## **Swiss Re Investors**



DFA and Allocation of Cost of Capital Brian White, ALM Manager Europe 2001 Joint GIRO/CAS Convention Glasgow, October 2001

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# **Agenda**

- Introduction
- What is a 'good' risk measure?
- Allocation of the cost of capital
- Conclusions

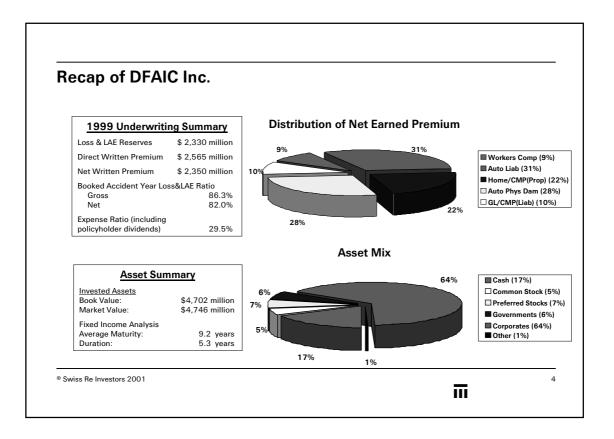
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## Introduction

- CAS DFA Seminar in Boston in Spring 2001
- CAS created an example company 'DFAIC Inc.' and asked a number of questions regarding:
  - Capital (allocation, adequacy)
  - Returns and risk for the lines of business
  - Suitability of reinsurance programme
  - Efficiency of asset allocation



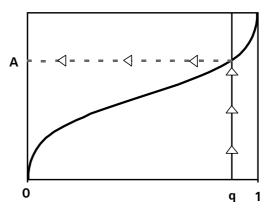
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# Risk measures - Probability of ruin

In the banking industry also known as Value at Risk (VaR)

<u>Specific Sense:</u> Probability capital falls below zero.

General Sense: The corresponding value, for a selected qth percentile tolerance, for a given financial variable.



The curved line is the inverse cumulative distribution function for a given risk variable

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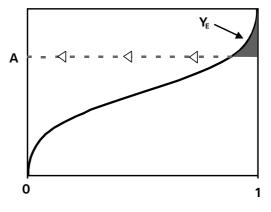
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# Risk measures - Expected policyholder deficit

Unlike VaR, EPD takes into account the magnitude of insolvency.

<u>Specific Sense:</u> The amount or percentage of total obligations that will not be met.

General Sense: The corresponding value, for a selected tolerance percentage Y<sub>E</sub> of all summed potential outcomes, for a given financial variable.



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## Coherent risk measures

#### **Definition of terms:**

• X<sub>i</sub> represent portfolios of risks (think of it as the liabilities of a particular insurance company)

•  $\alpha$  be some constant

•  $\rho(*)$  be a function that assigns (not the same as the variability a value of risk to a portfolio of  $X_i$ )

## Four axioms that define a coherent risk measure:

•  $\rho(X + \alpha) = \rho(X) + \alpha$  Translation invariance

•  $\rho(X_1+X_2) \le \rho(X_1) + \rho(X_2)$  Subadditivity

•  $\rho(\alpha X) = \alpha \rho(X)$  Positive homogeneity

• For  $X_1 < X_2$ ,  $\rho(X_1) < \rho(X_2)$  Monotonicity

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# Risk measures - Tail conditional expectation (TCE)

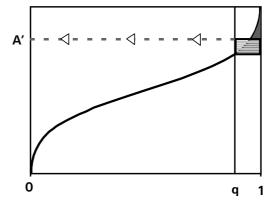
Also known as Tail Value at Risk

Unlike the standard deviation risk measure, TCE concentrates on the tail of the distribution.

Combination of VaR and EPD

Unlike VaR and EPD it is a Coherent Risk Measure

Expected value of the largest (1-q) of all possible outcomes



Graphically, the area in the box is the same as the area under the curve between q and 1

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# Allocation of cost of capital

- Capital allocation is an intermediate step to allocation of cost of capital
- Having chosen TCE as our risk metric, we explored a number of different capital allocation methods:
  - Marginal 'First-in'
  - Marginal 'Last-in'
  - Shapley value
- Selected Shapley value because it satisfied the axioms of a good allocation measure

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## 'Without' line of business definition

- For each of the lines of business, we selected to reinsure the new and existing business away
- Paying the reinsurance premium accomodates the asset impact of removing the line of business

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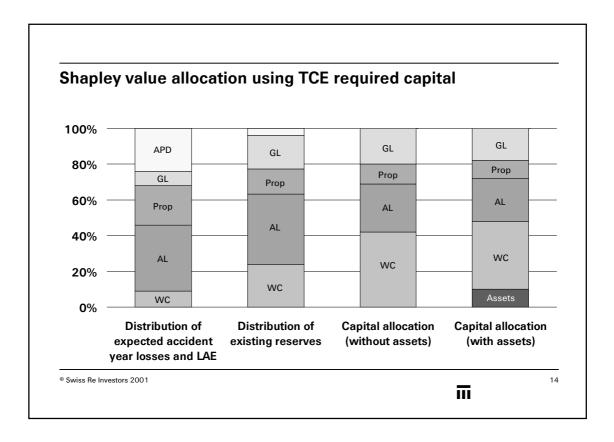
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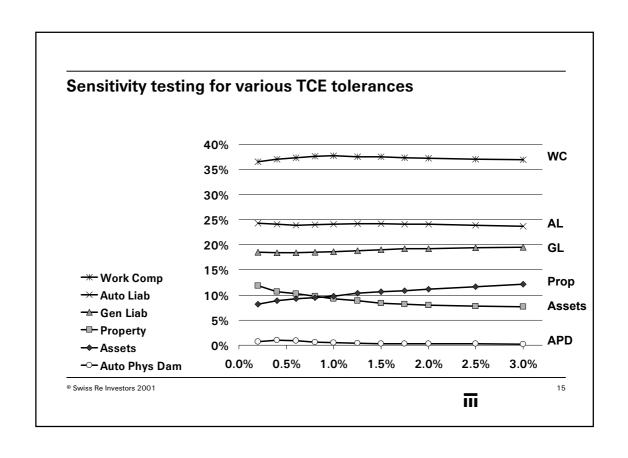
## 'Without' assets definition

- Determine the asset allocation that minimises TCE for all liabilities present (i.e. minimum risk efficient frontier portfolio)
- Advantages
  - Eliminates possibility of negative capital allocation
- Disadvantages
  - Much more complex
  - Need to find a solution for each marginal run
  - Very time consuming, especially for Shapley values

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## **Conclusions**

- TCE a useful method of considering risk
- Calculation of marginal contributions, especially Shapley value, time consuming and difficult
- A specific allocation of required return on capital to assets can be considered
- On our risk measure of TCE, we proposed DFAIC Inc. allocate a relatively greater proportion of required return on capital to its longer-tailed, more inflation sensitive lines of business

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