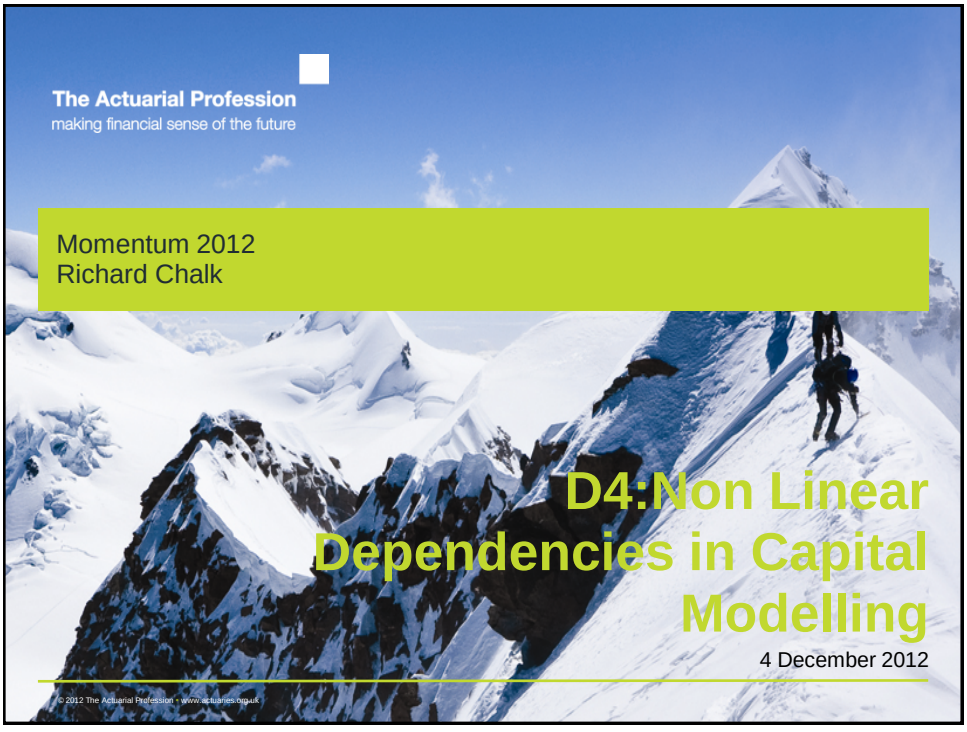


Disclaimer

- The views expressed in this presentation are those of the presenter, and do not necessarily reflect those of the employer

Assumed knowledge

- No knowledge beyond the CT3 syllabus is assumed
- In particular, no detailed knowledge of capital modelling is assumed

Agenda

- Objectives
- Dependency and its causes
- Relevance in capital models
- Copulas and capturing dependency
- Tail dependence

Objectives

What we seek to achieve

- An awareness of what is meant by dependency
- What a copula is, and why we use them
- In overview, what options we have in copulas
- What is meant by tail dependency, and why it is important

What we will NOT cover

- The detailed mathematics of copulas
- The properties of individual copulas

What is dependency?

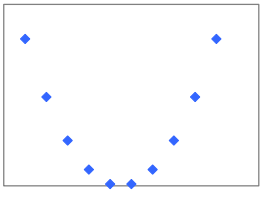
- Dependency describes how different risks (or different probability distributions) interact with each other
- For example:
  - between classes of business, say between motor and home losses
  - between risk types, say between insurance risk and market risk

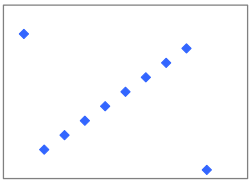
What is dependency?

- Dependency is not (just) correlation
  - Dependency refers to the complexity of how two risks interact throughout their distributions
  - However, correlation is just a single measure of the strength of that dependency
- As an analogy, summing up a dependency relationship with a correlation is like summing up a distribution using just the mean

What is dependency?

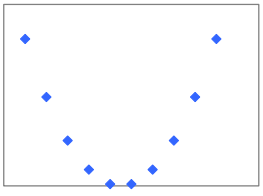
- What is the correlation of these scatter graphs?
- Would you say that a dependency relationship exists?

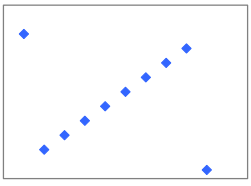




What is dependency?

- These graphs exhibit zero (or close to zero) correlation
- But clearly there is a relationship





What is dependency?

- So, what do I mean by non-linear dependency?
- This is the idea that the relationship between two risks can vary, sometimes dramatically, depending on what part of the distribution we are looking at
- So, our dependency structure should allow for this

What is dependency?

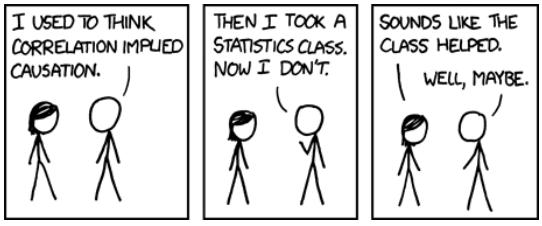
- Non-linear dependency is of particular interest in the *tails* of those risks
- The reason the credit crunch was so severe was that everything went bad together – a strong dependency relationship only emerged in the tails
- This tail dependency is often driven by severe economic stresses
- We'll discuss this in more detail in a minute

What can cause dependency?

- We're interested in capturing the effect of dependency relationships, but it's worth giving thought to what causes this
- Dependency can result from:
  - causation (a loss in one thing leading to a loss in the other)
  - common drivers (two losses being driven by the same thing)

What can cause dependency?

- Just because two things happen at the same time, doesn't mean one caused the other



Correlation doesn't imply causation, but it does waggle its eyebrows suggestively and gesture furtively while mouthing 'look over there'

http://xkcd.com/552/

What can cause dependency?

- As an example of causation…
- Imagine if we experienced large insurance losses from a major storm, following by significant market risk losses from a market drop
  - It might be accurate to conclude a causal relationship
- Imagine if the market drop came first
  - Obviously, market performances can't affect the weather

What can cause dependency?

- What are the common drivers that can lead to a dependency relationship?

What can cause dependency?

- What are the common drivers that can lead to a dependency relationship?
  - Weather
  - Catastrophe
  - Economic climate
  - Interest rates
  - Inflation
  - Legal environment
  - Firm specific factors
  - … and many others

How is this relevant in capital models?

- Here, we're talking about simulation based capital models
- That is, models which seek to capture the behaviour of the business mathematically, and generate a series of simulations in order to generate a distribution for the business as a whole

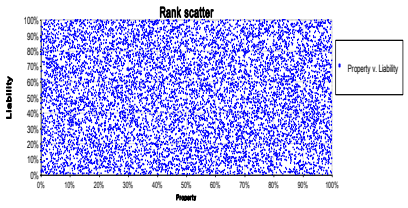
How is this relevant in capital models?

- Consider this simple example framework:
  - We are seeking to model two lines of business, property and liability
  - Suppose we model large losses for these two lines with a Pareto distribution
- What is the relationship between these two risks?

How is this relevant in capital models?

- We generate simulations from each distribution

	Property	Liability
1	1342	4633
2	1155	1043
3	2150	1252
4	1124	1065
5	1226	1366
6	1832	3113
7	5795	1263
8	1359	1750
9	1219	1100



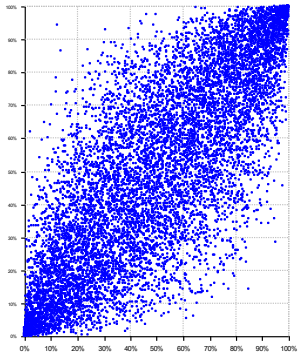
- From the rank scatter diagram, we can see no relationship between the two variables
- For this, we need a *copula*

What is a copula?

- Practically, a copula lets us describe how two risks (or distributions) interact, without having to worry about the structure of the risks themselves
- It determines the *ranks* of the losses we are modelling
  - For instance, if the 5th largest loss in one risk occurs at the same time as the 20th largest loss in another risk

What is a copula?

- This can be best demonstrated using a *rank scatter plot*

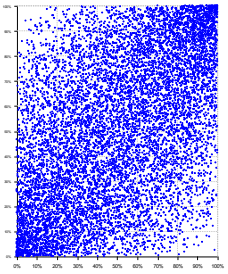


Capturing dependency with copulas

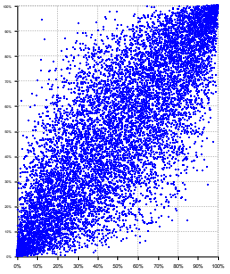
- When defining a copula, we need two things:
  - the *type* of copula, which describes the *shape* of the relationship; and
  - the *parameter(s)* of the copula (for instance, correlation), which describe the *strength* of the relationship
- One of the easiest ways to see what a given copula does is to look at its graph
- Let's look at some examples…

Capturing dependency with copulas

- Gaussian copula



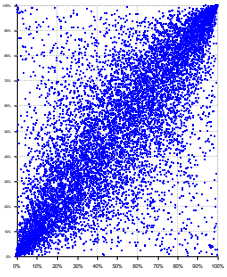
Gaussian copula,
50% correlation



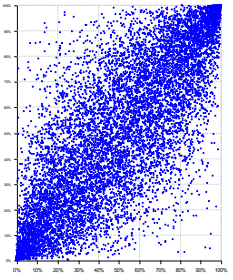
Gaussian copula,
80% correlation

Capturing dependency with copulas

- t copula



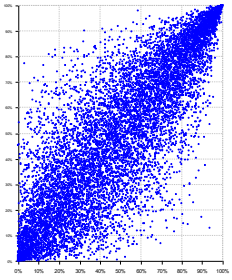
t copula, ν=2
80% correlation



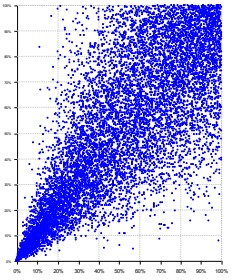
t copula, ν=10
80% correlation

Capturing dependency with copulas

- Archimedean family



Gumbel copula, θ=3

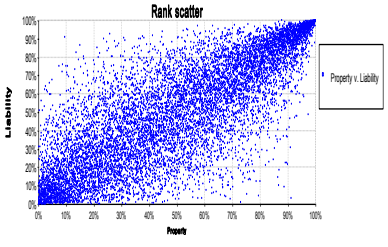


Clayton copula, θ=3

Capturing dependency with copulas

- So now, we can pair up our simulations from before according to the copula we've chosen (here, a Gumbel):

	Property	Liability
1	1342	4633
2	1155	1043
3	2150	1252
4	1124	1065
5	1226	1366
6	1832	3113
7	5795	1263
8	1359	1750
9	1219	1100



- This preserves the *rank correlation* of the copula

What is tail dependency?

- Tail dependency is a measure of how likely it is we will observe very large values in one risk, given that we have already observed very large values in another risk
- Mathematically, we can express this idea as:

$$P\left(X_1 > F_1^{-1}(p) \middle| X_2 > F_2^{-1}(p)\right)$$

- When looking at tail dependence, we see what happens as *p* tends to 1 (for upper tail dependence) or 0 (for lower tail dependence)

What is tail dependency?

- Whether a copula *needs* to exhibit strong tail dependency depends on the risks you're undertaking
  - For large numbers of smaller risks, or risks with smaller tails, tail dependency isn't quite as significant
  - For smaller numbers of potentially very large risks, tail dependency becomes much more important

What is tail dependency?

- Asymptotically, the t, Gumbel and Clayton copulas all exhibit tail dependency; however, the Gaussian does not
- Whether this is important is a very case specific question, but it is important to be aware of
- The choice of copula should be based on both theoretical considerations, and on the outputs of the model (such as output correlations)

Why do we care about tail dependence?

- So some copulas have tail dependence and others don't – why does that matter?
- In capital modelling, we are often interested in the tail; specifically, the 1-in-200 level
- If our model isn't reflecting the dependence correctly at the 99.5^{th} percentile, we could be understating our capital

Why do we care about tail dependence?

- Returning to our example
  - Say the liability and property losses are each distributed according to a Pareto distribution with alpha 2 and beta 1,000
  - Consider a Gaussian copula with correlation 50% or 80%
  - Consider a Gumbel copula with parameter 1.55 or 2.6 (which give rank correlations of approximately 50% and 80% respectively)
  - Let us look at the 99.5% Value at Risk (VaR)

Why do we care about tail dependence?

- This gives us the following results

Structure	50% correlation	80% correlation
Independence	21,000	21,000
Gaussian copula	23,600	24,600
Gumbel copula	25,200	25,900

- This is based on 10,000 simulations in Igloo
- Results to 3 significant figures

Conclusion

- So, what's the moral of the story?
  - Different copulas represent different behaviours, sometimes significantly
  - Especially when looking at the crucial tail, the choice and parameters can be material
  - Understand the implications, theoretical and practical, before making a decision

Questions and comments