Institute and Faculty of Actuaries

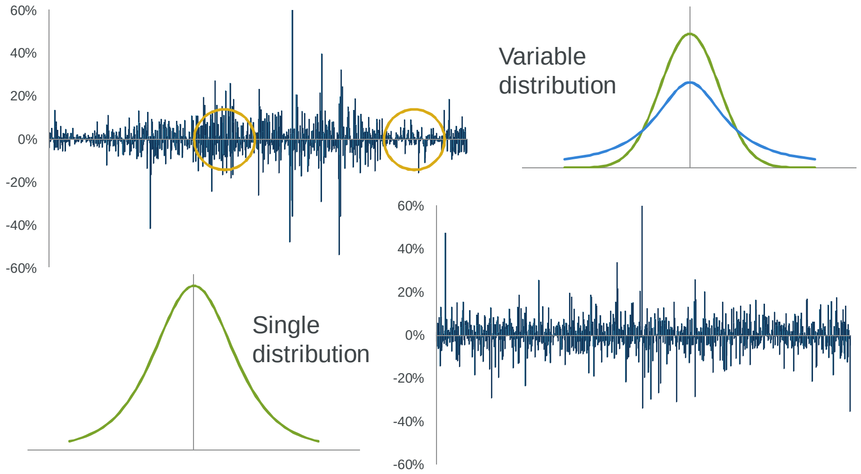
Time Series Models within a VaR framework

Peter Mason

10 November 2014

A time series suggests that the distribution of modelled variables is not I.I.D. through time but dependent on past values or behaviours. The last few years has seen a trend away from time series models towards single step distributions for SII and the purpose of this session is to consider the case for time series models.

Background



Top: Monthly changes in log AAA corporate credit spreads 1919-2012
Below: Simulated T_4 values calibrated to monthly changes in AAA credit spreads

10 November 2014 2

Your presenter today…

- Degree in Physics from Imperial
- Deloitte since 2010
- Part of the Capital Markets Group
- Background in risk modelling
- Banking and insurance industries
- Student member of IFoA

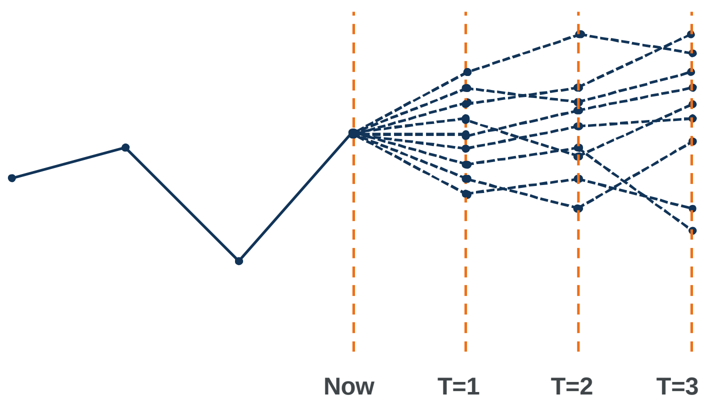
Agenda

- ➢ Why use time series?
- ➢ What sort of models?
- ➢ How to use them?
- ➢ Questions.

Why use time series

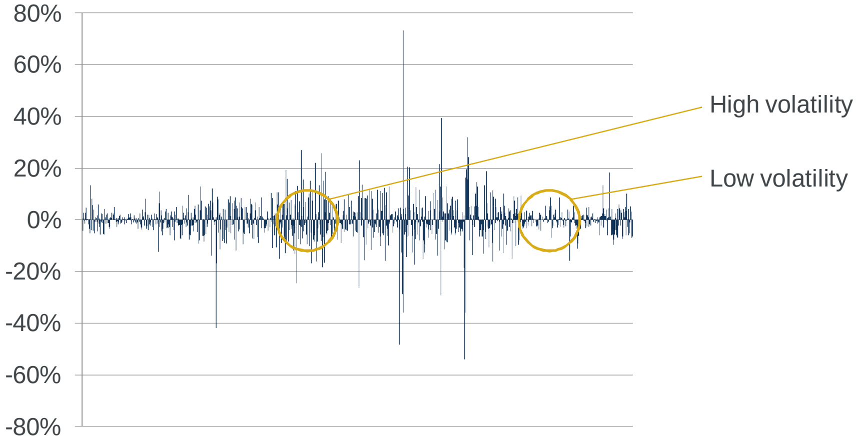
One use is multi-period modelling, perhaps for ORSA or business planning purposes.
In order to project over multiple time steps we need a time series model, even if this is just a series of independent steps such as a random walk.

Multi-period modelling



Perhaps we just want to better capture the risk over the next period given recent and past observations. This example of AAA credit spread movements shows periods of higher and lower volatility observed historically.

Current economic position



Monthly changes to log AAA credit spreads 1919-2012

This approach of modelling the current economic position is often referred to as a PIT model. There are arguments both for and against PIT models but the greatest is usually the subjectivity of the calibration.

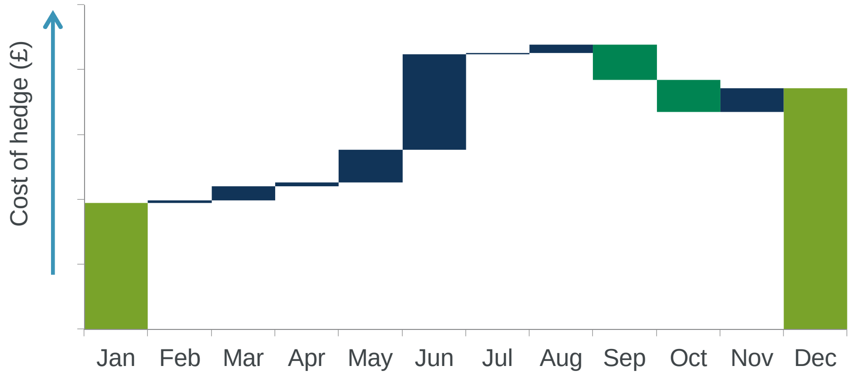
Current economic position

Point-in-Time (PIT):
- Conditional model
- Based on the “prevailing” risk environment

Through-the-Cycle (TTC):
- Unconditional model
- Based on “average” level of risk

Final example of use is intra-period modelling. Here we consider a period to typically be a year, so this is the modelling of risks on a monthly, weekly or even daily basis.
This could be used to quantify the cost or effectiveness of a particular hedging strategy.

Intra-period modelling



10 November 2014 9

What sort of models

Auto regression (including mean reversion) can be important for multi-year projections.
Efficient markets might suggest that such features should not exist but if they do then the impact on long-term projections could be significant.

Auto regressive

Wilkie's stochastic investment model: ARMA(1,0):

$$\nabla \ln Q(t) = QMU + QA(\nabla \ln Q(t-1) - QMU) + QSD.QZ(t)$$

AR with lag 1: Originally proposed for modelling inflation rates

Consider interest rate projections:

Might not detect regressive characteristics over short time horizons (1-2 years)…

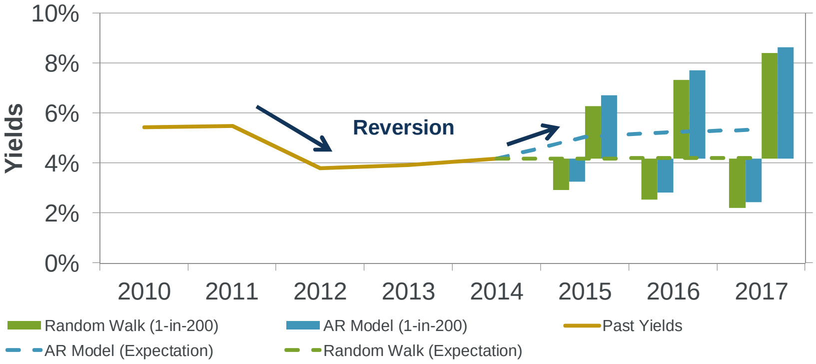
…But potentially very significant to projected stresses over longer time horizons (10-20 years)

We compare the "independent steps" or random walk approach to an autoregressive model.
10Y gov bond yields appear to exhibit significant auto regression with a 3 year lag. Over the first 3 years we see the AR model projecting higher expected yields than the random walk due to a recent decrease in yields.

Auto regressive

Model Yields as: $Y_t = Y_{t-1} e^{-X_t}$

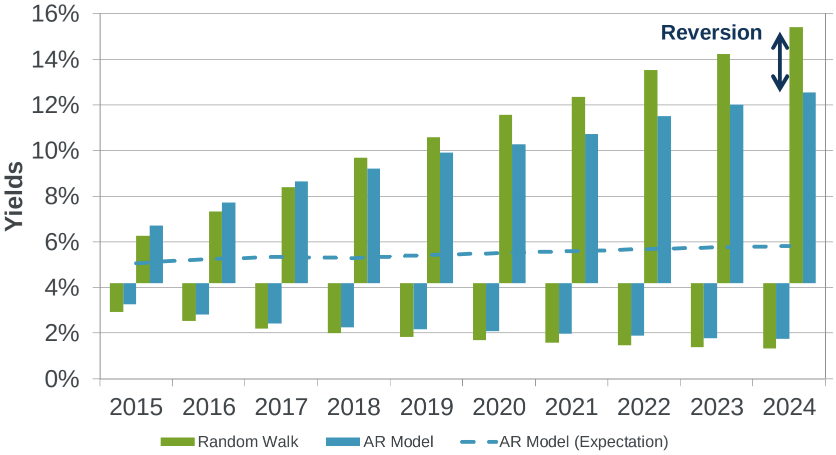
Random Walk: $X_t = 0.16z$ AR Model: $X_t = -0.24X_{t-3} + 0.15z$



Projections of expected and 1-in-200 stressed 10-year government bond yield.
Calibrated to Jan-Jan changes 1970-2014. Jan 2014 yield: 4.20%

If we consider the longer term projections we see a narrowing of the projected stresses due to the auto regression restricting the steady increase in stresses which is observed under the random walk model.
We also observe that the expected yield remains higher under the AR model than the random walk model.

Auto regressive



Projections of 1-in-200 stressed 10-year government bond yields.
Calibrated to Jan-Jan changes 1970-2014. Jan 2014 yield: 4.20%

10 November 2014 13

A strong mean reversion could explain big recoveries after big crashes, as have been observed historically, but so can a stochastic volatility.
Stochastic volatility can also explain the apparent fat-tails seen in a through-the-cycle model fit to historical data.

Stochastic volatility

Big crashes drive an increase in volatility
- Can lead to big recoveries
- Or further big losses

Apparent fat-tail distributions can also be described by stochastic volatility
- Even using a Normal distribution

Capturing stochastic volatility in capital models can lead to pro-cyclicality

GARCH(1,1) model fitted to monthly changes in log AAA credit spreads
- Can observe significant autocorrelation in volatility

$$x_t = \sigma_t z_t$$

$$\sigma_t^2 = 0.0133 + 0.4240\sigma_{t-1}^2 + 0.2809x_{t-1}^2$$

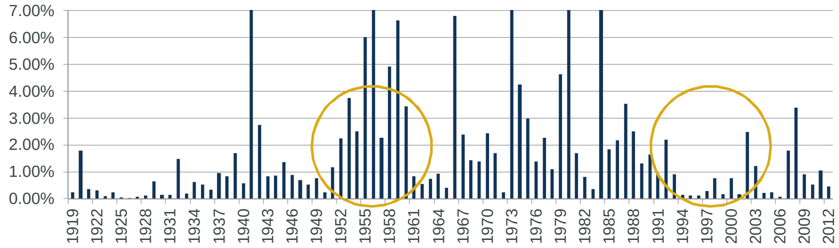
10 November 2014 14

We can consider these two different periods again. We can see that the 1950s appeared to have a much higher volatility than that of the late 1990s – a through the cycle capital model might have understated the risk during the 1950s and overstated it during the late 1990s.

Stochastic volatility

High volatility:
Expect larger changes (positive or negative)

Low volatility:
Expect more mundane movements



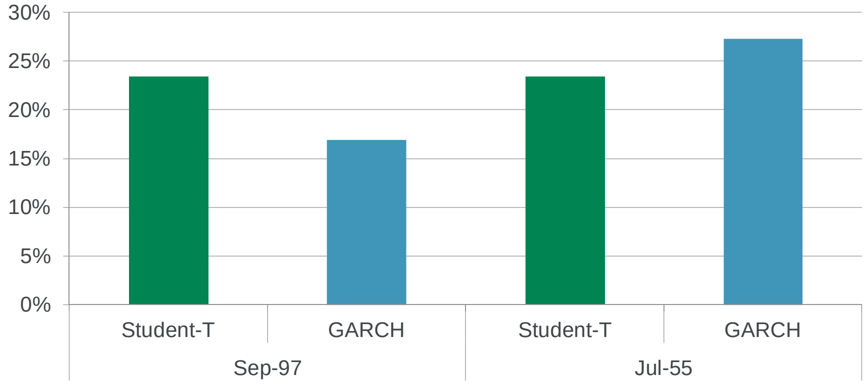
AAA credit spreads monthly log changes squared

10 November 2014 15

Using GARCH to backsolve for historical volatility we can see the 1-in-200 stresses that it would have predicted during those two contrasting period. A Student-T distribution closely matches empirical 1-in-200 but overstates stress in 1997 and understates stress in 1955.

Stochastic volatility

Backsolve for volatility at different historical time points:



Relative one month 1-in-200 stresses for AAA corporate credit spreads in 1997 & 1955 using T_4 and GARCH(1,1) models. Empirical 1-in-200 stress is about 23%

10 November 2014 16

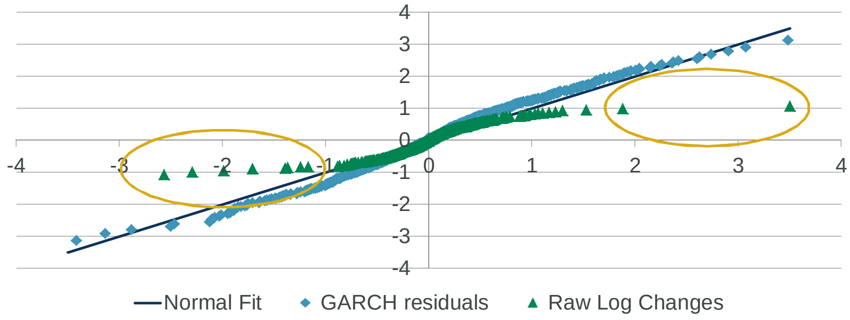
To demonstrate that a Normal distribution is appropriate, a Q-Q plot of the model residuals.
We observe a small skew in residuals which is not being captured but we have removed the fat-tails that you see in the raw data.

Stochastic volatility

Fit of the GARCH model using a Q-Q plot of the residuals (z) against a Normal (0,1) distribution: $x_t = \sigma_t z_t$

$$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2 + \beta x_{t-1}^2$$

Compare to fit of 'raw' log changes which have fat tails



Jumps in a process may be difficult to distinguish from stochastic volatility but if quantifying hedging efficiency then the inclusion of jumps instead of stochastic volatility may have a considerable effect.
In a BS world, hedging can be very efficient (perfect if continuously rebalanced).

Jump models

Difficult to distinguish empirically from stochastic volatility

Can lead to very different results

Paths exhibit discontinuities

Consider the impact on hedging efficiency

How to use them

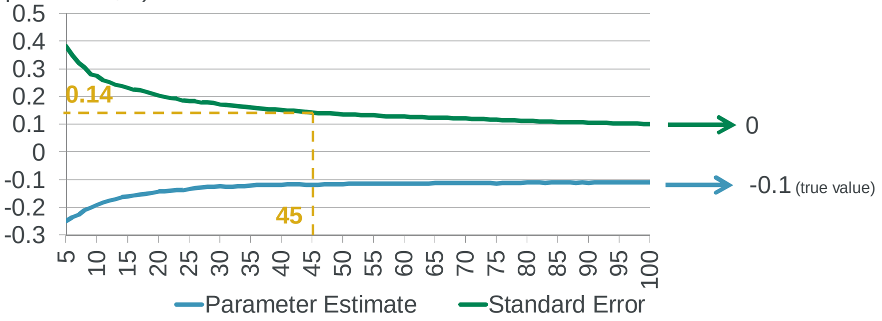
One of the most important aspects of calibration is whether you can rely on an estimate. Simulating an AR1 process of different lengths shows how the error in the parameter estimate reduces with the length of the time series available.

Parameter Error

AR1: $X_t = \alpha X_{t-1} + \sigma z$

Sample autocorrelation (method of moments estimate)

Error depends only on number of data points (not volatility, σ, or size of parameter, α) – but downward bias



Compare the method of moments approach to an MLE approach.
MLE provides an unbiased estimate but is no more accurate. Error is about 0.14 for about 45 reducing to 0.1 for time series of 100 steps.

Parameter Error

Instead consider MLE approach: $X_t = \alpha X_{t-1} + \sigma z$

Unbiased estimate but no more accurate

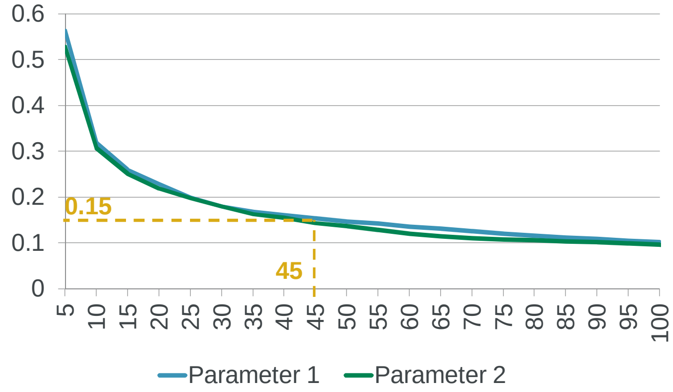
Might believe that a more parameterised model has higher error .
True for very short time series but still only about 0.15 for 45 and 0.1 for a time series of 100 steps.
Also observe that the error in each parameter is approximately the same.

Parameter Error

Error for AR2 parameters: $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \sigma z$

Slightly larger for small samples

Very similar standard error in each parameter estimate

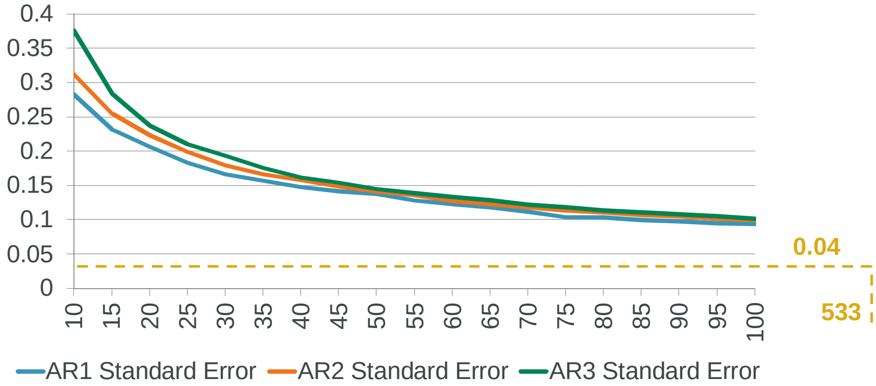


Just for completeness we compare the standard error for the parameters in AR1, 2 & 3.
As expected they diverge for small samples but the error is below 0.15 for sample sizes in excess of 50.

Parameter Error

Comparison of parameter error between AR1, AR2, AR3

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_3 X_{t-3} + \sigma z$$

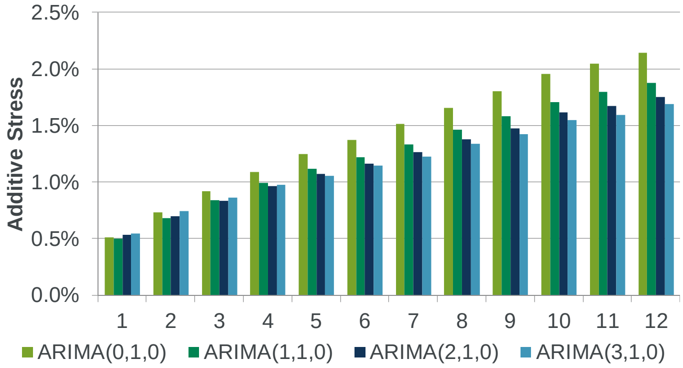


10 November 2014 23

Next we should consider the impact of using a time series – given that we can reliably estimate the parameters for a time series of 533 data points, we should compare the outcomes using different parameterisations.
We can see that each new parameter reduces 1-in-200 stress but how can we decide where to draw the line?

Model Choice

What is the impact of different model choices and how can we choose between them?



AR (x)	Param Value	Error
1	-0.11	±0.04
2	-0.13 -0.08	±0.04
3	-0.13 -0.09 -0.07	±0.04

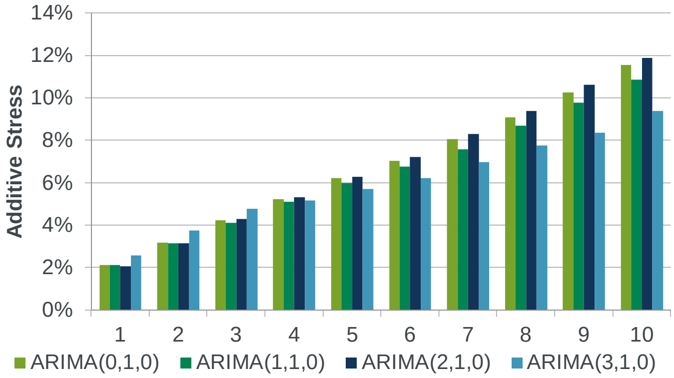
Monthly projections of 1-in-200 stress for 10-year government bond yields (calibrated to end-Sept using data from 1970)

10 November 2014 24

When considering annual projections the shorter sample makes parameter error more significant. It appears that the first significant parameter is the 3-year lag, but again the justification for this is not clear.

Model Choice

Differences between calibrating to monthly vs annual data:



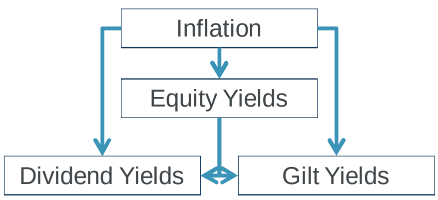
AR (x)	Param Value	Error
1	-0.04	±0.14
2	0.003 0.04	±0.15
3	0.05 0.001 -0.25	±0.15

Annual 1-in-200 stress projections of 10-year government bond yields calibrated to Jan-Jan changes 1970-2014

Most capital models, for example, are going to include more than a single source of risk. The challenge, therefore, is to not just model these risks in isolation, but identify a structure for modelling them consistently together.

Risk Combinations

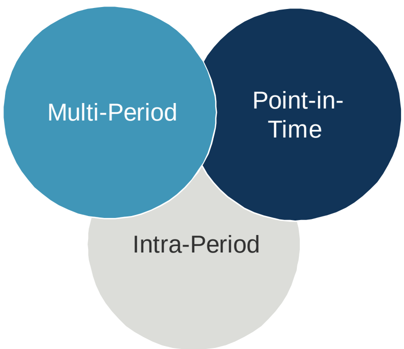
Perhaps use a ‘cascade’ structure similar to that proposed by Wilkie



Determining the structure requires a degree of judgement

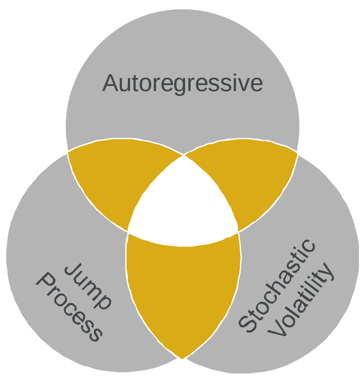
Summary

Summary

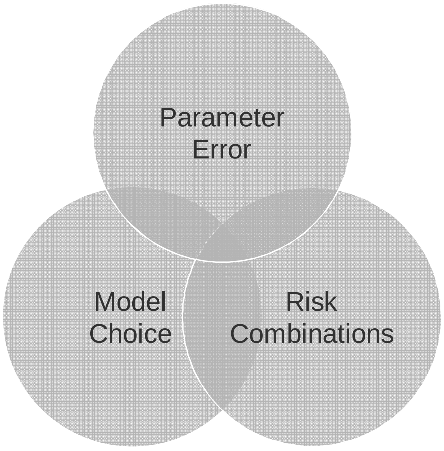


Different modelling purposes

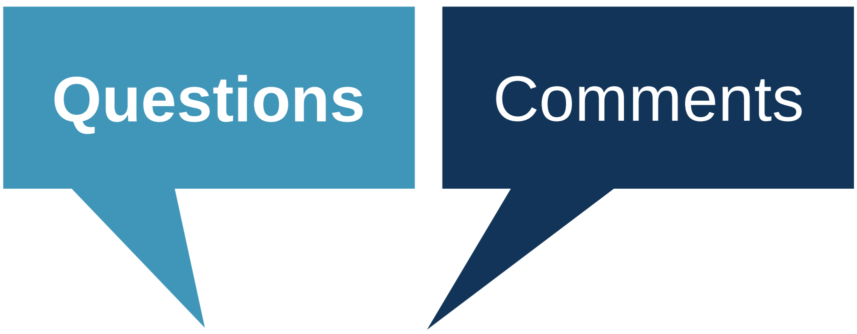
Models with appropriate features



Summary



Practicalities of using time series models



Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.