The Actuarial Profession
making financial sense of the future

GIRO Conference and Exhibition 2012

Triangle-free reserving

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Agenda

I. Preliminaries
II. The triangle-free approach
III. Performance comparison (triangle-free vs chain ladder)
IV. Advantages/disadvantages of the triangle-free approach

I. PRELIMINARIES

The triangle trick

Triangle trick: aggregate losses by accident year and by development year, identify development trends, project to ultimate

		Development year									
		0	1	2	3	4	5	6	7	8	9
	1	758,859	6,712,563	7,295,862	8,481,698	8,581,273	8,929,061	9,406,673	9,421,491	9,425,375	9,547,636
	2	588,009	1,786,021	2,187,149	2,365,737	2,474,465	2,842,739	2,842,739	2,882,701	3,398,944	
	3	514,089	1,532,487	2,331,175	8,377,877	8,954,659	9,117,566	9,138,301	9,147,275		
	4	419,422	2,882,030	4,009,785	4,413,923	4,468,089	4,616,335	4,823,964			
	5	261,482	2,089,735	3,050,709	3,684,369	4,130,221	5,036,548				
Accident year	6	893,053	2,121,944	4,368,448	4,546,849	6,942,262					
	7	481,366	954,766	2,026,609	2,481,851						
	8	696,678	1,505,950	2,283,808							
	9	4,336,497	5,355,547								
	10	433,625									

Pros: simplicity, snapshot view, a good point estimate even with CL

The main problem with development triangles: information compression

Size: 595 kB

Size: 16 kB

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Size: 595 kB

5,000 claims over 10 years

Size: 16 kB

… compressed into
10 x 11 / 2 = 55 points

Based on 55 points we extract: (i) a point estimate; (ii) some measure of volatility; (iii) the full reserving distribution!!!

In pricing, we face a similar problem when using burning cost analysis

Burning cost is (roughly) the calculation of expected losses based on an average of the losses in the past few years, with an allowance for claims inflation, changes in exposure, and possibly IBNR

Burning cost may give us a fair idea of the mean and possibly some idea of volatility, but is not adequate to estimate the full distribution of future losses

II. THE TRIANGLE-FREE APPROACH

A different approach

As in pricing, we intend to create a reserving distribution for IBNR based on

- the creation of a frequency model
- the creation of a severity model based on the individual severities
- the combination of the two with MC simulation or other methods

All this does not necessarily need to be done without triangles, but the method we propose here is triangle-free

II. THE TRIANGLE-FREE APPROACH

High-level methodology

A. Estimate the IBNR distribution
   i. Estimate the reporting delay distribution
   ii. Use the reporting delay distribution to estimate the IBNR claim count distribution → Output: frequency model
   iii. Estimate the severity distribution taking IBNER into account → Output: severity model
   iv. Combine the frequency and severity model with e.g. MC simulation to produce an aggregate loss model for IBNR
B. Estimate the IBNER distribution
   i. Can be done by traditional CL projection methods or GLM
C. Estimate the UPR distribution
   i. A pricing exercise!
D. Combine IBNR, IBNER and UPR to produce an overall aggregate loss model
   i. Straightforward (e.g. using the outputs of the MC simulation) if independent

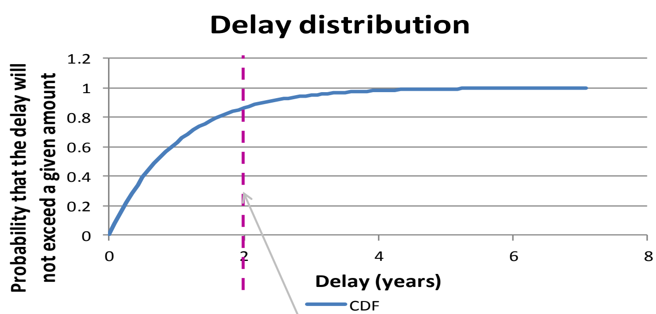
II. THE TRIANGLE-FREE APPROACH

A. ESTIMATING THE IBNR DISTRIBUTION

II.A.1 – Create a frequency model

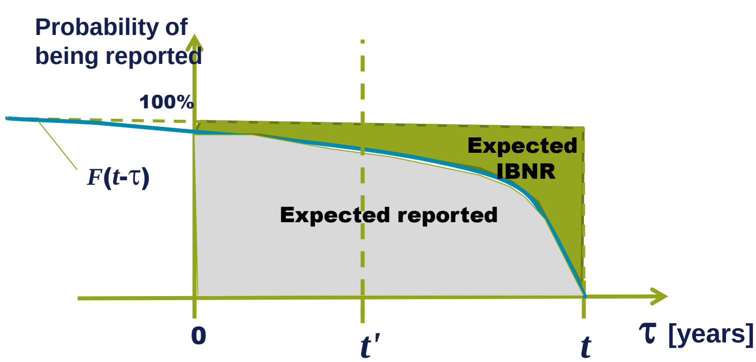
The delay distribution

The cornerstone of this method is the delay distribution. This gives the probability that the delay by which a loss is reported will not exceed a certain value



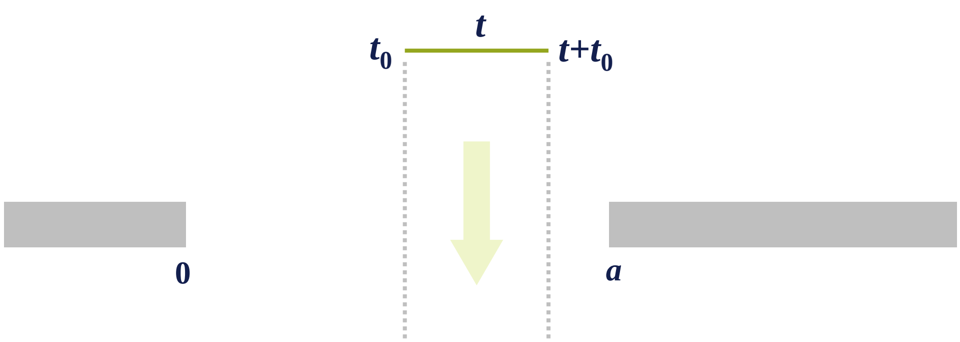
*Interpretation: a day x two years ago is "85% earned" (i.e., **on average** 85% of the claims occurred on that day will have been reported)*

If we know the delay distribution, we can work out the expected number of IBNR claims



$$\hat{\mu}_t = \frac{t}{\int_0^t F(t-\tau)d\tau} r_t = \frac{\text{\# of days in the period } [0,t]}{\text{\# of earned days in the period } [0,t]} r_t$$

However, we do not know the true delay distribution, but only a biased version of it



The effect of the bias

- The observed delay distribution is biased towards small delays
- Furthermore, the exposure/risk profile may not be constant and this causes a further bias in the distribution of observed delays
- Case where the exposure is constant (for illustration):

$$\hat{f}_a(t) = \begin{cases} \dfrac{\left(1 - \frac{t}{a}\right) f(t)}{G(a)} & \text{for } t < a \\ 0 & \text{elsewhere} \end{cases}$$

- By inverting the formula above we can easily work out the bias-corrected delay distribution

Tail factor

- We can easily calculate the tail factor by assuming a reasonable functional form for the delay distribution (typically, an exponential) and calculating its parameters
- If we assume an exponential decay, the tail factor is

$$\varphi_{\text{tail}} = \frac{1}{1 - \exp(-a/\tau)}$$

where a = size of observation window

- The mean decay time τ can be calculated from the observed average delay time τ_{obs} of the empirical distribution like this:

$$\tau_{obs} = \tau\left(1 + \frac{e^{-\frac{a}{\tau}} - \frac{\tau}{a}\left(1 - e^{-\frac{a}{\tau}}\right)}{1 - \frac{\tau}{a}\left(1 - e^{-\frac{a}{\tau}}\right)}\right)$$

→ Find τ numerically

Output example

PROJECTION TO ULTIMATE -- ALL YEARS TOGETHER

Period	Days elapsed	Earned days	Factor to ultimate	Exposure	Latest reported	Ultimate losses	Standard error
2001-10	3,652.00	1,967.62	1.86	10,000	352	653.33	31.85

PROJECTION TO ULTIMATE -- YEAR BY YEAR

$$\hat{\mu}_t = \frac{\text{\# of days in the period } [0, t]}{\text{\# of earned days in the period } [0, t]} r_t$$

Year	Days elapsed	Earned days	Factor to ultimate	Exposure	Latest reported	Ultimate losses	Standard error
2001	365	347.94	1.05	1,000	56	58.75	3.16
2002	365	302.13	1.21	1,000	59	71.28	6.06
2003	365	275.29	1.33	1,000	43	57.01	7.24
2004	366	254.25	1.44	1,000	41	59.02	8.07
2005	365	220.78	1.65	1,000	42	69.44	9.18
2006	365	190.62	1.91	1,000	41	78.51	10.10
2007	365	158.90	2.30	1,000	23	52.83	10.98
2008	366	119.16	3.07	1,000	30	92.15	12.00
2009	365	71.35	5.12	1,000	14	71.62	13.10
2010	365	27.19	13.42	1,000	3	40.27	14.05
					Mean	65.09	
					Variance-to-mean ratio	3.28	

Frequency model

- Based on the projections to ultimate, we can estimate the year-on-year volatility and hence decide which frequency model to adopt (e.g. Poisson, NB)

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II.A.2 – Create a severity model

The loss data set

- In pricing, one models the loss distributions by collating all *relevant* past losses (those that are thought to be a good guide to the future)
  - Revaluing them to the mid-term of the policy, and then fitting a distribution to them
  - Adjusting them for IBNER if open claims are also considered
- In determining the severity model for IBNR losses, we need to go through a similar process, with one complication: we are interested in the probability that a loss X be ≤ x *conditional to having occurred in a given year and being yet unreported.*
- Two (probably conflicting) effects: that of claims inflation and that of being reported after a number of years

A possible approximation

Estimate the **kernel distribution** by fitting it to the claims from all years, after revaluing them to current term (much in the same way we do in pricing)

$$X \sim F_X(x)$$

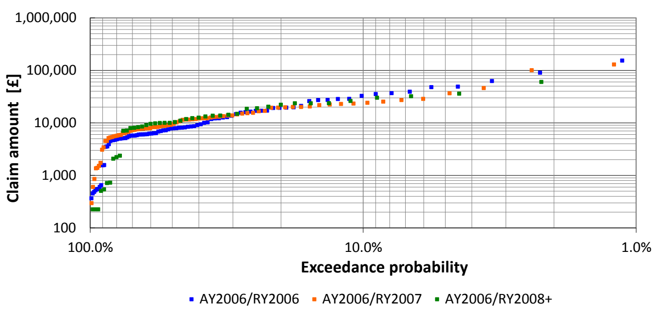
Assume that the severity distribution of the IBNR claims is the same for all years except for an inflation/deflation factor that applies homogeneously to all claims:

$$X(\text{reported after } \delta \text{ years, occurred at year } t_0) \sim F_X\left(\frac{(1+r)^{t-t_0}}{(1+s)^{\delta}}x\right)$$

You need evidence or very good judgment to support any method that is not a simple approximation!

You need good evidence to support anything which is not a simple approximation!

UK EL claims for policy year = 2006 reported in 2006, 2007 and 2008+ (unrevalued)



KS(2006,2007)=1.22 (p=10%)

KS(2006,2008+)=1.36 (p=5%),

KS(2007,2008+)=0.74 (p>>10%)

Both open and closed losses are considered in the graph above

Dealing with IBNER

For large losses, historical development of reserves is often available and IBNER can be estimated with chain-ladder-like (!) techniques

	2000	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*2007*	*2008*
Paid	19,792	363,306	487,648	1,735,328	1,922,504	1,922,504	1,922,504	1,922,504	1,922,504
O/S	967,500	877,200	753,360	147,060	0	0	0	0	0
Incurred	987,292	1,240,506	1,241,008	1,882,388	1,922,504	1,922,504	1,922,504	1,922,504	1,922,504
O/S ratio	98.0%	70.7%	60.7%	7.8%	0.0%	0.0%	0.0%	0.0%	0.0%
IBNER factor		1.256	1.000	1.517	1.021	1.000	1.000	1.000	1.000

Depending on data availability, the dependency of IBNER on a number of factors such as size of claim, outstanding ratio, type of claim, etc. can be studied with GLM-like techniques [triangle-free]

For small losses, there's a number of tricks one can use:

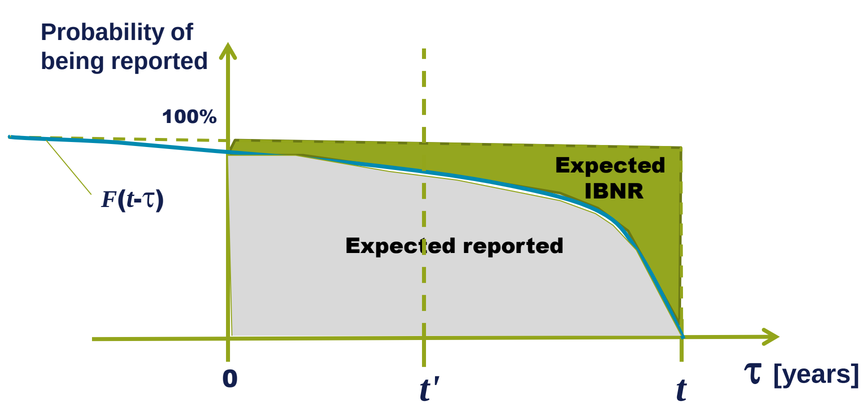
- use development of average claims [may contain triangles!]
- use closed claims only (at least below a certain threshold)
- use initial/final information

Overall method for creating a severity model for IBNR claims

Step 1 ***IBNER***	Adjust invididual past losses for IBNER
Step 2 ***Revaluation***	Revalue the individual past losses to current terms
Step 3 ***Kernel severity model***	Model the severity distribution in current terms, obtaining the *kernel severity distribution,* from which the distribution for different years can be obtained
Step 4 ***Modify severity model***	Estimate the severity distribution for losses that occurred at a given time and that will reported at another given time

A simulation protocol

(Case of uniform Poisson rate) Pick *N* points from the green area below, keeping track of the value of the horizontal axis (*t*)



A typical output of the IBNR simulation process

The typical output of the simulation process is similar to the output of a total gross loss model for pricing

Return Period (Years)	Percentile	Total Loss	Total Number
1 in 1.33	25%	14,205,625	230
1 in 2	50%	16,630,426	245
1 in 4	75%	19,631,593	260
1 in 5	80%	20,369,140	264
1 in 10	**90%**	**22,627,203**	**274**
1 in 20	95%	25,143,560	283
1 in 50	98.0%	29,441,708	292
1 in 100	**99.0%**	**32,382,767**	**298**
1 in 200	99.5%	35,057,817	304
1 in 500	99.8%	64,920,594	311
1 in 1000	99.9%	66,937,322	318
	Mean	**17,363,917**	**245.1**
	Std Dev	**4,948,653**	**22.2**

II. THE TRIANGLE-FREE APPROACH

B. ESTIMATING THE IBNER DISTRIBUTION

IBNER for reported claims

- The IBNER component can be calculated much in the same way as the IBNER component of IBNR claims
- Not only we can estimate an average IBNER factor for claims of a certain type (development year = d, size = x, o/s ratio = r…):

$$IBNER = \sum_{k=1}^{n} (IBNER(d_k, x_k, r_k \ldots) - 1) \times X_k$$

- …but also estimate an uncertainty around that, and produce an empirical IBNER distribution through a MC simulation

II. THE TRIANGLE-FREE APPROACH

C. ESTIMATING THE UPR DISTRIBUTION

Future losses: a pricing problem

Finding the distribution of future losses is, quite simply, a pricing problem, and it can be solved by the usual tools of pricing

When the aggregate loss distribution is based on past losses:

- The frequency model is basically the same as that which describes the past years…
- … and the severity model is the kernel model, revalued to the relevant point of the policy year

II. THE TRIANGLE-FREE APPROACH

D. OVERALL RESERVES

Combining IBNR, IBNER, UPR

Overall reserves = (Pure) IBNR + IBNER + UPR

If IBNR, IBNER and UPR can be considered roughly independent, it is straightforward to find the overall reserve distribution, e.g.

- by MC simulation (add the results of the individual simulations)
- by Fourier transform (the FT of the overall reserves is the product of FT of IBNR, IBNER and UPR)

IIII. PERFORMANCE COMPARISON (TRIANGLE FREE vs CHAIN LADDER)

Experimental set-up

- Based on artificial data sets (less meaningful, but possibility of large-scale experiments and comparison with "true" result)
- 10 years of simulated experience
- No of claims : Poisson with rate = 100
- Delay distribution: Exponential with mean = 1, 2, 3, 4, 5, 10, 20
- Severity distribution: Lognormal with mu = 9.52, sigma = 1.70
- No of simulations = 100 (for each simulation, a full reserving exercise must be carried out)

Assumptions

- No IBNER
- No claims inflation
- Constant exposure

Experiment #1 – Predicting ultimate claim count

- Frequency: Poi(100), Delay: Exp(various)
- Tail factor: the same for CL and TF (that calculated with TF, as there is no unique agreed-on way to calculate it for the CL)

USING CALCULATED TAIL FACTOR

Average delay [y]	TF mean squared error	CL mean squared error	Error reduction
1	0.38	0.55	31%
2	6.93	10.89	36%
3	10.14	14.86	32%
4	15.83	21.96	28%
5	22.51	28.94	22%
10	156.11	177.44	12%
20	148.12	188.56	21%

USING MARKET ASSUMPTION FOR THE TAIL

Average delay [y]	TF mean squared error	CL mean squared error	Error reduction
1	3.83	5.48	30%
2	5.65	9.58	41%
3	9.82	13.56	28%
4	11.15	17.33	36%
5	14.81	19.24	23%
10	22.14	32.86	33%
20	26.99	58.34	54%

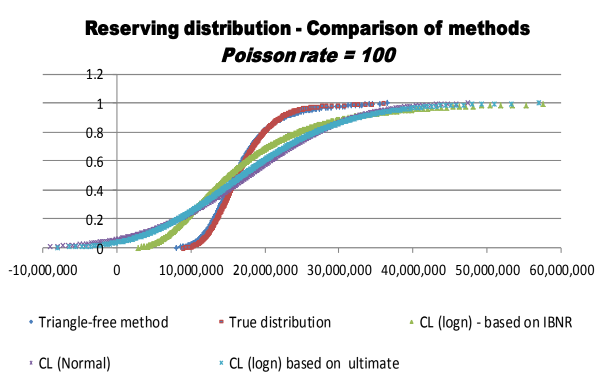
Experiment #2 – Predicting the expected IBNR (total claims)

- Frequency: Poi(100), delay: Exp(3), severity: LogN(9.52,1.70)
- In the "empirical" column, no attempt is made to model the severity during the analysis –resampling is used instead

					Chain ladder	Triangle-free (empirical)	Triangle-free (model)
				Prediction accuracy (MSE)	**7,392,262**	**4,423,237**	**3,842,561**
				Prediction accuracy (MSE) as a percentage of average true value	**44.7%**	**26.8%**	**23.3%**
Simul	*IBNR (true)*	*IBNR (Chain ladder)*	*IBNR (Triangle-free, empirical)*	*IBNR (Triangle-free, model)*	*Error (Chain ladder)*	*Error (Triangle-free, empirical)*	*Error (Triangle-free, model)*
1	10,773,575	19,477,811	15,443,006	16,104,318	8,704,236	4,669,431	5,330,743
2	17,839,076	11,163,722	13,864,553	14,578,474	- 6,675,354	- 3,974,522	- 3,260,601
3	20,073,695	17,343,084	15,841,802	15,691,210	- 2,730,611	- 4,231,893	- 4,382,485
4	16,519,312	17,671,032	22,341,455	22,659,106	1,151,719	5,822,143	6,139,794
5	15,622,367	19,756,994	15,493,358	15,027,101	4,134,627	- 129,009	- 595,265
6	16,807,437	9,557,564	14,331,857	12,685,124	- 7,249,873	- 2,475,580	- 4,122,313
7	16,309,016	26,602,474	19,518,622	19,587,276	10,293,459	3,209,606	3,278,260
8	21,486,134	15,839,159	20,258,625	22,039,740	- 5,646,975	- 1,227,509	553,606
9	11,863,136	16,661,276	15,250,191	16,031,506	4,798,140	3,387,055	4,168,371
10	14,938,406	11,782,679	13,048,958	13,355,460	- 3,155,727	- 1,889,448	- 1,582,946
...	...		...	...	...	...	...

Experiment #3 – Comparing the overall IBNR distribution

- Two samples from Exp #2 for which the point estimate was similar for CL and TF were chosen
- For CL, a lognormal distribution with mean and variance equal to those calculated based on Mack's method was used



	Normalised KS distance from "true" distribution
Triangle-free	2.3
Chain ladder (lognormal)	18.5
Chain ladder (normal)	18.2
Chain ladder (lognormal, based on ultimate)	17.6

Limitations of the experimental set-up

- Part of the difference in the mean squared error can be attributed to the fact that we know the models we've used (Poisson, exponential, lognormal)
- The comparison is only with the chain ladder and not with all triangle-based methods
- Experiment #3 is currently anecdotal and should be replaced with a large-scale experiment
- The experiment is based on artificial data sets. This is to some extent inevitable, but the question arises as to whether these data sets represent reality fairly
  - Ideally, the way in which these sets are generated should be pre-agreed upon by practitioners in a controlled experiment
  - The possibility of bootstrapping real-world data sets should be explored

IV. ADVANTAGES/DISADVANTAGES OF THE TRIANGLE-FREE APPROACH

Advantages of this approach

No loss of information!

The output is the full reserves distribution

- We can account for parameter, data and model uncertainty

Deals properly with the tail factor in claims count

Deals properly with calendar-year effects (changes in the severity distribution, in the reporting speed)

Deals properly with large losses

- The modelling of large losses is as good as the modelling of the severity distribution and can use extreme value theory, market severity curves, etc.

Can work when historical triangles are not available

The reserving stochastic model is fully aligned to the pricing model

Disadvantages of this approach

Increased complexity: the additional pain is roughly the same as that of going from *burning cost analysis* to *frequency/severity analysis* in pricing

Increased data requirements: a total claims triangle is not sufficient

Lack of good visualisation: the method doesn't have the at-a-glance feel that triangles have

- The reason of course is that the information is not compressed before doing the analysis
- However, one can visualise the delay distribution, the frequency distribution, the severity distribution, etc.
- Triangles can *still* be used to visualise the results and also can be run for comparison purposes

General comments

- Despite the label “triangle-free”, this method can be hybridised to whatever extent necessary with triangle-based methods.
  - The key features of this approach are: (i) the full(er) use of the information in the loss data set; (ii) the creation of a frequency and severity model as in pricing
  - Perhaps a better name would be “compression-free reserving”!
- **This is a framework, not a unique protocol**
- Many of the issues that arise in practice in applying this methodology are the same that we have in pricing

Questions or comments?

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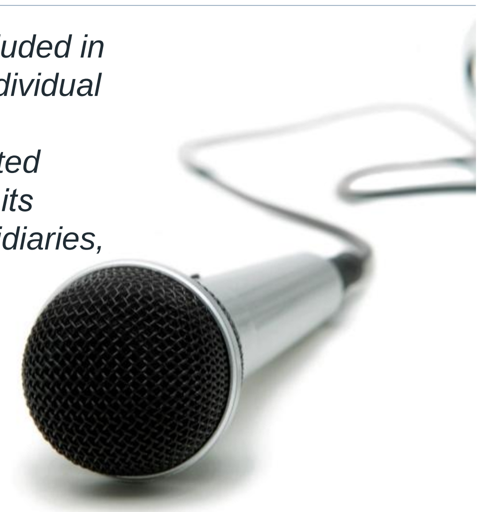

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