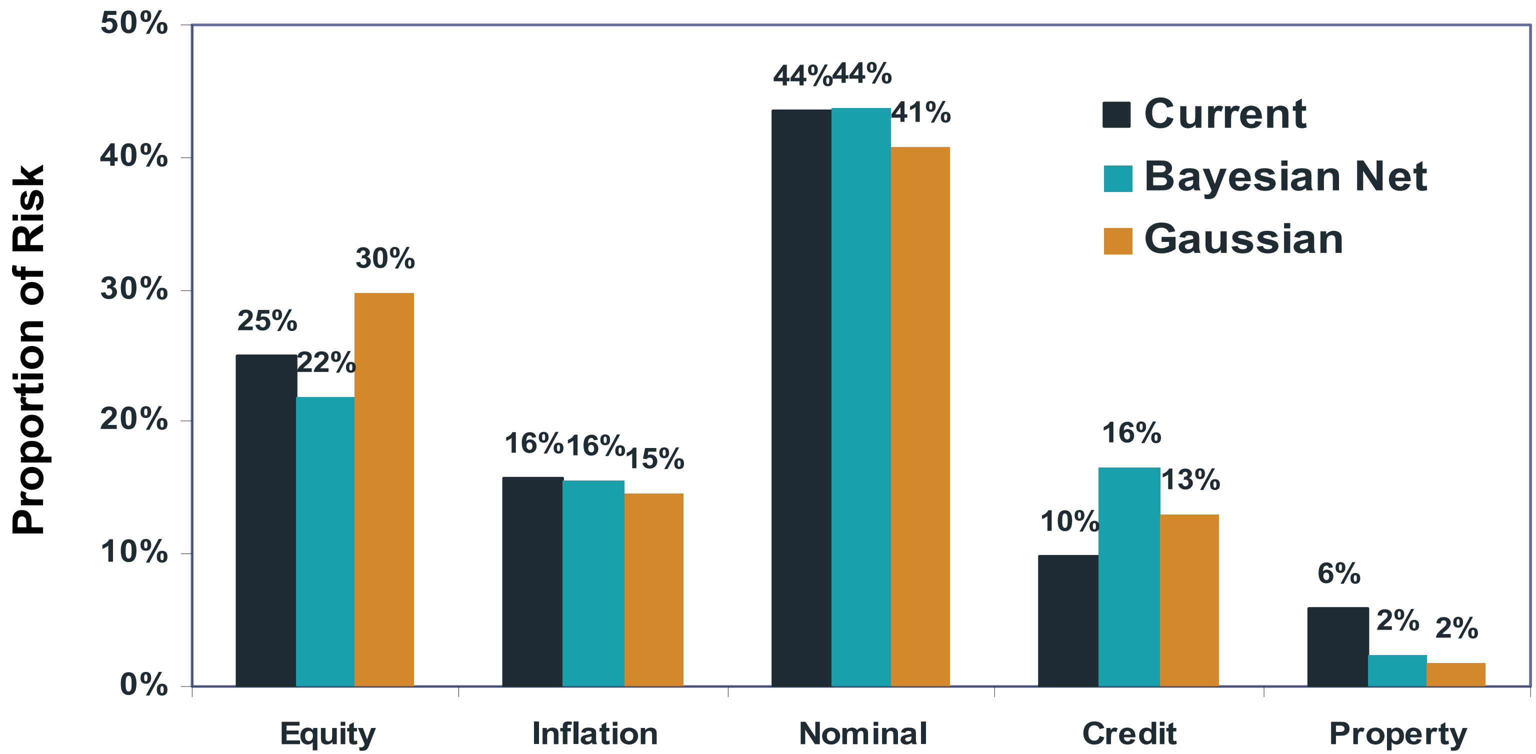
	Current	Gaussian Optimised	Bayesian Net Optimised
Expected F.R.	81.63%	81.69%	81.20%
Downside F.R.(1)	59.7%	60.2%	**64.9%**
Expected Surplus £m	(19.65)	(19.56)	(19.86)
Downside Surplus £m (1)	(52.09)	(51.52)	(45.30)
Asset Allocation	£m	£m	£m
UK Gilts	5.0 (2)	0.0	0.0
UK Linkers	5.0 (2)	8.5 (3)	11.6 (3)
Corporate bonds	30.0 (2)	34.2 (3)	46.3 (3)
Equity	28.0	33.3	18.1
Property	12.0	4.0	4.0
	80.0	**80.0**	**80.0**

Using fat-tailed sims

(1) CTE at 5th percentile; (2) with 2/3rds hedged; (3) with 75% hedged

Optimisation results compared:
Risk decomposition

- Bayesian Net approach emphasises move into Credit



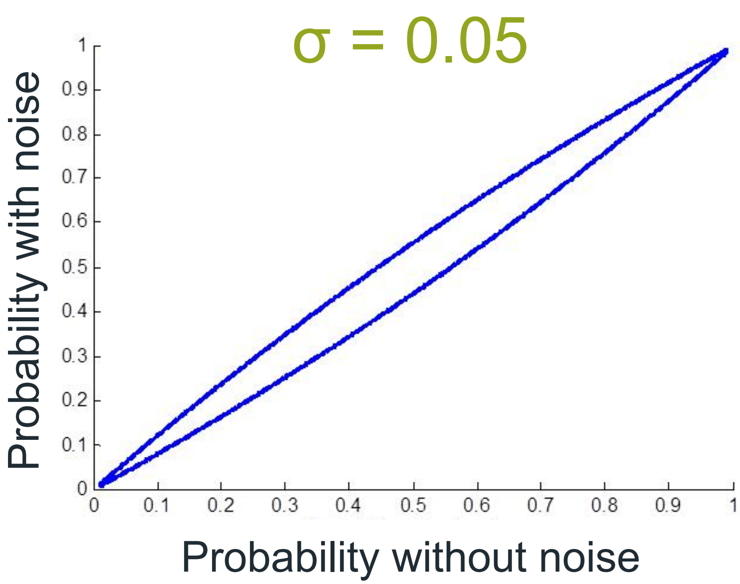
Performance under different outcomes

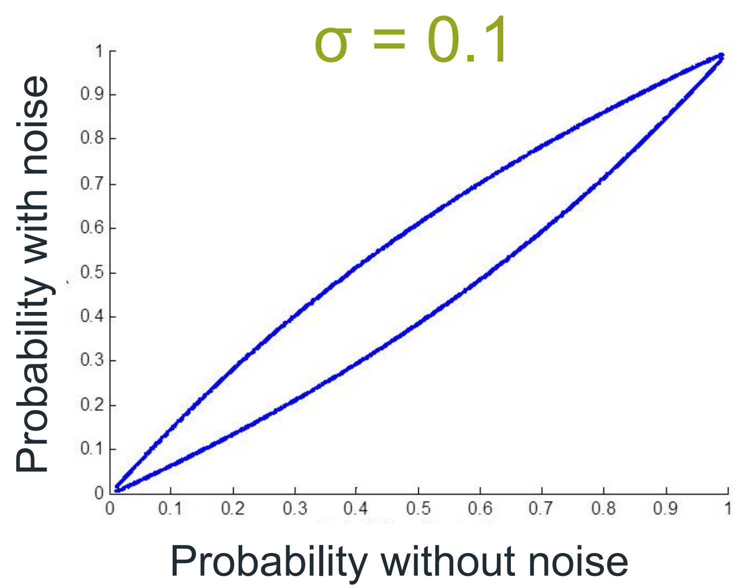
- Bayesian Net solution more resilient to Equity shocks, but underperforms in event of Corporate returns falling

Simulation	Projected Surplus and Funding Ratio					
	Current		Gaussian Optimised		Bayesian Net Optimised	
	£m	%	£m	%	£m	%
1) Liabs +28%, Equity -25%, Corps -3%	(50.7)	60.5	(50.8)	60.5	(44.7)	65.2
2) Liabs +3%, Equity +24%, Corps +7%	(12.4)	87.9	(12.3)	88.1	(15.1)	85.4
3) Liabs -10%, Equity -25%, Corps -12%	(20.0)	77.8	(23.5)	73.8	(22.0)	75.5

Sensitivity Analysis - Method

- We perturb the Conditional Probability Tables by adding different levels of noise and recalculate the allocation and efficient frontiers
- For each level of noise given by the parameter σ the variation of the perturbed probabilities with respect to the unperturbed is given in the figures below





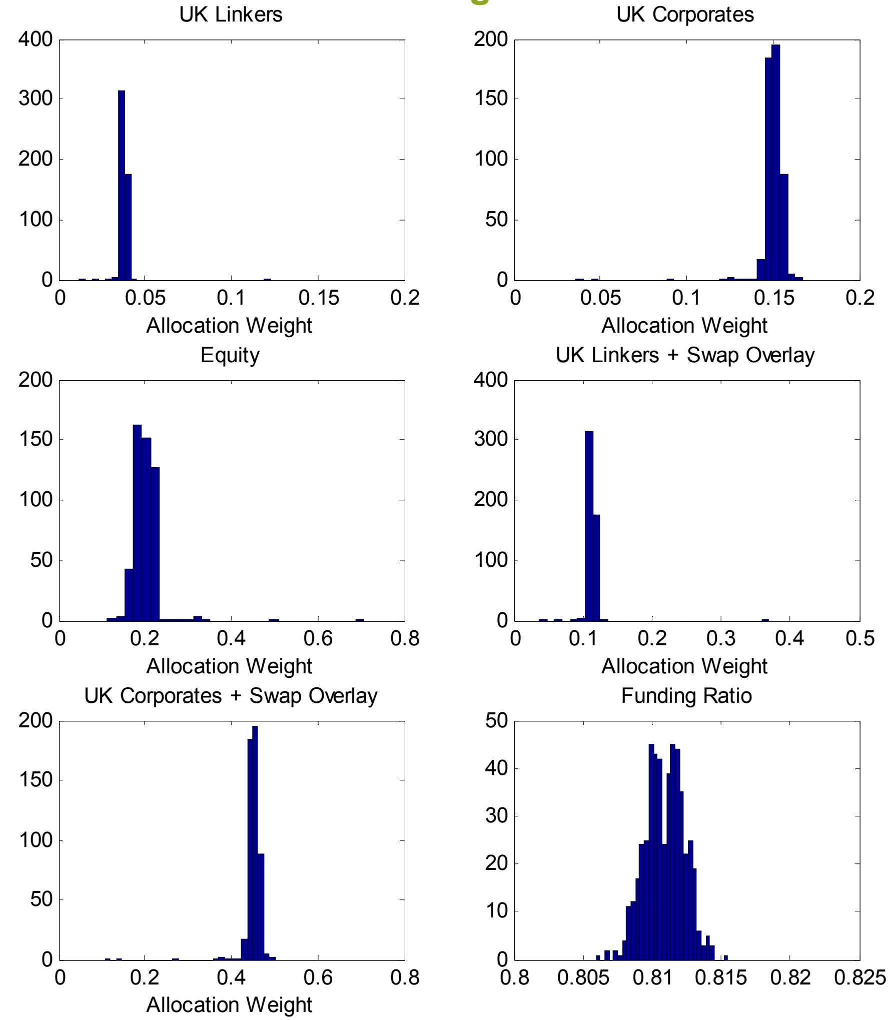
Sensitivity Analysis - Results

- $\sigma = 0.05$
- Allocations calculated at 65% downside Funding Ratio

Conclusions

- The same asset classes are suggested by the optimiser as the unperturbed case
- Allocations are stable with respect to the unperturbed case

Distribution of Weights and Funding Ratio



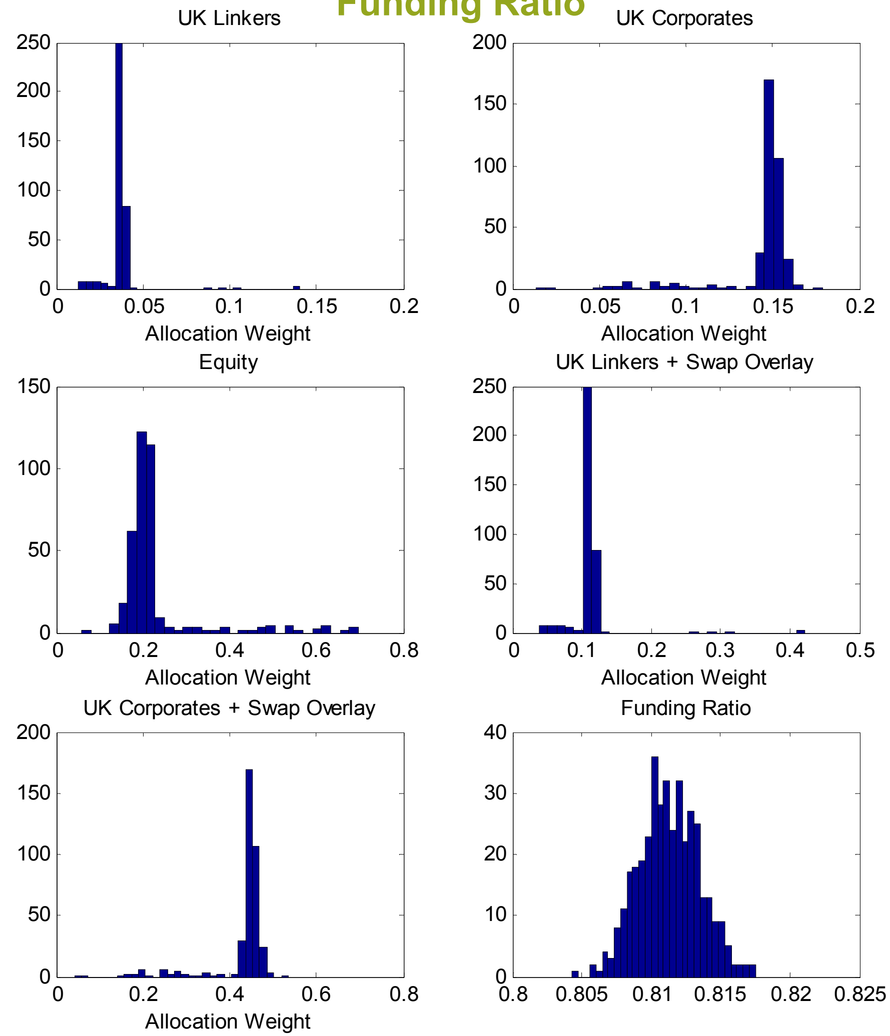
Sensitivity Analysis - Results

- σ = 0.1
- Allocations calculated at 65% downside Funding Ratio

Conclusions

- The same asset classes are suggested by the optimiser as the unperturbed case
- Allocations are stable with respect to the unperturbed case
- Allocations more dispersed compared to case σ = 0.05

Distribution of Weights and Funding Ratio



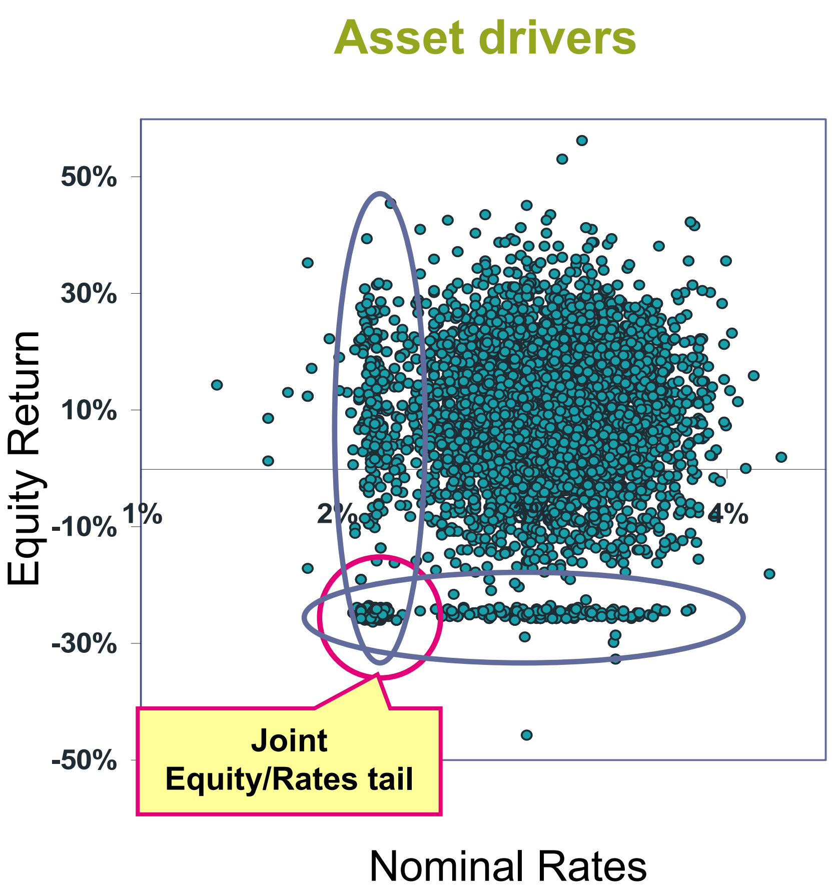
Coherent asset allocation in the presence of stress events

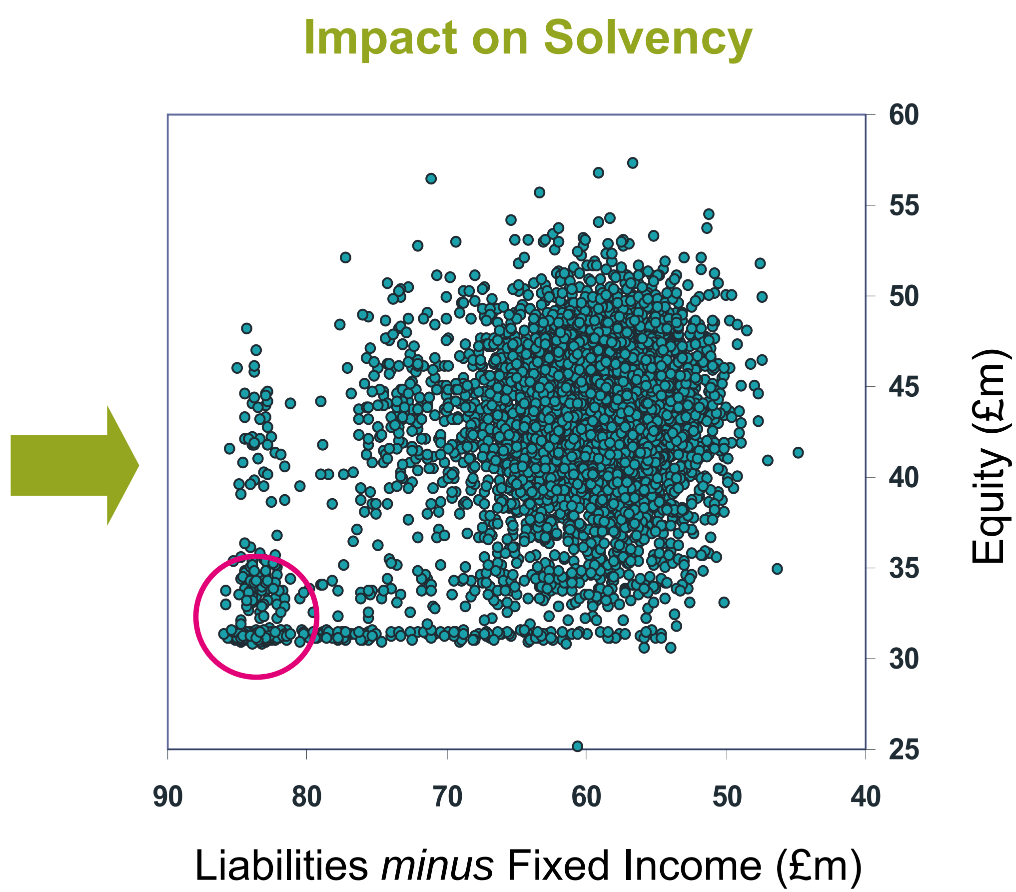
Extensions

Extensions

- Application to Macro-hedging
- Application to “Efficient hedging”
- Future development

Macro-Hedging using Bayesian Net
Motivation: from our Case Study

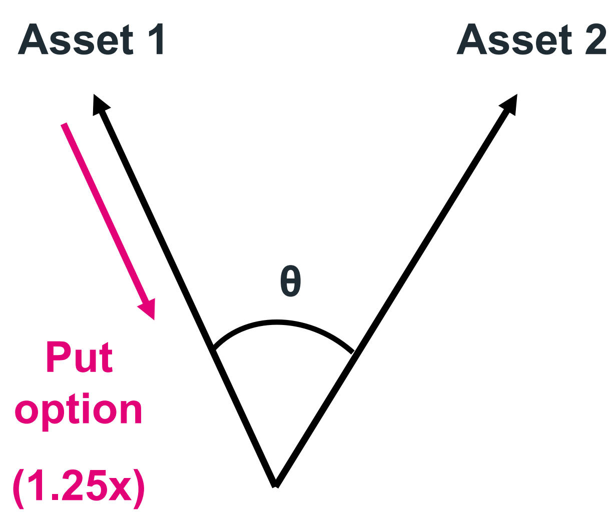




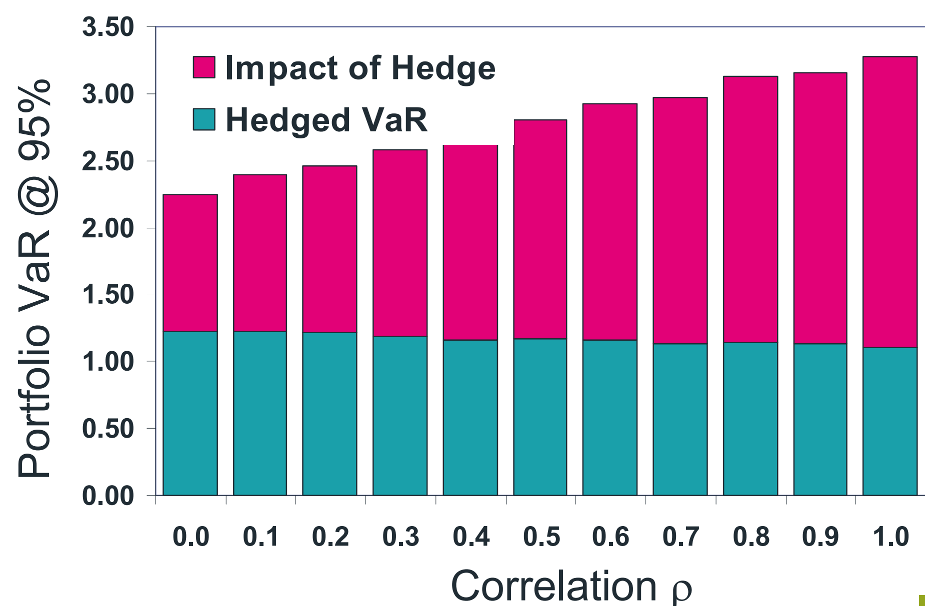
Macro-Hedging using Bayesian Nets
Motivation: some simple theory

- Consider two assets, modelled as ~ N(0,1), correlation ρ, equal weight
- Overlay a "Put Option", strike 0.1, on asset 1, 1.25x Notional
- By how much does this hedge reduce the portfolio VaR at 95%?

Simple two-asset + Put option



Hedge efficiency



Macro-Hedging instruments

- **Instruments considered:**

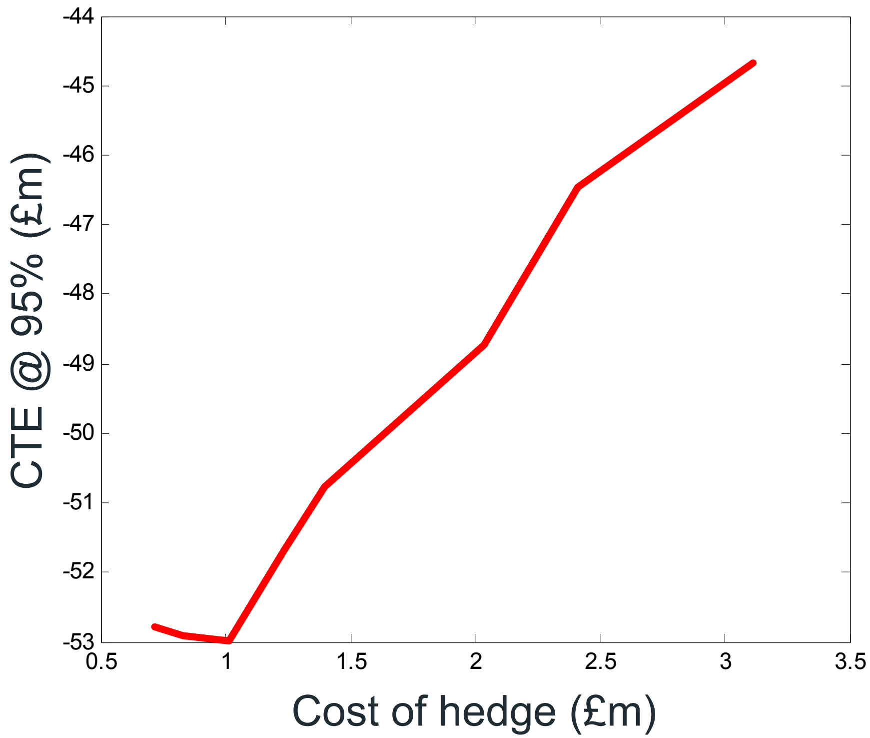
Instrument	Comment
Swaption	
1) Receiver (30y)	Different strikes below par
2) Receiver spread plus Payer Swaption (30y)	Different notionals of a put spread below par and short a payer above par
Equity	
3) Put	Different strikes below ATM-forward FTSE level

Macro-Hedging using Bayesian Nets

Equity Put

- **Method**

  Buy 10,000 Put option contracts on FTSE 100 at different strikes

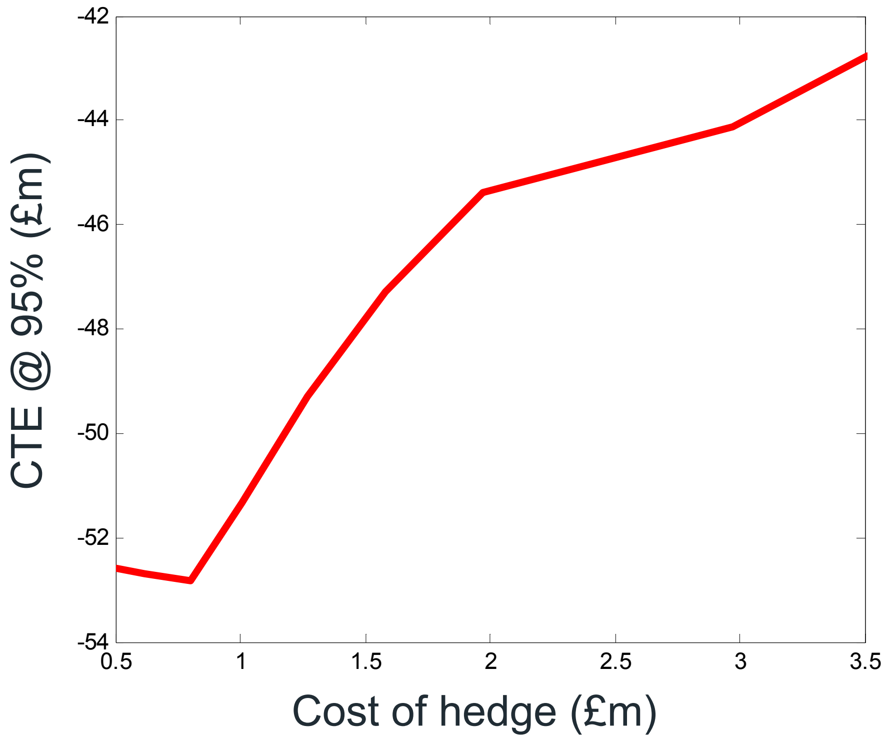


Macro-Hedging using Bayesian Nets

Receiver Swaption

- **Method**

  Buy Receiver Swaptions (30y) GBP 100 m notional



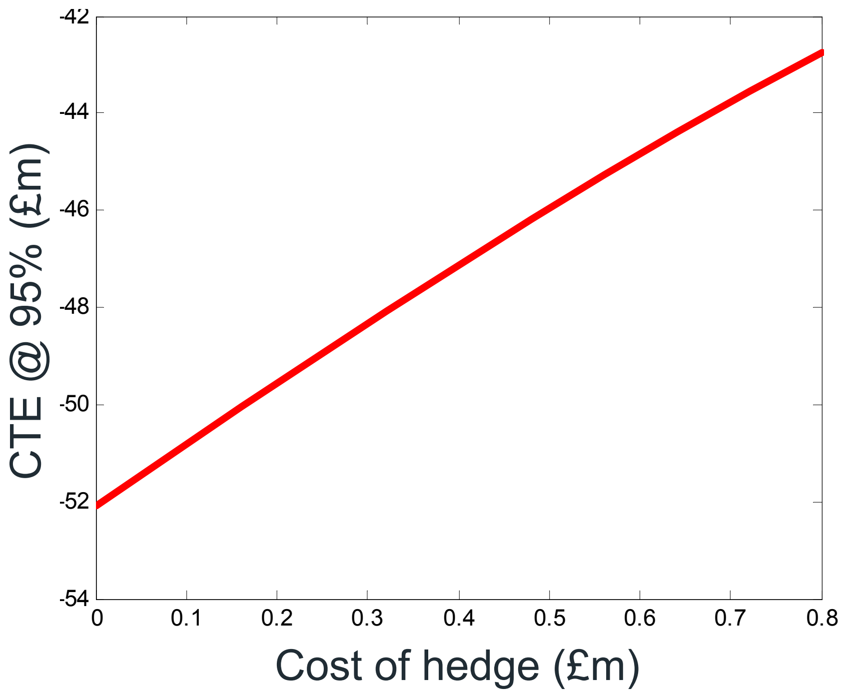
Macro-Hedging using Bayesian Nets

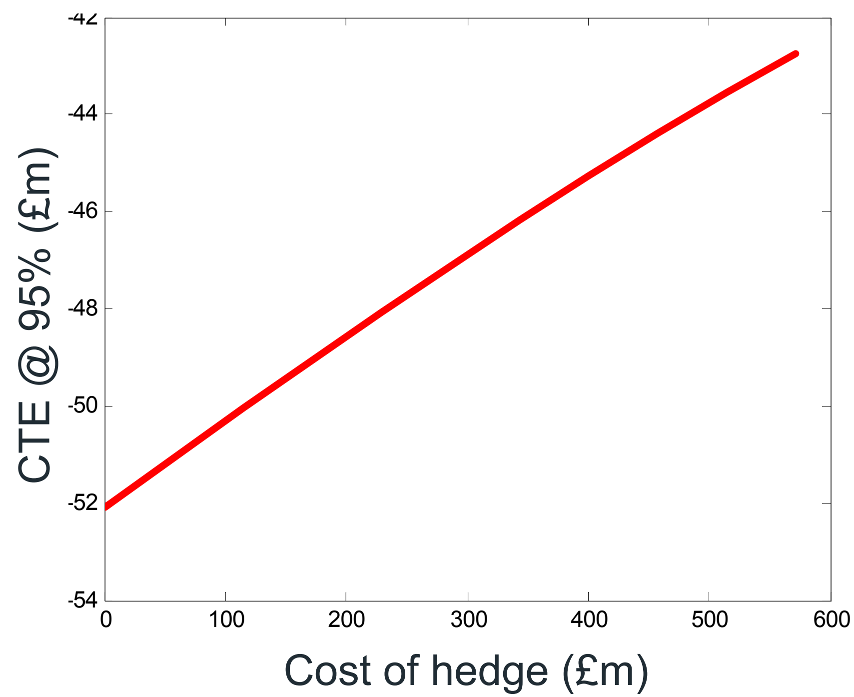
Receiver Swaption Spread plus Payer Swaption

- **Method**

  Buy Receiver Swaptions Spread (30y) at 2.3% and 2.6%; and

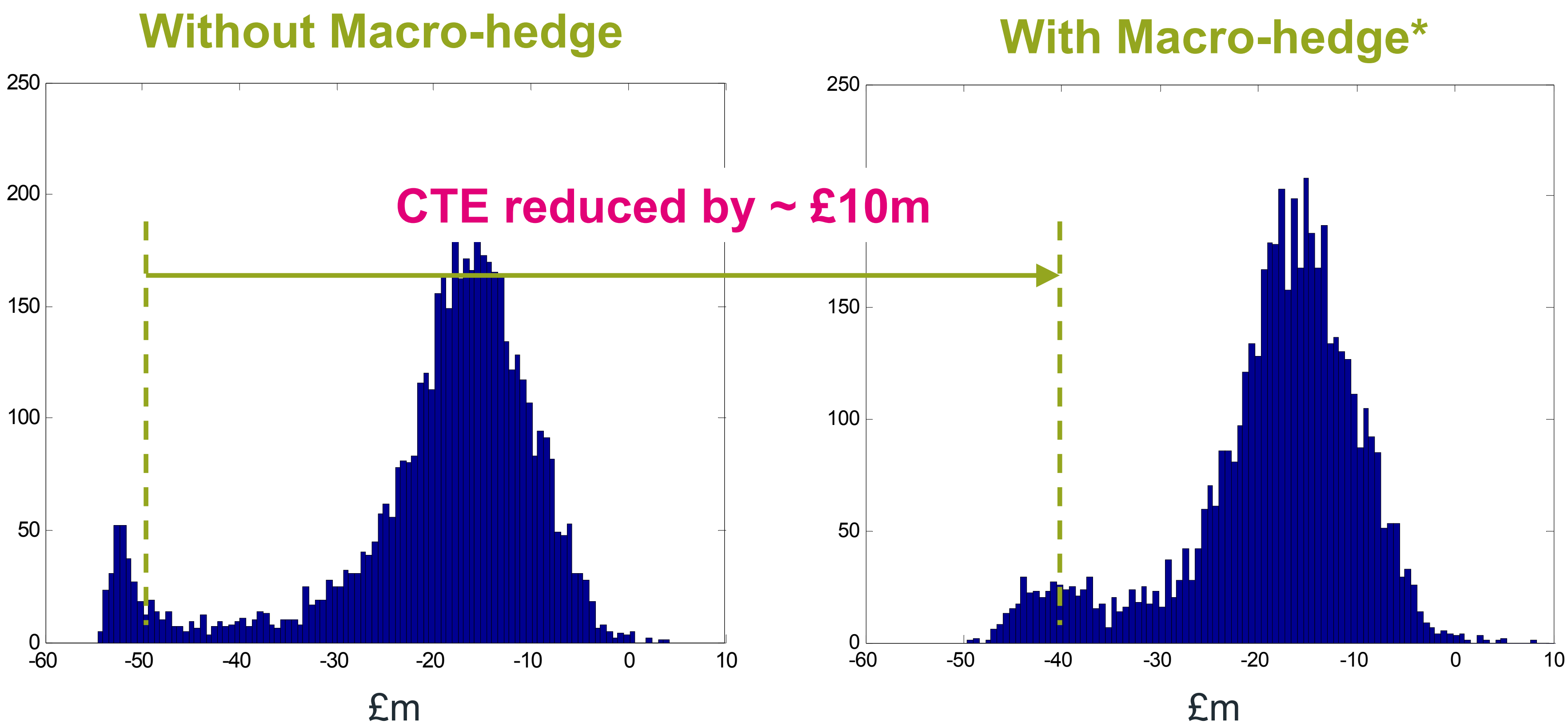
  Sell Payer Swaption at 4.5%





Macro-Hedging using Bayesian Nets

Receiver Swaption Spread plus Payer Swaption

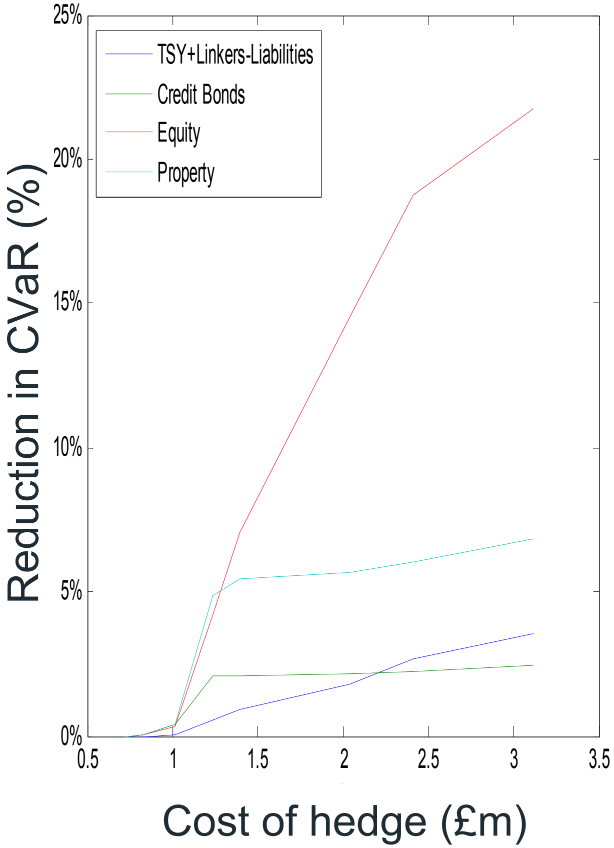


*** Hedge cost = £0.8m**

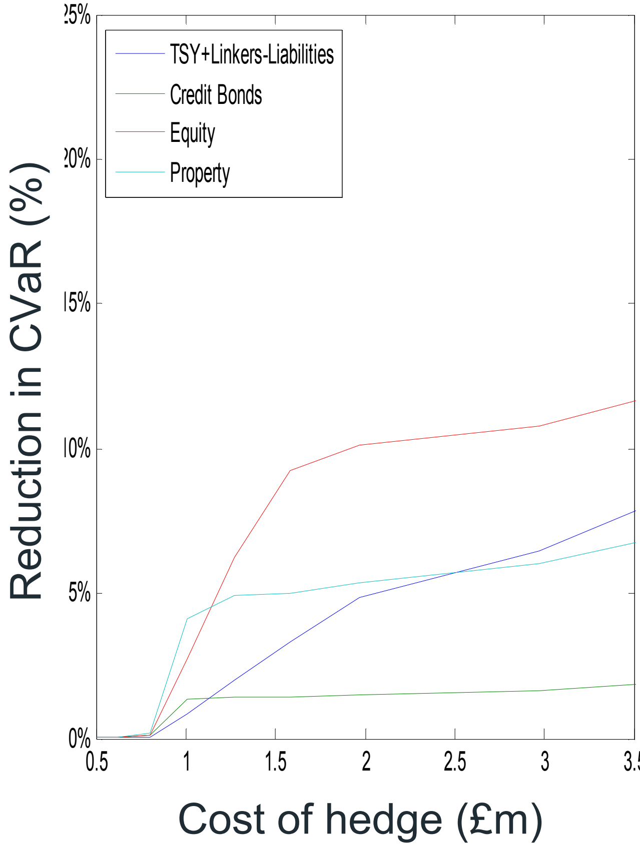
Macro-Hedging using Bayesian Nets – Effect on different asset classes

Macro-hedging reduces risk along several dimensions

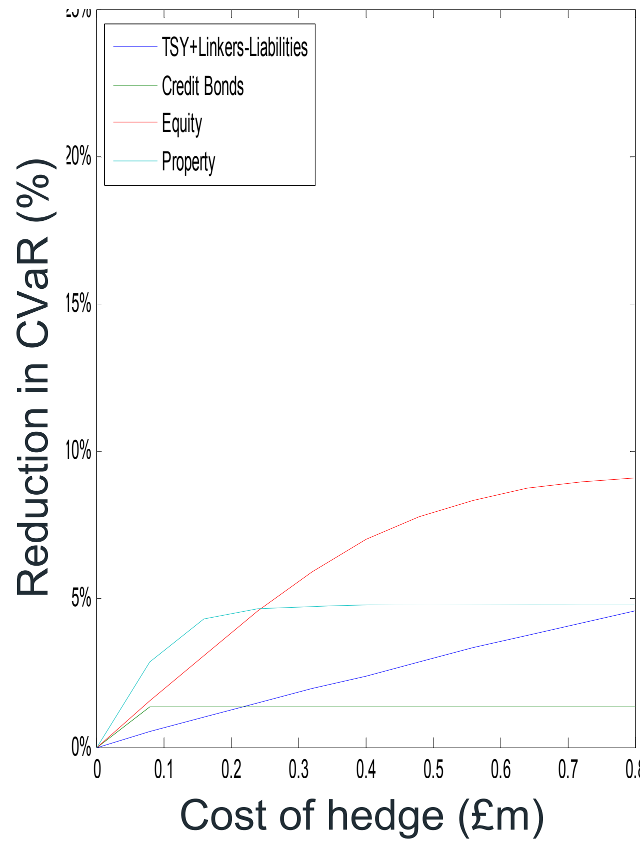
Equity



Receiver Swaption



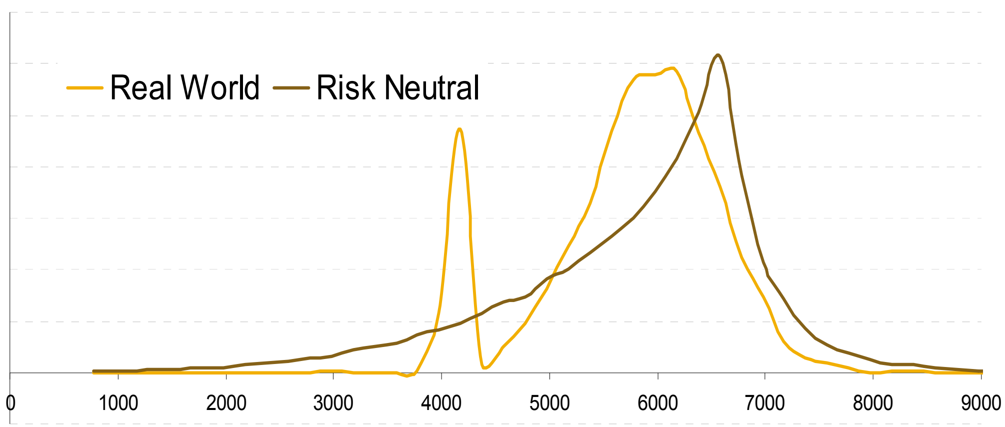
Receiver Swaption Spread + Payer Swaption



Efficient Hedging:
Exploiting Differences vs. Risk Neutral Distribution

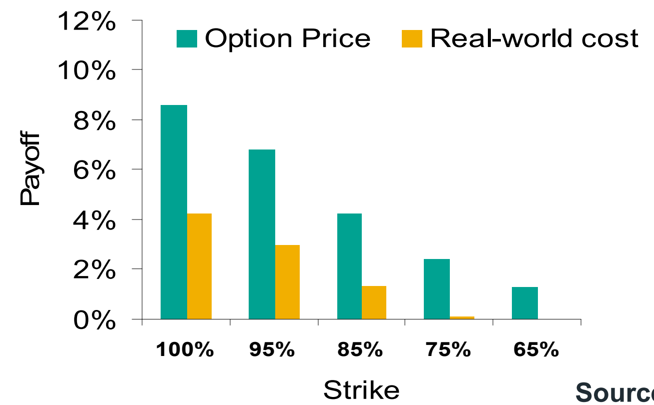
- Differences between the market-implied and real world (Bayesian Net) distributions can be exploited to reduce risk and / or enhance return
- Puts are expensive to purchase relative to real world value and calls are expensive at very high strikes

The Risk Neutral Distribution overemphasises extremes



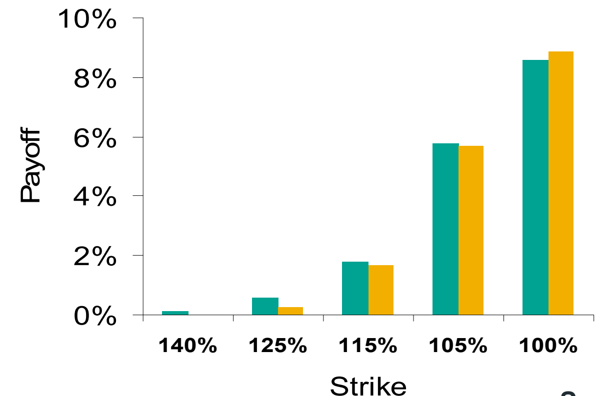
Source: RBS

Put Options



Source: RBS

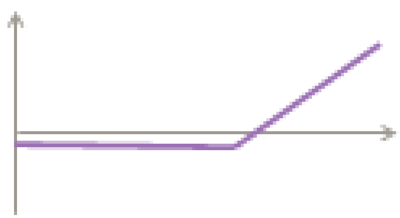
Call Options



Source: RBS

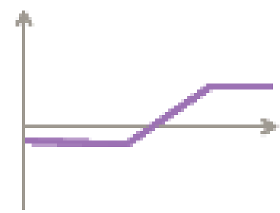
Efficient Hedging:
Exploiting Differences vs. Risk Neutral Distribution

Put
Equity fall – protected (strike dependent)
Equity rises – capture upside, less premium



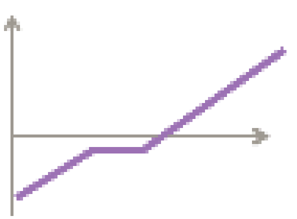
Long equity
Long put

Collar
Equity fall – protected (strike dependent)
Equity rise – some upside lost



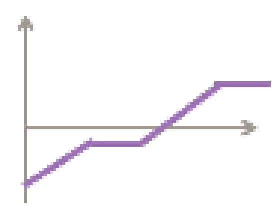
Long equity
Long put
Short Call

Put spread
Equity fall – partial protection
Equity rises – capture upside, less net premium



Long equity
Long high strike put
Short low strike put

Put Spread Collar
Equity fall – partial protection
Equity rise – some upside lost

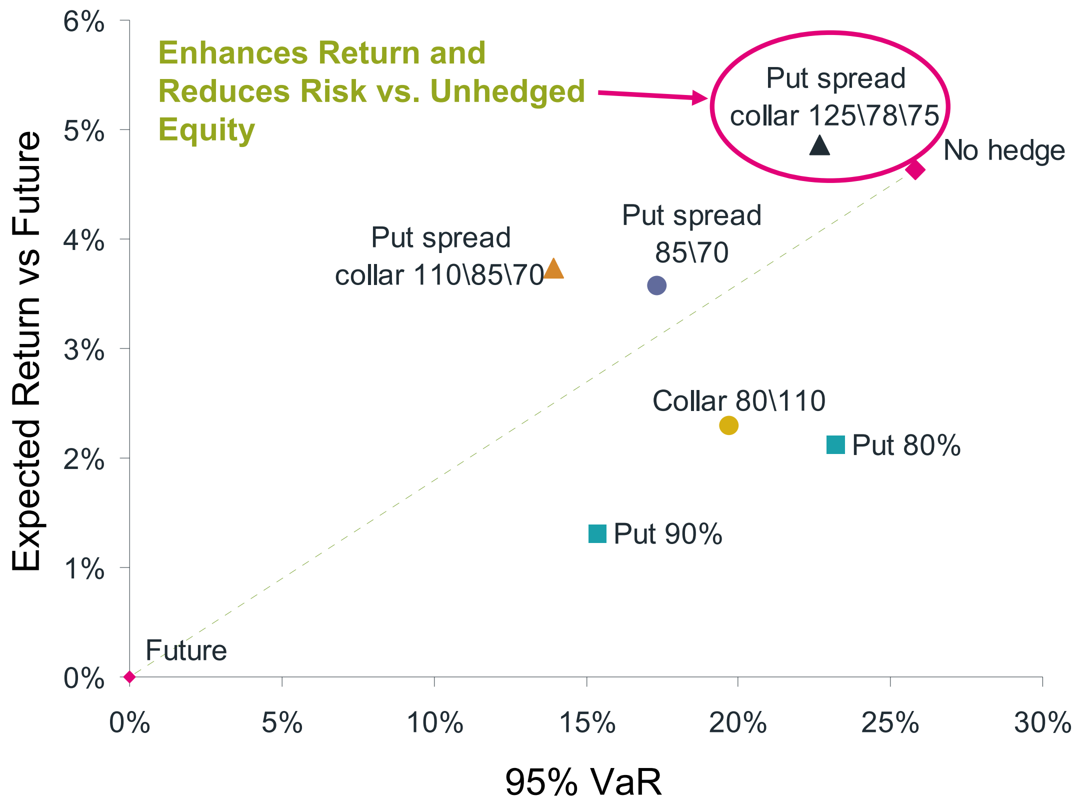


Long Equity
Long high strike put
Short low strike put
Short Call

Efficient Hedging:
Exploiting Differences vs. Risk Neutral Distribution

- However under current market conditions, put spreads and / or put spread collars are cheap

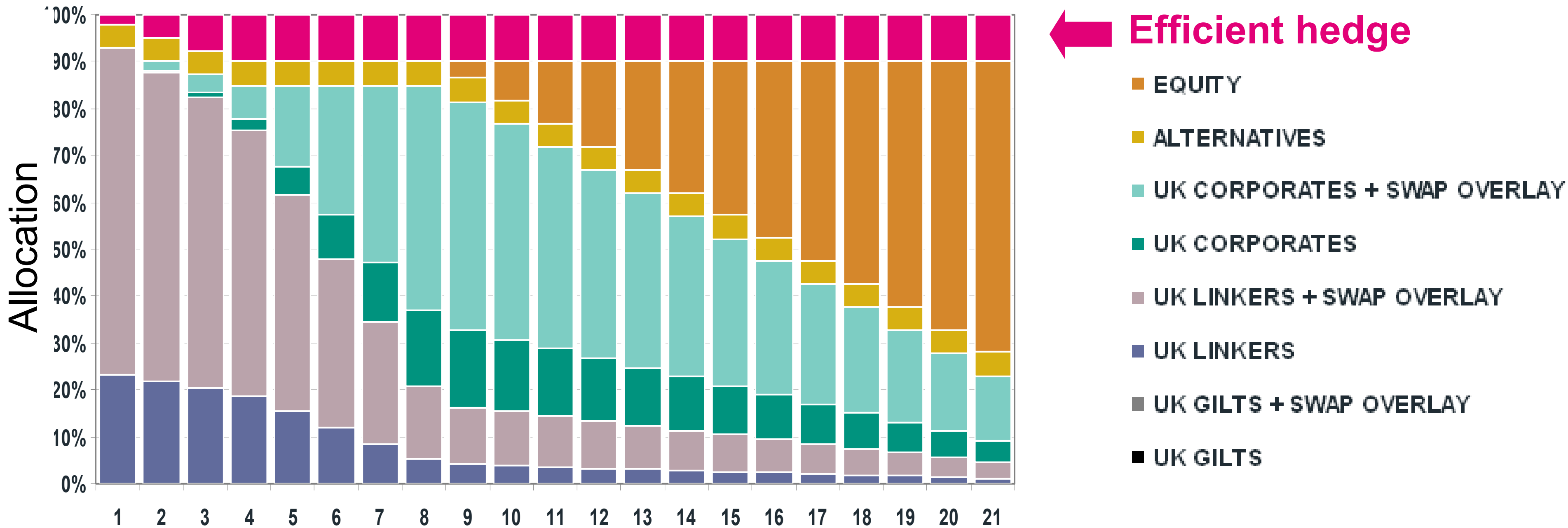
Enhancing Return and Reducing Risk through Efficient Hedging



Source: RBS

Efficient Hedging:
New Asset Allocation

- **Efficient Hedge = Equity + Put spread + Short Call**
  - Constrained to max 10%



Future developments

Completeness condition: The “2^N check”

- Must ensure complete set of stress nodes …
- … such that no materially detrimental cases are missed …
- … for the intended application
- For N drivers, need evaluate 2^N possible extremities

Blending with existing approaches

- May already have existing processes for generating marginals (TSM, BH, proprietary, etc …)
- Use the Bayesian networks approach to build the desired tail dependence properties (in terms of joint probabilities)
- Impose this ranking upon own marginals

Additional Tools

- Having a subjective component the approach could benefit form some additional calibration tools such as:
  - The Black-Litterman approach adapted to Bayesian Networks
  - CAPM to infer loss returns on asset classes with the help of an equilibrium theory
  - Maximum Entropy
  - Linear Programming
  - Techniques for separating complex nets into simpler ones

- On the cognitive side, and numerically
  - Gaussian approximations
  - Monte Carlo simulations when the number of nodes becomes big

Summary

- Stress tests, scenario analysis and tail risk management can be tackled by:
  - Constructing alternative Bayesian networks for tail events
  - Varying tail event probabilities, loss estimates, return distributions
- Asset allocation decision now incorporates views, but inputs are required:
  - Some probabilities easier to assess and assign than others
  - Cognitive biases can lead one to mis-specify probabilities
- Potential weaknesses of the method are also its strengths:
  - Tail events occur rarely by definition, so it is hard to conduct a statistical or completely scientific (objective) analysis
  - This method encourages the practitioner to be explicit and careful about the subjective parts of the analysis, leading to increased transparency
  - The method can be used easily to engage senior management for their views regarding tail events
  - Quants and risk managers play a vital role in the methodology's success

Coherent asset allocation in the presence of stress events

Q&A

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenters.