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GENERAL INSURANCE PRICING SEMINAR

13 JUNE 2008, LONDON

Demand Modelling in Personal Lines

James Tanser
Watson Wyatt Limited

Agenda

- Motivation
  - What makes a demand model special?
  - Why are we interested?
- Tools
  - Linear and non-linear models
  - Continuous variables
- Challenges
  - Market price
  - Aggregators

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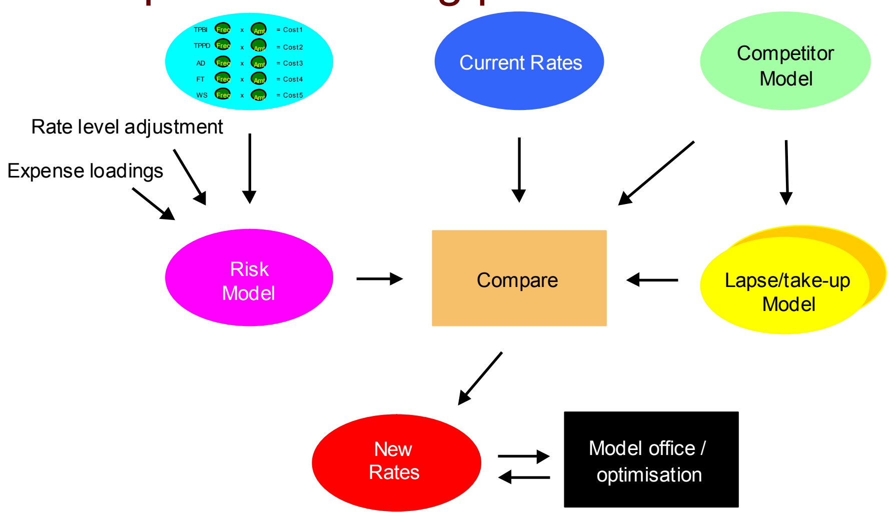
The premium rating process



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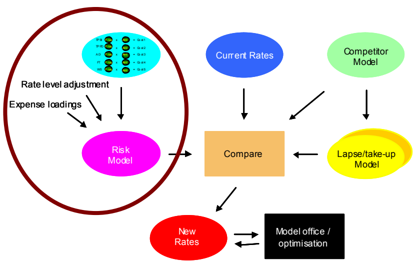
What makes demand models special?

- A demand model:
  - Looks at customer purchasing behaviour
  - Key element is market price
- Retention models are easier
  - Can get away with only old and new premiums
  - New business as proxy for market premium
- Probability of purchase may be low

Why are we interested?

- We understand risk
  - GLM modelling over 10 years old
  - Data is clean and reliable
  - We know the interactions to look for



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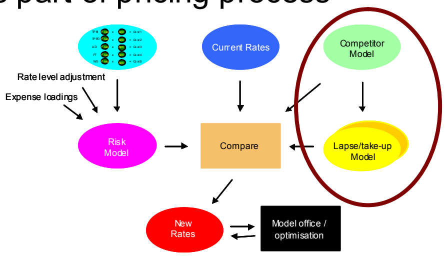
Why are we interested?

- We have a handle on retention
  - Data is collected
  - Standard models used
  - Price change understood
  - Integrated as part of pricing process



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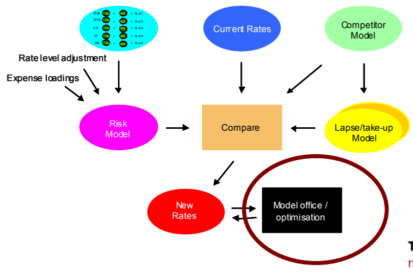
Why are we interested?

- Customer demand is last ingredient
  - Some data is collected – some is missing
  - “Hot money”
  - Lifestyle changes
- Price optimisation requires model which are:
  - Accurate
  - Up to date



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GIRO working party

- Chairman
  - James Tanser (Watson Wyatt)
- Members
  - John Light (RSA)
  - Owen Morris (NU)
  - Sophia Mealy (AON)

GIRO Working party

- Provide an introduction to the topic describing the terms used
- Summarise the current methodologies used in the market
- Summarise possible alternate methodologies identified by a search of available literature
- Investigate several methods using agreed methodology to determine the descriptive and predictive power of the methods when applied to actual insurance data
- Provide a brief conclusion and highlight areas for further work.

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Generalised linear models

$$E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi})$$

$$Var[\underline{Y}] = \phi.V(\underline{\mu}) / \underline{\omega}$$

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Typical model forms

Y	Claim frequency	Claim number	Average claim amount	Probability (eg lapses)
g(x)	ln(x)	ln(x)	ln(x)	ln(x/(1-x))
Error	Poisson	Poisson	Gamma	Binomial
ϕ V(x)	1 x	1 x	estimate x^2	1 x(1-x)
ω	exposure	1	# claims	1
ξ	0	ln(exposure)	0	0

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Models

- Generalised linear models cope well with most common requirements
- A logistic model is most appropriate
  - considers log(p / [1-p]) with binomial error
  - maps [0,1] to [- ∞, ∞]
  - invariant to whether you model success or failure
- If lapses are low and results not to be used directly, a Poisson multiplicative model can help
  - theoretically wrong (can predict multiple lapses), but easier to communicate

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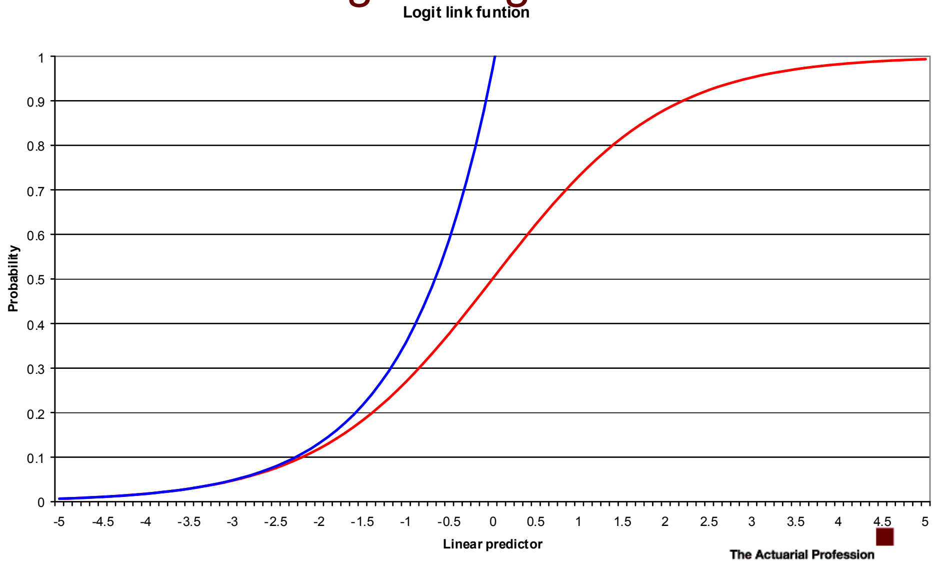
Understanding the logit transform



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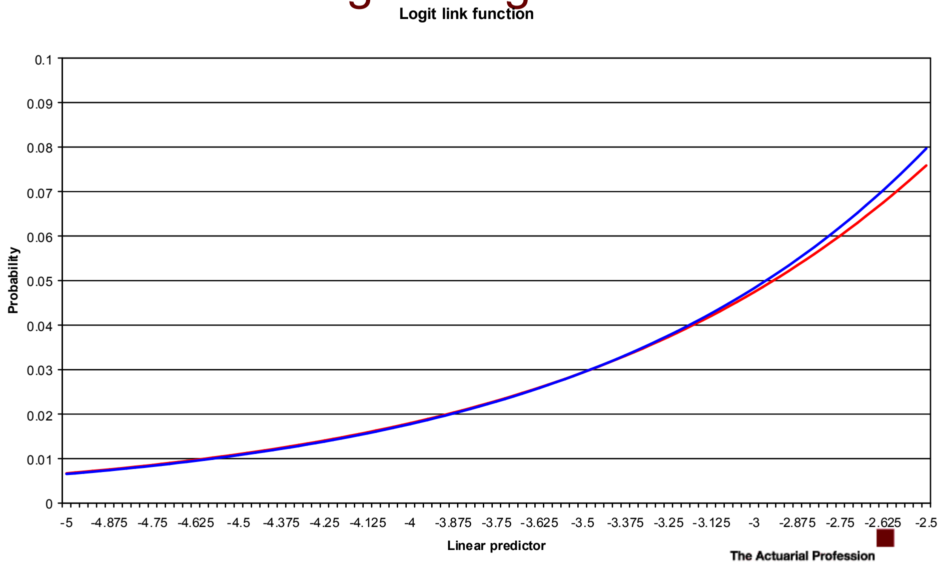

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Other models

- Could try:
  - Binomial / log link
  - Binomial / probit link
  - Binomial / complementary log-log link
- Transform the data
  - Sampling
- Working party is looking at these

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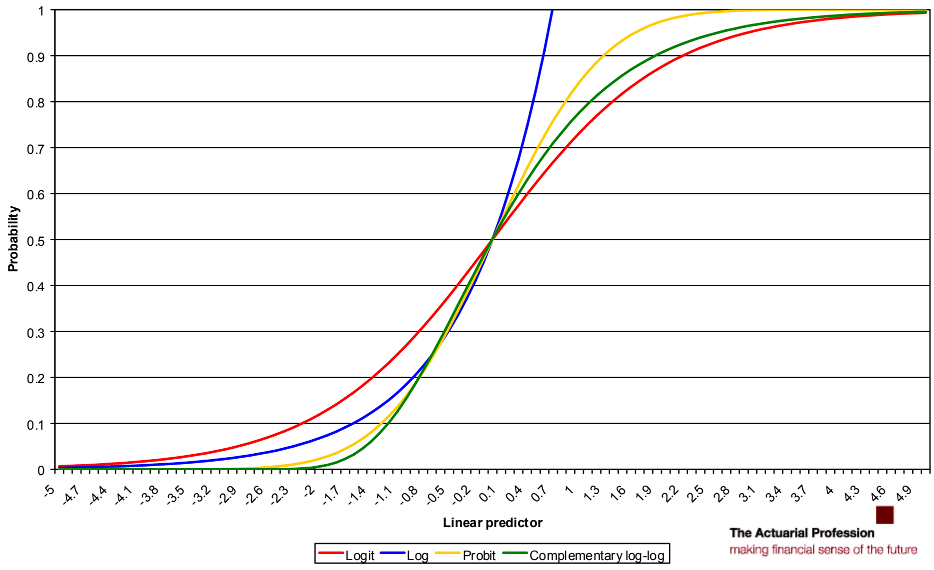
Link functions



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Sampling

- Take 100% of conversions and x% of others
- In theory, makes not difference to binomial/logit models
- Questions:
  - What rate should be targeted?
  - Predicative versus Descriptive
  - What about other links?

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Continuous variables

- Why continuous?
- Model form
- Continuous variables in a GLM framework

Why continuous?

- Key drivers of demand:
  - Own price
  - Market price
  - Interaction between the two
- Best modelled continuously
- Price sensitivity not necessarily the same everywhere…

Model form:
Linear versus non-linear

- Varying views:
  - Simplistic
  - Complex linear
  - Non-linear
- Consider relative competitiveness as an example
  - Our price / market price

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Simplistic

- Treat as variate, assume linear

$$\mu = g^{-1}(\Sigma X_j \beta_j + c(p/m))$$

- Assumes "same" prices sensitivity everywhere
  - Logit link => Lower probability individuals more sensitive ($\Delta p/p$ larger)
- No-one does this, but helpful to understand issues

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Complex linear

- Use a continuous function of competitiveness

$$\mu = g^{-1}(\Sigma X_j\beta_j + c_k.f_k(p/m))$$

- Function is polynomial or spline
- Can interact with other variables to achieve range of shapes
- Simple to apply with existing tools

Non-linear

- Linear in competitiveness, gradient varies by segment

$$\mu = g^{-1}(\Sigma X_j\beta_j + (p/m).\exp(\Sigma Z_j\gamma_j))$$

- Similar issues to simplistic, but locally OK
- Hard to fit due to co-linearity of parameters

What is best approach?

- More research needed
- Working party looking at variations to test predictive power
- In our survey, 4 times as many people (12) used complex linear than non-linear (3)

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Continuous variables in a GLM framework

- Variates allow each unique data value to have a different effect on the linear predictor, but force some smoothness
- Even detailed discrete treatment can produce odd results
- In practice implemented via:
  - Polynomials

  or
  - Splines

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Polynomials

- Include powers of the variate in the model
  - One parameter for each power
  - Can scale variates to avoid large (small) values
  - Can defined orthogonal polynomials to reduce correlation
- Extrapolated values may not be sensible

Design matrix
Polynomial

18	M
20	F
22	F
24	M
26	M
28	M
30	F
32	F
34	M
36	F

1	18	324	5832	104976	…	1
1	20	400	8000	160000	…	0
1	22	484	10648	234256	…	0
1	24	576	13824	331776	…	1
1	26	676	17576	456976	…	1
1	28	784	21952	614656	…	1
1	30	900	27000	810000	…	0
1	32	1024	32768	1048576	…	0
1	34	1156	39304	1336336	…	1
1	36	1296	46656	1679616	…	0

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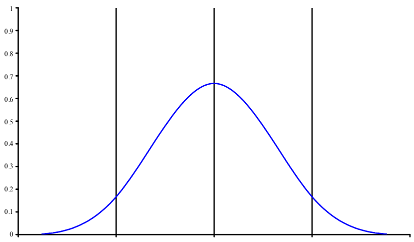
Regression splines

- Include a number of spline basis function in the model
  - Number of parameters depends of type of spline and number of knots
  - Sensible choice of basis function (eg B-splines) ensures values in [0,1]
- Can specify type
  - Order of spline
  - Type of extrapolation

B-Splines

- Set of basis functions usually covering four segments (defined by five knots)
- Each function is itself a cubic spline



- Each basis function has the same shape, except for the three basis functions at each extreme which occupy fewer than four segments

Spline formula

$$\frac{(x-t_i)^3}{(t_{i+1}-t_i)(t_{i+2}-t_i)(t_{i+3}-t_i)}$$

$$\frac{(x-t_i)^2(t_{i+2}-x)}{(t_{i+2}-t_i)(t_{i+2}-t_{i+1})(t_{i+3}-t_i)} + \frac{(x-t_i)(x-t_{i+1})(t_{i+3}-x)}{(t_{i+3}-t_i)(t_{i+2}-t_{i+1})(t_{i+3}-t_{i+1})} + \frac{(x-t_{i+1})^2(t_{i+4}-x)}{(t_{i+2}-t_{i+1})(t_{i+3}-t_{i+1})(t_{i+4}-t_{i+1})}$$

$$\frac{(x-t_i)(t_{i+3}-x)^2}{(t_{i+3}-t_i)(t_{i+3}-t_{i+2})(t_{i+3}-t_{i+1})} + \frac{(x-t_{i+1})(t_{i+3}-x)(t_{i+4}-x)}{(t_{i+3}-t_{i+2})(t_{i+3}-t_{i+1})(t_{i+4}-t_{i+1})} + \frac{(x-t_{i+2})(t_{i+4}-x)^2}{(t_{i+3}-t_{i+2})(t_{i+4}-t_{i+2})(t_{i+4}-t_{i+1})}$$

$$\frac{(t_{i+4}-x)^3}{(t_{i+4}-t_{i+3})(t_{i+4}-t_{i+2})(t_{i+4}-t_{i+1})}$$

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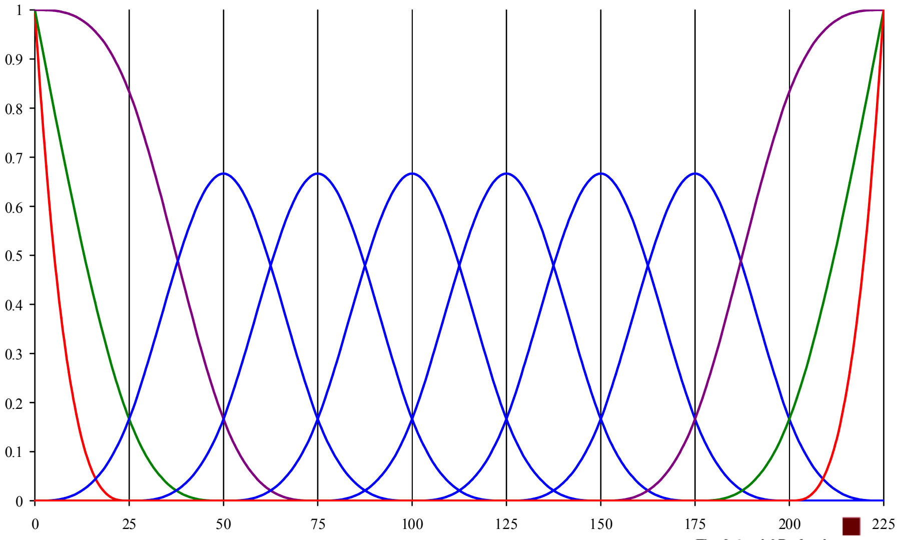
B-Splines – cubic example



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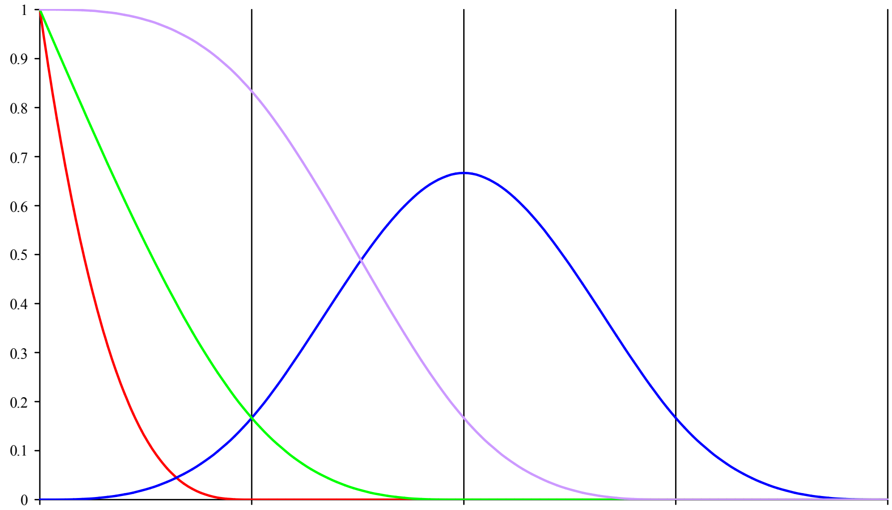
B-Splines – quadratic extrapolation



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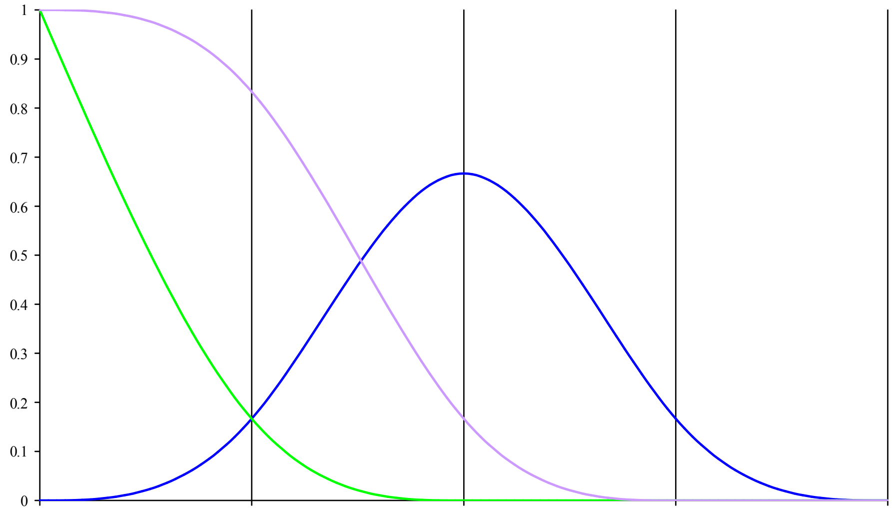
B-Splines – linear extrapolation



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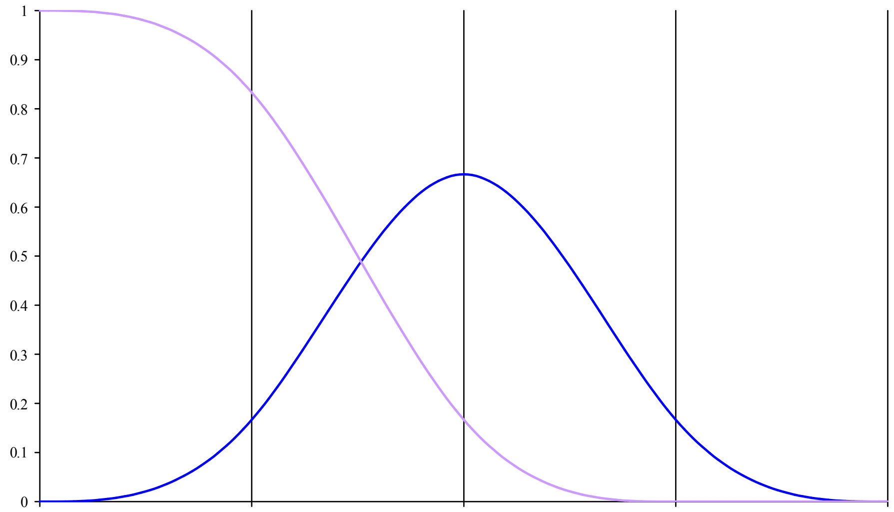
B-Splines – constant extrapolation



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Design matrix
Spline

18	M
20	F
22	F
24	M
26	M
28	M
30	F
32	F
34	M
36	F

$\Rightarrow$

1	1	1	0	0	…	1
1	0.52	0.98	0.02	0	…	0
1	0.17	0.83	0.17	0	…	0
1	0.02	0.5	0.48	0.02	…	1
1	0	0.17	0.67	0.17	…	1
1	0	0.02	0.48	0.48	…	1
1	0	0	0.17	0.67	…	0
1	0	0	0.02	0.48	…	0
1	0	0	0	0.17	…	1
1	0	0	0	0.02	…	0

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Splines

- Practical way of modelling continuous variables
- Often better than polynomials
- Increases complexity, therefore best used
  - when it is important that rates vary continuously with a variable
  - when modeling elasticity to be used in price optimisation analyses

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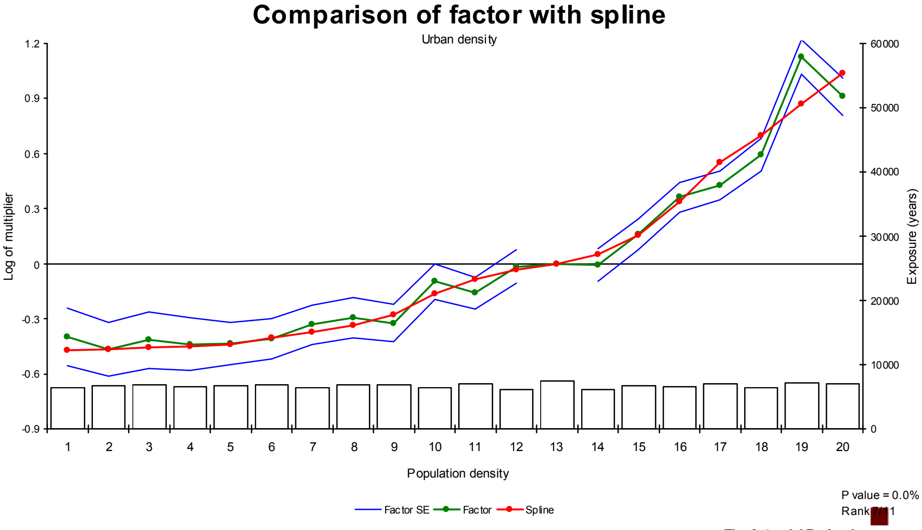
Example



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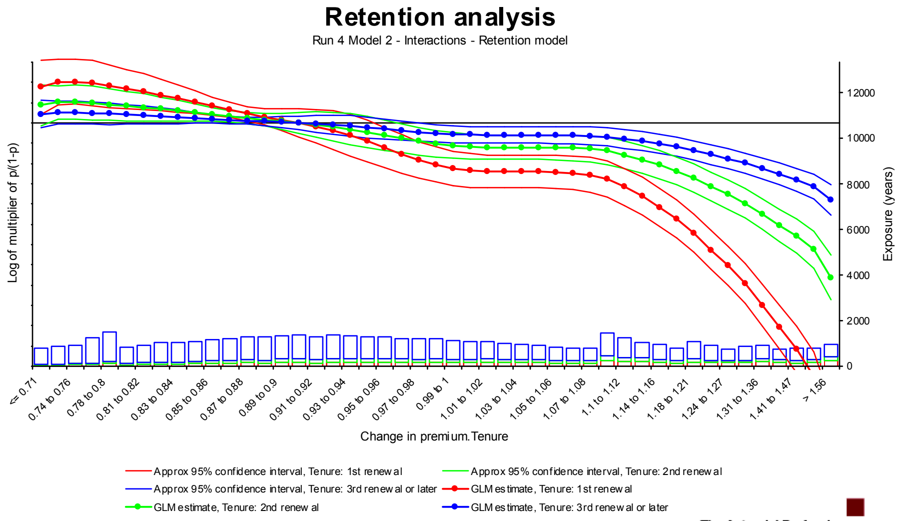
Example retention elasticity curve



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Market price

- Key to getting good model
- Hard to get hold of
  - Rates are not published
  - Rates change – daily!

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Sources of information

- WhatIf?
- Direct questioning of callers
- Mystery shopping
- Conversion rates (market temperature)
- Ranking from aggregator sites

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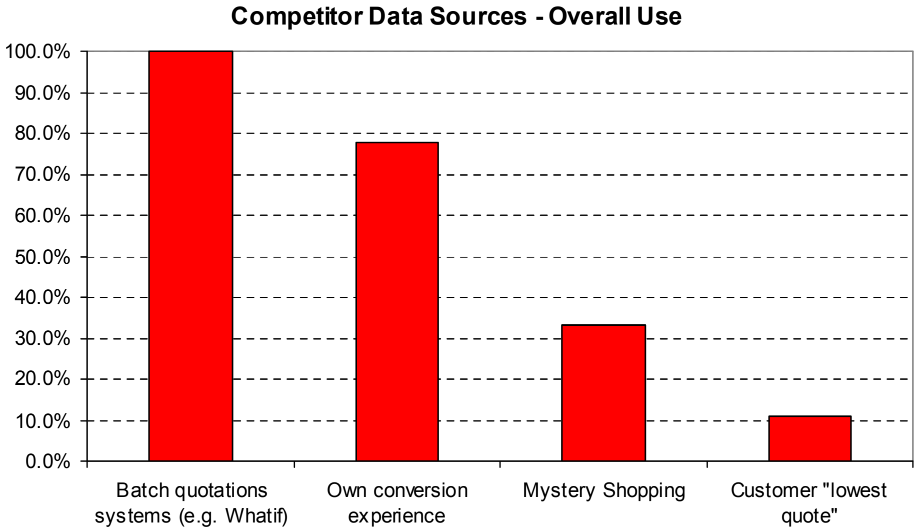
Sources of information



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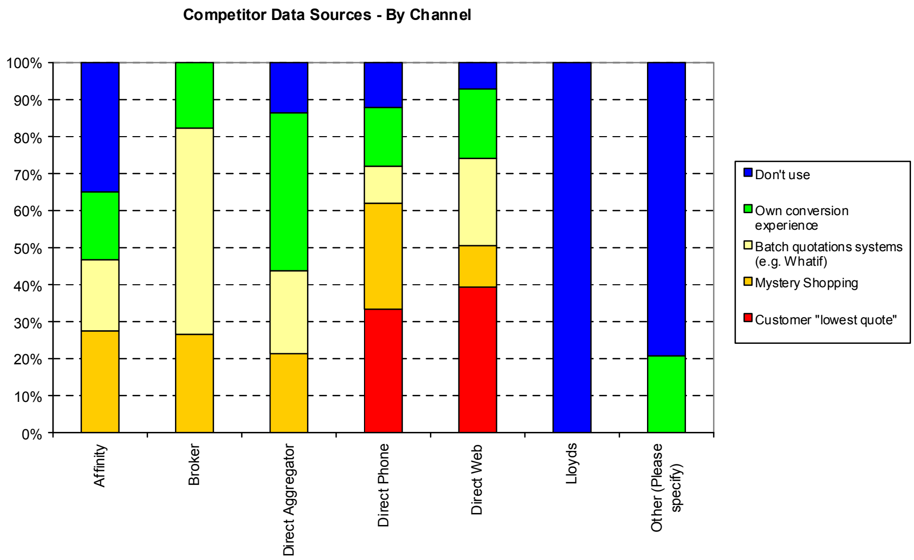

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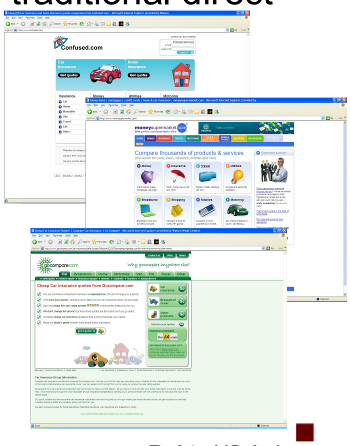
Aggregators

- Drives cart and horse through traditional direct model
  - Fixed cost per converted policy
  - Looks like brokerage?
  - Two sets of commission
- Data issues:
  - Limited information
  - Time constrained
  - Cannibalisation
  - Low conversion rate (1%?)



Aggregators

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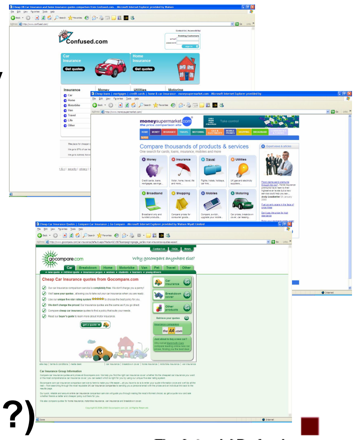

Aggregators

- Low probability models present special challenges
  - Selection of model
  - Data volumes
- Working party is looking at these issues
- Ranking is of paramount importance…
- … but so is brand

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Conclusion

- Interesting area with many challenges
- If you get it right, it can give a significant competitive advantage

- Come and see the GIRO workshop on Demand Modelling!

Contact details

- James Tanser
- Watson Wyatt Limited
- +44 1737 274249
- james.tanser@watsonwyatt.com

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