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COMPOUND INTEREST IN THE SEVENTEENTH CENTURY

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RICHARD WITT'S *Arithmeticall Questions*, published in 1613, was a landmark in the history of compound interest. In a previous paper (*J.I.A.* **96,** 121) I have described the book and attempted to show why it is so important.

This article sets out some information about Witt's life, which has come to light as a result of research done since the previous article (when very little was known about him). Then follows a discussion of some of the books on compound interest which were published later in the seventeenth century. These not only help to fill in the practical background but also show how the techniques came to be more widely known and how problems arose when interest rates were reduced. It is worth noting that there was an average inflation rate of around 1% p.a. during the first half of the century and an average inflation rate of nil, or even a slight deflation, during the second half (Birks & Morrell, 1976; Mitchell & Deane, 1962). Thus, to a very considerable extent, the interest rates used in calculations were real rates of interest.

Witt's book delved deeply into compound interest in a very practical way, with clarity of expression and accuracy in calculation. Tables of standard compound interest functions were given at 10% p.a. (the then legal maximum rate of interest) and $6\frac{1}{4}$% (which appears to have been the normal rate used for the purchase of freehold property, although Witt's worked examples do suggest that 5% was sometimes used for this purpose). Although $(1+i)^n$ was also tabulated for 5%, 6%, 7%, 8% and 9%, the other standard functions were not printed for these rates of interest, which were therefore probably less commonly used in practice. Many worked examples were given, some with very elegant solutions, showing that Witt had a considerable understanding of the subject, and thought processes like those of modern actuaries. His book can perhaps be regarded as the first ever actuarial textbook.

He came from a Northamptonshire farming family. His grandfather was probably William Wytt of Dene, who died in February 1555 leaving his farm to his wife and his two sons, James and Richard. The latter, father of our author, appears to have left Northamptonshire to live in London. In 1563 he was married at St Clement, Eastcheap, in the City of London, to Anne Dickinson, a butcher's daughter.

Richard Witt senior died in November 1567, leaving his wife, Anne, pregnant with their only child. In his will, made while he was ill and only a day or two before his death, he described himself as 'Citizen and Salter of London'. He was in debt and he asked his creditors to give his executors time to pay. He left his goods to his wife and child.

Our author was born four months later. He was baptized Richard, after his father, on 27 March 1568 at St Clement, Eastcheap.

His mother married again on 5 February 1570. Her new husband was William Smyth, an embroiderer. They lived in the parish of St James Garlickhithe. Their first child was a daughter, Christian, who was born in December that year, and a son, Wentworth, was born in March 1572. Another son, Nathaniel, was born in 1574. The family now moved to the parish of St Michael Bassieshaw (near London Wall), where two more daughters, Helen and Katheryn, were baptized in October 1576 and March 1579 respectively.

The family may by now have been living in some degree of poverty, because in 1580 Wentworth Smith, our author's step-brother, was admitted at the age of 8 to Christ's Hospital, which was a charitable foundation for helping orphans and poor families. In 1583, Nathaniel Smith, the other step-brother, was also admitted to Christ's Hospital.

However, the records of Christ's Hospital appear to contain no record of Richard Witt, so it must be assumed that he was educated elsewhere. It is not clear whether he studied at a university. There is an entry in the matriculation register of Oxford for a Richard Whit or White of Magdalen Hall who was admitted in February 1584, but whether it relates to our author (who would then have been nearly 16) cannot now be established. As we shall see later, Richard Witt left some of his books to Oxford University in his will, so it may be that he was educated there.

There is now a considerable gap in my information about him and this is particularly unfortunate since these were the formative years during which he must have acquired most of his practical mathematical knowledge and experience.

The next news of him comes in December 1605, when his mother, Anne Smith, made her will. Her second husband had died previously and the main beneficiary of the will was Richard Witt:

> First I do give and bequeath unto my loving son Richard Witt, citizen and salter of London, one cypress chest which was my father's, three turkey carpets, two thrum cushions of my own making, a great brass pot, my best limbeck, and three gold rings, one of them being a seal ring with the salters' arms engraven on it, one other of five stems, and the other a hoop ring which was my mother's.

After some bequests to her other children she went on:

> And whereas I do owe and am indebted unto my said son, Richard Witt, in the sum of seven pounds, which he at divers and sundry times disbursed and lent unto me in my want and necessity, and hath been alway a careful loving and kind son towards me, I do therefore give unto him one great iron mortar with a pestle, together with all other my goods. . . .

She also appointed him as her executor.

She died in April 1609 and was buried at St Clement, Eastcheap. It appears that she had been living in a tenement adjacent to the church of St Michael Bassieshaw, and although this was let after her death to someone else, Richard Witt bought the lease in March 1613 (though apparently not for his own occupation). This may have been the tenement in which he spent part of his childhood.

Richard Witt was now becoming well-known as a mathematical practitioner. In 1610 he joined with seven other London mathematicians in a recommendation of a new book on accountancy and arithmetic written by William Colson. This was entitled *A General Tresury, a Perpetual Repertory, or a Common Councel-Place of accounts for all Countries in Christendome*. It seems as though there was difficulty in finding a printer, because the book did not appear until 1612, when it was described as being printed "at the expences of the Author". It contained some of the earliest tables of compound interest to appear in an English book, i.e. tables of $(1+i)^n$, $s_{\overline{n}|}$, $a_{\overline{n}|}$, at 10% p.a., for $n=1$ to 21, with one of two elementary examples of their use. Perhaps this is what inspired Witt to write his own book? The seven other mathematicians who recommended the book were Arthur Hopton, Thomas Bretnor, Henry Brigges, Gerard Malynes, John Speidall, Nathaniel Withers and Mr Burton, and it is probably reasonable to presume that Witt would have been at least acquainted with most of them. It is worth mentioning that Hopton, Brigges, Malynes, Speidall and Witt all later published works dealing at least in part with compound interest.

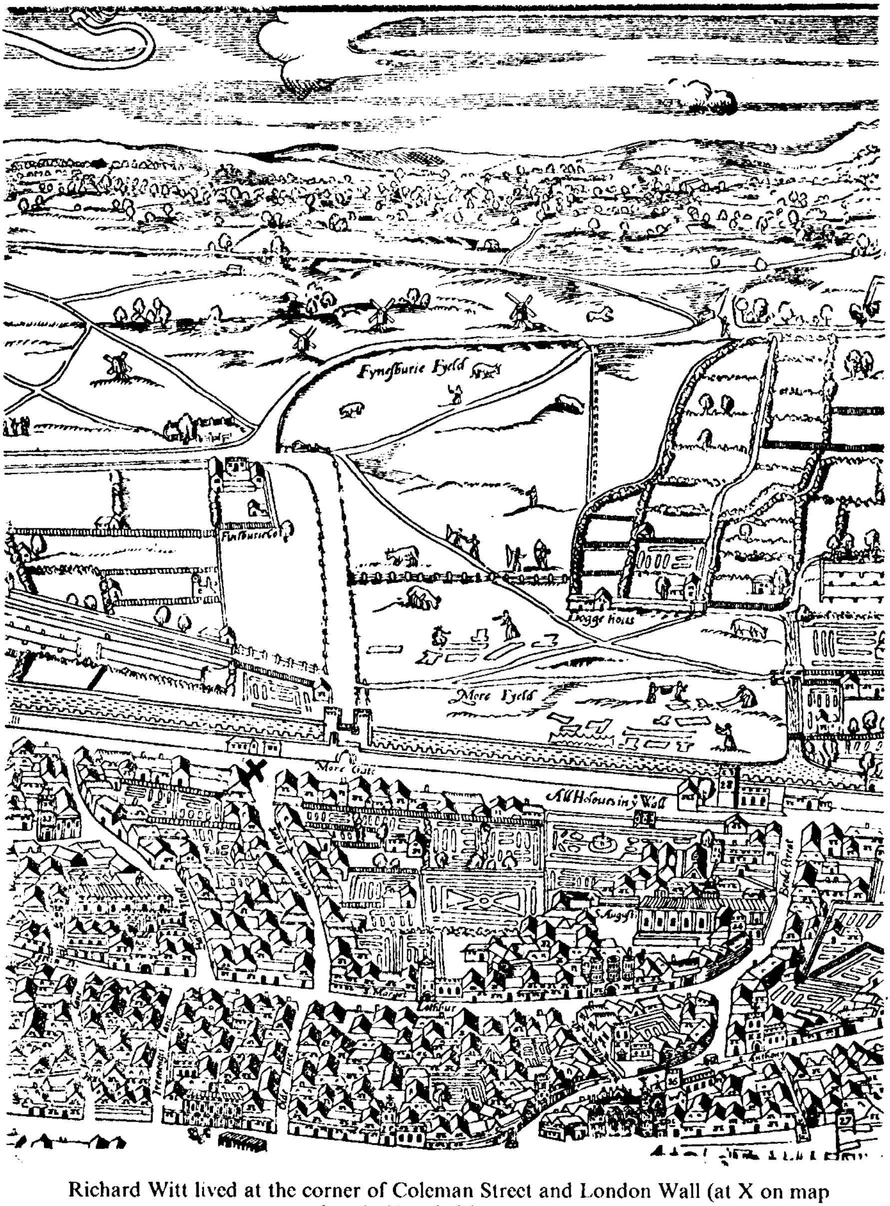
In 1613 Witt's own book was published. He was aged 44, and living in the parish of St Mary Woolchurch as a bachelor. This parish covered the area round what is now the Mansion House but which was then a meat and fish market. The book is dedicated to The Right Worshipful Mr John Herdson, Esquire, who apparently loved the science of arithmetic and respected the practitioners of it, including Witt, who had done some work for him.

In August of the same year Richard Witt married Helen Pennaunt, a widow about two years younger than himself. They were married in the parish church of St Stephen, Coleman Street, and Richard Witt then moved into his wife's home in Coleman Street, which was held on a lease from the Drapers' Company. It is probable that her mother also lived there, until she died six months later.

Helen Witt, as she now was, could not read or write. She had married her previous husband in 1597 and had had three children, all of whom had died young. Her husband had died in 1606, and she had then gone on living with her mother in the house in Coleman Street. She had no further children by Richard Witt.

The house was on the corner of Coleman Street and London Wall, and was described as "a small low messuage or tenement". We can get some idea of what it was probably like from Agas's pictorial map of London of around 1560 (see illustration), though the details may not be wholly reliable. We learn from Stow's *History of London* that there was a conduit of fresh water just outside, in London Wall. In 1615 the Drapers' Company granted Richard Witt and his wife a 21-year lease on the property at the old rent (i.e. £1 p.a.) in return for a 'fine' (i.e. a capital sum) of £50.

Unfortunately the house had to be rebuilt around 1619. This was the tenant's liability and it cost Richard Witt and his wife almost all their estate, i.e. £290. They therefore asked their landlords, the Drapers' Company, to allow them to surrender the old lease, which still had 16 years to run, and to be granted a new



Richard Witt lived at the corner of Coleman Street and London Wall (at X on map above). (Agas's Map, *c.* 1560.)

lease of 40 years at the old rent. While not going quite as far as this, the Company had some sympathy for their position, because they were granted a new lease of 31 years at the old rent, "for which they gave humble thanks unto this Court". Nevertheless it must have been a financial disaster for Richard Witt.

What did a "practitioner in the art of numbers" (as Witt describes himself) actually do? If Witt produced handbills or other advertisements for his services, none appear to have survived. However, we know that his interests were much broader than commercial arithmetic alone, because of the references in his will to his books and instruments of geometry, mentioned below. Probably his repertoire was similar to that of Robert Hartwell (described as 'Philomathematicus'), whose advertisement appears at the end of a mathematical textbook published in 1618. Hartwell, who lived in Fetter Lane, "near the Hand and Pen", taught arithmetic, geometry, the doctrine of triangles, the use of mathematical instruments, navigation, the use of globes, dialling, and finally three financial items, "accounts for merchants by order of debitor and creditors, accounts for retailers of all sorts, and perfection of accounts in controversy".

In the preface to his book Witt makes some remarks about arithmetic in general:

> If thou find any profit, or shalt perceive how much thyself or any others have been deceived for want of the knowledge in this little book expressed: consider and judge, since if this little creek of a river (as I may term it) is profitable and of use, how much more profitable and useful the main Ocean of Arithmetic is.
>
> Experience itself shows that men cannot hold any trading one with another without great use of numbers: whereupon it follows that the losses are not few which men incur by ignorance of the art of numbers.
>
> It is true (I confess) that most men, having but small skill therein, do yet make a shift (such as it is) to pass their manifold business and accounts without sense or feeling of their losses: it is as true that the blind drink many a fly. If there be error in the making of an account, is it not because there is some question or questions of numbers depending upon some things contained in the books of accounts which ought to have been answered before the accounts could be truly made? And is it not likely that the more skill a man hath in the art of numbers, with so much the more facility and certainty of truth he shall be able to resolve questions of numbers?
>
> Persuade thyself, Arithmetic is profitable, not only to men in their private affairs, but also in the Commonwealth businesses, as well in time of war as peace.

In April 1623 Witt wrote his will (although in good health at the time). He wrote it himself, apparently without assistance from a lawyer, and it still survives (in his own handwriting) in the Guildhall Library. He left various small bequests to his step-brothers and his step-sister. Many of his relations were left a copy of a theological work called *Calvin's Institutions* or a bible. (He was clearly a religious man, and on one occasion he is recorded as attending a church vestry meeting, in May 1623.) He left his servant, Thomas Painter, "forty shillings to buy him a mourning cloak and all my books and papers of Arithmetic and Geometry in English and all my copies written and printed and all mine instruments of geometry".

He bequeathed all his books of arithmetic, geometry and divinity in Latin, or other languages except English, to the University of Oxford, but unfortunately I have been unable to trace the bequest there.

He left forty shillings to Mr Volier, preacher at St Peter's by the Cross in Cheap (i.e. Cheapside) to preach at his burial on these words from Jeremiah:

Thus saith the Lord, let not the wise man glory in his wisdom, neither let the mighty man glory in his might, let not the rich man glory in his riches. But let him that glorieth glory in this, that he understandeth and knoweth me, that I am the Lord which exercise loving kindness, judgment and righteousness in the earth, for in these things I delight, saith the Lord.

He also left £8 to the Salters' Company, to which he belonged, for a supper on the day of his burial. He left his leases and goods to his wife, Helen.

He died on 4 November 1624, at the age of 56. The year 1624 was a plague year and, although the worst of the outbreak was over by November, it is possible that this may have been the cause of his death, though there is no evidence on the point. He was buried, according to the wish expressed in his will, next to his mother in the church of St Clement, Eastcheap.

His wife, Helen, died seven years later, in June 1631. In her will she left forty shillings for a ring to Thomas Painter, to whom her husband had left his books and instruments and who had now become a schoolmaster.

Let us now leave Richard Witt and discuss how the subject developed after his book was published.

First it is worth noting a very interesting little piece of work among the papers left by Thomas Harriot (1560–1621), the mathematician and astronomer. The papers are undated but were perhaps written about 1615. He calculates the binomial expansion

$$b^2\left(1+\frac{1}{nb}\right)^{7n}$$

$$=b^2+\frac{7bn}{n}+\frac{7n(7n-1)}{2n^2}+\frac{7n(7n-1)(7n-2)}{6n^3b}+\frac{7n(7n-1)(7n-2)(7n-3)}{24n^4b^2}+\ldots$$

and then puts $b=10$ and derives

$$\lim_{n\to\infty} £100\left(1+\frac{1}{10n}\right)^{7n} = 100+70+\frac{49}{2}+\frac{343}{60}+\frac{2,401}{2,400}+\ldots$$

$$= £201.7\text{s}.6\text{d}$$

i.e. "the sum of interest upon interest continually for every instant in seven years with the principal of £100 after the rate of 10 in the 100 for a year".

It is slightly breathtaking to find such a sophisticated calculation being done at this early date. The reason for it is not known, but it is clear that mathematicians were well aware of the value of receiving interest more frequently than annually.

In 1624 Henry Brigges (1561–1630), the mathematician, published his *Arithmetica Logarithmica*. This contained extensive tables of logarithms, which had first been invested 10 years earlier, and showed how to apply them to a variety of problems, including compound interest. Brigges demonstrated how to calculate isolated values of $(1+i)^n$, v^n, $s_{\overline{n}|}$, *and* $s_{\overline{n}|}$, however unusual the rate of interest, and whether n is integral or not. He also showed how to calculate accurately the yield i on an investment of a lump sum for a given number of years. However, he made no attempt to find the yield in problems involving a series of payments, which, of course, is much more complicated.

Also in 1624 there appeared the third edition of a book by Thomas Clay, which had first appeared in a more slender form six years earlier. Clay, who described himself as "surveyor, and student in the mathematics", lived in Fountain Court, within the end of Shoe Lane next to Fleet Street. There is a commendatory verse at the beginning (by one Adam Smith). Would that we had something similar at the beginning of every modern actuarial textbook! It runs:

> Revenues, order, officers and laws,
> With each man's duty, how estates to raise:
> Of every loss and profit, the true cause
> Thou here describ'st in plain yet useful phrase.
> Thy precepts brief, yet pithy in each part,
> Declare thy judgment and extol thine art.
> In purchase, sale, exchange or leasing out
> (Be it in present or reversion)
> This little tract so cleareth every doubt
> As shows itself in such conversion,
> For Lords and owners all, a work so rare
> That none yet extant can with it compare.

Clay gives tables of the standard compound interest functions at 10% for up to 40 years. He also gives tables of v^n/i for $i = 5\%$, $6\frac{1}{4}\%$ and $8\frac{1}{3}\%$, the purpose being to find the value of freehold purchases in reversion.

Some useful background information about contemporary views on interest and usury is given by Gerard Malynes, a merchant who wrote in 1622 (and details of whose life are given in the *Dictionary of National Biography*). As mentioned earlier, Malynes had joined with Richard Witt and others in 1610, to recommend a new book on commercial arithmetic.

Malynes draws attention to various biblical references to usury, including God's words in Exodus 22: "If thou lend money to any of my people that is poor by thee, thou shalt not be to him as an usurer, neither shalt thou lay upon him usury."

He also recalls that Saint Jerome said, "There is no difference betwixt usury, fraud and violent robbery", and quotes numerous similar opinions. However, he then points out that nevertheless the practice of usury "is most usual in many Kingdoms and Common-weales, and the laws are also made accordingly; for this sin is rather in the conscience than in the act, and therefore there is no penalty imposed upon it by God's law. True it is that the Statute Law of England doth tolerate ten upon the hundred and so do some other laws twelve and more . . ." (i.e. 10% or 12% p.a. interest).

Malynes takes the view that, while it is sinful to take interest from the poor who can ill afford it, the same does not apply to money lent to rich merchants. He writes:

As there are three sorts of dealings among men, that is, gift, bargaining and lending, so are there three sorts of men, the stark beggar, the poor householder and the rich merchant or gentleman. To the first you ought to give freely, not only to lend freely; to the second you ought to lend either freely or mercifully, and not to feed upon him with excessive usury; but with the third you may deal straightly, and ask your own with gain, especially when he gaineth by your money; using in all these a conscience with discretion.

At the time Malynes was writing, interest had been allowed by law at up to 10% p.a. in England since 1571, and he states that "King James himself is contented to allow the said rate to Londoners for some moneys borrowed of them". Clearly there was a public sector borrowing requirement even in those days! However, Malynes had misgivings about the universal application of such a high rate as 10% to rich and poor alike and he states:

Wherein time and occasions do alter things, and as the case for the present standeth with England and foreign nations, we have usury like a wolf by the ears, dangerous to be kept, and more dangerous to abandon the same. This usury being indeed exercised to the poor or to the rich, without respect of damage ensuing, becometh intolerable. But in case of interest . . . simply to disallow it is to cut off all trade and commerce, or reparation of damages, and to go about to remedy a mischief with a greater inconvenience.

Malynes gives an interesting example of how the rate of interest could be used as an economic regulator:

In Poland, Lithuania, Prussia and other countries adjacent, when they do abound in corn, money is commonly very scarce, and the price of corn thereby much abated; at which time they will rather tolerate or proclaim the moneys to be enhanced in price, or to be delivered at interest after fifteen, twenty and sometimes twenty and five upon the hundred for a year, or a lesser time; hereupon presently, great store of money cometh from all places thither, which maketh the price of money [presumably should read: "the price of corn"] to rise. Afterwards, when many ships are laden and the fleet departed from Amsterdam and other places, then the interest beginneth to fall accordingly.

Malynes reports the contents of *A tract against usury* which had been presented to the English Parliament in 1621 (and had been written by Sir Thomas Culpeper, 1578–1662). It argued for a reduction in the interest rate of 10% on the following grounds:

1. A high rate of usury leads rich tradesmen to give over trade and undoes beginners, who are discouraged because their industry serves but to enrich others and beggar themselves.
2. Many trades are decayed because they cannot afford to borrow at 10%.
3. Other nations, which only charge 6%, out-trade and undersell us, and find it cheaper to undertake capital works, e.g. shipbuilding.
4. A high rate makes the land itself of small value, so that men do not seek by industry any more to improve it.

The tract then discussed objections to reducing the rate:

1. The long continuance of 10% and things are well enough—but "the practice of usury hath not been so generally used as it is now, when men's consciences are hardened unto it, without any scruple or indifferent consideration .... And the rule that innovations are dangerous holds true, where the body natural or politic is in perfect state of health; but where there is a declining, there to make no alteration is a certain way to run to destruction."
2. That sudden changes are dangerous—but there could be transitional arrangements for two years.
3. That money will suddenly be called in and the borrowers much prejudiced—but again, this could be covered by transitional arrangements.
4. That money will be harder to come by, and commerce much hindered—but a high rate of interest does not increase the quantity of money (whereof having plenty causes commerce to flourish); moreover, lenders will still have the same amount of money to lend.
5. That merchant strangers' money now coming here at use will be carried away again—but do we want their money invested here anyway, bearing in mind that £100 at 10% p.a. multiplies itself to £100,000 in 70 years and "it is therefore compared to the butler's box: for even as men when they are at play feel not what they give to the box, but at the end of Christmas it makes all or near all gamesters losers, so there are not few which continue in usury that are not ruined".

Hence the tract concluded "ten in the hundred to be biting usury". Some of the arguments are, of course, familiar in present-day debates about the level of interest rates.

Malynes then discusses pawnbroking and he points out that the rate of interest often charged is between 30% and 60% p.a. Indeed "there is taken the shilling penny by the week of the fish-wives and other women selling small wares up and down streets, which is above 400 upon the hundred by the year". The law prescribing 10% as the maximum rate of interest was evaded by having a 'bill' under which the pawnbroker purchased the goods and they were resold to the former owner when he completed his repayments. Malynes suggests that the evil could be remedied by the establishment of official Pawn Houses throughout the country, which would charge only 10%.

There is an interesting, though somewhat muddled, short chapter on life annuities in Malynes' book. Apparently Continental merchants wishing to raise capital would pay an annuity rate of 12½% p.a. for a single life annuity without return of capital on death. They only paid 6¼% p.a. if the capital was to be returned on death, and this is in line with Malynes' remark elsewhere that in other countries "monies are plentifully to be had at five, six and seven in the hundred". Malynes gives an example of a contract which may seem somewhat curious to us but which, he says, "is much used in divers cities beyond the seas, to draw monies into their hands". The investor pays £100 to the city and, if eight nominated lives all survive to the end of one year, the city grants the investor a perpetual rent of £12.10s.0d p.a. This is based on a notional capital value of £200, and when all the lives are dead, the city pays the capital sum of £200 itself. The City of Amsterdam "was wont to give good consideration" and their annuity rates were somewhat generous, being as follows for similar contracts on different numbers of lives all surviving one year:

Number of lives	Annuity rate per £100 purchase price	Equivalent purchase price per £1 p.a. annuity (not quoted by Malynes)
6	£13.6s.8d	£7.50
7	£14.16s.4d*	£6.75
8	£16.13s.4d	£6.00
9	£19.1s.0d.	£5.25
10	£22.4s.5d	£4.50
11	£26.13s.4d	£3.75
12	£33.6s.8d	£3.00

* Misprinted as £14.6s.4d.

Malynes points out that many foreign cities allow investors to put up money for property development, in order to increase the city's wealth and inhabitants. The rent payable is $6\frac{1}{4}\%$ p.a. on the capital invested. He advises that "because the valuation of their money doth often alter and is enhanced, whereby all things become dearer, the parties are advised to have their rents paid in specie", i.e. in gold and silver coins. The investor thereby achieved inflation and currency protection—without index-linking!

Elsewhere in his book Malynes discusses various kinds of marine insurance policy, but he also deals briefly with life assurance. He quotes two examples. In the first case a person who had an interest in the life of Sir Richard Martin, Master of the Mint, who was aged about 90, took out an insurance of £300 at a premium of 25%: Sir Richard died within the year and the insurers paid up. In the second case, one master Kiddermaster had borrowed money and for his creditors' assurance took out a policy for £2,000 at a premium of £4 and £5% for many years until he had repaid the money. All kinds of insurance were made in the office in the west end of the Royal Exchange in London, through Commissioners acting under the authority of the Lord Mayor of London, who had authority to commit to prison any assurers who did not pay up when required. There seems to have been some kind of elementary underwriting in determining the premium for life assurance: "The Assurance upon the lives of men (whether aged or young, of good qualities and diet, of disposition gentle or quarrelsome, a traveller or a dweller) being somewhat extraordinary, every man is best able to consider of it by the acquaintance of the persons."

In 1625, only three years after Malynes wrote his book, the English law was changed so as to reduce the maximum legal rate of interest from 10% to 8%. Presumably the tract presented to Parliament, discussed above, had had its effect. The reason given in the preamble to the Act was that there was a very great abatement in the value of land and merchandise both at home and abroad, and many people had need to borrow money in order to continue following their trade, but they could not afford 10% interest and were thereby forced to sell their land and stocks at very low rates, and so become unprofitable members of the Commonwealth, "to the great hurt and hindrance of the same".

In 1628 there appeared a book of *Tables of Leases and Interest* printed by William Jones dwelling in Red-cross street (near the Barbican). The author's name was not given. The booklet was clearly intended as a practical ready reckoner for everyday use by valuers and others who were concerned in the renewal of leases. There were eleven principal tables, as follows:

Table	Function tabulated	*Rate of interest*
1	$v^{21-n}\, a_{\overline{n}\rceil}$	19/170 (corresponding to $a_{\overline{21}\rceil} \doteqdot 8$)
2	$v^{20-n}\, a_{\overline{n}\rceil}$	19/170 (corresponding to $s_{\overline{21}\rceil} \doteqdot 8$)
3	$a_{\overline{n}\rceil}$	8%
4	$s_{\overline{n}\rceil}$	8%
5	$(1+i)^n$	8%
6	$v^{21-n}\, a_{\overline{n}\rceil}$	9/68 (corresponding to $a_{\overline{21}\rceil} \doteqdot 7$)
7	$v^{20-n}\, a_{\overline{n}\rceil}$	9/68 (corresponding to $a_{\overline{21}\rceil} \doteqdot 7$)
8	$a_{\overline{n}\rceil}$	10%
9	$s_{\overline{n}\rceil}$	10%
10	$(1+i)^n$	10%
11	v^n	19/170; 8%; 9/68; 10%

The text points out that table 1 can be used to find the value of leases in possession rather than in reversion by means of the formula

$$a_{\overline{r}\rceil} = v^{21-21}\, a_{\overline{21}\rceil} - v^{21-r}\, a_{\overline{21-r}\rceil}$$

and also that it can be used to find the values of reversions where the term in possession plus the term in reversion falls short of 21 years, by the formula

$$v^s\, a_{\overline{t}\rceil} = v^{21-p}\, a_{\overline{p}\rceil} - v^{21-q}\, a_{\overline{q}\rceil}$$

where $p = (21-s)$ and $q = 21-(s+t)$.

There are some interesting subsidiary tables at the end, showing how the principal tables referred to above were calculated. For example, table 1 is based on a subsidiary table having a 'root' of 170/189 (which equals v). Three columns are tabulated in this subsidiary table, for values of n from 1 to 21, as follows:

$$A_n = 170^n$$

$$B_n = 189^n$$

$$C_n = \sum_{r=1}^{n} 170^r \, . \, 189^{n-r}.$$

(It will be noted that $C_n = C_{n-1} \times 189 + A_{\overline{n}|}$ and this was probably how the column of C_n was built up, by an iterative process.)

Then, as the text points out,

$$a_{\overline{n}|} = C_n/B_n$$

$$v^n = A_n/B_n$$

$$(1+i)^n = B_n/A_n$$

$$s_{\overline{n}|} = C_n/A_n$$

$$v^t \, a_{\overline{n}|} = C_n \, A_t/B_{n+t}$$

It would be interesting to know how the author of the tables discovered what rate of interest corresponded to a 21-year annuity worth seven or eight years' purchase. Presumably it must have been done by trial and error—no mean feat in those days. Once the rate of interest had been found, it would have been necessary to select an appropriate numerator and denominator for the fraction to represent v, and this must have involved more trial and error.

Clearly 21-year leases were still being sold on the basis of seven or eight years' purchase, corresponding to a rate of interest of over 13% or 11% p.a. respectively, despite the fact that the legal maximum rate of interest for loans was only 8%. This was legally possible because the transaction was a purchase rather than a loan.

It seems that these tables went on being used for many years and they were apparently still in use in 1686 when they are mentioned in a work published in that year (referred to below) as having been "for a long time made use of for renewing of leases" (and where they are referred to as "Mr Aecroid's Tables").

In 1629 appeared the second edition of William Webster's *Tables of Compound Interest*, which had apparently been expanded considerably since the first edition. The author dedicated his book to the Company of Salters, of which he was a brother. (It is curious that both he and Witt belonged to the same Company.) He wrote from his house in Bearebinder Lane, London, which was adjacent to the Woolchurch Market (and hence very near where Witt lived before his marriage), and offered to "give instruction in interest calculations and in multiplication and division". He gave tables of the four standard compound interest functions at 8% p.a. for up to 31 years. He also gave tables for half-yearly and quarterly payments, but based these not on 8% p.a. but on 4% per half year and 2% per quarter. (By the time the fifth edition of his book came out, in 1647, he had been appointed cashier of the Worshipful Company of Merchants of London, Adventurers to the Gold Coast in the parts of Africa, though he still lived in Bearebinder Lane.)

In 1630 Edmund Wingate (1596–1656), mathematician and legal writer, published a standard arithmetic textbook, where he quotes Briggs's method of finding isolated values of compound interest functions by logarithms. He gives worked examples, including the following problem:

> A, having a daughter of the age of 3 years, delivers at the same time a thousand marks, or £666.13s.4d, upon condition that B shall redeliver unto his daughter at the age of 15 years two thousand marks, or (what is all one) £1333.6s.8d. Now the Question is, at what rate B enjoys the £666.13s.4d that it may augment to £1333.6s.8d in 12 years?

He uses logarithms to find the answer (£5.19s.0d%).

Another well-known textbook used in schools was *The Ground of Arts...* (originally written by Robert Record around 1540). Whereas the 1618 edition had included only two brief tables of $a_{\overline{n}|}$ and v^n at 10% for $n = 1$ to 21, taken from Stevin's *Arithmetique* of 1585, the 1632 edition had been expanded by the same Robert Hartwell as is mentioned earlier, who was by now living at Great Saint Bartholomew's in the new street, to include tables of all the four standard compound interest functions at 8% and 10%. There is also a table of $a_{\overline{n}|}^{-1}$—but it is very erroneous!

Books on the subject were now coming out in profusion. In 1631 Richard Lever procured a small number of copies of some Dutch compound interest tables and published them in London with an English title page and an introduction translating the headings to the tables. They gave standard functions, at various rates of interest between 5% and 8%, for up to 33 years and for fractions of a year.

In 1633 Robert Butler of Vine Court in Golding Lane (near the Barbican) published extensive tables of the standard compound interest functions at 8%, including tables based on half-yearly and quarterly payments of an annuity. Less comprehensive tables based on 5%, 6% and 7% interest were also included. He argued strongly that, in order not to exceed the statutory rate of 8% p.a., the rate of interest for periods of less than a year must be calculated accurately in accordance with his own tables, rather than using the common approximation of multiplying 8% by the fraction of a year concerned.

In 1634 Thomas Fisher published a second edition of Witt's book, which had first appeared in 1613; this second edition is described in my 1970 article (*J.I.A.* **96,** 121). Thomas Fisher had a shop in Lothbury.

In the same year William Purser, a mariner and practitioner in the mathematics, published some tables of the standard compound interest functions for rates of interest between 5% and 10% p.a. He pointed out that custom sometimes played a large part in determining the purchase price of land and said that some land is better worth 20 years' purchase than other land which carries great inconvenience is worth 16 years' purchase. "Again, houses which are in the nature of annuities (candle rents termed) have a danger of casualty, besides the continual charge of reparation, which doth much diminish the value thereof." He also states that annuities for a short period are based on 8% p.a., but for longer annuities lower rates are used, down to 5% p.a., "because when men have an assurance of their estates for many years they are the willinger to take less gains".

He excuses himself for giving certain tables at higher than the legal maximum rate of interest as follows:

> Here is likewise tables at 9 and 10 per cent. per annum, which I will not commend to thee for thy ordinary use, but for thy necessity, except thou art an unconscionable and hardened usurer, then I leave these rates to thy desires, withall warning young gentlemen to take heed what bargains they make with thee, setting them up as beacons to avoid danger, lest you shipwreck their estates.

Finally he makes a few remarks on leases for lives, stating that the custom of the place is the only rule thereof, "for some say that three lives and 21 years, others 24 years, have commonly the same fine: and again, one life and 11 years are equal".

Some interesting information about the London business world during the early seventeenth century is given by Tawney (1925). For example, he emphasizes the importance of the scriveners, who drew up the legal documents but who had developed into brokers or financial middlemen, and who sometimes even lent money themselves. He discusses also some of the ways in which people tried to evade the law against charging more than the prescribed maximum rate of interest, as well as many other questions relating to usury.

With the advent of the Civil War, there seems to be a gap of some years in the publication of new books on compound interest, although editions of some of the earlier works, e.g. Webster's *Tables*, continued to appear. At last, in 1651, there appeared a book by Roger Clavell, gentleman and student in the mathematics. He restricted himself to tables based on 6%, to which the legal maximum rate of interest had now been reduced. The major part of his book is a ready reckoner of simple interest for periods of less than one year, but he also includes the standard compound interest functions for periods of up to 31 years.

We come now to an important and interesting work, which gives a considerable amount of information about commercial practice in valuing leases. This was written by Henry Phillippes and published in 1654. He gives tables of $a_{\overline{n}|}$ at 5%, 6%, 8%, 9%, 10% and 12%. He also has a table showing the number of years it would take an investment of £1 to multiply itself by 2, 4, 8, 16, 32, 64, 128, 256, 512 or 1,024 at 5%, 6%, 8%, 10% or 12%.

Phillippes appears to be the first English writer to discuss in any depth which is the most appropriate rate of interest to use for different kinds of transactions. There was now a practical need for consideration of this point, because of the reductions in interest rates during the past 30 years, and the considerable differences in the results obtained from valuations at different rates.

He points out that land purchase transactions had always been based on a lower rate of interest. For example, when interest rates were 8% p.a., land was worth 18 years' purchase, i.e. it was valued at about $5\frac{1}{2}$% p.a. (Witt, writing when interest rates were 10% p.a., gave examples based on land being valued at either $6\frac{1}{4}$ or 5% p.a., and it therefore seems likely that there was no single universally accepted rate; it no doubt depended in part on the quality of the land.) Phillippes suggests that now interest rates are 6% p.a., land "is well worth 20 years' purchase", i.e. it should be valued at no more than 5% p.a. He gives four reasons why the initial yield obtainable on the purchase of land ought to be less than the rate of interest:

(a) land is a safe investment;
(b) not only might the price of land rise if there are further reductions in interest rates, but it might be possible to raise the tenants' rents in a short time;
(c) because of dislike of usury, many investors prefer to purchase land instead of lending money at interest;
(d) it is equitable, because large landowners can afford to live on their substantial rents even though these are based on a relatively low yield, whereas those who lend out at interest are normally less wealthy and need the higher yield!

Phillippes considers that whatever is the current rate of interest should be used for valuing leases of land (as opposed to purchases) and for valuing all other annuities which are certain and assured by lands.

He then discusses the rate of interest to be used when letting and selling houses. He points out that houses are subject to loss from fire, tempests and wars. Hence it is usual to allow a relatively high rate of interest in capitalizing the rents to arrive at the value of the house. He says that when interest was at 8% p.a. it was the usual custom to let 21-year leases of houses for seven years' purchase (implying a rate of interest of about 13% p.a.). He suggests that now that interest has been reduced to 6% p.a., such leases might be calculated at 10% p.a. He makes the point that "some houses being new and strongly built, need little or no reparations, and others, being old and decayed, need great and costly reparations, and many times must be partly built" (i.e. partly rebuilt); "since these things lie upon the tenant, therefore the better sort of houses will be worth more, and the other less. But the prices of all these kind of houses may be reckoned after the rates of 8, 9, 10 or 12, either for their leases or for the purchase of them outright."

Presumably the cost of rebuilding or repairs for a house which was rented out on a week-to-week basis remained the responsibility of the landlord, but, clearly, once a lease was granted the responsibility passed to the tenant. Hence landlords were content with the use of a relatively high rate of interest to capitalize a rental which might otherwise be depleted by the cost of repairs (as well as by the cost of collecting the rent and the possibility of vacancies, though these points are not mentioned by Phillippes).

Phillippes then says that "there is another way of purchasing land or houses, by buying lives therein". The purchaser paid a lump sum and could occupy the house rent-free or at a low rent until the death of a named life (often his own). It was the custom to calculate the lump sum as the amount which would be paid for a lease of seven years. Sometimes the lease continued until the death of the last survivor of two or more lives, when the lump sum was calculated as the amount which would be paid for a lease of 14 years (for two lives) or 21 years (for three lives) and so on. Phillippes suggests that this method of calculation is unfair and proposes that the lump sum should be the amount which would be paid for a lease of 12 years (for one life), 23 years (for two lives), 33 years (for three lives), 42 years (for four lives), and so on, up to 78 years (for twelve lives).

Phillippes also shows how to find the approximate yield in a proposed leasehold transaction by inspection of the tables for different rates of interest.

Two later books provide further evidence that transactions were now commonly taking place at a wide range of different rates of interest. John Newton (1668) gives tables of $a_{\overline{n}|}$ for rates of interest from 5% to 14% (in 1% steps). Sir Samuel Morland (1679) does the same, except that he excludes rates of interest over 10%; he gives headings to one of his tables that indicate the purpose for which each rate was used:

5%: The purchase of freehold land.
6%: The purchase of copyhold land, or leases of land.
8%: The purchase of very good houses.
10%: The purchase of leases of ordinary houses.

In 1677 Michael Dary, philomath (of King Harry's Yard, near the Hermitage, Wapping), published a little booklet demonstrating how to use logarithms to solve compound interest problems. For example, he accurately finds the yield in an annuity transaction, using logarithms and a systematic trial and error approach. I am not aware of any previous writer having tackled this problem. He also gave a few tables based on 6% interest.

In 1686 a booklet of *Tables for Renewing and Purchasing of the Leases of Cathedral-Churches and Colleges* was published at Cambridge, with a note of commendation by Sir Isaac Newton. It went through many subsequent editions, until well into the eighteenth century. It contained tables for renewing leases, based on the lower rates of interest now in use, which produced higher 'fines' or capital values than the amounts commonly charged. Not surprisingly a controversy developed on what was the correct rate of interest to use. It was in the interests of the churches and colleges to use as low a rate of interest as possible but their tenants saw matters quite differently! This controversy rose to a height in 1729–31, but that period is outside the scope of this article.

In 1692 John Locke, the philosopher, published a book arguing against reducing the rate of interest by law again from its present level of 6%. An unsuccessful attempt to get such a reduction had been made some 25 years previously by Sir Thomas Culpeper junior, whose father's tract on the same theme had had such success in 1621, leading to the Act of Parliament in 1625 which reduced interest rates from 10% to 8%. Presumably the question was now being discussed again, with some people apparently aiming to reduce interest rates to 4%. Locke felt that people who were desperate for money would find ways of evading such a law, and went on:

High interest is thought by some a prejudice to trade; but if we will look back, we shall find that England never throve so well, nor was there ever brought into England so great an increase of wealth since, as in Queen Elizabeth's and King James I and King Charles I time, when money was at 10 and 8 per cent. I will not say high interest was the cause of it; for I rather think that our thriving trade was the cause of high interest, every one craving money to employ in a profitable commerce. But this I think I may reasonably infer from it, that lowering of interest is not a sure way to improve either our trade or wealth.

He added:

Perhaps it is now at six per cent in as good a proportion as is possible, six per cent being a little higher than land at twenty years' purchase, which is the rate pretty near that land has generally carried in England, it never being much over nor under.

However, Locke was here perhaps rather overstating the stability of land values. I would guess that, although land bringing in good rents was commonly valued at 18–20 years' purchase for most of the century, it was probably down to 16 years' purchase, or even less, around 1620. (See Culpeper's treatise of 1621 and the preamble to the Act of 1625 already mentioned.)

Locke also points out that there are considerable regional variations and that land is in some parts constantly sold for four or five years' purchase more than in others, e.g. 17 or 18 years' purchase in some places but 22 or 23 years' purchase in other places where there are profitable manufactures.

In 1699 Edward Hatton gave some useful background information:

Some tenants are at will, as those that have to lease, but hold their estates from year to year, so long as the landlord pleases. Others hold by lease, which leases are either made for a term of years, as most of the houses in the City of London are let, or else for three lives, as most of the lands in the North of England are let. . . . The value of estates fee-simple [i.e. freehold] are worth in most part of England about 18, 19 or 20 years' purchase. Estates for three lives are generally valued at 11, 12 or 13 years' purchase; and in case a life fall, they will renew it for about one year's purchase.

He also gave a table of the standard compound interest functions at 6%.

Thus we can say that the seventeenth century saw the introduction and gradual spread of compound interest techniques in England. At the start of the century almost all transactions were apparently based on the legal maximum rate of 10% p.a., except for land purchase which was based on a much lower rate. At that time no books on the subject had been published in English, although a few Continental works were probably in limited circulation among the mathematical practitioners. As the century wore on, several books of tables and examples of their use were published, and the subject also started to be included in the standard school arithmetic textbooks. Not all of the published tables were accurate, however! Later the legal maximum rate of interest was reduced, first to 8% and then to 6%, and this gave rise to the use of a much wider variety of interest rates than formerly, and the need to consider carefully which was the appropriate rate to use for any transaction. Some leases were held during the lifetime of one or more named individuals, but the value of such leases was determined on a 'rule of thumb basis' rather than scientifically.

Finally, this account would not be complete without a reference to the most important development in the subject during the whole of the century, namely the publication in January 1693 (in the *Philosophical Transactions* of the Royal Society) of Edmund Halley's paper on the valuation of life annuities. This was the first published work which combined probability and compound interest, in order to produce a method for valuing life annuities—basically the same method as is used today. He derived the following table of a_x at 6% (based on a mortality table for the City of Breslaw which he constructed on similar lines to the mortality table constructed by Graunt in 1662):

Age	a_x	Age	a_x
1	10·28	40	10·57
5	13·40	45	9·91
10	13·44	50	9·21
15	13·33	55	8·51
20	12·78	60	7·60
25	12·27	65	6·54
30	11·72	70	5·32
35	11·12		

He pointed out that British Government annuities (then being sold on the basis of seven years' purchase for a life) were therefore very advantageous for purchasers of annuities on young lives. He also showed how to value last survivor and reversionary annuities.

Although no immediate practical use seems to have been made of this paper, it was often mentioned in later years and things were never the same again. Modern actuarial science had been born.

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