Institute and Faculty of Actuaries

Through the Cycle and Point in Time 16 October 2014

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Context and Background

- We live in a changing world. The market dynamics of yesterday don't apply today.
- What we are looking to explore is about how to use our data about the past to inform our view of the future based on where we are at the present.
- The methods we will be looking at do have limitations, some of which we shall consider.
- Change comes in many forms.

Context and Background

Change comes in many forms:

- Cycles – Economic Cycle or Other Recurring Cycle?
- Trends – Permanent or Temporary?
- Shocks – Reversing or New Paradigm?
- The past may be a poor predictor of the future, but it's the only thing we've got to work with.
- History may seem to, but doesn't, repeat itself. Some features recur, others don't.

Recent Global Change:
New paradigm or part of a cycle?

- 1944 conclusion of the Bretton Woods Conference
- c1970 demise of the Bretton Woods agreement
- c1987 emergence of volatility skew
- c1997 Central Bank inflation targeting
- 2007/8 Financial Crisis – Bank Liquidity
- 2011 Quantitative Easing

Recent Local Change: New paradigm or part of a cycle?

- 1990 fall of the Berlin Wall
- 1992 UK leaves the Exchange Rate Mechanism
- 1998 Russian debt crisis
- 1999 launch of the Euro
- c2000 Japanese low interest rates environment
- 2010 Greek debt crisis

A few longer term examples of change:

- $3^{rd}/4^{th}$c Rome Empire Collapse
- 1066 Battle of Stamford Bridge
- 1340’s Edward III of England default / Venetian Banking Crisis
- Spain (1557), France (1558) and Portugal (1560) defaults
- 1634 - 1637 Amsterdam Tulip Bubble / 1640 Massachusetts panic
- Early 1720’s South Sea Bubble
- 1835-1842 USA 1st Great Deflationary Depression
- 1929-1932 Great Depression

A few other long term examples of change:

- Invention of the Wheel
- Domestication of the Horse
- Bubonic Plague
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- Exhaustion of Fossil Fuel
- Global Warming
- Next Asteroid Collision

Context and Background

- When confronted with paradigm change, what does the calibrator do?
- Is the change permanent or temporary?
- If permanent there is no “new paradigm” data to work with.

Post Shock Calibration Options

Short Term Options post shock:

- Assume reversion and maintain “old” calibrations
- Assume new level and maintain “old” calibrations
- Try to model the “new world” with no data except the start point!

Short-Medium Term Options post shock:

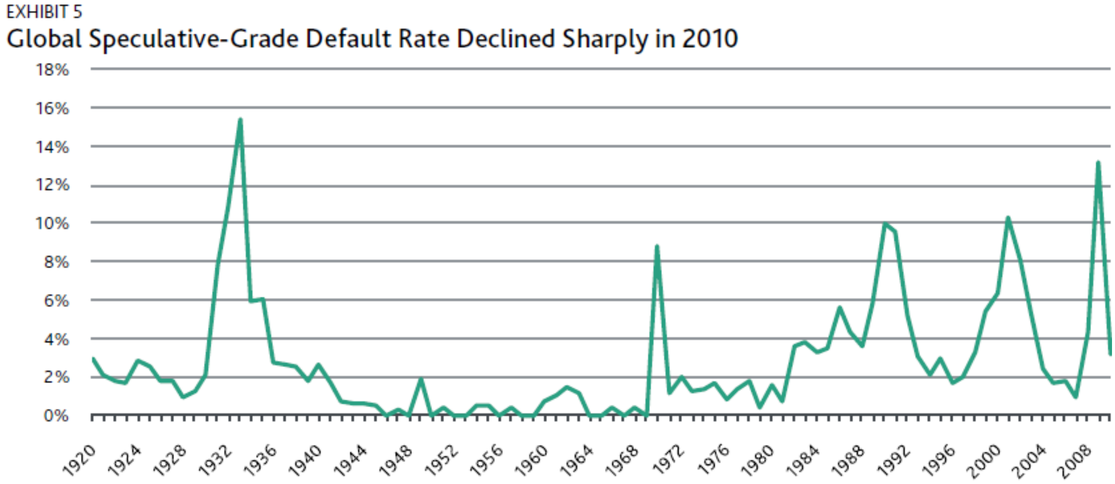
- Allow post shock data to emerge in calibrations
- Only use post shock calibration data.

When the past is a valid source for the future

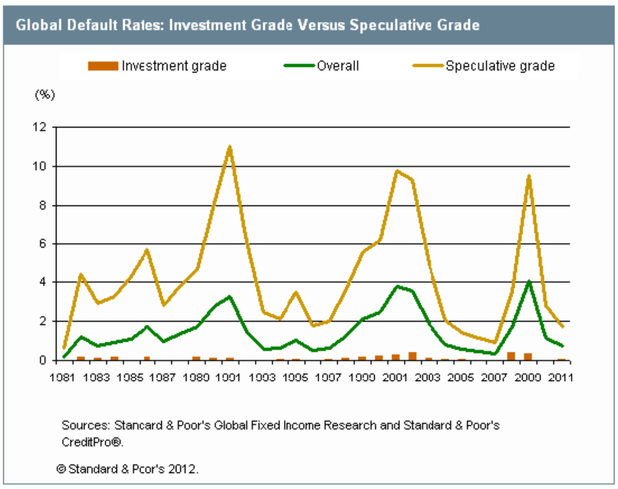
The rest of this presentation considers situation where we decide we can attempt to use the past to derive our probability distributions for the future.

- Probability distributions are widely used for communicating financial risk and uncertainty. For example, a 95% prediction interval for the future profit over a period, should have (at least) a 95% chance of containing the actual outcome.
- Particularly in the context of credit risk, the literature distinguishes two types of probability laws: point-in-time and through-the-cycle. We give some example definitions.

Cyclical Behaviour in Defaults (Moody's)



Cyclical Behaviour in Defaults (S&P)



Point-in-Time Default Probabilities (Moody's)

- A point-in-time credit risk measure is one which utilizes all available and pertinent information as of a given date to estimate a firm's expected probability of default over some time horizon
- The information set includes not just expectations about a firm's own long-run credit risk trend, but sectoral, geographic, macroeconomic, and macro-credit trends as well
- PIT PDs are suited for situations where the cost of defaults or credit spread changes is high, and early detection of changes in credit risk at both the single name and portfolio level is important
- Examples include a variety of market-price based quantitative models, including Merton-style PD models and CDS spread based models

Through-the-Cycle Probabilities (Moody's)

- A through-the-cycle credit risk measure primarily reflects a firm's long-run, enduring credit risk trend. Transient, short-run changes in credit risk that are likely to be reversed with the passage of time are filtered out
- The predominant features of TTC credit risk measures are their high degree of stability over the credit cycle and the smoothness of change over time. Compared to PIT risk measures, TTC risk measures display much less volatility and procyclicality over the cycle
- The value of the trade-off of accuracy/timeliness for stability clearly shows up in applications where portfolio adjustment costs or regulatory compliance costs are high, such as meeting required capital and in fixed income portfolio investment guidelines
- Examples include credit ratings and financial ratios based models

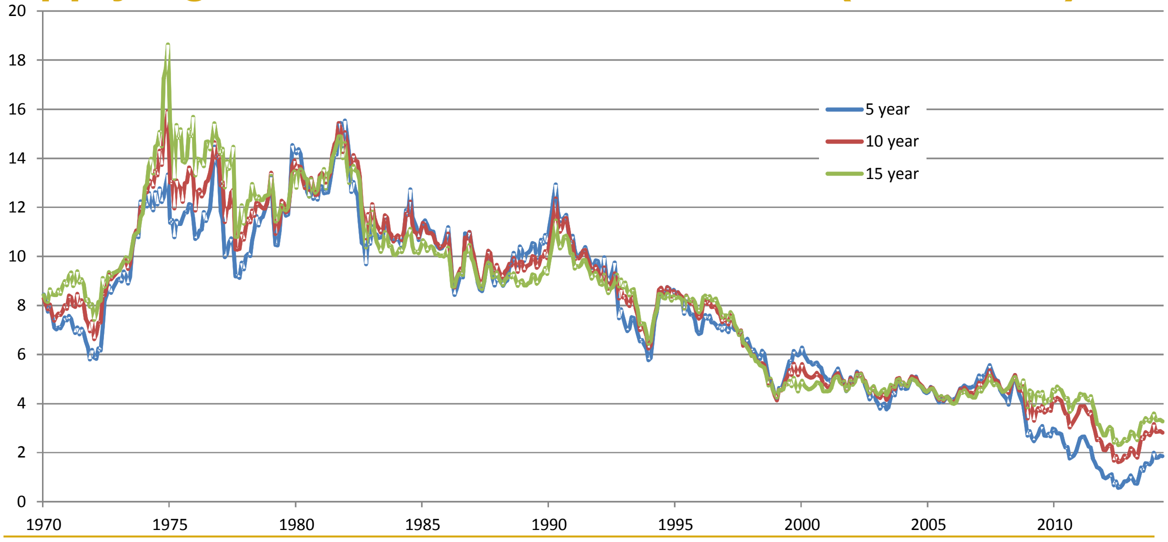
Complicating Factor: Break out of the Cycle

- Firstly, there is an underlying assumption that credit experience is cyclical. While historic default experience is clearly not a perfect sine wave, there is an apparent tendency for years of bad experience to cluster together, separated by years of more benign experience. The concept of a through-the-cycle estimate is an average over the cycle, from which short term movements have been filtered out. However, the really severe credit events are downward spirals of default contagion, when the hoped-for cyclical upturn fails to materialise. We would not want the imposition of cyclical assumptions to assume away the events with the biggest impact.

Implicit Efficient Market Assumption

- Secondly, in using market prices (such as spreads or CDS premiums) in PIT estimates, there is an assumption that these price moves anticipate changes in default probabilities, which is a form of the efficient market hypothesis. However, spreads could reflect many other things, such as the liquidity in an underlying bond, default risk on the CDS itself, noise from irrational traders or data artefacts in thinly traded markets.

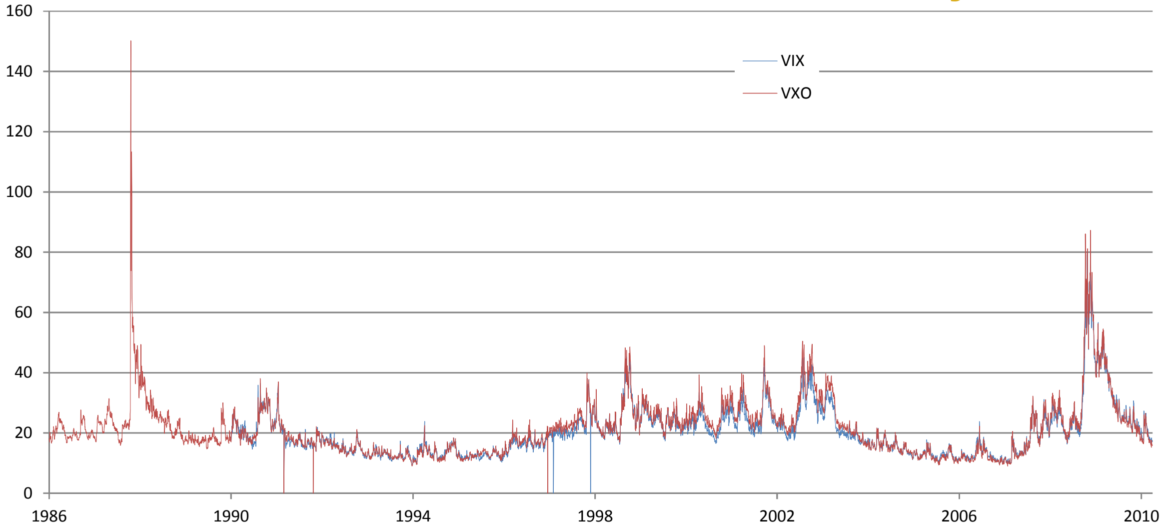
Applying PIT / TTC to Interest Rates (BoE Data)



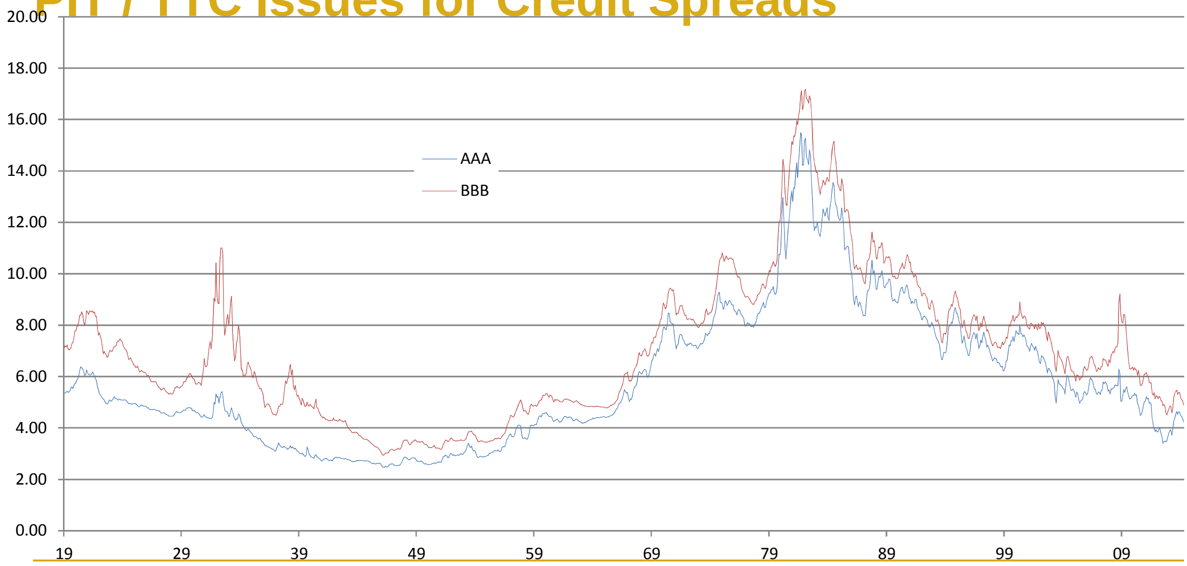
PIT / TTC issues with Interest Rates

- How does the conditional distribution of future interest rate changes vary according to the level of interest rates?
- Could assume a constant volatility, volatility proportional to rate level, volatility proportional to square root of level …
- Historically low (absolute) volatility 1995-2005 when rates within 4%-8% range. Higher historic volatility before 1995 and after 2005. Implies a U-shaped conditional volatility. Does that make sense?
- How could we introduce implied volatilities of caps / floors / swaptions?

PIT / TTC Issues for Interest Rate Volatility



PIT / TTC Issues for Credit Spreads



PIT / TTC Issues with Equity Market Level

- Difference stationary process
- Drift depends on level of interest rates
- Stochastic volatility or regime switching
- Role of equity implied volatilities

Why not Just Build a Full Time Series Model?

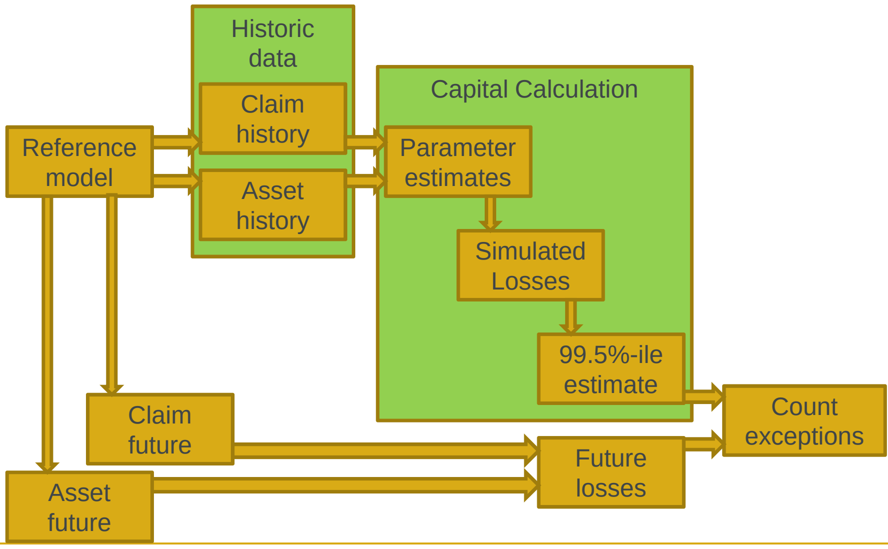
- If we can guess the correct model form, then fitting the model directly might be a good idea

Historic monthly returns → subset → Historic annual returns

fitting ↓ validation ↕

Model of Monthly Returns → subset → Annual returns from model

Monte Carlo Back-Test for TTC Stresses



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How to test a PIT Model? An ARMA(1,1) case study

- Institutions can choose to calculate capital on a point-in-time, basis, or on a through-the-cycle basis.
- The calculation may be articulated in the context of fitted time series models, in which case the point-in-time estimate is determined from the conditional forecast, while a through-the-cycle estimate comes from an unconditional forecast
- Both methods derive a value at risk as a statistic that depends on historic data. Thus, in each case, the stated value-at-risk is a random variable itself (as the input to these - the observed data - is drawn from a random sample).

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Assessing TTC/PIT VaR estimators

- As VaR estimates derived from both PIT and TTC methods are random variables, we can base our analysis on both methods by considering their sampling distribution properties using different measures, for example:
  - What is the average estimated stress required for a given VaR level?
  - What is the sampling distribution of the confidence level achieved?
- We now present a simple framework from **very early** work within the EEWP to assess PIT/TTC VaR estimators.

Experiment 1: ARMA(1, 1) process

- As a thought experiment, we assume that we have index data spanning 20 years
- We know that the index that we are trying to model follows an ARMA(1,1) process
- Our aim is to specify a stressed such that the probability of the index breaching this level over one year is set at a given confidence level (e.g. 0.5%)

Two estimators

- We decide to set VaR using three estimators: a 'PIT' estimator, a 'TTC' estimator, and a 'simple TTC' estimator
- The PIT estimator VaR will be based on the conditional expectation and variance of the index (conditional on its previous history) derived from the estimated parameters of the ARMA(1,1) model
- The TTC estimator will be based on the *unconditional* expectation and variance of the index derived from the estimated parameters of the ARMA(1,1) model
- The simple TTC estimator will be based on the sample mean and sample variance

Formally…

- We assume the following process:

$$\tilde{y}_t = \mu + \phi_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \tilde{\epsilon}_t, \quad \tilde{\epsilon}_t \sim N\left(0, \sigma^2\right)$$

- In addition, we assume that the parameters have the following prior distributions:

$$\mu \sim N\left(0, 0.2^2\right)$$
$$\phi_1 \sim N\left(0.3, 0.1^2\right)$$
$$\theta_1 \sim N\left(0.3, 0.1^2\right)$$
$$\sigma^2 \sim N\left(1, 0.2^2\right)$$

Why specify prior distributions?

- We specify a prior distribution for the parameters because, in the absence of an analytical solution, the estimators which give an unbiased confidence interval have to be determined by Monte Carlo analysis. This presents a problem:
  - knowledge of the true parameters also provides knowledge of the true prediction interval at each point in time, removing the need to simulate;
  - however, without knowledge of the true parameters we cannot derive an unbiased estimator for the prediction interval since using a different set of parameters will yield a different estimator to that required.

Solution – introduce prior distributions

- To get round this problem, we therefore assign prior distributions to the parameters, since knowledge of these prior distributions does not provide knowledge of the true prediction interval.

- We assume that the prior distribution is set by expert judgement, informed perhaps by the initial parameter estimates.

PIT vs TTC estimates

	PIT estimator	TTC estimator	Simple TTC estimator
Mean	$(E(y_{t+1}\|Y_t)) = \hat{\mu} + \hat{\varphi}_1 y_t + \hat{\vartheta}_1[y_t - E(y_t\|Y_{t-1})]$	$(E(y_{t+1}) = \hat{\mu}/(1-\hat{\varphi}_1)$	$\hat{y} = \sum_{t=1}^{T} y_t/T$
Stressed value	$\hat{v}_1 = E(y_{t+1}\|Y_t) + \hat{c}_1\hat{\sigma}$	$\hat{v}_2 = E(y_{t+1}) + \hat{c}_2\widehat{\sigma'}$ $\widehat{\sigma'} = \hat{\sigma}\sqrt{\left(1+\hat{\vartheta}_1{}^2\right)/\left(1-\hat{\varphi}_1{}^2\right)}$	$\hat{v}_3 = \hat{y} + \hat{c}_3\hat{s}$ $\hat{s} = \sqrt{\sum_{t=1}^{T}(y_t-\hat{y})^2/(T-1)}$
Stress	$\hat{v}_1 - y_t$	$\hat{v}_2 - y_t$	$\hat{v}_3 - y_t$

- Y_t is the process history
- $\widehat{\sigma'}$ is the estimate of the unconditional standard deviation of the estimated ARMA(1, 1) process, $\hat{y}$ is the sample mean, $\hat{s}$ is the square root of the sample variance
- $\hat{c}_1$, $\hat{c}_2$, and $\hat{c}_3$ are the constants required such that:

$$Pr(y_{t+1} < \hat{v}_1) = Pr(y_{t+1} < \hat{v}_2) = Pr(y_{t+1} < \hat{v}_3) = 0.005$$

Experiment procedure

- Simulate 25,000 different sample datasets of the process assumed, each with 20 observations
- For each sample dataset, estimate the ARMA(1,1) model parameters, the sample mean, and the sample variance
- Using all 25,000 datasets, estimate the values of $\hat{c}_1$, $\hat{c}_2$, and $\hat{c}_3$ by simulating an additional year of data (y_{t+1}) and solving numerically so that there are 0.005 * 25000 = 125 exceptions for each estimator (where an exception is an instance where y_{t+1} is less than the estimated stressed value)

Experiment results – expected stress applied

- Expected stress:
  - PIT: $\mathrm{E}(\hat{c}_1\hat{\sigma}) = -3.08$
  - TTC: $\mathrm{E}\left(\hat{c}_2\widehat{\sigma'}\right) = -4.17$
  - Simple TTC: $\mathrm{E}(\hat{c}_3\hat{s}) = -3.47$

- In this example, the PIT estimator results in a lower *expected* stress.
- It is interesting to note that, although the simple TTC estimator assumes the wrong model, it is a more efficient estimator of the stress
- This is not necessarily a general result, and will depend on the process assumed (e.g. if the priors of θ_1 and φ_1 are set to a constant zero, then the simple TTC estimator results in a lower expected stress than the PIT estimator).

Experiment results – sample distribution of confidence level achieved

- Firms will also be interested in the distribution of the level of cover achieved under each estimation method for example, statements along the lines of
  - Q1: what is the confidence level achieved in the most prudent 25% of cases?
  - Q2: what is the confidence level achieved in the least prudent 10% of cases?

- Our experiment can help answer questions like these.

Experiment results – sample distribution of confidence level achieved

Question	PIT estimator	TTC estimator	Simple TTC estimator
Confidence level in most prudent 25% of cases?	<0.43%	<0.12%	<0.29%
Confidence level in least prudent 10% of cases?	>1.3%	>0.7%	>1.1%

Conclusions

- PIT/TTC VaR estimators can be thought of as random variables and can be evaluated using the properties of their sample distributions
- Sometimes, a simple TTC estimator can be a better solution than the unconditional VaR estimator based on estimated parameters of the 'true' model
- There is no 'right' answer – firms have to make a decision which aligns with their own risk appetite on a case-by-case basis

Some suggestions for future work

- Test a range of estimators for a range of processes
- Use more realistic parameters based on real-world data
- Assess estimators on a greater number of measures.

Any thoughts from the audience?