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Deloitte.

Calibrating Scenario Generators for Pensions Asset Liability Modelling

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Introduction

- Purpose of Scenario Generators
- Calibrating average returns
  - Calibration methods
  - Different return statistics (coin toss example)
  - Communicating results
- Calibrating stochastic behaviour
  - Interest rates: 1 factor Vašíček
  - Inflation & risk assets
- Wrap up and questions

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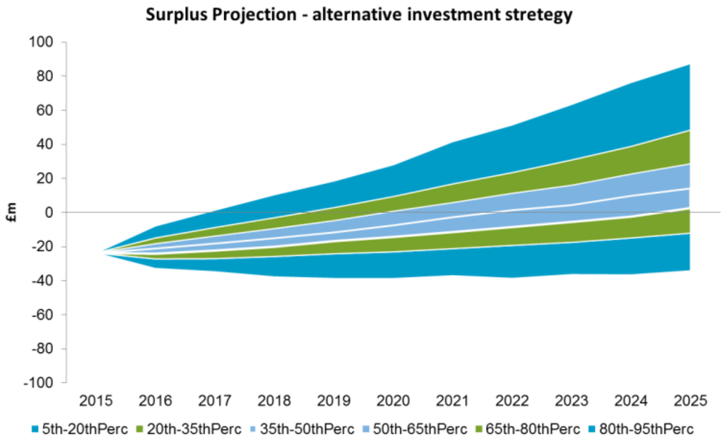
Introduction

- A key tool that has been used in the pension scheme industry, particularly by investment consultants, is the Asset Liability Model (ALM)
  - Used to compare risk and return profile of different strategies
- One approach to ALM involves Monte Carlo Modelling



- As such, the output of an Economic Scenario Generator (ESG) becomes a key input into such a model
  - In particular, we would like real world economic scenarios with risk premiums
- What approach can we take to calibrate each parameter within an ESG for the purpose of an ALM described above
  - Assuming historic data is used
- The particular parameters will depend on the underlying models used, however a number of themes are likely to reoccur irrespective of the precise nature of the ESG
  - We will explore some of these by building up a basic ESG model and considering each parameter as it is introduced.

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Process

Calibrate stochastic behaviour

- Interest rate behaviour (risk free rate)
- Risk asset behaviour
- Correlations

Calibrate average returns

- Risk premiums

Input to ALM

- Apply to multi-asset portfolios

- We will discuss risk premium calibration initially…
- …and then move on to calibrating the stochastic behaviour of interest rates and risk assets

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Risk Premiums

Introduction

- We interpret a risk premium as the additional expected return in excess of the (nominal) risk free rate, to compensate an investor for taking exposure to some sort of risk

- If we know the risk free rate over a particular time horizon, we can convert between risk premiums and expected returns

- The RPs we might want to include in our model are:
  - RPs of risk assets (equity, property, credit etc)
  - Interest rate RP (term premium)
  - Inflation RP

- Why are risk premiums important, and what properties do we want to capture?
  - Modelling likelihood of being fully funded requires asset class growth rates
  - Portfolio optimisation exercises require arbitrage free scenarios

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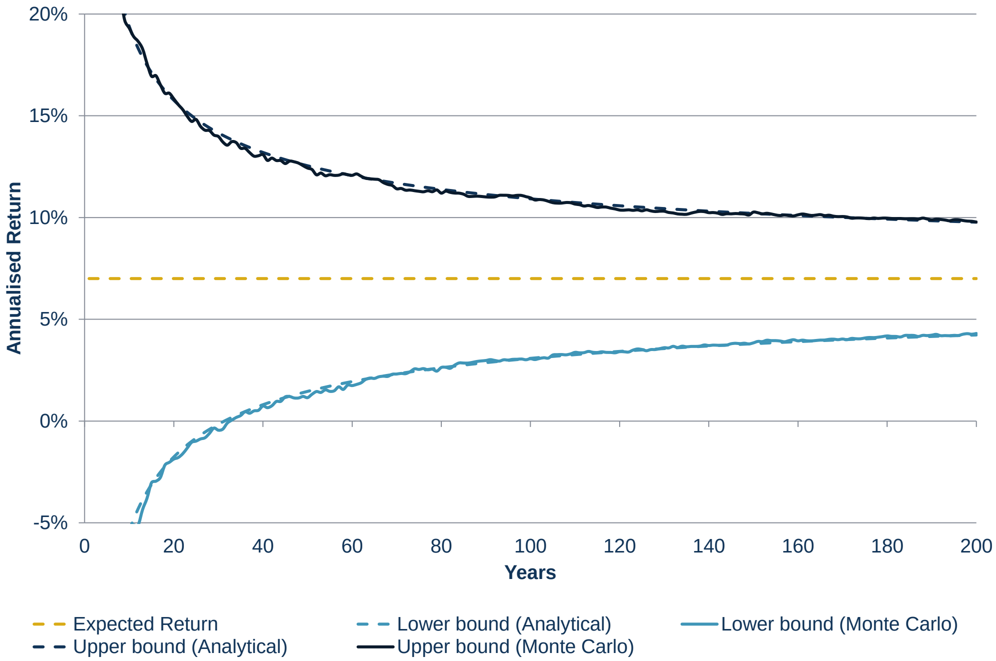
Risk Premiums (continued)

Option 1: Use historic data for calibration

- If we simulate returns from a known distribution, how quickly do the annualised sample returns converge?



Parameter	
Expected return	7%
Volatility	20%
Significance	5%

- Convergence takes a long time
- Past performance is not an indicator of future results
- This may generate arbitrage or near arbitrage opportunities.

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Risk Premiums (continued)

Option 2: Derive from fundamentals

- We can use economic theory or judgement to set a particular RP
- However, if we do this in isolation for many asset classes we are increasingly likely to introduce near arbitrage opportunities
- We can set some RP relative to others to avoid these opportunities
  - Black-Litterman
  - The tradeoff is losing direct control over calibrating particular RP

Our preference is option 2, deriving from risk premiums from fundamentals, whilst limiting the maximum efficiency an optimised portfolio can achieve

Modelling in a Pension Scheme context

- We need to focus on important RPs and let secondary RPs vary to avoid unrealistic portfolio efficiency
- What RPs do we care about most?
  - Equity risk premium
  - Term premium

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Risk Premiums (continued)

Equity Risk Premium

- What do we use the equity risk premium (ERP) for?
  - Model the projected asset growth rate in excess of gilts
  - Determine the discount rate spread
  - The discount rate spread will depend on covenant strength; from a modelling perspective we only want a single ERP

- We can estimate this from fundamentals
  - Estimate the expected return on equity from a fundamental model
  - Subtract the risk free rate

- This can be interpreted as an **arithmetic risk premium**

Communicating return assumptions

Different measures of return

Geometric versus arithmetic risk premium

Property	Arithmetic Risk Premium	Geometric Risk Premium
Definition 1	The arithmetic return of an asset above the risk free rate	The geometric return of an asset above the risk free rate
Definition 2	The instantaneous expected return above the risk free rate	The difference between the long run growth rate of an asset and the risk free rate
Can be blended…	Cross-sectionally	Longitudinally

- If an asset has variable returns, the geometric return will be lower than the arithmetic return
- Specifically;

$$G \approx A - 0.5\sigma^2$$

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Example

Heads or tails

- You start with £1. You can stake part of this money on a coin toss (E) whereby heads you win and I triple your stake, and tails you lose and I get your stake. The remainder of your money is safe. We repeat this process 10 times.

- **What is the expected return on the coin toss?**

Lottery	Heads	Tails	A?	G?
E	3x	0x	50%	-100%
Cash	1x	1x	0%	0%

- If you stake all of your capital, there is only a 1/1024 probability that you will be left with any money at all!

Therefore, is it misleading to say the lottery has an expected return of 50% p.a.?

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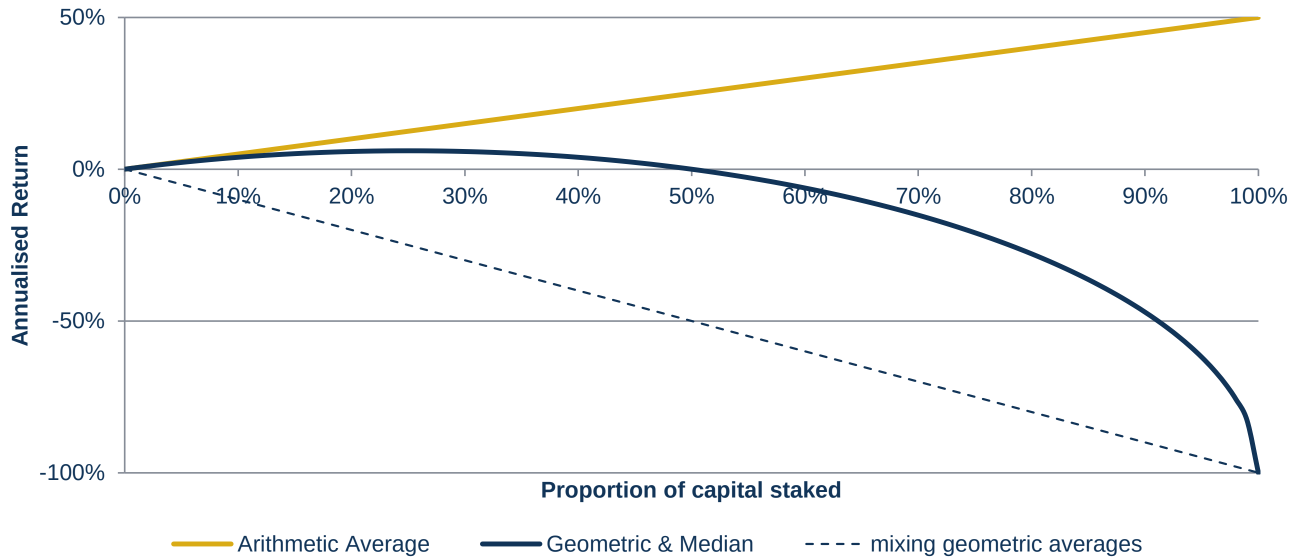
Example



Trial			1	2	3	4	5	6	7	8	9	10
Outcome			H	T	H	T	H	T	H	T	H	T
Proportion of capital staked	100%	£1.00	£3.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00
	50%	£1.00	£2.00	£1.00	£2.00	£1.00	£2.00	£1.00	£2.00	£1.00	£2.00	£1.00
	25%	£1.00	£1.50	£1.13	£1.69	£1.27	£1.90	£1.42	£2.14	£1.60	£2.40	£1.80

- The geometric average of a blended portfolio will be greater than the weighted geometric averages of its constituent parts.

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Communicating return assumptions

- Even if we use arithmetic RPs to calibrate our Economic Scenario Generator, the arithmetic return might not be the most appropriate metric to communicate to Trustees
- It could give too optimistic a view if misinterpreted as a growth rate
- We also need to take care to avoid mixing geometric returns cross-sectionally across asset classes
- For asset classes with lognormally distributed total return indices, the geometric return and the median return tend towards the same value

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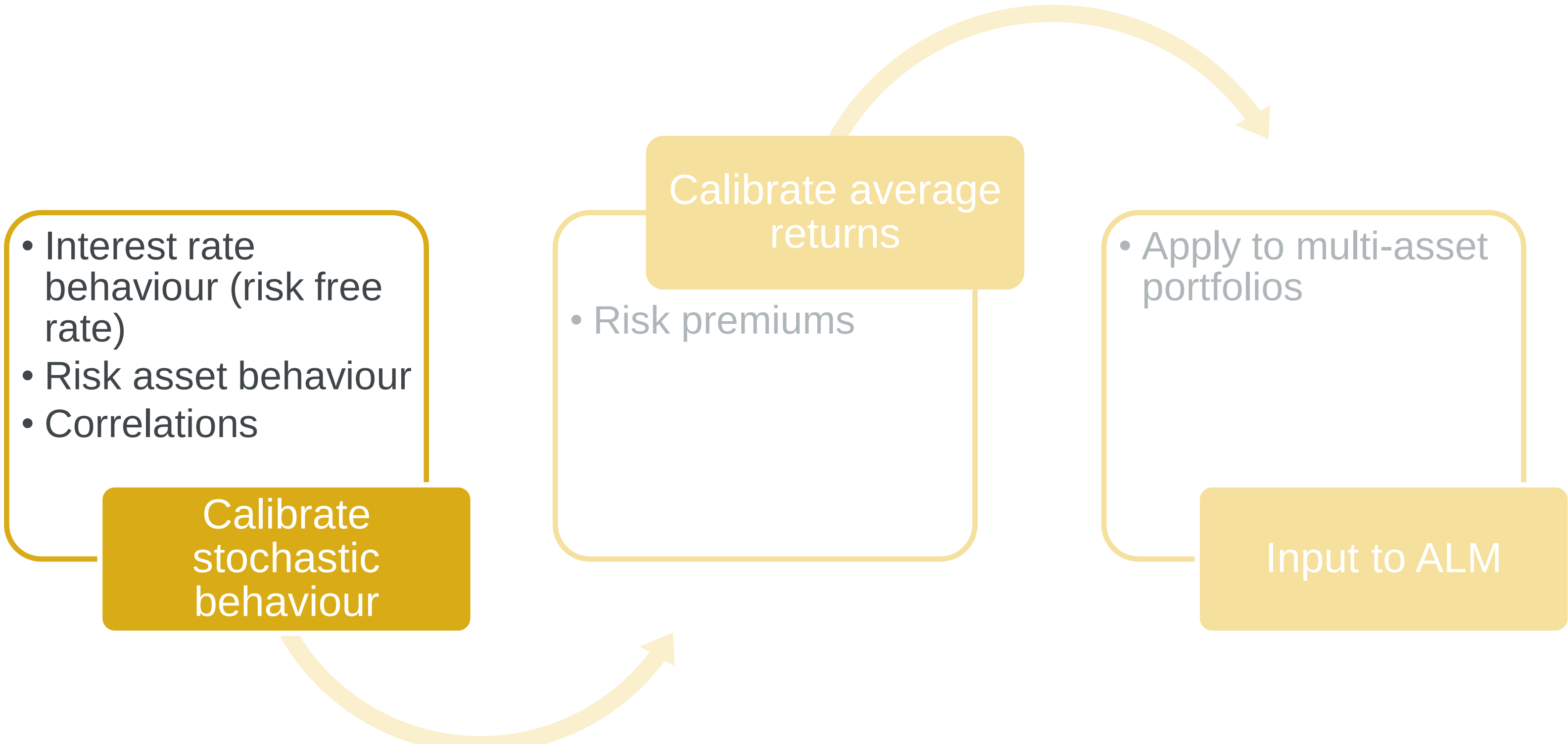

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Interest rates – 1 factor Vašíček

- An interest rate model is foundational
  - We need a risk free rate
- Being able to model bonds is probably the most important aspect of a model for our purpose
  - Bond yield movements (of various terms) will be the driving force behind changes in the present values of pension scheme liabilities
  - Pension scheme asset portfolios are often bond heavy (of one type or another)

- Let's begin with a basic interest rate model – one factor Vašíček
- In its most commonly quoted form, the Vašíček model is a description of the short rate over time
  - This would imply that the best method to calibrate the model would be to look at the characteristics (historic or otherwise) of cash
- However, for our purpose it is not really the behaviour of cash that we are interested in
  - Pension schemes tend not to really be sensitive to the behaviour of the short rate
- The model can also be restated as a model of any particular rate
- So if we are most interested in behaviour of the twenty year rate, for our purposes we feel that stating the model in terms of this point (and calibrating to twenty year behaviour) makes the most sense
- We can follow a similar process for real and nominal yields

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1 factor Vašíček – continued

- The model has the following parameters that we need to consider:
  - Starting level (of chosen yield)
  - Risk premium (or term premium)
  - A mean reversion level
  - Speed of mean reversion
  - A volatility parameter
- All of these parameters deserve consideration as to how best to calibrate them
  - We will be concentrating on the final two of these for the next several minutes - before we do, let us briefly discuss the first three
- **Starting level**
  - In practice this will likely be calibrated simply to today's market rate
  - This is the simplest starting point and is market consistent
- **Risk premium – this is some measure of how additional volatility (i.e. longer dated bonds) are rewarded**
  - A key (but not only) determinant of the slope of yield curve
- **Mean reversion level**
  - This determines the level to which our chosen rate reverts back to over the very long run
  - One option is to take a historical average of this rate
  - The Hull-White model is an extension of the Vašíček with a time varying reversion level
  - This allows the initial yield curve observed in the market to be replicated in the model.

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1 factor Vašíček – continued

Speed of mean reversion

- We might expect yields to revert back to a ‘natural’ level over time
  - Theory: An interest rate spike will reduce spending and investment, depressing economic activity and inflation, and leading to looser monetary policy down the line
  - Vice versa for a sudden fall in interest rates (although this hasn’t necessarily played out in practice over the last decade!)
- This process will make it less likely that interest rates drift to unrealistically high / low levels within our scenarios over time
- What is the strength of this pull back to the long term mean reversion level?
  - This is captured by the ‘speed of mean reversion’ parameter

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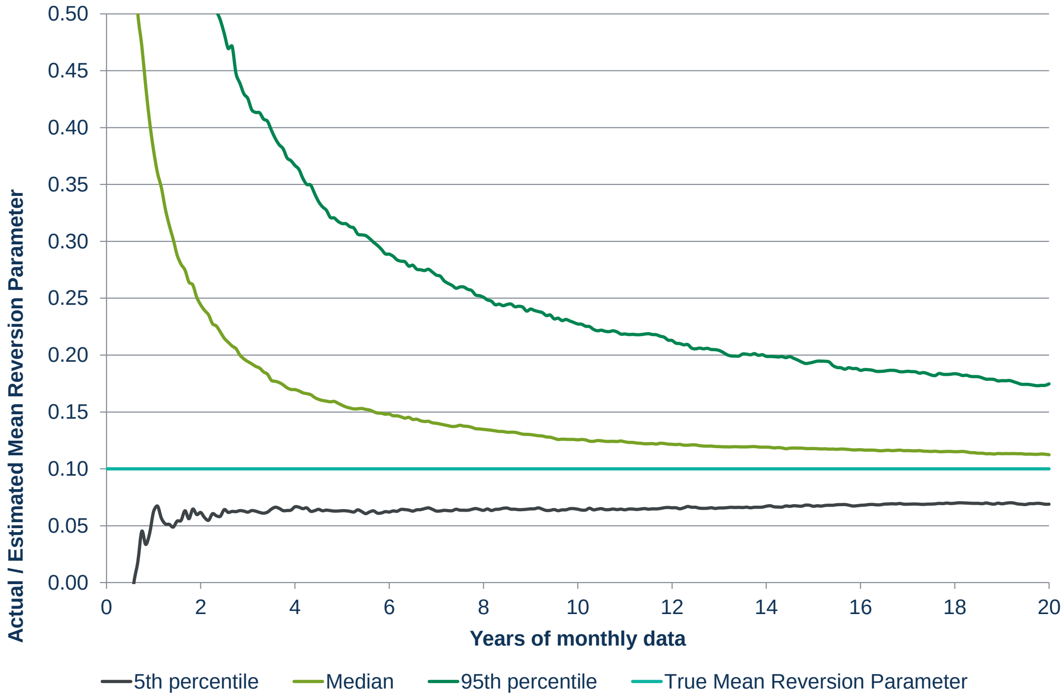
1 factor Vašíček – continued

Speed of mean reversion

- Option 1 – we can look at the behaviour of our chosen yield over time, and measure the degree to which it reverts
- However, there are a couple of statistical issues
  - Stability
  - Bias



Parameter	
Simulated Term	Short rate
True mean reversion parameter	0.1
Volatility (annual)	5%
Mean reversion level	3%
Initial yield	1%
Data periodicity	Monthly
Number of sims	2000

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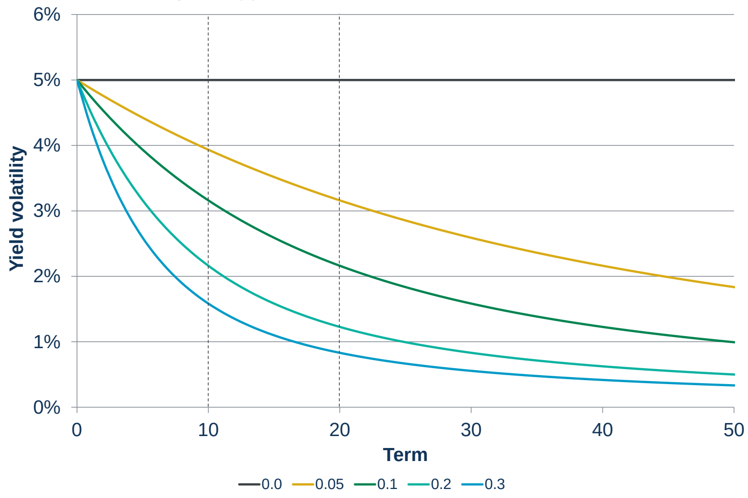
1 factor Vašíček – continued

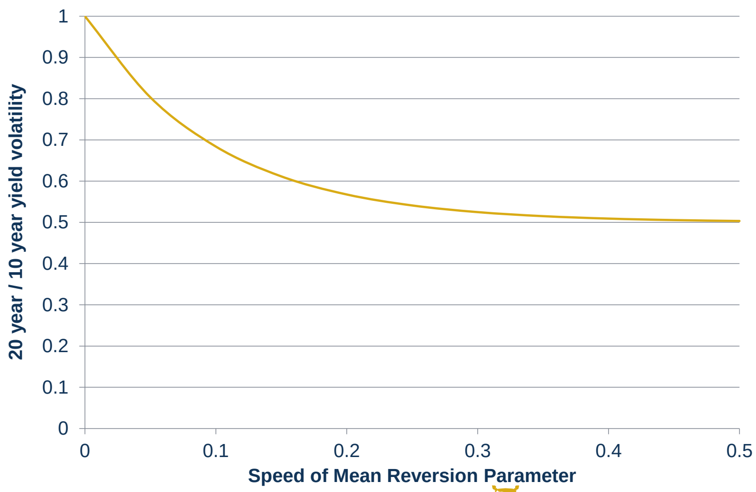
Speed of mean reversion (continued)

- Option 2 – we can look at the relative volatilities of yields of different terms
  - If the speed of mean reversion parameter is higher, long term yields will be less volatile relative to short term yields, because short term interest rate shocks will dampen more rapidly
  - Specifically;

$$yield\ volatility \times term \propto 1 - e^{-speed\ of\ mean\ reversion\ parameter \times term}$$

  - It is more efficient to estimate volatility than autocorrelation from limited data (more information can be extracted by sampling more frequently)





Our preference is for option 2 given the lack of bias and statistical efficiency

This is likely to calibrate weaker mean reversion, and we are more likely to generate negative rates as a result.

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1 factor Vašíček – continued

Volatility

- There are two primary options for calibrating our volatility parameter

Option 1: derive from historic volatility

- We can choose a particular term of interest, calculate historic yield volatility, and calibrate our parameter so our simulations replicate this
- This is simple and easy to explain to Trustees
- Future volatility may differ significantly from past volatility

Option 2: derive from market implied volatility

- We can use market implied volatility from traded interest rate derivatives (such as swaptions)
- The implied volatility at which market participants are willing to hedge may differ from their true volatility expectations
- Implied volatility will also be impacted by many other factors (for example, transaction costs, counterparty default, capital raising costs etc)

Our preference is for option 1 for real world pension modelling.

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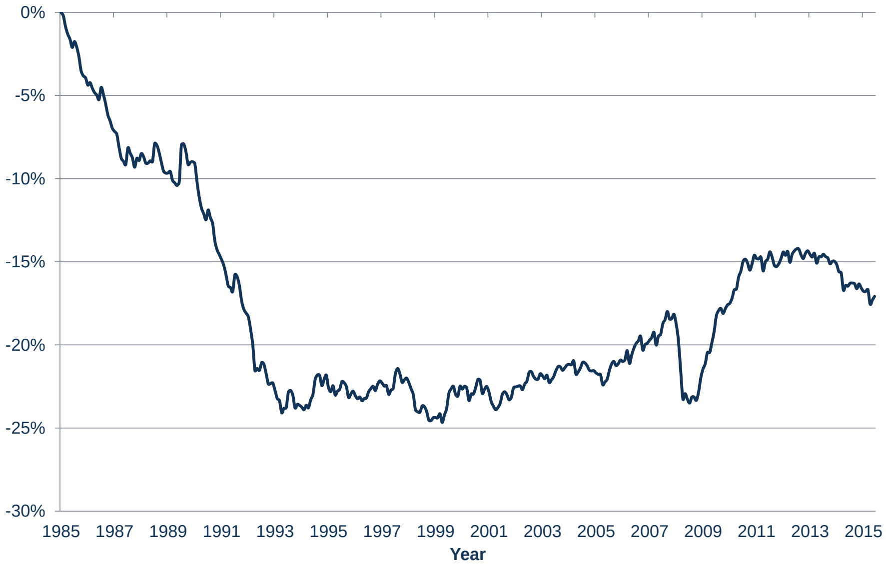
Inflation

Introduction

- We have implicitly calibrated breakeven inflation by calibrating real and nominal rates, but not RPI itself
- Does RPI differ from the short breakeven inflation rate?
- We have recorded the cumulative return on realised inflation minus breakeven inflation over time



- What does this mean?
  - The return clearly isn't flat – RPI and short breakeven inflation are not identical
  - This implies another risk factor exists for RPI.

Inflation - continued

Significance

- How important is this in a Pension Scheme context?
  - Breakeven inflation rates are potentially more significant than realised RPI
  - However, we may require RPI for some Asset Liability Modelling (for example, if we are using a cashflow framework)

Calibration

- We can model the difference between RPI and the short breakeven rate as a random walk, with a volatility and risk premium parameter
- However, in practice the process might not have independent increments (it may trend)
- Therefore, the sampling frequency for calibrating volatility becomes significant.

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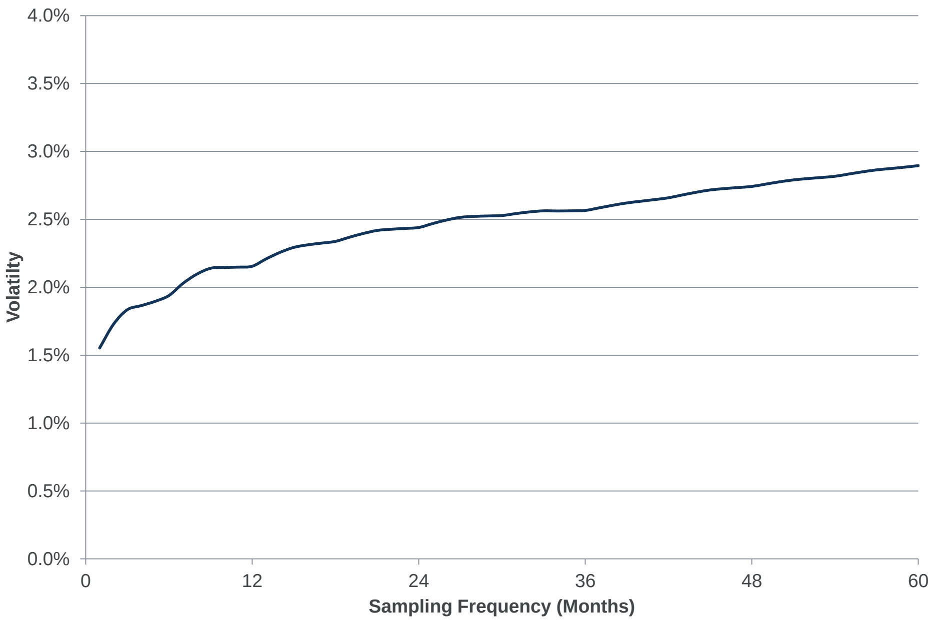
Inflation - continued

Term Structure of Volatility

- There is a tradeoff between calibrating using the shortest data available whilst avoiding the distorting impact of autocorrelation
- The term structure of volatility for actual minus breakeven inflation is shown below



- This is a wider consideration when calibrating volatility, particularly where pricing is stale
- As in the case of gilts, we will defer discussion of the associated risk premium.

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Risk asset(s) – Introduction

- To complete our basic model we need to add risk assets
  - In practice we would want several of these given the current nature of pension scheme investing
  - However for our purposes today lets just pick one – say UK equities
- We will model the risk asset class as geometric Brownian motion
  - Independent increments imply an efficient market
  - We will assume there is a static Equity Risk Premium (ERP) that is relative to the short rate
    - i.e. avoiding negative ERP in scenarios with high interest rates
- We have the following parameters to calibrate:
  - Volatility
  - Risk premium

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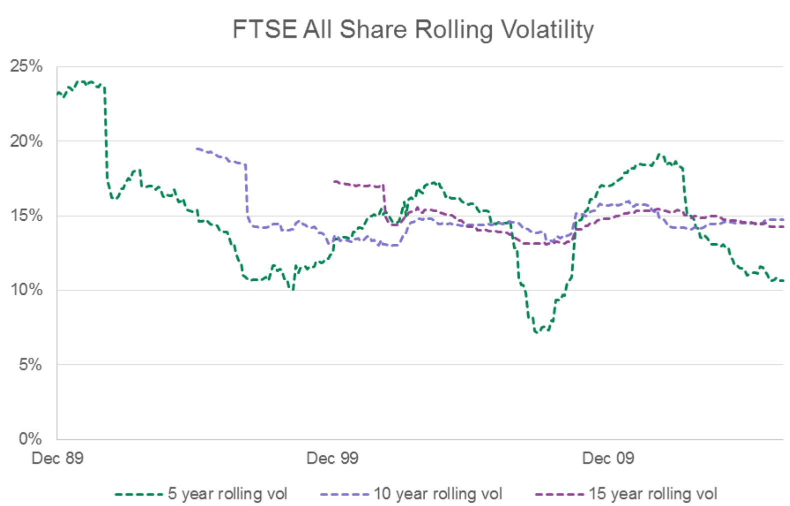
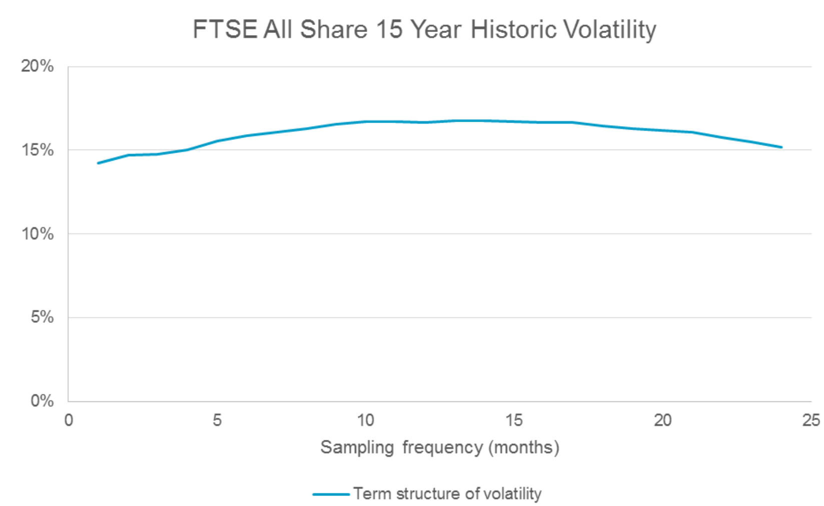
Risk asset(s) – Volatility

- As with bonds and RPI we want to know what period and sampling frequency to use
- For our dataset (FTSE All Share) short term volatility has significant fluctuations, but longer term fairly stable
  - And term structure less of an issue, so using as frequent sampling as possible (within reason) makes sense





- Any other asset classes used would require similar analysis

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Correlations

- For the sake of completeness it is worth briefly considering correlations
- We will require an nxn matrix where n is the number of factors in the model
- Once a methodology has been chosen for all other parameters we can calibrate this historically
- As with volatility some consideration around sampling frequency is required
- For Vašíček, as noted earlier we have more correlation parameters than are actually required
  - In practice this means discarding one bond term when calibrating correlations
  - Term to discard would be less “useful” term, e.g. keep 20 year correlation but ignore longer term bond
- Finally due to sampling error comparison of model output correlations with inputs helps identify stability issues

Conclusion

- There are alternative approaches to calibrating ESGs, and final outputs could differ widely depending on the choice of calibration methodology

- Engaging in this process can:
  - Build intuition around what aspects of modelling are more or less statistically noisy
  - Understand trade-offs in the calibration process…
  - …avoiding the misapplication of final asset liability model…
  - … and helping us understand where the model might be less realistic (for example, the behaviour of cash if we calibrate our model with reference to the 20 year point of the yield curve)

- Communicating summary statistics is an important aspect of the process
  - To do this, we need to understand the different measures ourselves
  - We should consider which measure of average return is most helpful for the particular advice we are giving
  - We should communicate clearly how clients should be interpreting these statistics

Questions

Comments

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Appendix: Risk Premiums (continued)

- In the Vašíček model, the risk premium parameter is arithmetic
- Therefore, the arithmetic risk premium of a bond will be proportional to its volatility

$$A \propto Bond\ Volatility$$

- This implies:

$$G = Constant \times Bond\ Volatility - 0.5 \times (Bond\ Volatility)^2$$

- The expected long run growth rates of different term bonds (rolled continuously) will differ
  - This is the case even if the arithmetic risk premium is set equal to zero

- If geometric risk premiums were constant across term, this would imply arbitrage
  - Portfolios of many bonds of different terms would have a higher geometric risk premiums than each standalone bond
  - If geometric risk premiums are constant, arithmetic risk premiums would be non linear across term
  - A long position of long term bonds with a short position in short term bonds could eliminate risk whilst generating excess returns.

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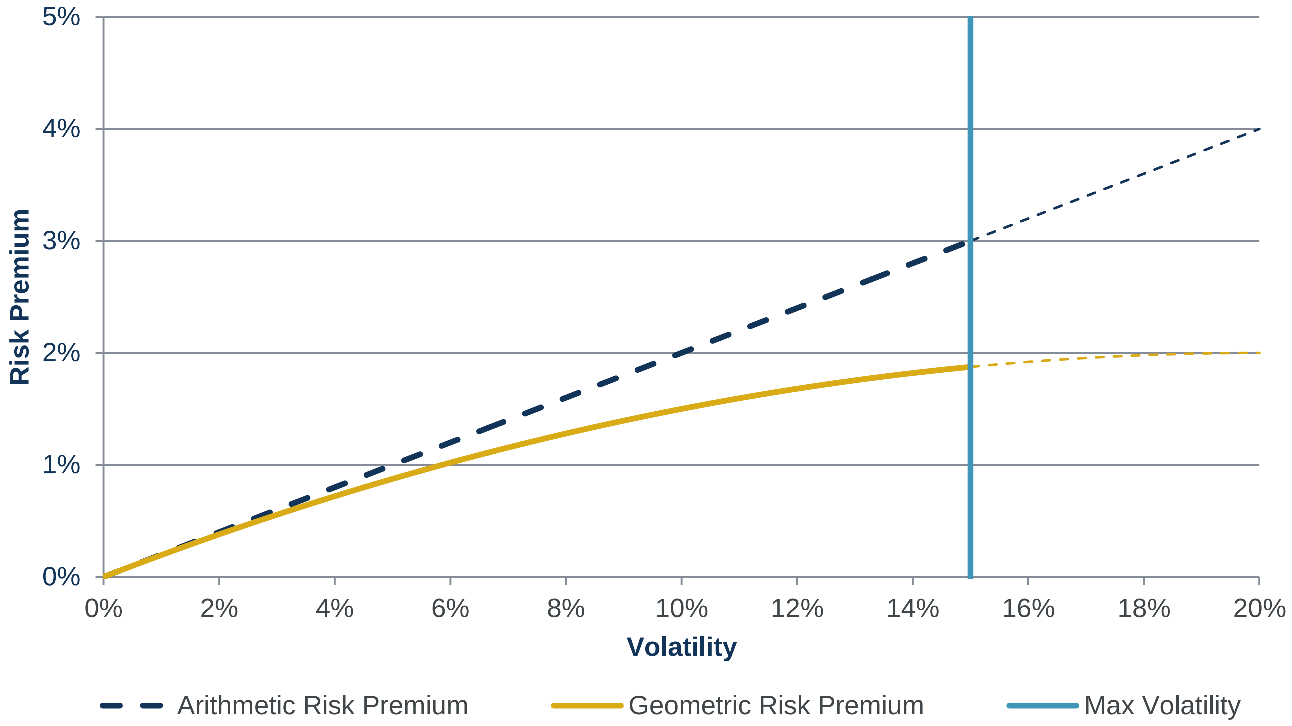
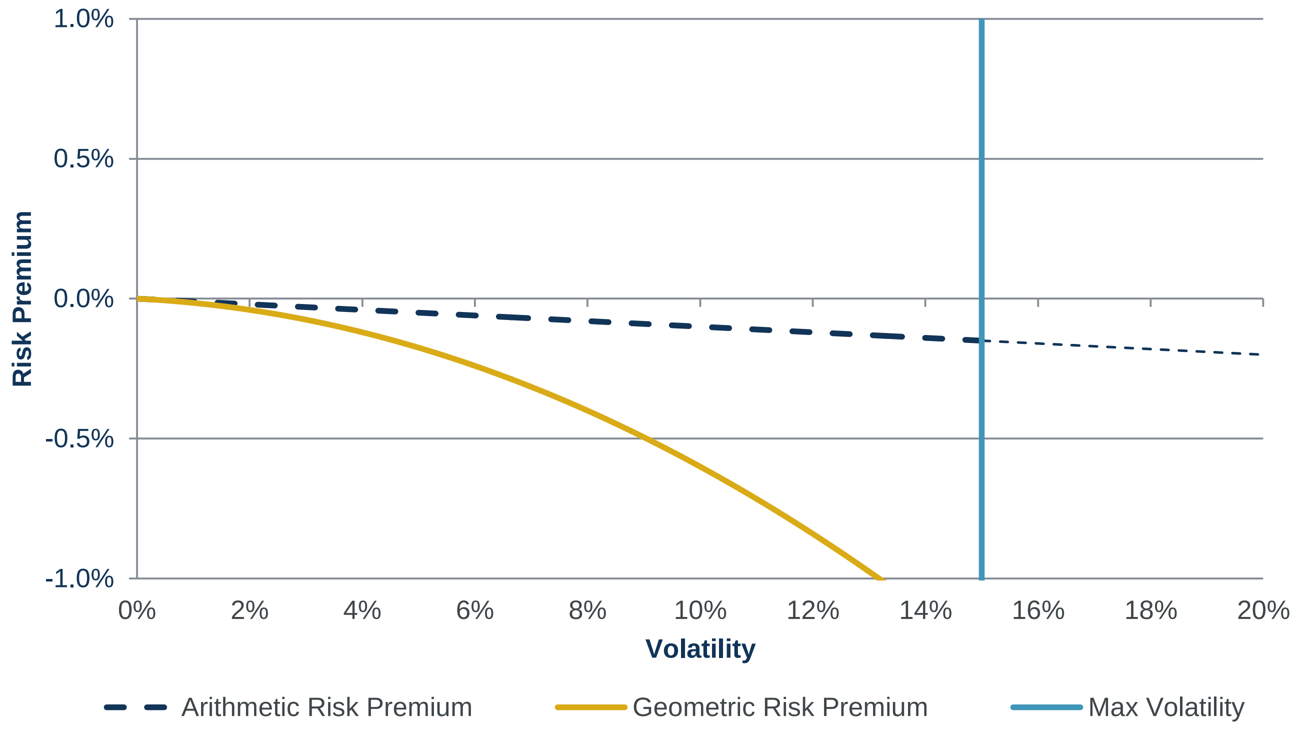
Appendix: Risk Premiums (continued)

Positive Arithmetic Risk Premium



Appendix: Risk Premiums (continued)

Negative Arithmetic Risk Premium



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Appendix: Risk Premiums (continued)

Term premium – Calibration

- Setting the (arithmetic) risk premium will be judgemental
  - We can set to zero, implying the expected wealth at any given point in time from investing in gilts will be equal regardless of the specific mixture of gilts held
  - We can choose an expected growth rate of bonds of two terms over the long run, and set the arithmetic risk premium so this is reflected in our model (which will in also determine the growth rate of all other standalone bonds)
  - We can choose a non – zero arithmetic risk premium to broadly target the slope of the initial yield curve
    - Taking into account the impact of the mean reversion level on the initial slope of the curve

Our preference is to set the arithmetic risk premium to zero (or a low number) as a default

In general, at the 20 year point in the curve market participants (predominantly Pension Schemes) are buying and selling to hedge rather than to generate excess growth. With no strong rationale to set this premium significantly above or below zero, we think it is safer to take no credit for a term premium and to discuss the potential return implications of leveraged LDI (for example) outside the model.

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