The Actuarial Profession
making financial sense of the future

2003 Pensions Convention

1 - 3 June
Grand Hotel, Brighton

Overview

- What the equity risk premium is?
- How to derive a risk-adjusted discount rate?
- Why do financial economists assume equities return the same as bonds?
- How you would value an LPI liability without the assistance of GN27?
- Why don't insurers give much better annuity terms?

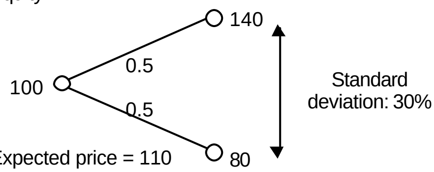
Why do this?

- Pensions are about valuing [illegible]
- We (usually) hold [illegible] liabilities
    - These [illegible] rates
- What is t[illegible] ties
    - Take acc[illegible] ed.

deflators
deflators
deflators

Simple model

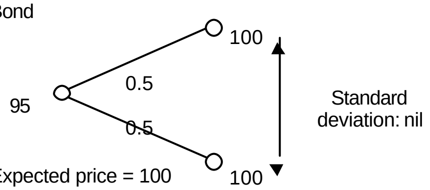
- Equity



- Expected price = 110
- Expected return = 10%

Certain payout

- Bond



- Expected price = 100
- Expected return = 5%

Pricing (valuation) methodology

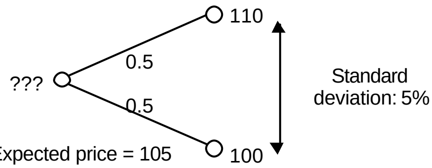
- Estimate expected return
- And discount expected payout at this rate, ie

$$100 = \frac{110}{(1+10\%)}, \qquad 95 = \frac{100}{(1+5\%)}$$

- Problem - how do we estimate the expected return for a particular cashflow profile?

Example

- Cashflow



- Expected price = 105
- Expected return = ??

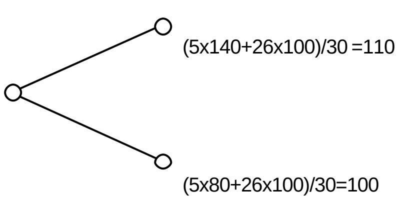
Solution

- Can find a solution by interpolation
- Standard deviation nil gave expected return 5%
- Standard deviation 30% gave expected return 10%
- So estimated price is

$$\frac{105}{\left(1+5\%+\frac{5}{30}5\%\right)}=99$$

Arbitrage approach

- 5/30 x Equity + 26/30 x Bond is:

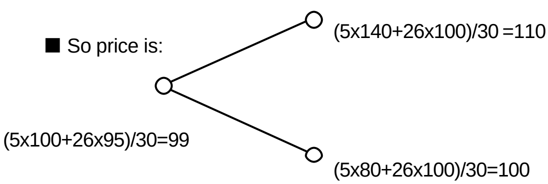


Arbitrage argument

- Have two portfolios which give identical payouts
  - The example asset (payouts = 110, 100)
  - The equity/bond portfolio (5/30 : 26/30)
- Price must be the same
  - Otherwise you're placing a negative value on a portfolio which always gives positive payouts
  - (Sell the dearer portfolio and buy the cheaper)
  - Which is silly QED

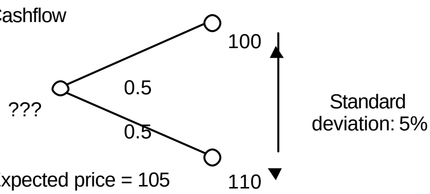
Arbitrage approach

- 5/30 x Equity + 26/30 x Bond is:
- So price is:



Change the example

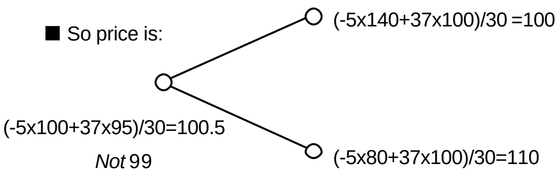
- Cashflow



- Expected price = 105
- Expected return = ??

Arbitrage approach

- -5/30 x Equity + 37/30 x Bond is:
- So price is:



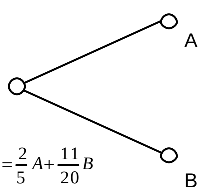
Conclusions so far

- Expected returns are difficult to predict
- …so valuation appears to be a hard problem
- But arbitrage technique is very powerful
- …so valuation becomes an easy problem

General solution for binomial example

- Can construct any cashflow from a suitable equity / bond portfolio
  - (A-B)/60 equities
  - (7B-4A)/300 bonds
- Arbitrage argument implies:

$$\text{Price} = 100\frac{(A-B)}{60} + 95\frac{(7B-4A)}{300} = \frac{2}{5}A + \frac{11}{20}B$$



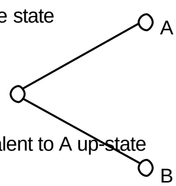
Multiple viewpoints for solution

- State prices
- State price deflator
- Risk neutral pricing

State prices

- Price of a cashflow in exactly one state
- State prices here are:
  - 0.4
  - 0.55

0.4A + 0.55B

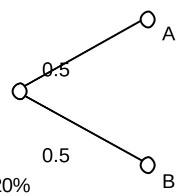


- A cashflow profile (A,B) is equivalent to A up-state assets and B down-state assets
  - So price is 0.4A + 0.55B
- Easy to calculate, understand
- Problem when moving to continuous-states

State price deflators

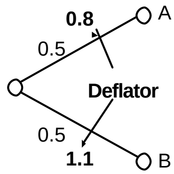
- Reintroduce the probabilities

0.4A + 0.55B



- Effective up-state discount rate 20%
- Effective down-state discount rate -10%

State price deflators



- Deflator = State price ÷ Probability
- Can re-write price formula
- Price = expected value of (Deflator x Cashflow)

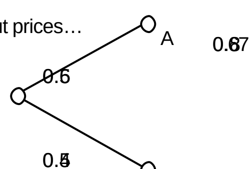
Deflator formalism

- Started with Expected return same as Discount
  - Ie value now = Discount x Expected value (Cashflow)
- Now have Expected value (Discount x Cashflow)
- Deflator is a *stochastic discount function*
  - Deflator takes a different value in each future state
- Stochastic scenario generator
  - Generates asset prices etc in each scenario
  - Generate deflators as well…
  - … and then any cashflow can be valued

Different investors

- Investors agree about prices…
- …and scenarios

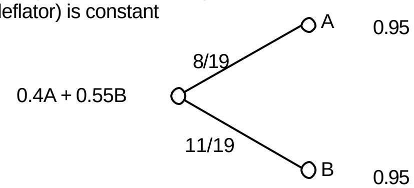
0.4A + 0.55B



- But not probabilities
- So they will use different deflators

Risk-neutral investor

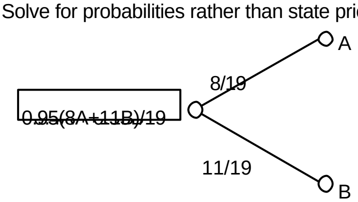
- For a particular choice of probabilities, discount factor (deflator) is constant



- Apply *same* discount rate to *all* cashflows
- Hence such an investor is *risk neutral*

Risk neutral valuation

- Solve for probabilities rather than state prices

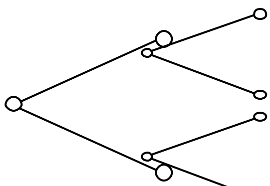


- Value = expected payout *under risk neutral probabilities*, discounted *at risk-free rate*

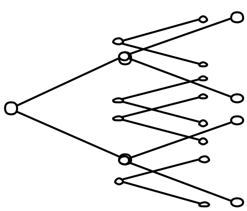
Risk-neutral valuation: notes

- ‘Real-world’ probabilities are lost
- Expected return on all assets the same under risk-neutral probabilities
- Mathematical trick, *not* equivalent to a claim that all assets expected to give same return in real world

Bigger models

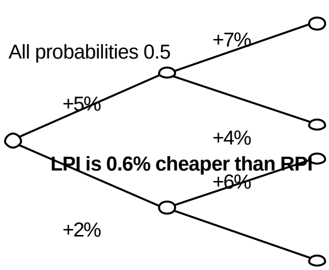


Bigger models



Inflation example

All probabilities 0.5



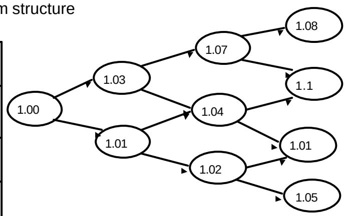
	RPI		LPI
	12%	0.83	10%
	9%	0.86	9%
	8%	0.91	7%
	4%	0.96	4%
Price of RPI-linked cashflow:	**0.962**		**0.956**

LPI is 0.6% cheaper than RPI

And bigger trees…

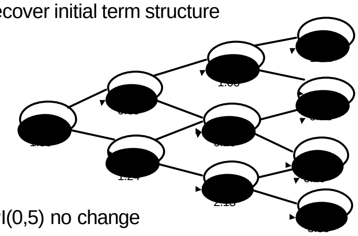
- RPI cumulative index
- Calibrate using term structure

Time	Real	Nominal
1	2%	3.75%
2	2%	4%
3	2%	4%



And bigger trees…

- Recover initial term structure



RPI index deflator

- LPI(0,5) no change
- LPI(0,3) max payout 9.3% => LPI .1% cheaper
- LPI(3,5) min payout 9.3% => LPI 3.4% more expensive

In the limit move to normal distribution

- Use term structures to set implied inflation
- Calculate prices relative to full RPI over 3 years

LPI	tree	LN model
(0.5)	0.0%	0.3%
(0.3)	-0.1%	-0.5%
(3.5)	+3.6%	+2.9%

...and then can generalise

- Any time period
- Cumulative LPI or annualised LPI

- …but will always need an inflation volatility assumption

Overview

- What the equity risk premium is? Don't care
- How to derive a risk-adjusted discount rate?
  Use risk free rate or deflators
- Why do financial economists assume equities return the same as bonds? A maths trick, they don't believe this
- How you would value an LPI liability without the assistance of GN27? Use a deflator model
- Why don't insurers give much better annuity terms?
  Because they are aware of the market consistent prices for providing annuities.

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