Economic Capital Models for Basel/Solvency II, Pillar II

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Contents

1. Basel II, Solvency II & Economic Capital

- 1988. First Basel Accord takes first steps toward international minimum capital standard. Approach fairly crude and insufficiently differentiated.

- 1993. The birth of VaR. Seminal G-30 report addressing for first time off-balance-sheet products (derivatives) in systematic way. At same time JPMorgan introduces the Weatherstone 4.15 daily market risk report, leading to emergence of RiskMetrics.

- 1996. Amendment to Basel I allowing internal VaR models for market risk in larger banks.

- 2001 onwards. Second Basel Accord, focussing on credit risk but also putting operational risk on agenda. Banks may opt for a more advanced, so-called internal-ratings-based approach to credit.

Basel II

- **Rationale** for the new accord: more flexibility and risk sensitivity

- **Structure** of the new accord: three-pillar framework:

  ❶ Pillar 1: minimal capital requirements (risk measurement)

  ❷ Pillar 2: supervisory review of capital adequacy

  ❸ Pillar 3: public disclosure

Solvency II

The European Commission and the Committee of European Insurance and Occupational Pensions Supervisors are carrying out a fundamental review of the regulatory capital regime for insurance companies, known as Solvency II, with the aim of establishing an improved solvency system that protects the interests of policyholders by reducing the likelihood of prudential failure.

There are clear parallels between the approaches being taken in Basel II and Solvency II, in particular, the emphasis on risk modelling and the use of a three-pillar system: Pillar 1 sets out the minimum capital requirements (MCR) for insurance, market, credit and operational risk; Pillar 2 defines the supervisory review process and Pillar 3 the disclosure and transparency requirements. Implementation is currently planned for 2010.

The UK Treasury on Solvency II

“There is a strong economic rationale for a reformed EU-wide solvency framework which is forward-looking in its assessment of risk and brings regulatory capital into line with economic capital. However, Solvency II cannot just be about capital requirements; no amount of capital can substitute for the capacity to understand, measure and manage risk and no formula or model can capture every aspect of the risks an insurer faces. The new framework should promote higher quality risk management, working with the grain of industry developments, and ensure that the assessment of regulatory capital is integrated with firms’ wider capital management processes.” [Treasury and FSA, 2006]

Economic Capital: What Is It?

- Economic capital is the capital required by a bank/insurer to limit the probability of insolvency to a given level over a given horizon.

- Whereas regulatory capital is based largely on external rules that are intended to ensure a level playing field, economic capital is an attempt to measure risk in terms of economic realities.

- Many companies see economic capital models as part of their response to Pillar II (supervisory review) of the regulatory regime.

- At its most general, economic capital should offer a firm-wide language for discussing and pricing risk and assessing the return on risk capital. A bank with a good economic capital model would hope to be able to use its capital more efficiently.

Economic Capital: How Far Have We Got?

A 2007 study states:

> "there is significantly increased experience in using Economic Capital across the whole financial services sector (e.g. for banking, frameworks have been in place an average of over 6 years and for insurance 4) and firms now feel broadly comfortable with the accuracy of outputs (75%+ for both insurance and banking). This in turn has meant that far more institutions feel sufficiently comfortable with their Economic Capital results to use them in dicussions with external stakeholders, and there is increased use in business applications, albeit often as supplementary information rather than as a core driver." [IFRI Foundation and CRO Forum, 2007]

The 2006 IFRI/CRO Forum survey

17 banks taking part:

- Europe: ABN Amro, Allied Irish Banks, Barclays, Credit Suisse, Deutsche Bank, Dresdener Bank. EuroClear, Fortis. America: Citigroup, Goldman Sachs, Royal Bank of Canada, State Street. Asia: DBS Group, Macquarie Bank, National Australian Bank, Standard Chartered, Westpac.

17 insurance companies taking part:

- Europe: Aegon, Allianz, Assicurazioni Generali, Aviva, AXA, Fortis, ING, Munich Re, Prudential, Royal & Sun Alliance. Switzerland: Converium, Swiss Re, Winterthur, Zurich. America: American International Group, Royal Bank of Canada. Australia: Insurance Australia Group.

2. Correlation and Diversification

The pooling of risks across portfolios, business lines, organisations achieves diversification. The extent of the diversification benefit depends on the degree of dependence between the pooled risks. Aggregate economic capital should reflect the diversification benefit.

In [Kuritzkes et al., 2002] three levels of aggregation are identified:

1. stand-alone risks within a single risk factor (e.g. underwriting risk in each contract of a domestic motor portfolio);

2. different risk factors within a single business line (e.g. combining asset, underwriting and operational risks in non-life or life insurance);

3. different business lines within an enterprise.

No Single Approach for Diversification

"One of the major areas for further discussion within the industry (currently in Basel II and shortly in Solvency II) - the treatment of diversification effects - shows little sign of a single approach holding sway. Estimates of the impact of diversification on the capital estimate differ significantly driven by two main factors:

- The approaches institutions use differ, for instance variance-covariance matrices are most popular in banking, while simulation approaches and copulas are more popular in insurance
- Correlation estimates used vary widely, to an extent that is unlikely to be solely attributable to differences in business mix"

[IFRI Foundation and CRO Forum, 2007]

No Single Approach for Diversification II

"Over 70% of participants characterise their inclusion of inter-risk diversification as top-down. Interestingly, 20% of participants use a simple summation approach (which might be considered a Pillar 1 as opposed to a Pillar 2 approach) and of those quantifying diversification benefits, over three quarters use an analytical variance/covariance approach. The estimated impact of inter-risk diversification is significant - with an overall reduction in aggregate capital in the range of 15–20% being most common." [IFRI Foundation and CRO Forum, 2007]

Mathematical Framework

An enterprise may be split into d sub-units (business lines, risk factors by business line, contracts/investments). Each sub-unit generates a loss or a (negative) change-in-value L_i over the time horizon of interest. The aggregate change-in-value distribution is given by

$$L = L_1 + \cdots + L_d \, .$$

Ideal goal:

Determination of risk capital should be based on a stochastic model for $(L_1, \ldots, L_d)$ that accurately reflects the dependence structure.

Diversification and Correlation

> " Diversification benefits can be assessed by correlations between different risk categories. A correlation of $+100\%$ means that two variables will fall and rise in lock-step; any correlation below this indicates the potential for diversification benefits." [Treasury and FSA, 2006]

The last statement is not true of ordinary linear (Pearson) correlation! But true of rank correlation.

Lock-step

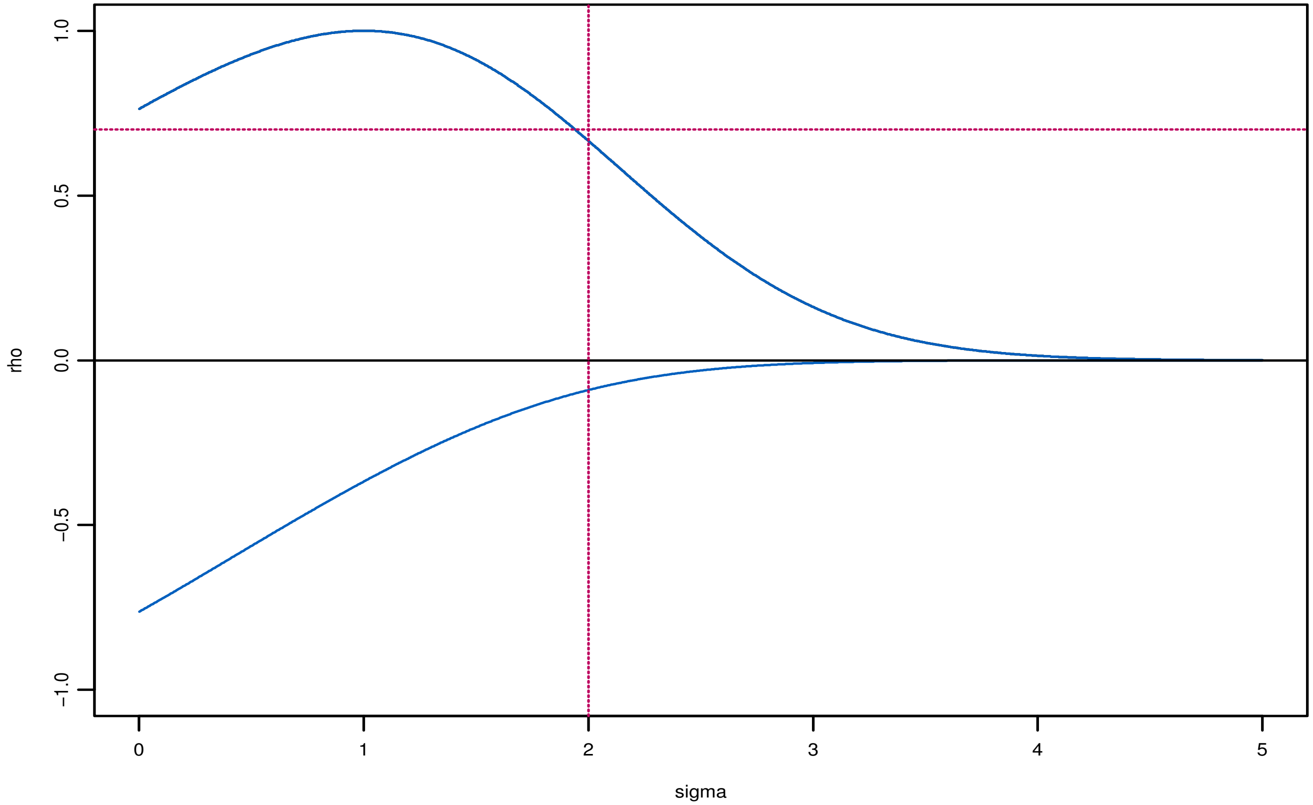
The mathematical term for this is comonotonicity. It means all risks are increasing functions of a common underlying risk: $(L_1, \ldots, L_d) = (v_1(Z), \ldots, v_d(Z))$. Such risks would be considered undiversifiable.

Comonotonicity and Correlation

$$(\text{linear correlation} = 1) \;\Rightarrow\; \text{comonotonicity}$$
$$\text{comonotonicity} \;\not\Rightarrow\; (\text{linear correlation} = 1)$$

- We can create models where individual risks move in lock-step (are undiversifiable), but have an arbitrarily small correlation.
- For two given distributions, attainable correlations form a sub-interval of $[-1, 1]$.
- Upper bound corresponds to comonotonicity, lower to countermonotonicity (negative lock-step)
- Our intuition about linear correlation is in fact very faulty!

Example of Attainable Correlations



Take $X_1 \sim$ Lognormal$(0, 1)$, and $X_2 \sim$ Lognormal$(0, \sigma^2)$. Observe how interval of attainable correlations varies with σ. Upper boundary represents comonotonicity. See [McNeil et al., 2005] for details.

Correlation Confusion

"Among nine big economies, stock market correlations have averaged around 0.5 since the 1960s. In other words, for every 1 per cent rise (or fall) in, say, American share prices, share prices in the other markets will typically rise (fall) by 0.5 per cent."

The Economist, 8 November 1997

"A correlation of 0.5 does not indicate that a return from stockmarket A will be 50% of stockmarket B's return, or vice-versa...A correlation of 0.5 shows that 50% of the time the return of stockmarket A will be positively correlated with the return of stockmarket B, and 50% of the time it will not."

The Economist (letter), 22 November 1997

3. Some Issues in Bottom-Up Capital Calculation

The standard formula for the solvency capital requirement in Solvency II is an example of a bottom-up or modular approach.

Individual risks (sub-units) are transformed into capital charges $\mathsf{SCR}_1, \ldots, \mathsf{SCR}_d$. These are then combined to calculate the overall solvency capital requirement SCR. ([CEIOPS-06, 2006], page 71)

The combination operation may involve a calculation of the following kind:

$$\mathsf{SCR} = \sqrt{\sum_{i=1}^{d} \sum_{j=1}^{d} \rho_{ij} \mathsf{SCR}_i \mathsf{SCR}_j}$$

where the ρ_{ij} are the “correlations” between the risks. (for, example [CEIOPS-06, 2006], page 98)

What Principle Underlies This?

Suppose

- we measure risks with a quantile-based (Value-at-Risk) approach ($\text{SCR}_i = \mathrm{VaR}_\alpha(L_i)$, $\text{SRC} = \mathrm{VaR}_\alpha(L)$, $\alpha > 0.5$);

- the risks $(L_1, \ldots, L_d)$ are jointly normal with zero mean and correlations given by ρ_{ij}.

(More generally, we could consider any positive-homogeneous risk measure (such as cVaR/expected shortfall) in first assumption and any centred elliptical distribution (such as multivariate Student t) in second.)

Short Derivation of Aggregation Rule

$$\mathrm{sd}(L) = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathrm{sd}(L_i)\mathrm{sd}(L_j)}$$

Now $\mathrm{VaR}_\alpha(L) = \lambda_\alpha \mathrm{sd}(L)$ and $\mathrm{VaR}_\alpha(L_i) = \lambda_\alpha \mathrm{sd}(L_i)$ where λ_α is the α-quantile of standard normal. This yields

$$\begin{aligned}
\mathrm{VaR}_\alpha(L) &= \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathrm{VaR}_\alpha(L_i)\mathrm{VaR}_\alpha(L_j)} \\
\mathsf{SCR} &= \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathsf{SCR}_i\mathsf{SCR}_j} \quad .
\end{aligned}$$

Issues with this style of bottom-up

- It is only underpinned by theoretical principles in a very specific and unrealistic model of the risk universe.

- It is dependent on the widely misunderstood concept of correlation.

- The kinds of risks where we have reliable empirical experience of typical values are in the minority (e.g. financial market risks, and even then only at shorter time horizons)

- Can we trust "experts" to deliver correlations in other cases? There are consistency requirements: every ρ_{ij} should be compatible with the distribution of L_i and L_j. The matrix (ρ_{ij}) must be positive semi-definite. It is quite easy to specify nonsensical correlation matrices.

How to Account for Tail Dependence?

"Further analysis is required to assess whether linear correlation, together with a simplified form of tail correlation may be a suitable technique to aggregate capita requirements for different risks." [CEIOPS-06, 2006] (page 75)

"When selecting correlation coefficients, allowance should be made for tail correlation. To allow for this, the correlations used should be higher than simple analysis of relevant data would indicate." [CEIOPS-06, 2006] (page 142)

Is the sum of capital charges a bound for SCR?

Suppose again that risk capital charges have the quantile interpretation so that $\mathsf{SCR}_i = \mathrm{VaR}_\alpha(L_i)$ and $\mathsf{SCR} = \mathrm{VaR}_\alpha(L)$.

In the case where we have no diversification (comonotonic risks $L_i = u_i(Z)$, $i = 1, \ldots, d$) we can compute that

$$\mathsf{SCR} = \sum_{i=1}^{d} \mathsf{SCR}_i$$

Fallacy:

"this is an upper bound for the solvency capital requirement under all dependence assumptions for $(L_1, \ldots, L_d)$."

Superadditive Capital

Actually, it is possible to construct models for $(L_1, \ldots, L_d)$ with unusual dependence structures such that

$$\mathrm{VaR}_\alpha(L) > \sum_{i=1}^{d} \mathrm{VaR}_\alpha(L_i) = \mathsf{SCR}$$

It is also possible to find violations for independent risks when individual loss distributions are strongly skewed.

- To rectify this problem we would have to base risk measurement and capital charges on a subadditive risk measure (like expected shortfall).
- Many argue that the models leading to superadditivity are too implausible to consider, but they do undermine our principles!

Better Bottom-Up

- Copulas are a better theoretical tool for combining the individual capital charges. They avoid tricky consistency requirements imposed by working with linear correlations.

- Implicitly aggregation based on the Gauss copula has been used in insurance for years. For example @RISK by Palisade software implicitly uses the Gauss copula to perform Monte Carlo risk analysis.

- However, calibration remains a problem. Copula parameters are usually inferred from matrices of rank correlations, but are we expert enough to set these?

- Bottom-up approaches require the exogenous specification of parameters determining the dependence model.

4. Integrated Models for Capital Calculation

In a fully integrated approach the correlations are endogenous and result from specifying the mutual dependence of risks across the enterprise on common risk drivers or factors.

$$L_i = f_i(\text{common factors}, \text{idiodyncratic errors}), \quad i = 1, \ldots, d.$$

Generally these models are handled by Monte Carlo, i.e. the generation of scenarios for the common driving factors.

They appeal because they are structural and explanatory.

Advantages

- For an internal solvency capital model, this would be the more "principles-based" way to proceed.

- A natural framework for a top-down risk-based allocation of solvency capital to business units which opens door to risk-based performance measurement (RORAC).

- A framework for actual computation of the diversification benefit and attribution of that benefit to sub-units.

- Framework for sensitivity analyses with respect to common factors and model risk studies with respect to model assumptions.

- Tail correlation may be studied in terms of extreme outcomes in key risk drivers.

Capital Allocation

We require a method of breaking up the overall solvency capital requirement into a vector of capital allocations $(\mathrm{EC}_1, \dots, \mathrm{EC}_d)$ such that

$$\mathsf{SCR} = \sum_{i=1}^{d} \mathrm{EC}_i$$

If we base our capital adequacy computation on a risk measure, such as VaR, it is known that a rational and fair way of doing this is Euler allocation [Tasche, 1999]. In the case of VaR we have $\mathsf{SRC} = \mathrm{VaR}_\alpha(L)$ and the capital allocations are given by

$$\mathrm{EC}_i = E\left(L_i \mid L = \mathrm{VaR}_\alpha(L)\right),$$

where EC_i is known as the VaR contribution of business unit i. Contributions can be estimated by Monte Carlo.

Diversification Scoring

Tasche [Tasche, 2006] defines diversification factors as follows:

$$
\begin{aligned}
\mathsf{DF} &= \frac{\mathsf{SRC}}{\sum_{i=1}^{d} \mathsf{SRC}_i} \\
\mathsf{DF}_i &= \frac{\mathrm{EC}_i}{\mathsf{SRC}_i}
\end{aligned}
$$

The former measures portfolio diversification - overall benefit in terms of reduction in solvency capital that the business units achieve by being together within the enterprise.

The latter measures effect of diversification for unit i - the benefit to business unit i in terms of reduction in solvency capital achieved by belonging to enterprise.

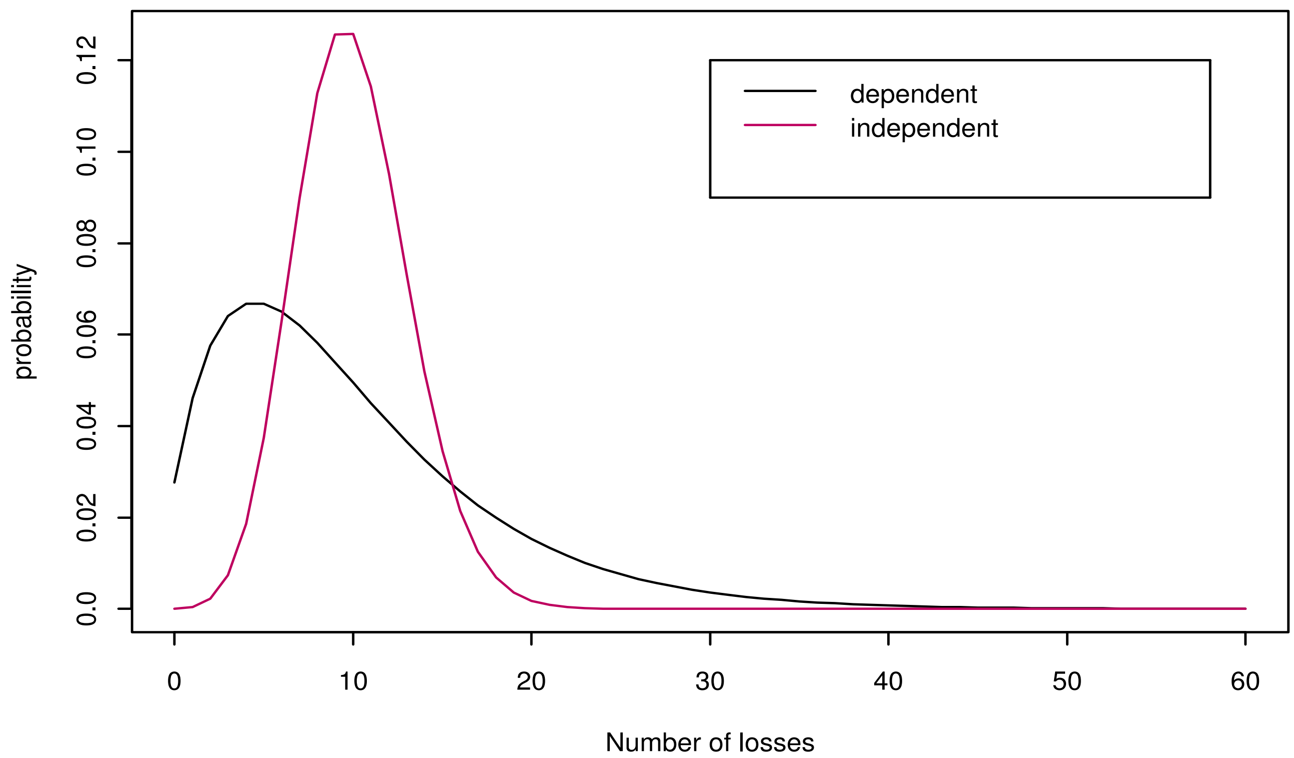
Issues

The conclusions about capital adequacy and risk-based performance comparison are only as good as the underlying models, which need to be built by skilled craftsmen. The biggest issue is the sensitivity of the results to the model inputs, in particular the model components specifying the dependence of risks on common factors.

Seemingly innocuous assumptions about correlations can have large effects.

Consider following example from credit risk. By adding a common factor that induces a default correlation of 0.005 between every pair of counterparties, we inflate tail of loss distribution.

Impact of Dependence on Credit Loss Distribution



Comparison of the loss distribution of a homogeneous portfolio of 1000 loans with a default probability of 1% assuming (i) independent defaults and (ii) a default correlation of 0.5%.

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