The Actuarial Profession
making financial sense of the future

Chasing the Tail

Correlations and Dependencies in Economic Capital Models

36th Annual GIRO Convention

Richard Shaw, William Diffey and Grigory Spivak

6-9 October 2009
EICC, Edinburgh, Scotland

Correlations and Dependencies in Economic Capital Models Working Party

FIRM Working Party Members

- Richard Shaw (Chair)
- William Diffey
- Lorenzo Fattibene
- Grigory Spivak

FIRM Sponsored Paper

- The paper “Correlations and Dependencies in Economic Capital Models“ is available on both the FIRM 2009 and GIRO 2009 Websites
  - Covers in more depth the many practical issues that arise in this presentation

Topics

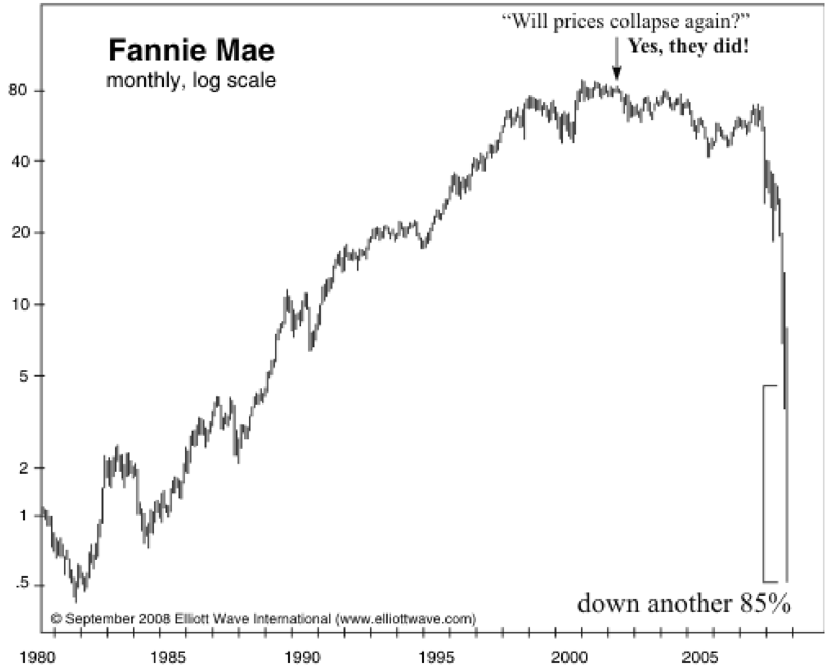
- Why Diversification is Important
- Definition and Types of Dependency
- Aggregation Techniques
- Modelling Issues
- Impact of Dependency Modelling on Economic Capital
- Communication of Economic Capital Dependency Impacts
- Conclusions

Why Diversification is Important

Diversification – Time to take stock

- Fate of the banks should be a warning to insurers
- Mis-pricing of diversification risk within asset backed structures
- Diversification is the reason why insurance companies exist
- Management of diversification should be owned by the Board and Senior Management
- An embedded ERM Framework needs to consider diversification benefits

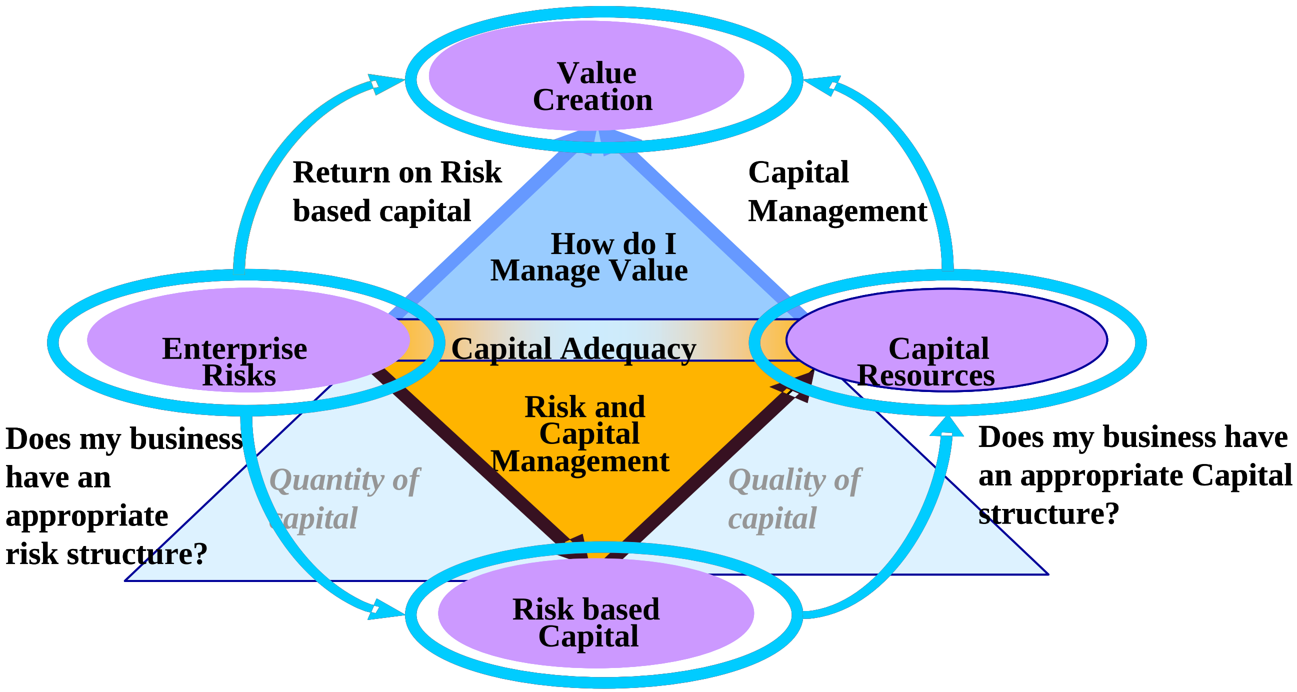
Updated from Prechter's August 20, 2008 TV Appearance



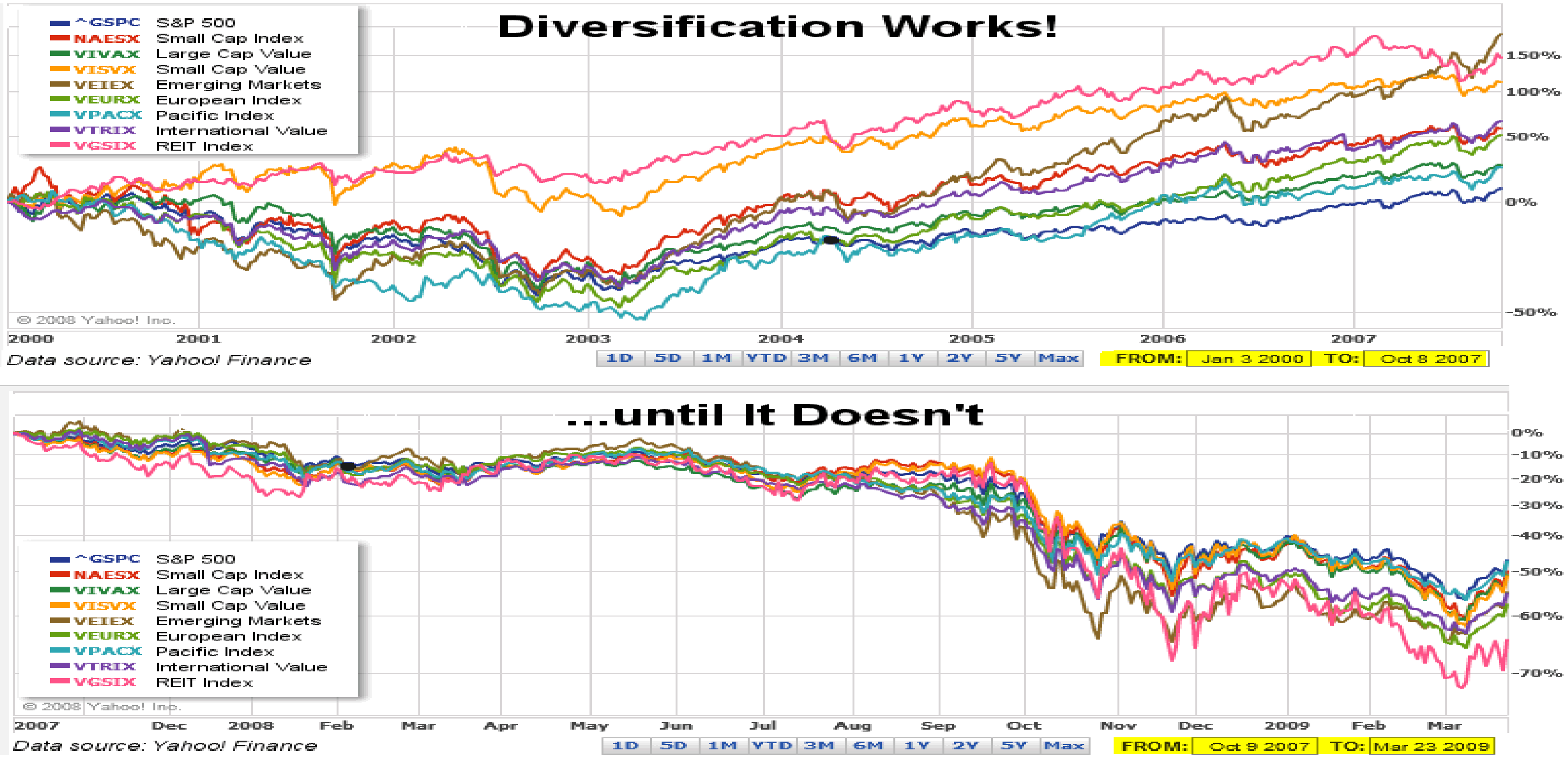
Why Diversification is Important

Diversification – A core benefit within Economic Capital

- Diversification needs to be a key feature of an effective ERM Framework

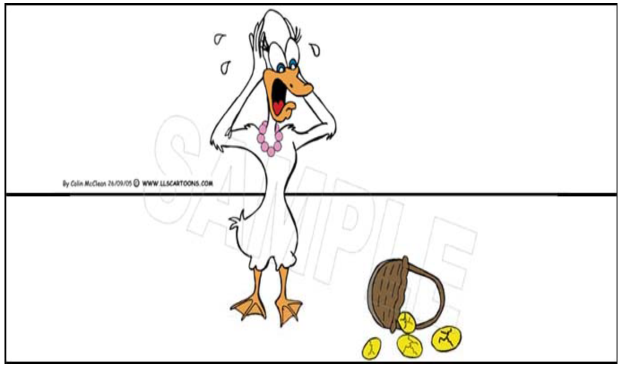


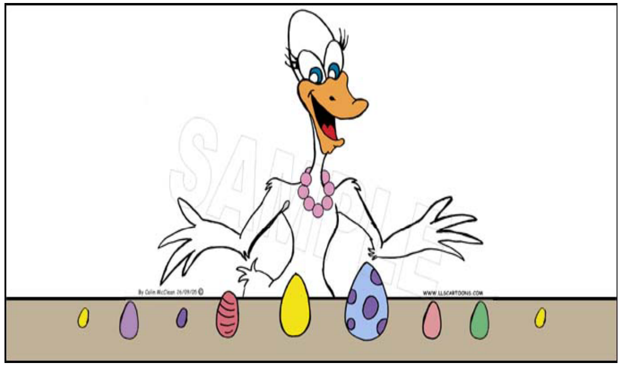
Why Diversification is Important
Diversification works until it doesn't



Why Diversification is Important
Aspects of Diversification

- There are many practical considerations
- Diversification benefits need to be managed
  - Too valuable to overlook or neglect
  - Managed within the same risk appetite framework as earnings or volatility
- Diversification benefits may arise by accident
  - Geographical or LOB expansion
  - However, diversification rarely drives strategy
- Communication of diversification is key
  - Mixture of qualitative and quantitative methods
  - Need for risk management tools to aid this communication
  - Internal vs external communication





Why Diversification is Important

Financial Management aspects of Diversification

- Regulatory Compliance
- Regulatory Capital
- Economic Capital
- Management of Risk
- Liability Valuation
- Asset Valuation (including ESGs)
- Pricing Strategic Planning & Decisions
- Capital Management
- Reinsurance Strategy

- There are many inter-dependencies

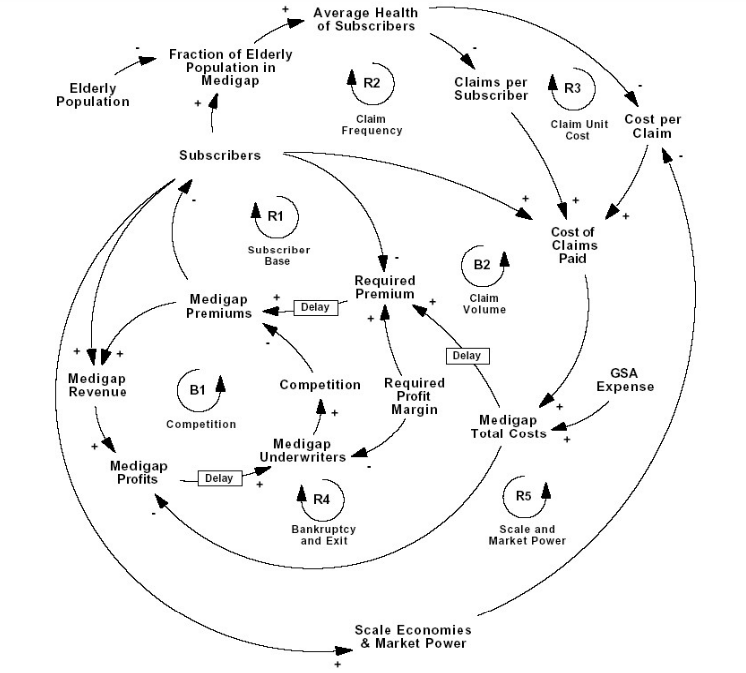


Exhibit 5-19 Adverse selection and the Medigap death spiral

Why Diversification is Important
Governance aspects of Diversification

- Correlation coefficients a common language
  - But a language that can trap the non specialist

- The Credit Crunch has identified issues:
  - Need for Diversification KRIs and KCIs
  - ESG implicit correlation transparency
  - Improvements in diversification 'Auditability'
  - Diversification trading through securitisation

- Solvency II and diversification disclosure
  - Already a feature of ICAs (UK)
  - Solvency II will require disclosure

- Investors need to be reassured

Why Diversification is Important
Regulatory aspects of Diversification

- ICA:
  - ICG for ‘excessive’ diversification benefits has reduced in recent years
  - However, the FSA still not convinced by some companies’ claims

- Solvency II:
  - Group support regime is crucial to the success of SII
  - Diversification benefits between parent companies and subsidiaries
  - Diversification benefits between Life and Non-Life QIS 4 modules
  - Risk Margins at portfolio, company or line of business level
  - Uncertainty of diversification benefits recognisable within internal models
  - Regulatory disclosure of diversification required for the first time (Pillar III)
  - Impact on M&A and Composites

- IFRS

Why Diversification is Important
Rating Agencies aspects of Diversification

- Need to identify “True” diversification
  - Natural scepticism of some claims
  - Supposedly diverse institutions experienced financial stress during the credit crunch
- Remote ‘tail’ events happen too frequently:
  - Black Swans
- Focus on:
  - Monoline insurers
  - Fungibility of capital
  - Level of commitment to subsidiaries
- How useful are rating agency models with standard correlation matrices

Why Diversification is Important

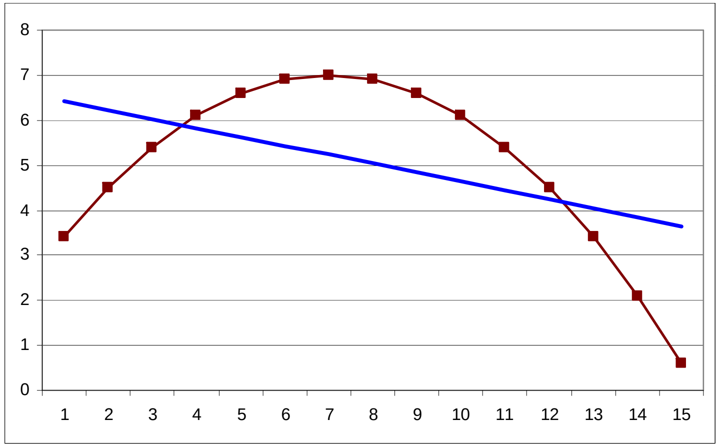
Solvency II – An internal model is a lot more than a model for capital

… it is an integral part of the risk management process

Definition and Types of Dependency

What do we mean by dependency

- The value of one risk factor gives an indication of the value of another risk factor.
- One extreme is perfect dependence: if you know the value of one risk factor, you know exactly the value of another risk factor.
- The other extreme is independence: the value of one risk factor does not enable you to make any predictions about the other risk factor
- Dependence and Correlation – NOT the same thing



Definition and Types of Dependency
Types of dependency

- Explicit Method: Dependency between random variables is expressed via common risk factors which these random variables depend on
  - Risk1 = Function1(X1, …, Xn, and other Risk1-specific factors) + Residual1
  - Risk2 = Function2(X1, …, Xn, and other Risk2-specific factors) + Residual2

  - Popular approach in modelling non-life risks
  - aka Causal Modelling, Common Risk Drivers etc

- Implicit method: Dependency structure is Specified directly by:
  - Correlation matrix
  - Copula

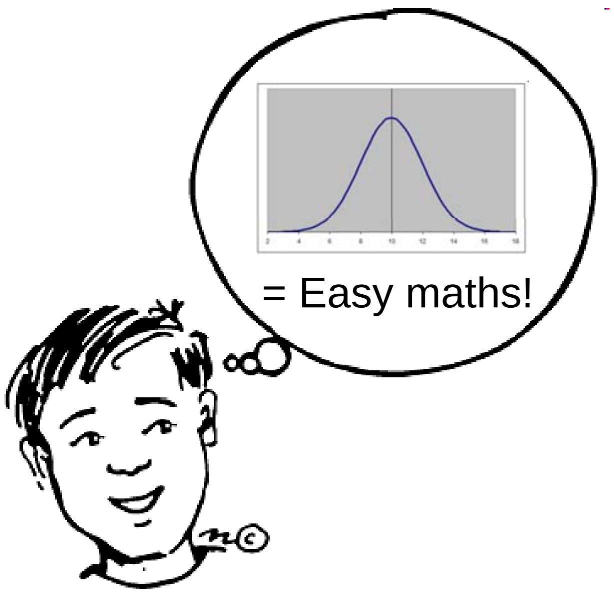
- Implicit Method – Economic Capital Aggregation:
  - Variance Covariance approach – Correlation Matrix, Marginal Risk Capitals
  - Copula approach – Correlation Matrix, Copula, Marginal Risk Distributions

Aggregation Techniques
Variance Covariance Matrices

- Correlation matrix is associated with variance-covariance approach to aggregating dependencies:

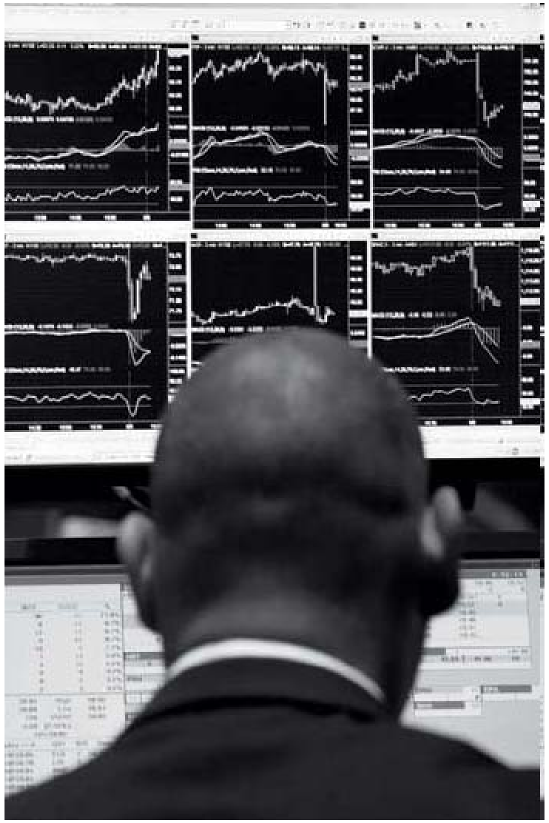
$$Total\_Capital = \sqrt{\sum_{i,j} Corr(i,j) \times Capital(i) \times Capital(j)}$$

- Mathematically linked to the dependency structure of the multivariate Normal distribution
- Standard approach to modelling dependency for many companies
- Widely used in insurance and credit markets
- Solvency II: QIS4 Technical Specification



Aggregation Techniques
(Mis)use of correlations in finance

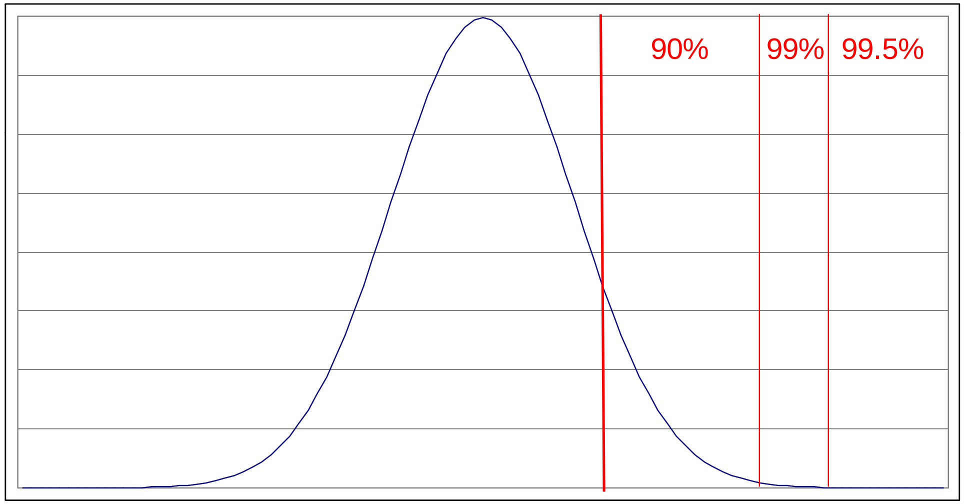
- Financial risks are NOT Normal:
  - High probability of a large loss ('negative skewness')
  - High probability of extreme outcomes ('extreme kurtosis')
  - 'Heavy tails'



- Nassim Taleb: "The thing never worked. Anything that relies on correlation is charlatanism."

Aggregation Techniques
Why NOT to use "Tail" (or "Stressed") Correlations

- Common misconception: "I need to use a Variance Covariance approach with higher tail-end correlations to capture tail dependence"
- Even if correlations were calibrated adequately to describe the level of dependency at 99.5%, what about 99%, 95%, 90% or 75% etc
- Need continuous distributions, not just another point estimate

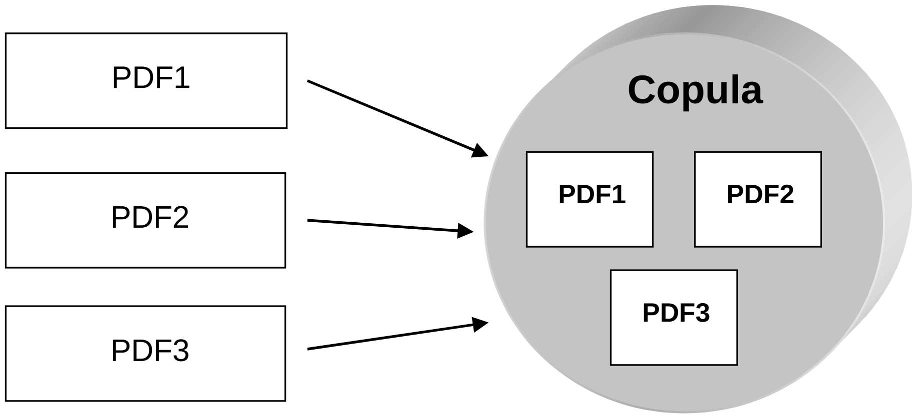


Aggregation Techniques
Concept of Copulas – Definition

Individual probability density functions….

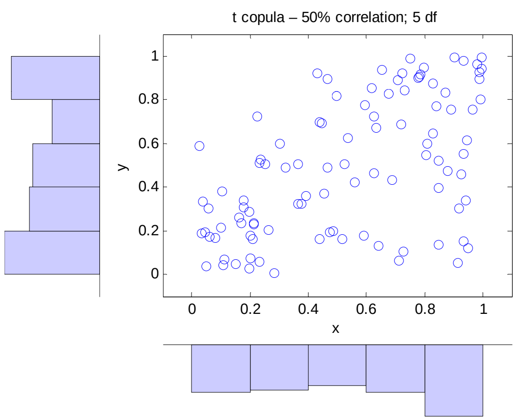
…joined together by a copula….

…into a multi-dimensional joint distribution.



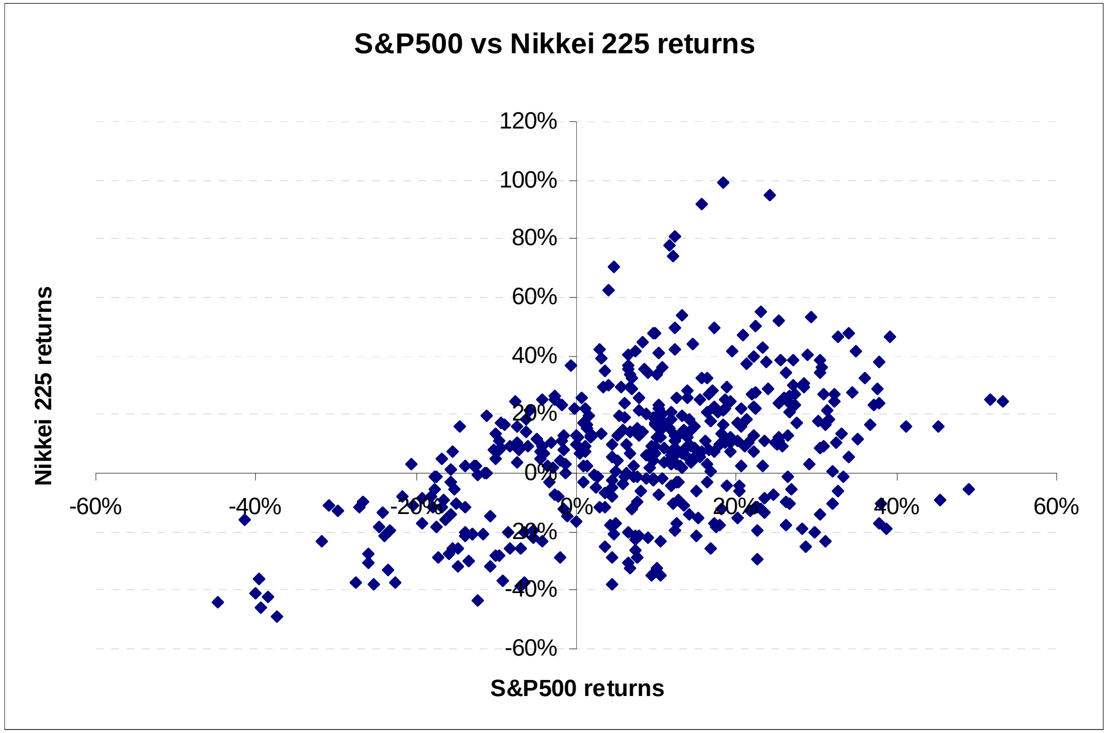
For 2 risks a copula can be viewed as inducing a greater joint likelihood of large values of U(0,1) for each risk.

The assumed marginal risk distributions for each risk are important when inverting loss amounts to values in the [0,1] space for fitting copulas by maximum likelihood



Aggregation Techniques
Concept of Copulas – Tail Dependency

- Probability of one random variable taking a very large/ small value given that the other random variable takes a very large/small value
- Example: dependence between 2 indices is high in period when returns are extremely low



Aggregation Techniques
Concept of Copulas – Tail Dependency Mathematical Definition

- Upper / Lower tail dependency:

$$\lambda_U(X,Y) = \lim_{u \uparrow 1} P(Y > F_Y^{-1}(u) \mid X > F_X^{-1}(u))$$

- Lower tail dependency:

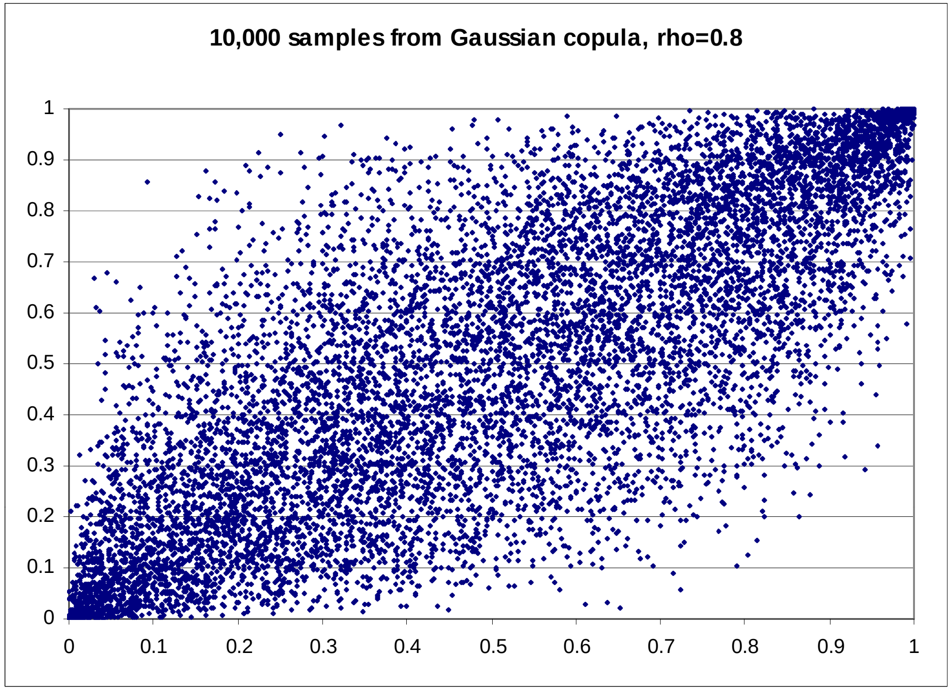
$$\lambda_L(X,Y) = \lim_{u \downarrow 0} P(Y \leq F_Y^{-1}(u) \mid X \leq F_X^{-1}(u))$$

- Probability, not a correlation coefficient. Takes values (0, 1) not (-1, 1)

Aggregation Techniques
Concept of Copulas – Gaussian Copula

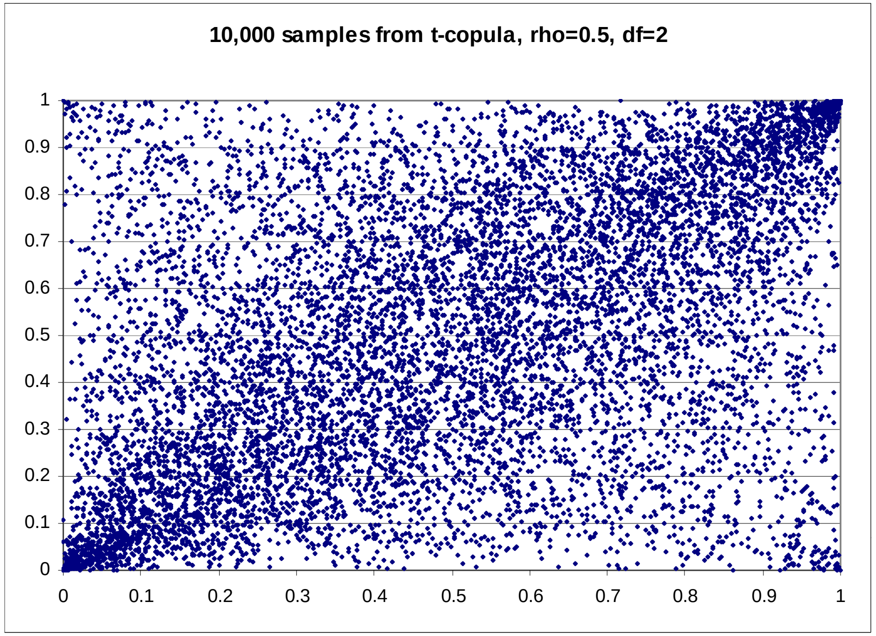
- Copula of Multivariate normal distribution with correlation matrix R
- Key problem: **NO tail dependence**:

$$\lambda_U = \lambda_L = 0$$



Aggregation Techniques
Concept of Copulas – T Copula

- Mathematically convenient: easily extended to multidimensional case, easily simulated.
- Can model “Tail” Dependency
- Parameters: correlation matrix R & degrees-of-freedom (DF) parameter.
- Symmetric: left and right tail dependencies are equal.
- One DF parameter for all risks. Can be extended to have individual DFs for each pair of risks



Aggregation Techniques
Concept of Copulas – Other Copulas

- Archimedean: Gumbel, Clayton and Frank
  - Allow for heavy, non-symmetric tails
  - Difficult to extend to a multi-dimensional case
  - Copula characteristics determined by one risk parameter
  - Parameter selection less intuitive than the T Copula

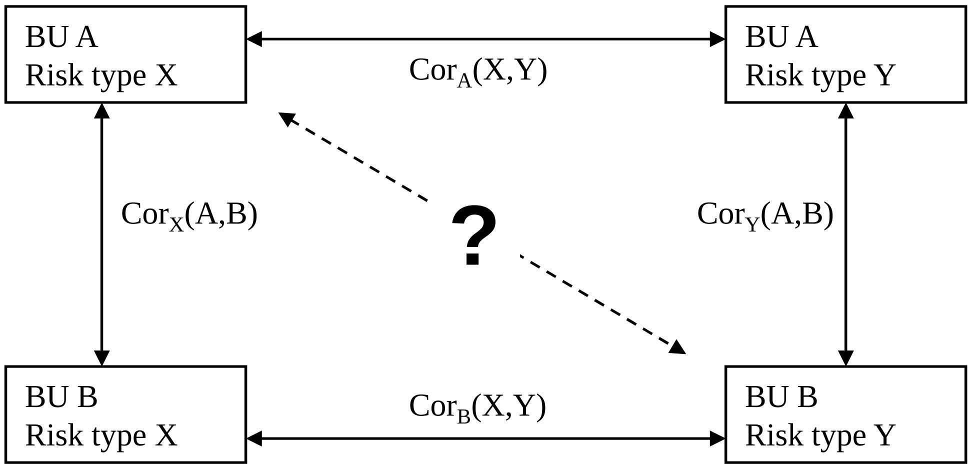
- Other types: Vine copulas
  - Allow to combine different types of copulas for pairs of risk into one copula
  - More difficult to model

Modelling Issues

Practical Issues with Correlation Matrices

- Filling in the Cross terms
  - A common situation for Insurance Groups with many BUs which each have a common number of Risk Categories
  - E.g. What is the correlation for France BU Equity and UK BU Fixed Interest
- Is the Matrix Positive Semi-Definite (PSD)?
- Huge matrices for large companies
- What type of correlations to use for calibrating copulas?
- How to estimate tail dependency parameters?

Modelling Issues
Filling in the Cross terms



- What is the Algorithm for Cross terms ?
- Groupe Consultatif approach: $\frac{Cor_X(A,B)+Cor_Y(A,B)}{2} \times \frac{Cor_A(X,Y)+Cor_B(X,Y)}{2}$
- In some cases this can lead to internally inconsistent values

Modelling Issues
Is the Matrix Positive Semi-Definite (“PSD”)

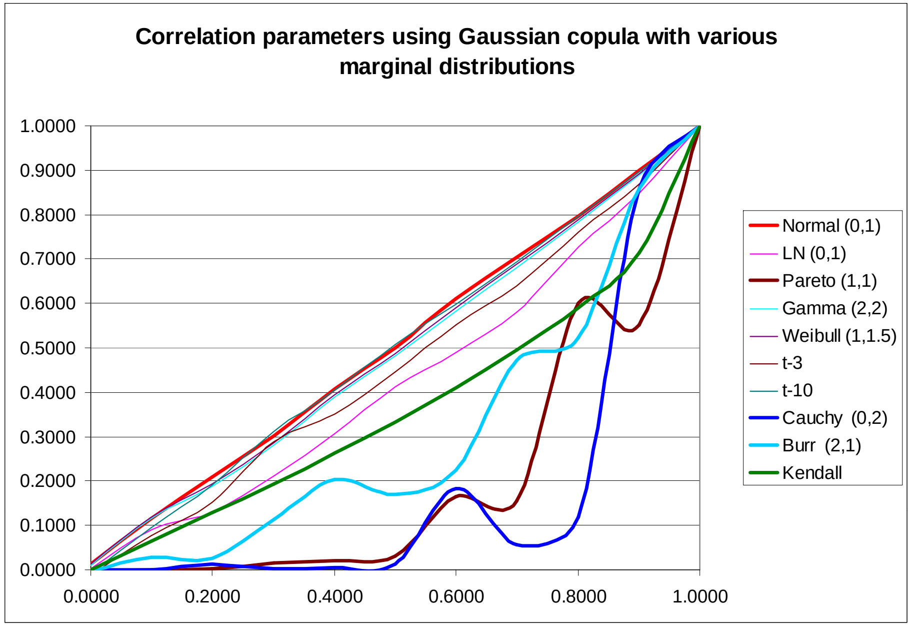
- PSD property is matrix analogue of positive numbers
- Can only perform a Cholesky Decomposition on PSD matrix: matrix version of square root
  - A Cholesky Decomposition of a starting correlation matrix is often used to simulate correlated U(0,1) values from the multi-variate normal distribution
- Required when working with Copulas
- Finding the nearest PSD matrix can be a very complex problem in the practice area of “Semi-definite” programming (“SDP”)

Modelling Issues
What type of correlations to use for Calibrating Copulas

- Input R into Gaussian and T copulas is not a correlation matrix estimated from raw data
- Need to estimate Kendall Tau correlation, and then convert into copula parameter using formula:

$$\rho_{Gaussian} = \sin\left(\frac{\pi\rho_{Kendall}}{2}\right)$$

- Makes a difference for some marginal distributions, including Cauchy, Burr, Pareto



Impact of Dependency Modelling on Economic Capital
ABC Insurance Company – Introduction

- ABC Insurance Company is a non-life ("P&C") insurer
- Capital is the aggregation of risks from 10 different risk categories
  - Risk distributions are assumed to be identical
  - Separate Lognormal and Normal risk distribution scenarios
  - Correlation coefficients are the same between risk-pairs – 10,% 25% or 50%
- 2 different aggregation techniques are considered:
  - Copulas – Correlation Matrix (10 x 10) and 10 Marginal Risk distributions
    - Gaussian Copula and T Copula with 10, 5 and 2 d.f.
  - Variance Covariance Matrix approach
  - 25,000 simulations per copula using Matlab
- Capital is measured as VaR over 12-months = Loss (%) – E(Loss)

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Distribution Assumptions

Risk Type	Distribution	Mu	Sigma	E(X)	SD(X)	CV(X)
Equity	Lognormal	7.5706	0.2462	2,000	500	25%
Property	Lognormal	7.5706	0.2462	2,000	500	25%
Interest Rate	Lognormal	7.5706	0.2462	2,000	500	25%
Credit Spread	Lognormal	7.5706	0.2462	2,000	500	25%
Credit Default	Lognormal	7.5706	0.2462	2,000	500	25%
UW - Cat	Lognormal	7.5706	0.2462	2,000	500	25%
UW Non-Cat	Lognormal	7.5706	0.2462	2,000	500	25%
Reserve	Lognormal	7.5706	0.2462	2,000	500	25%
Expenses	Lognormal	7.5706	0.2462	2,000	500	25%
Operational	Lognormal	7.5706	0.2462	2,000	500	25%

Risk Type	Distribution	Mu	Sigma	E(X)	SD(X)	CV(X)
Equity	Normal	2,000	500	2,000	500	25%
Property	Normal	2,000	500	2,000	500	25%
Interest Rate	Normal	2,000	500	2,000	500	25%
Credit Spread	Normal	2,000	500	2,000	500	25%
Credit Default	Normal	2,000	500	2,000	500	25%
UW - Cat	Normal	2,000	500	2,000	500	25%
UW Non-Cat	Normal	2,000	500	2,000	500	25%
Reserve	Normal	2,000	500	2,000	500	25%
Expenses	Normal	2,000	500	2,000	500	25%
Operational	Normal	2,000	500	2,000	500	25%

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Lognormal (25% CV, 25% Correlation)

Economic Capital - 25% Correlation						
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	1,760	1,685	1,578	1,421	1,658
90%	10	3,688	3,610	3,582	3,418	3,763
95%	20	4,928	4,906	5,004	4,889	5,182
99%	100	7,423	7,916	8,177	9,049	8,212
99.5%	200	8,391	9,087	10,031	11,052	9,455
99.95%	2,000	11,082	13,926	14,929	18,544	13,468

% change cf Gaussian Copula						
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-4.2%	-10.3%	-19.3%	-5.8%
90%	10	0.0%	-2.1%	-2.9%	-7.3%	2.0%
95%	20	0.0%	-0.4%	1.6%	-0.8%	5.2%
99%	100	0.0%	6.6%	10.2%	21.9%	10.6%
99.5%	200	0.0%	8.3%	19.5%	31.7%	12.7%
99.95%	2,000	0.0%	25.7%	34.7%	67.3%	21.5%

- Variance-Covariance (“V CV”) capital approach similar to T Copula (n d.f.)
- Example: 99% V CV ~ T Copula 5 d.f.
- Percentiles Increase → Implied T Copula n d.f. increases (lower tail dependency)
- Note: T Copula Capital < Gaussian Capital for lower percentiles (e.g. 75%)

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Lognormal (25% CV, 10% - 50% Correl)

Economic Capital - 10% Correlation					% change cf Gaussian	
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-7.6%	-10.4%	-23.1%	-9.3%
90%	10	0.0%	-1.4%	-2.3%	-6.4%	1.9%
95%	20	0.0%	2.0%	2.1%	4.4%	6.2%
99%	100	0.0%	7.7%	13.0%	23.5%	13.1%
99.5%	200	0.0%	11.2%	18.5%	31.7%	15.5%
99.95%	2,000	0.0%	21.8%	32.6%	62.9%	25.5%

Decreasing % change trend

Economic Capital - 25% Correlation					% change cf Gaussian	
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-4.2%	-10.3%	-19.3%	-5.8%
90%	10	0.0%	-2.1%	-2.9%	-7.3%	2.0%
95%	20	0.0%	-0.4%	1.6%	-0.8%	5.2%
99%	100	0.0%	6.6%	10.2%	21.9%	10.6%
99.5%	200	0.0%	8.3%	19.5%	31.7%	12.7%
99.95%	2,000	0.0%	25.7%	34.7%	67.3%	21.5%

Economic Capital - 50% Correlation					% change cf Gaussian	
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-1.0%	-5.9%	-12.8%	-2.2%
90%	10	0.0%	-2.5%	-2.4%	-7.2%	1.1%
95%	20	0.0%	-1.7%	0.2%	-2.1%	2.6%
99%	100	0.0%	3.7%	7.6%	14.3%	6.6%
99.5%	200	0.0%	6.7%	10.6%	18.5%	8.7%
99.95%	2,000	0.0%	11.7%	11.7%	32.7%	7.9%

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Lognormal (25%, 50% CV, 25% Correl)

Economic Capital - 25% Correlation						CV 25%
Percentile	**Return**	**Gaussian**	**t - 10 df**	**t - 5 df**	**t - 2 df**	**V CV**
75.0%	4	0.0%	-4.2%	-10.3%	-19.3%	-5.8%
90%	10	0.0%	-2.1%	-2.9%	-7.3%	2.0%
95%	20	0.0%	-0.4%	1.6%	-0.8%	5.2%
99%	100	0.0%	6.6%	10.2%	21.9%	10.6%
99.5%	200	0.0%	8.3%	19.5%	31.7%	12.7%
99.95%	2,000	0.0%	25.7%	34.7%	67.3%	21.5%

Economic Capital - 25% Correlation						CV 50%
Percentile	**Return**	**Gaussian**	**t - 10 df**	**t - 5 df**	**t - 2 df**	**V CV**
75.0%	4	0.0%	-4.7%	-11.2%	-24.4%	-17.0%
90%	10	0.0%	-0.6%	-3.3%	-5.5%	0.8%
95%	20	0.0%	3.4%	0.4%	3.7%	7.3%
99%	100	0.0%	6.0%	10.9%	23.5%	18.2%
99.5%	200	0.0%	11.5%	14.1%	32.9%	24.0%
99.95%	2,000	0.0%	13.8%	29.1%	57.1%	38.9%

- Larger CV → V CV approach gives a larger % margin over the Gaussian copula
- Larger CV → The V CV approach is equivalent to T Copula with a lower n d.f. (i.e. larger tail dependency)

Impact of Dependency Modelling on Economic Capital
ABC Insurance Company – Normal vs Lognormal Distributions

Economic Capital - 25% Correlation					Normal	CV 25%
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-2.7%	-7.4%	-15.0%	0.9%
90%	10	0.0%	-1.7%	-3.3%	-7.4%	0.8%
95%	20	0.0%	-1.1%	-0.2%	-3.3%	0.6%
99%	100	0.0%	3.7%	5.4%	13.1%	0.2%
99.5%	200	0.0%	5.5%	12.1%	19.5%	1.0%
99.95%	2,000	0.0%	19.6%	19.5%	35.8%	2.2%

Economic Capital - 25% Correlation					LogNorm	CV 25%
Percentile	Return	Gaussian	t - 10 df	t - 5 df	t - 2 df	V CV
75.0%	4	0.0%	-4.2%	-10.3%	-19.3%	-5.8%
90%	10	0.0%	-2.1%	-2.9%	-7.3%	2.0%
95%	20	0.0%	-0.4%	1.6%	-0.8%	5.2%
99%	100	0.0%	6.6%	10.2%	21.9%	10.6%
99.5%	200	0.0%	8.3%	19.5%	31.7%	12.7%
99.95%	2,000	0.0%	25.7%	34.7%	67.3%	21.5%

- Variance Covariance (V CV) ~ Gaussian Copula Capital (Normal Distribution)
- Sampling error present even with 25,000 simulations

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Implied 'Tail Correlations'

Implied Correlation = V CV Sum				LogNorm	CV 25%
Percentile	**Return**	**Gaussian**	**t - 10 df**	**t - 5 df**	**t - 2 df**
75%	4	29.6%	26.2%	21.6%	15.4%
90%	10	23.6%	22.1%	21.6%	18.7%
95%	20	21.5%	21.2%	22.6%	21.0%
99%	100	18.4%	22.4%	24.7%	32.7%
99.5%	200	17.3%	22.2%	29.5%	38.2%
99.95%	2,000	13.3%	27.5%	33.3%	57.4%

- Some companies use higher than average correlations, referred to as 'tail correlations', in the variance covariance matrices to reflect their views about tail dependence
- This is often done on the basis of a guess or prudent margin without any theoretical foundations
- Table shows the implied equal 'tail correlation' to be used with a variance covariance matrix such that the capital is equivalent to the capital from the use of a correlation matrix with 25% pairwise correlations and the respective copulas
- **i.e. Correlation x% such that V CV Capital (x%) = Copula Capital (25%) at %ile**.

Impact of Dependency Modelling on Economic Capital

ABC Insurance Company – Implied 'Tail Correlations'

Implied Correlation = V CV Sum				Normal	CV 25%
Percentile	**Return**	**Gaussian**	**t - 10 df**	**t - 5 df**	**t - 2 df**
75%	4	24.3%	22.4%	19.3%	14.5%
90%	10	24.4%	23.2%	22.1%	19.4%
95%	20	24.5%	23.8%	24.4%	22.2%
99%	100	24.9%	27.6%	28.9%	34.9%
99.5%	200	24.3%	28.3%	33.3%	39.4%
99.95%	2,000	23.5%	38.4%	38.2%	52.7%

25% Correlation

Implied Correlation = V CV Sum				LogNorm	CV 25%
Percentile	**Return**	**Gaussian**	**t - 10 df**	**t - 5 df**	**t - 2 df**
75%	4	29.6%	26.2%	21.6%	15.4%
90%	10	23.6%	22.1%	21.6%	18.7%
95%	20	21.5%	21.2%	22.6%	21.0%
99%	100	18.4%	22.4%	24.7%	32.7%
99.5%	200	17.3%	22.2%	29.5%	38.2%
99.95%	2,000	13.3%	27.5%	33.3%	57.4%

25% Correlation

- Copula Dynamics → Difficult to 'second' guess 'tail correlations'
- Sampling error is present e.g. "Normal" Gaussian values should equal 25%

Communication of Economic Capital Dependency Impacts
Possible Risk Metric Measures

- Economic Capital Aggregation
- Joint Probability Density Function
- Scatter Plot
- Joint Excess Probability
- Tail Concentration Function
- Kendall Tau Correlation
- Coefficient of Tail Dependence
- Implied Gaussian Correlation – Targeting a Risk Metric (e.g. R(z))

  Possible use in determining copulas if empirical calculations made

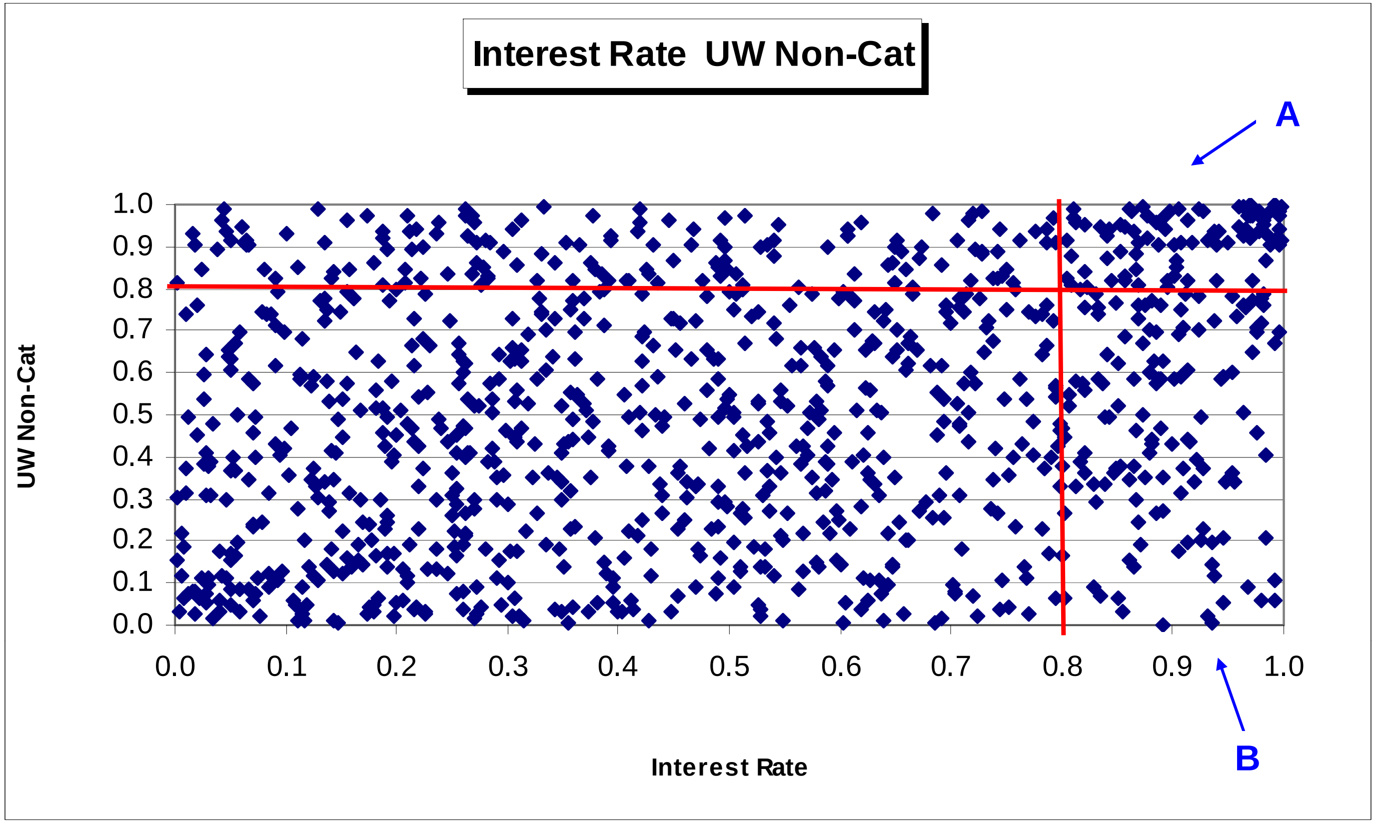
- 2 possible Methods for the Risk Metric of interest:
  - Between key pairs of risks e.g. UW Cat Risk and Equity
  - Construct a matrix of values for the risk metric of interest

Communication of Economic Capital Dependency Impacts
R(z) – Right Tail Concentration Function

- Description:
  - The tail strength of a copula can be defined using the Right and Left Tail Concentration Functions R(z) and L(z) * respectively as follows:
    - Right Tail Concentration Function: $R(z) = P(u>z / v>z)$
    - Left Tail Concentration Function: $L(z) = P(u<z / v<z)$
  - where: u and v are defined by $F_X(x) = u$ and $F_Y(y) = v$; x and y are values from X and Y respectively and u and v are values on the unit interval [0,1].

* Venter, Gary G. "Tails of Copulas". Proceedings of CAS LXXXIX (2002) pp. 68 – 113

Communication of Economic Capital Dependency Impacts
R(z) – Right Tail Concentration Function



- R(0.8) = A / (A + B)
- Only 1,000 data points from 25,000

Communication of Economic Capital Dependency Impacts
R(z) – Right Tail Concentration Function

- Advantages:
  - It is practical and the concept is relatively easy to understand
  - The calculation is relatively easy to perform
  - It provides a consistent methodology for comparing the relative strength of 2 or more different copulas
  - It is possible to represent the information either as a matrix of values for all risks or a pair of risks

- Disadvantages:
  - It is a relatively new concept
  - It is difficult to translate a value of R(z) (or L(z)) into a number that is commonly understood e.g. linear correlation, or its equivalent at the 'tails'.
  - Sampling error may distort the 'tail' dependency strength

Communication of Economic Capital Dependency Impacts

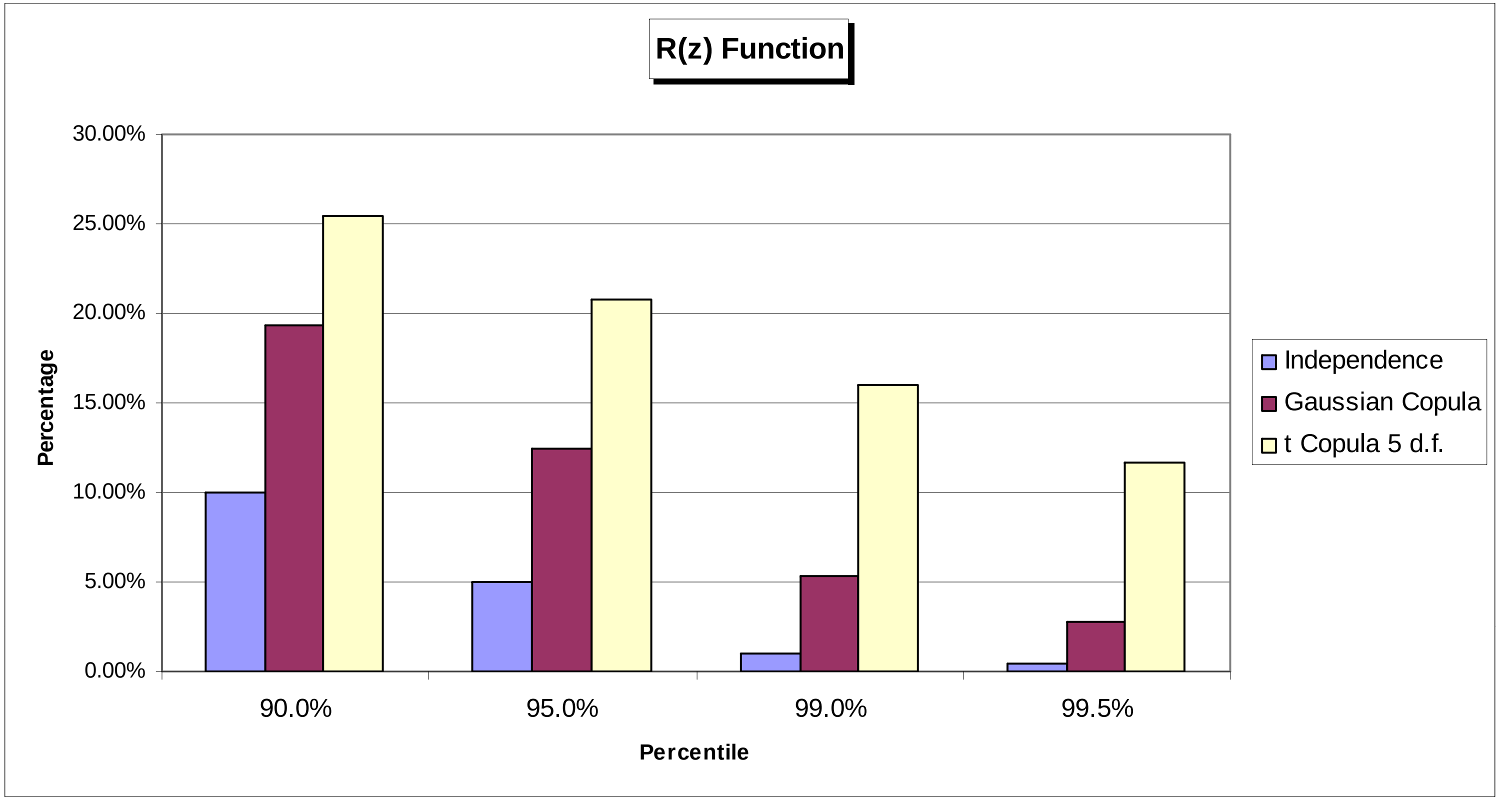
R(z) – Right Tail Concentration Function

R(Z): t Copula 5 d.f.				z	95.0%						
	No.	1	2	3	4	5	6	7	8	9	10
Equity	1		17.58%	21.85%	21.61%	20.89%	19.68%	20.16%	17.98%	17.58%	19.68%
Property	2			21.18%	20.03%	19.87%	19.38%	20.03%	20.36%	19.38%	19.71%
Interest Rate	3				21.20%	19.98%	19.45%	20.82%	20.67%	19.68%	20.89%
Credit Spread	4					19.06%	18.46%	22.20%	18.91%	19.96%	21.82%
Credit Default	5						19.13%	20.71%	20.16%	19.61%	19.76%
UW - Cat	6							18.87%	18.46%	19.35%	21.62%
UW Non-Cat	7								19.27%	19.51%	21.74%
Reserve	8									21.06%	21.38%
Expenses	9										19.48%
Operational	10										
									Independence		5.0%

RJEP(Z): t Copula 5 d.f.				z	95.0%						
	No.	1	2	3	4	5	6	7	8	9	10
Equity	1		0.87%	1.08%	1.07%	1.04%	0.98%	1.00%	0.89%	0.87%	0.98%
Property	2			1.04%	0.98%	0.97%	0.95%	0.98%	1.00%	0.95%	0.96%
Interest Rate	3				1.12%	1.06%	1.03%	1.10%	1.09%	1.04%	1.10%
Credit Spread	4					1.02%	0.99%	1.19%	1.01%	1.07%	1.17%
Credit Default	5						0.97%	1.05%	1.02%	1.00%	1.00%
UW - Cat	6							0.93%	0.91%	0.96%	1.07%
UW Non-Cat	7								0.97%	0.98%	1.09%
Reserve	8									1.04%	1.05%
Expenses	9										0.98%
Operational	10										
									Independence		0.25%

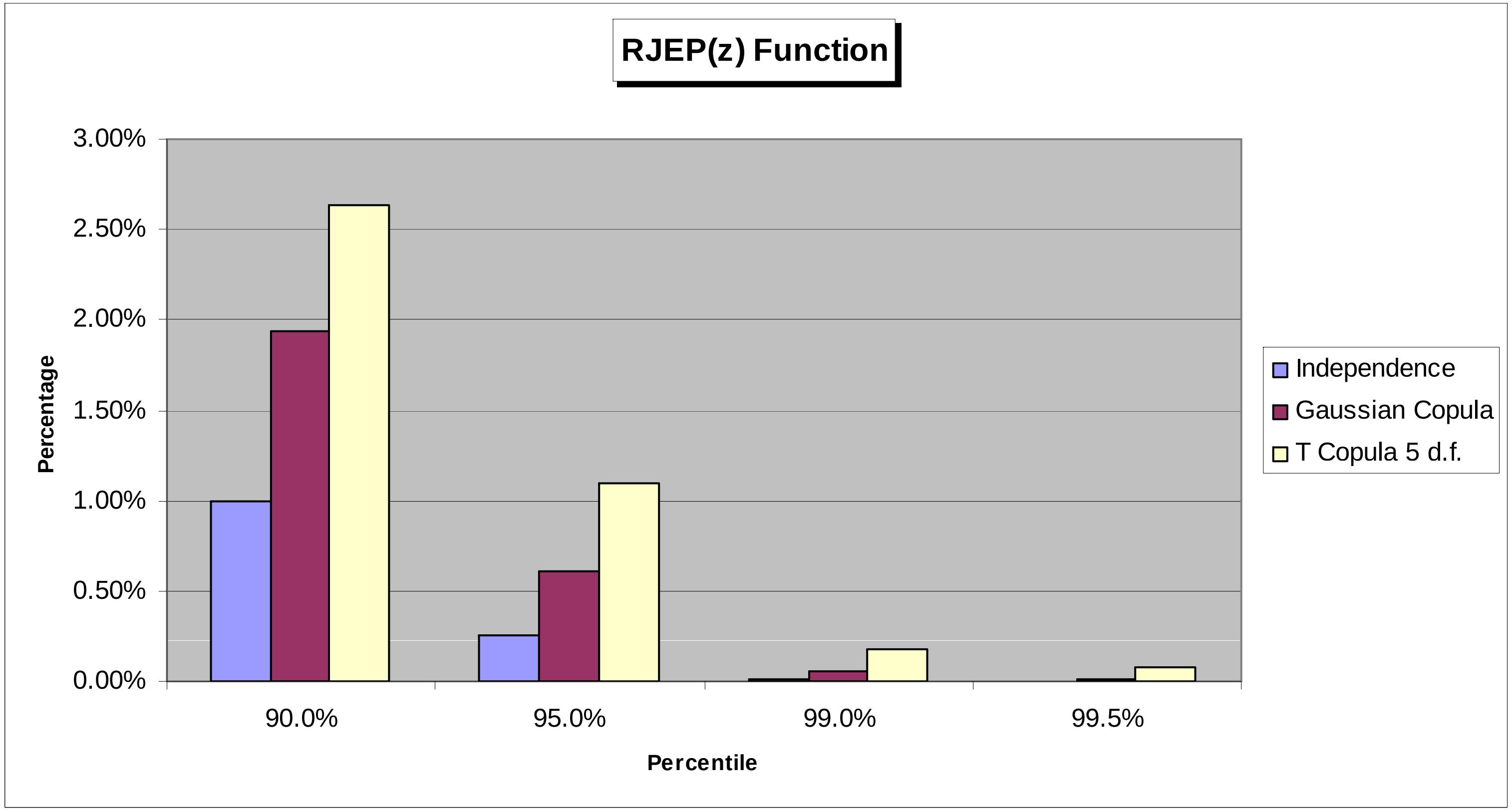
- Correlation = 25%; T Copula 5 d.f.; Calculated from 25,000 simulated outputs

Communication of Economic Capital Dependency Impacts
R(z) – Right Tail Concentration Function Copula Comparison



- Interest Rate vs UW Non-Cat
- Implied Gaussian copula correlation @ 99.0% (99.5%) = 54% (62%) etc

Communication of Economic Capital Dependency Impacts
RJEP(z) – Right Joint Excess Probability Copula Comparison



- Interest Rate vs UW Non-Cat

Conclusions
Concluding Remarks

- Managing Diversification benefit a key component of capital management
  - including effective communication to internal and external stakeholders
- Key Modelling Challenges:
  - Correlation – spurious relationships, parameterisation and variation over time
  - Copulas – selection and parameterisation
  - Model risk and parameter risk
- Communication Challenges:
  - What do we understand and mean by ‘tail correlation’
  - Copulas – Communication to non-technical people
- ‘Tail correlations”:
  - Should really be using correlation matrices together with copulas
  - Often a selection without theoretical foundations – a ‘Guess’ or ‘Margin’
  - However, practical issues – QIS 4, allow use of V CV approach to capital
  - A difficult trade-off