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Credibility Theory in UK PMI Market

Theory, applications and advice.

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- Applications in UK PMI Market
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- Discussion and Q & A

Overall aim is to share ideas, debate issues, and iron out oversights.

Credibility Theory Recap

Credibility Theory is a method that helps answer the question:-

How much "trust" should we place on the claims experience of an individual PMI Scheme versus data from all schemes?

- **Bayesian Underpinnings – see assumptions later.**
- **Strong Insurance Applications – Motor NCD, BF method.**

The Basic Formula

$$RP_i = Z\mu_i + (1-Z)\mu$$

- RP_i = Risk premium for group i.
- μ_i = Average claims experience for group i.
- μ = Average claims experience for all groups in portfolio.
- Z = Credibility factor where $0 \le Z \le 1$.

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Simple Example

- Let μ_i = £1000 and μ = £650.

If *Z = 0.4:-*

$$RP_i = 0.4 \times 1000 + 0.6 \times 650 = £790$$

- Higher Z implies more trust on a scheme's actual data. The risk premium is correspondingly closer to actual claims.

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Buhlmann & Straub and Calculating Z

- Empirical solution that defines Z using a company's own data.

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\mathrm{var}[m(\theta)]}}$$

- i = 1 to N; the number of groups in a company's portfolio. Hence each group has its own Z factor.
- j = 1 to n; the number of years of data the company has for each of the groups in the portfolio.

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Policy Volume

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$$

- Σ P(i,j) represents the risk volume for a particular risk i. This could be for example:-
  1. Premium income.
  2. Group size (members, covered lives etc).
  3. Claims

Inter Group Variance

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$$

- This is a measure of the variance between the different risks. I.e., how much risk i's data varies versus the other n-1 risks in the portfolio?
- If this figure is high, this suggests that the collective data is less trustworthy.

Intra Group Variance

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$$

- This is a measure of the variance between the different years of the same risk. I.e., how much company i's data varies over the N years of data used.
- If this figure is high, this suggests that the individual risk's data is less trustworthy as it varies greatly from year to year making predictive inference less reliable.

OVERALL EFFECT

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + k}$$

- Keeping E[.]/VAR[.] constant, say K; Increasing ΣP(i,j) causes Z to tend towards 1. This implies that a higher exposure or policy volume supports the decision to give more credibility to the individual risk.
- Keeping ΣP(i,j) constant; Z will decrease if K increases and *vice versa.* Hence, if variance between the risks is high relative to variance within the risks, K will be higher and Z smaller.
- Note that K does not depend on risk i.

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Some Assumptions

1. The distribution of each X_j depends on a parameter θ, whose unknown value is the same for each j – "two urn" model.
   - Is the risk parameter θ the same for each year? For example, a HR drive to encourage policyholders to claim on their PMI policy. A change in the risk characteristics of the group due to an acquisition. Change in economic conditions leading to redundancy fears.
2. Given θ, the X_j's are independent but not necessarily i.i.d.
3. $E[X_j|\theta]$ is independent of j.
   - Are claims independent year on year? Probably not due to inflation and other time trends such as durational effects or any change in insurance conditions.
4. $P_j V[X_j|\theta]$ is independent of j.

$X_j|\theta$ for example could be the BC of a PMI group in year j with coefficient θ.

θ for example may be a "health" coefficient. Gives the different PMI groups differing risk characteristics.

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Applications

- One of our aims was to be able to calculate Z's for group sizes of which we had little or no data. Essentially answer the question:-

" What weight should we give to a PMI group scheme with 2000 members?"

- To do this we looked at simulating the group claims and then apply the credibility formulae to the simulated results.

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Approach 1

Method:
- Model stochastically 1000 simulations of 4 year claims figures of groups with say 500 members.

Problem:
- Because we were only considering groups of size 500, inter-group variance became very small – usually negative suggesting Z = 0.
- This was the case for most runs looking at the same groups size – obvious error.
- Unlikely to have a 1000 groups of 500 members in our portfolio. Hence the base premium (average of collective risks) is not representative of our base experience.

Lessons:
- Credibility factors will change depending on the collective portfolio of risk a company has – since this will affect inter risk variance.

Approach 2

Method:
- Model stochastically 1000 simulations of 4 year claims figures of groups of varying sizes that mimic the mix of group sizes in our experience.

Problem:
- Simulation has to be very sophisticated to ensure that it mimics reality. Factors such as lapse rates, inflation, anti – selection, underwriting and duration will alter the inter and intra variances.
- Need to incorporate the effect of joiners and leavers. This will weaken the durational effect that will change the intra risk variance.

Lessons:
- Does the benefit of simulating the data, which would be time consuming and complicated (especially if model needs to be built), outweigh using actual data?

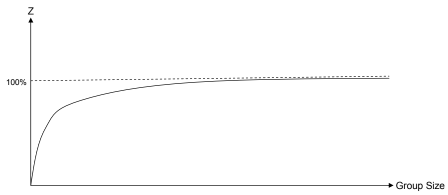
Approach 3

Method:
- Fit the B&S method to actual company data. This can be done by looking at a combination of company's own data and industry data from quotes. Quotes will usually give the necessary information to calculate Z.

Problem:
- The credibility factors seem high – from a "hunch" point of view.
- We will have determined a K that is based on data that we may have quoted for but have not actually written. To calculate RP_i we need to use μ that relates to the K.
- If the mix of group sizes in our portfolio changes and we continue to use the K based on the old mix, results may be misleading. Inter group variance is likely to have changed.

Lessons:
- Need to be pragmatic when it comes to credibility theory. Cannot re – calc Z every time we have get some new group data.
- Periodic reviews may be sensible to ensure that the K is not massively unrepresentative.

Results of approach 3



- If K is large convergence speed is slower. This is due to fact that the intra - group variance is higher and individual data less reliable.

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Some factors based on market data

Average Group Size	Four years of policy volume	Z
10	40	33%
20	80	50%
500	2000	86%
700	2800	90%
900	3600	92%

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Unknowns

- Is using market data a valid approach? Is credibility theory not a method that helps you use what you've got in the best way?
- If we use market data to calculate z, is the calculation assuming we have the same base experience as the market and a similar mix of new business.
- If we haven't, e.g. our business attracts smaller group sizes or has different underwriting practices, could we be giving too little or too much credibility?
- Does this mean that we have to use the market experience as our base rate? Is this valid, or again will differing sales channels or UW make our premium biased?
- Does it matter? Should we take a more pragmatic approach? Perhaps scenario testing may highlight possible issues.

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Questions/Discussion!

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