
EDMVND. HALLEIVS LL.D.
GEOM. PROF. SAVIL. & R.S. SECRET.

JIA 112 (1985) 279-301

EDMOND HALLEY: ASTRONOMER AND ACTUARY

By Geoffrey Heywood, M.B.E., F.I.A., F.F.A., F.R.A.S.

Edmond Halley, who was born in 1656 and died in 1742, is well known generally because of the Comet which bears his name. He is not, however, associated to any great extent with other activities and if it were not for the Comet it is likely that most people today would never even have heard of him. Outside the actuarial profession, and perhaps to some extent even within it, it is not generally known that in 1693 he constructed a life table from the bills of mortality in the German city of Breslau and then went on to calculate from that table annuity rates on one, two and three lives. Reference to this was formerly included in the introduction to the *Year Book*, but this has now been shortened and the reference to Halley has disappeared.

Halley's Comet is due to return to the neighbourhood of the Earth and the Sun at the end of 1985 and the beginning of 1986. The Comet moves around the sun in a highly elliptical orbit which takes it out beyond the planet Neptune and it returns to the Earth/Sun area on average every 75 or 76 years. The extremes of its return are 79 years and 74 years, the variation being due in the main to the gravitational perturbations of the giant planets Jupiter and Saturn when the Comet is in their vicinity. Its last appearance was in 1910. In 1682, during Halley's lifetime, the Comet appeared as an object visible to the naked eye and Halley computed its orbit and took the view that it was identical with the Comet which was known to have been seen in 1607, some 75 years previously. It is interesting to note that he consulted Sir Isaac Newton on the subject, and the latter, surprisingly, took the view that the orbit was not elliptical and that the two comets were not one and the same. Halley, however, was confident of his theory and he therefore went on to predict that the Comet would return again in 1759 and this it did some 17 years after his death. He said, perhaps somewhat modestly, that if the Comet should re-appear in 1759, when he knew he would no longer be alive, he hoped that posterity would not refuse to acknowledge that this was first discovered by an Englishman. When the Comet did appear in 1759 it was immediately called Halley's Comet and has been referred to as such ever since and is by far the most famous periodic comet in the solar system.

Subsequent research has been carried out by many astronomers, who have examined astronomical records of China, India, the Eastern Mediterranean and Europe and it has been discovered that a comet has been recorded as appearing at intervals of 75 or 76 years since 240 BC. A comet is recorded as appearing in 1059 BC and this again fits in with the periodic return of some 75 or 76 years. If the appearance in 1059 BC is taken as the first recorded appearance then the return in 1985/86 is no less than the 42nd appearance, of which, since that in 240 BC, only one return has been unobserved. Interesting returns are as follows:

Year	Remarks
12 BC	Contrary to popular myth, Halley's Comet was certainly not the 'Star of Bethlehem'.
66 AD	Halley's Comet could be the sword hanging in the sky which foretold the destruction of the Temple in Jerusalem.
684 AD	The earliest recorded drawing of the Comet, on its 684 AD visit, is recorded in the Nurnberg Chronicles, published in 1493.
837 AD	The most spectacular return of Halley's Comet, when its brightness was equal to that of the Planet Venus and its tail extended 93 degrees across the sky.
1066 AD	The Comet is recorded in the Bayeux Tapestry, woven to commemorate the Battle of Hastings.
1301 AD	It was seen by Giotto, a Florentine artist, who painted 'The Adoration of the Magi' and included the Comet in the painting. It is interesting that one of the Halley's Comet Probes launched this year has been called 'Giotto'.
1682 AD	Observed by Edmond Halley himself between August and September.
1759 AD	The first predicted return.
1835 AD	Widely observed.
1910 AD	Again widely observed and the last appearance.

The Comet, which even in 1910, was not located until a few months before it became visible to the naked eye, has already been found as long ago as 16 October 1982 by the sophisticated instruments and photographic procedures which are now available. It is expected to become visible to the naked eye about October 1985 and remain visible until March 1986. Unfortunately, this return is not particularly good for observation from the Northern Hemisphere and astronomers south of the Equator will be much better placed to see it.

The purpose of this note, however, is not primarily to discuss Halley as an astronomer, or his Comet, but to refer to the investigation into the bills of mortality of the City of Breslau, resulting in the publication of his life table and also the first calculation using correct formulae of annuities on one, two and three lives.

Before coming on to the Breslau tables, it is appropriate to record many other activities with which this remarkable man was involved. These may be summarized as follows:

At the early age of about 21 he promoted a voyage and sailed to St Helena, where he spent a year cataloguing for the first time the stars of the Southern Hemisphere. On his return he was awarded an M.A. from Oxford University and was also elected a Fellow of the Royal Society.

In 1685 he became Clerk, as it was then designated, of the Royal Society, an appointment which corresponded to that of Secretary. At about this time he visited Cambridge to see Sir Isaac Newton, who was preparing his memorable work 'Philosophiae Naturalis Principia Mathematica' and although it was then almost complete, Newton seemed reluctant to publish this work. Halley, in his capacity of Clerk of the Royal Society, impressed upon Newton the importance of making his discovery available to the scientific world and he eventually persuaded him to publish the Principia, and also met some of the cost of publishing from his own private resources.

Although he was unfortunate in being overshadowed by the great genius of Newton, he was nevertheless the first person to use Newton's methods to calculate the orbits of planets and comets and he could not have done this without a thorough understanding of these new ideas.

From 1696 to 1698 he spent an unhappy period as Deputy Controller of the Mint at Chester.

He returned to London in 1698 and immediately became interested in one of the important problems of his day, namely that of determining one's longitudinal position at sea and, closely allied with this, the variation of the Earth's magnetic field in the South Atlantic. Very unusually, for a civilian, he was commissioned as a Captain of one of His Majesty's Ships and set sail in 1698, in HMS *Paramore*, for a year's voyage to research these matters. On his return he undertook a second voyage in 1699, in the same ship, again to the South Atlantic, and not only charted the magnetic variation in those areas, but also the various currents which he discovered. His third and last voyage, in 1701, again in HMS *Paramore*, was much shorter and was confined to making soundings, charting the direction of currents and determining the magnetic variation in the English Channel.

He was also fascinated by the idea, as he called it, of "walking under water" and he wrote papers describing how a diving bell could be constructed and this was eventually made and experiments carried out successfully.

Another area in which he was interested was that of archaeology and history and he wrote a paper entitled 'A Discourse Tending to Prove at What Time and Place Julius Caesar made his First Descent upon Britain'. As a result of his research into the records of Caesar's expedition and noting the details of an eclipse of the Moon which occurred at that time, he reached the conclusion that Caesar's statement of when and where he landed was not correct.

In 1703 he was appointed Savilian Professor of Geometry at Oxford.

He became the second person to hold the appointment of Astronomer Royal, a post which was created by King Charles II in 1675 and Halley was appointed in 1721 and held it until his death in 1742.

Before discussing the Breslau mortality table, it is important to record that while he was Professor of Geometry at Oxford, he wrote a paper on compound interest and this was subsequently published in Sherwin's *Mathematical Tables* in 1761 and reproduced in *J.I.A.* **9**, 259. This paper sets out the basic principles of calculating the fundamental functions of compound interest on correct principles.

It is, however, with his compilation of the Breslau mortality table and the calculation by a correct method of annuity values that he will be remembered by the actuarial profession. The two original papers which he wrote were published in *Philosophical Transactions* in 1693, Vol. 17, pages 596 and 654. Although these papers were reproduced in modern English in *J.I.A.* **18,** 251–62, they are not very readily available in their original form and so are reproduced again at the end of this note in facsimile form, with the kind permission of the Royal Society.

Halley was not the first to produce statistics from bills of mortality. Earlier work had been carried out by John Graunt on bills of mortality of the City of London and his results were published in the year 1662. He did not, however, attempt to calculate annuity values, but his publication, namely 'Natural and Political Observations made upon The Bills of Mortality', may be regarded as the foundation of the science of demography. Full details of John Graunt are set out in *J.I.A.* **90,** 1–61.

Annuity values had been calculated earlier, in 1671 by Johannes de Wit, who is described as the Grand Pensionary of Holland and West Friesland, an office virtually equivalent to that of Prime Minister. A tile plaque of Johannes de Wit was presented to the Institute, on the occasion of the Centenary Celebrations in 1948, by the Actuarial Genootschap of Holland and is now displayed in the Redington Room. The method which he used is set out at some length by Frederick Hendriks in *J.I.A.* **2,** 121 & 222 and *J.I.A.* **3,** 93. Suffice it to say here that the formula which de Wit used to calculate annuity values, and which is theoretically inaccurate, is not precisely correct unless l_w is 0. The formula is

$$a_x = \frac{1}{l_x} \sum_{t=0}^{w-t-x} d_{x+t}\, a_t$$

Moreover the annuity values are cumbersome to calculate and the mortality rates which he used were empirical, thereby giving satisfactory results.

There is also an extensive reference to the efforts of de Wit and Halley in an article by T. B. Sprague on annuities, which appeared in the 9th Edition of the *Encyclopaedia Britannica* in 1875. Again, for those who want to pursue the history of the subject in greater depth, reference should be made to that article, but this note may conveniently end by quoting Sprague himself, when he says: "de Wit's report being thus of the nature of an unpublished state paper, although it contributed to its author's reputation, did not contribute to advance the exact knowledge of the subject; and the author to whom the credit must be given of first showing how to calculate the value of an annuity on correct principles is Dr. Edmond Halley, FRS".

Finally, it is interesting to note that the premium rates used by the Amicable Society, in its prospectus published in 1790, were based on the Breslau table and in the prospectus it was reproduced in full.

An Eſtimate of the Degrees of the Mortality *of Mankind, drawn from curious* Tables *of the* Births *and* Funerals *at the City of* Breſlaw ; *with an Attempt to aſcertain the Price of* Annuities *upon* Lives. *By* Mr. E. Halley, *R.S.S.*

THE Contemplation of the *Mortality* of *Mankind*, has beſides the *Moral*, its *Phyſical* and *Political* Uſes, both which have been ſome years ſince moſt judiciouſly conſidered by the curious Sir *William Petty*, in his *Natural* and *Political* Obſervations on the Bills of *Mortality* of *London*, owned by Captain *John Graunt*. And ſince in a like Treatiſe on the Bills of *Mortality* of *Dublin.*

Dublin. But the Deduction from thoſe Bills of *Mortality* ſeemed even to their Authors to be defective: Firſt, In that the *Number* of the People was wanting. Secondly, That the *Ages* of the People dying was not to be had. And Laſtly, That both *London* and *Dublin* by reaſon of the great and caſual Acceſſion of *Strangers* who die therein,(as appeared in both, by the great Exceſs of the *Funerals* above the *Births*) rendred them incapable of being Standards for this purpoſe; which requires, if it were poſſible, that the People we treat of ſhould not at all be changed, but die where they were born, without any Adventitious Increaſe from Abroad, or Decay by Migration elſewhere.

This *Defect* ſeems in a great meaſure to be ſatisfied by the late curious Tables of the Bills of *Mortality* at the City of *Breſlaw*, lately communicated to this Honourable Society by Mr. *Juſtell*, wherein both the *Ages* and *Sexes* of all that die are monthly delivered, and compared with the number of the *Births*, for Five Years laſt paſt, *viz.* 1687, 88, 89, 90, 91, ſeeming to be done with all the Exactneſs and Sincerity poſſible.

This City of *Breſlaw* is the Capital City of the Province of *Sileſia*; or, as the *Germans* call it, *Schleſia*, and is ſcituated on the Weſtern Bank of the River *Oder*, anciently called *Viadrus*; near the Confines of *Germany* and *Poland*, and very nigh the Latitude of *London*. It is very far from the Sea, and as much a *Mediterranean* Place as can be deſired, whence the Confluence of Strangers is but ſmall, and the Manufacture of Linnen employs chiefly the poor People of the place, as well as of the Country round about; whence comes that ſort of Linnen we uſually call your *Seleſie Linnen*; which is the chief, if not the only Merchandize of the place. For theſe Reaſons the People of this City ſeem moſt pro-

C 2 per

per for a *Standard*; and the rather, for that the *Births* do, a ſmall matter, exceed the *Funerals*. The only thing wanting is the Number of the whole People, which in ſome meaſure I have endeavoured to ſupply by compariſon of the *Mortality* of the People of all Ages, which I ſhall from the ſaid Bills trace out with all the Acuracy poſſible.

It appears that in the Five Years mentioned, *viz.* from 87 to 91 incluſive, there were *born* 6193 Perſons, and *buried* 5869; that is, born *per Annum* 1238, and *buried* 1174; whence an *Encreaſe* of the People may be argued of 64 *per Annum*, or of about a 20th part, which may perhaps be ballanced by the Levies for the *Emperor*'s Service in his Wars. But this being contingent, and the Births certain, I will ſuppoſe the People of *Breſlaw* to be encreaſed by 1238 *Births* annually. Of theſe it appears by the ſame Tables, that 348 do die *yearly* in the *firſt Year* of their *Age*, and that but 890 do arrive at a full *Years Age*; and likewiſe, that 198 do die in the *Five Years* between 1 and 6 compleat, taken at a *Medium*; ſo that but 692 of the Perſons *born* do ſurvive *Six* whole *Years*. From this *Age* the Infants being arrived at ſome degree of Firmneſs, grow leſs and leſs *Mortal*; and it appears that of the whole People of *Breſlaw* there die *yearly*, as in the following Table, wherein the upper Line ſhews the *Age*, and the next under it the *Number* of Perſons of that Age *dying yearly*.

(599)

7 . 8 9. . 14 . 18 . 21 . 27 . 28 . . 35 .
11 . 11 . 6 . $5\frac{1}{2}$. 2 . $3\frac{1}{2}$ 5 6 $4\frac{1}{2}$ $6\frac{1}{2}$ 9 . 8 . 7 . 7 .

36 . 42 . 45 49 54 . 55 . 56 . 63
8 . $9\frac{1}{2}$ 8 . 9 . 7 . 7 . 10 11 . 9 . 9 . 10 . 12

70 71 . 72 77 81 84 . 90 91 .
$9\frac{1}{2}$ 14 9 . 11 $9\frac{1}{2}$ 6 . 7 . 3 . 4 . 2 . 1 . 1 . 1 .

98 . 99 . 100.
0 . $\frac{1}{5}$. $\frac{3}{5}$

And where no *Figure* is placed over, it is to be underſtood of thoſe that die between the Ages of the preceding and conſequent *Column*.

From this Table it is evident, that from the Age of 9 to about 25 there does not die above 6 *per Annum* of each *Age*, which is much about one *per Cent.* of thoſe that are of thoſe *Ages:* And whereas in the 14, 15,16, 17 *Years* there appear to die much fewer, as 2 and $3\frac{1}{2}$, yet that ſeems rather to be attributed to Chance, as are the other Irregularities in the Series of Ages, which would rectifie themſelves, were the number of Years much more conſiderable, as 20 inſtead of 5. And by our own Experience in *Chriſt-Church Hoſpital*, I am informed there die of the *Young Lads*, much about one *per Cent. per Annum*, they being of the foreſaid *Ages*. From 25 to 50 there ſeem to die from 7 to 8 and 9 *per Annum* of each Age; and after that to 70, they growing more *craſie*, though the number be much diminiſhed, yet the *Mortality encreaſes*, and there are found to die 10 or 11 of each Age *per Annum*: From thence the number of the *Living* being grown very ſmall, they gradu-

(600)

gradually decline till there be none left to *die*; as may be ſeen at one View in the Table.

From theſe Conſiderations I have formed the *adjoyned Table*, whoſe Uſes are manifold, and give a more juſt *Idea* of the *State* and *Condition* of *Mankind*, than any thing yet extant that I know of. It exhibits the *Number* of *People* in the City of *Breſlaw* of all Ages, from the *Birth* to extream *Old Age*, and thereby ſhews the Chances of *Mortality* at all *Ages*, and likewiſe how to make a certain Eſtimate of the value of *Annuities* for *Lives*, which hitherto has been only done by an imaginary *Valuation*: Alſo the *Chances* that there are that a *Perſon* of any *Age* propoſed does live to any other *Age* given; with many more, as I ſhall hereafter ſhew. This *Table* does ſhew the *number* of *Perſons* that are living in the *Age* current annexed thereto, as follows:

Age. Curt.	Perſons.	Age. Curt.	Perſons	Age. Curt.	Perſons	Age. Curt.	Perſons	Age. Curt.	Perſons	Age. Curt.	Perſons
1	1000	8	680	15	628	22	585	29	539	36	481
2	855	9	670	16	622	23	579	30	531	37	472
3	798	10	661	17	616	24	573	31	523	38	463
4	760	11	653	18	610	25	567	32	515	39	454
5	732	12	646	19	604	26	560	33	507	40	445
6	710	13	640	20	598	27	553	34	499	41	436
7	692	14	634	21	592	28	546	35	490	42	427
Age Curt.	Perſons.	Age. Curt.	Perſons	Age. Curt.	Perſons	ge Curt.	Perſons	Age. Curt.	Perſons	Age. Curt.	Perſons
43	417	50	346	57	272	64	202	71	131	78	58
44	407	51	335	58	262	65	192	72	120	79	49
45	397	52	324	59	252	66	182	73	109	80	41
46	387	53	313	60	242	67	172	74	98	81	34
47	377	54	302	61	232	68	162	75	88	82	28
48	367	55	292	62	222	69	152	76	78	83	23
49	357	56	282	63	212	70	142	77	68	84	20

Age.	Perſons.
7	5547
14	4584
21	4270
28	3964
35	3604
42	3178
49	2709
56	2194
63	1694
70	1204
77	692
84	253
100	107
	34000
Sum Total.	

Thus it appears, that the whole People of *Breſlaw* does conſiſt of 34000 *Souls*, being the Sum *Total* of the Perſons of all Ages in the *Table*: The firſt uſe hereof is is to ſhew the Proportion of *Men* able to bear *Arms* in any *Multitude*, which are thoſe between 18 and 56, rather than 16 and 60; the one being generally too weak to bear the *Fatigues* of *War* and the Weight of *Arms*, and the other too craſie and infirm from *Age*, notwithſtanding particular Inſtances to the contrary. Under 18 from the *Table*, are found in this City 11997 Perſons, and 3950 above 56, which together make 15947. So that the Reſidue to 34000 being 18053 are Perſons between thoſe *Ages*. At leaſt one half thereof are Males, or 9027 : So that the whole Force this City can raiſe of *Fencible Men*, as the *Scotch* call them, is about 9000, or $\frac{9}{34}$, or ſomewhat more than a quarter of the *Number* of *Souls*, which may perhaps paſs for a Rule for all other places.

The *Second Uſe* of this Table is to ſhew the differing degrees of *Mortality*, or rather *Vitality* in all *Ages* ; for if the number of Perſons of any *Age* remaining after one year, be divided by the difference between that and the number of the Age propoſed, it ſhews the *odds* that there is, that a Perſon of that Age does not die in a *Year*. As for Inſtance, a Perſon of 25 *Years* of *Age* has the odds of 560 to 7 or 80 to 1, that he does not *die* in a *Year* : Becauſe that of 567, living of 25 years of Age, there do die no more than 7 in a *Year*, leaving 560 of 26 Years old.

So likewiſe for the *odds*, that any Perſon does not die before he attain any propoſed *Age*: Take the *number* of the remaining Perſons of the Age propoſed, and divide it by the difference between it and the number of thoſe of the *Age* of the Party propoſed; and that ſhews the *odds* there is between the Chances of the Party's living or dying. As for Inſtance ; What is the *odds* that a Man of 40 lives 7 Years : Take the number of Perſons of 47 years, which in the Table is 377, and ſub-

ſubſtract it from the number of Perſons of 40 years, which is 445, and the *difference* is 68: Which ſhews that the *Perſons dying* in that 7 years are 68, and that it is 377 to 68 or $5\frac{1}{2}$ to 1, that a Man of 40 does live 7 Years. And the like for any other *number* of *Years*.

Use III. But if it be enquired at what number of *Years*, it is an even Lay that a Perſon of any *Age* ſhall die, this Table readily performs it: For if the *number* of Perſons *living* of the *Age* propoſed be *halfed*, it will be found by the *Table* at what Year the ſaid *number* is reduced to half by *Mortality*; and that is the Age, to which it is an even Wager, that a Perſon of the *Age* propoſed ſhall arrive before he *die*. As for Inſtance; A Perſon of 30 Years of *Age* is propoſed, the number of that Age is 531, the half thereof is 265, which number I find to be between 57 and 58 Years; ſo that a Man of 30 may reaſonably expect to live between 27 and 28 Years.

Use IV. By what has been ſaid, the *Price* of *Inſurance* upon *Lives* ought to be regulated, and the difference is diſcovered between the *price* of enſuring the *Life* of a *Man* of 20 and 50, for Example: it being 100 to 1 that a Man of 20 dies not in a year, and but 38 to 1 for a Man of 50 Years of Age.

Use V. On this depends the Valuation of *Annuities* upon *Lives*; for it is plain that the *Purchaſer* ought to pay for only ſuch a part of the value of the *Annuity*, as he has Chances that he is living; and this ought to be computed yearly, and the Sum of all thoſe yearly Values being added together, will amount to the value of the *Annuity* for the *Life* of the Perſon propoſed. Now the preſent value of Money payable after a term of years, at any given rate of Intereſt, either may be had from Tables already computed; or almoſt as compendiouſly, by by the Table of Logarithms: For the Arithmetical Complement of the Logarithm of Unity and its yearly Intereſt (that is, of 1, 06 for Six *per Cent.* being 9, 974694.) being multiplied by the number of years propoſed, gives the preſent value of One Pound payable after the end of ſo many years Then by the foregoing Propoſition, it will be as the number of Perſons living after that term of years, to the number dead; ſo are the Odds that any one Perſon is Alive or Dead. And by conſequence, as the Sum of both or the number of Perſons living of the Age firſt propoſed, to the number remaining after ſo many years, (both given by the Table) ſo the preſent value of the yearly Sum payable after the term propoſed, to the Sum which ought to be paid for the Chance the perſon has to enjoy ſuch an Annuity after ſo many Years. And this being repeated for every year of the perſons Life, the Sum of all the preſent Values of thoſe Chances is the true Value of the Annuity. This will without doubt appear to be a moſt laborious Calculation, but it being one of the principal Uſes of this Speculation, and having found ſome ***Compendia*** for the Work, I took the pains to compute the following Table, being the ſhort Reſult of a not ordinary number of Arithmetical Operations; It ſhews the Value of Annuities for every Fifth Year of Age, to the Seventieth, as follows.

Age.	Years Purchaſe.	Age.	Years Purchaſe.	Age.	Years Purchaſe.
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

D This

This ſhews the great Advantage of putting Money into the preſent *Fund* lately granted to their Majeſties, giving 14 *per Cent. per Annum*, or at the rate of 7 years purchaſe for a Life; when young Lives, at the uſual rate of Intereſt, are worth above 13 years Purchaſe. It ſhews likewiſe the Advantage of young Lives over thoſe in Years; a Life of Ten Years being almoſt worth $13\frac{1}{2}$ years purchaſe, whereas one of 36 is worth but 11.

Uſe V. Two Lives are likewiſe valuable by the ſame Rule; for the number of Chances of each ſingle *Life*, found in the Table, being multiplied together, become the Chances of the Two Lives. And after any certain Term of Years, the Product of the two remaining Sums is the Chances that both the Perſons are living. The Product of the two Differences, being the numbers of the Dead of both Ages, are the Chances that both the Perſons are dead. And the two Products of the remaining Sums of the one Age multiplied by thoſe dead of the other, ſhew the Chances that there are that each Party ſurvives the other: Whence is derived the Rule to eſtimate the value of the Remainder of one Life after another. Now as the Product of the Two Numbers in the Table for the Two Ages propoſed, is to the difference between that Product and the Product of the two numbers of Perſons deceaſed in any ſpace of time, ſo is the value of a Sum of Money to be paid after ſo much time, to the value thereof under the Contingency of Mortality. And as the aforeſaid Product of the two Numbers anſwering to the Ages propoſed, to the Product of the Deceaſed of one Age multiplied by thoſe remaining alive of the other; So the Value of a Sum of Money to be paid after any time propoſed, to the value of the Chances that the one Party has that he ſurvives the other whoſe number of Deceaſed you made uſe of, in the ſecond Term of the proportion. This perhaps may

may be better underſtood, by putting N for the number of the younger Age, and n for that of the Elder; Y, y the deceaſed of both Ages reſpectively, and R, r for the Remainders; and $R + Y = N$ and $r + y = n$. Then ſhall $N n$ be the whole number of Chances; $Nn - Yy$ be the Chances that one of the two Perſons is living, Yy the Chances that they are both dead; $R y$ the Chances that the elder Perſon is dead and the younger living; and $r Y$ the Chances that the elder is living and the younger dead. Thus two Perſons of 18 and 35 are propoſed, and after 8 years theſe Chances are required. The Numbers for 18 and 35 are 610 and 490, and there are 50 of the Firſt Age dead in 8 years, and 73 of the Elder Age. There are in all 610 x 490 or 298900 Chances; of theſe there are 50 x 73 or 3650 that they are both dead. And as 298900, to 298900 − 3650, or 295250: So is the preſent value of a Sum of Money to be paid after 8 years, to the preſent value of a Sum to be paid if either of the two live. And as 560 x 73, ſo are the Chances that the Elder is dead, leaving the Younger; and as 417 x 50, ſo are the Chances that the Younger is dead, leaving the Elder. Wherefore as 610 x 490 to 560 x 73, ſo is the preſent value of a Sum to be paid at eight years end, to the Sum to be paid for the Chance of the Youngers Survivance; and as 610 x 490 to 417 x 50, ſo is the ſame preſent value to the Sum to be paid for the Chance of the Elders Survivance.

This poſſibly may be yet better explained by expounding theſe Products by Rectangular Parallelograms, as in *Fig.* 7. wherein AB or CD repreſents the number of perſons of the younger Age, and DE, BH thoſe remaining alive after a certain term of years; whence CE will anſwer the number of thoſe dead in that time: So AC, BD may repreſent the number of the Elder Age ; *A F,B I* the Survivors after the same term; and *C F, D I,* those of that Age that are dead at that time. Then shall the whole Parallelogram *A B C D* be *Nn*, or the Product of the two Numbers of persons, representing such a number of Persons of the two Ages given ; and by what was said before, after the Term proposed the Rectangle *H D* shall be as the number of Persons of the younger Age that survive, and the Rectangle *A E* as the number of those that die. So likewise the Rectangles *A I, F D* shall be as the Numbers, living and dead, of the other Age. Hence the Rectangle *H I* shall be as an equal number of both Ages surviving. The Rectangle *FE* being the Product of the deceased, or *Y y*, an equal number of both dead. The Rectangle *G D* or *R y*, a number living of the younger Age, and dead of the Elder : And the Rectangle *A G* or *r Y* a number living of the Elder Age, but dead of the younger. This being understood, it is obvious, that as the whole Rectangle *A D* or *N n* is to the *Gnomon FA B D E G* or *Nn— Y y*, so is the whole number of Persons or Chances, to the number of Chances that one of the two Persons is living : And as *A D* or *N n* is to *FE* or *Y y*, so are all the Chances, to the Chances that both are dead ; whereby may be computed the value of the Reversion after both Lives. And as *A D* to *G D* or *R y*, so the whole number of Chances, to the Chances that the younger is living and the other dead ; whereby may be cast up what value ought to be paid for the Reversion of one Life after another, as in the case of providing for Clergy-mens Widows and others by such Reversions. And as *A D* to *A G* or *r Y*, so are all the Chances, to those that the Elder survives the younger. I have been the more particular, and perhaps tedious, in this matter, because it is the Key to the Case of Three Lives, which of it self would not have been so easie to comprehend.

VII. If Three Lives are proposed, to find the value of an Annuity during the continuance of any of those three Lives. The Rule is, *As the Product of the continual multiplication of the Three Numbers, in the Table, answering to the Ages proposed, is to the difference of that Product and of the Product of the Three Numbers of the deceased of those Ages, in any given term of Years ; So is the present value of a Sum of Money to be paid certainly after so many Years, to the present value of the*

same

ſame Sum to be paid, provided one of thoſe three Perſons be living at the Expiration of that term. Which proportion being yearly repeated, the Sum of all thoſe preſent values will be the value of an Annuity granted for three ſuch Lives. But to explain this, together with all the Caſes of Survivance in three Lives: Let N be the Number in the Table for the Younger Age, n for the Second, and ν for the Elder Age; let Y be thoſe dead of the Younger Age in the term propoſed, y thoſe dead of the Second Age, and υ thoſe of the Elder Age; and let R be the Remainder of the younger Age, r that of the middle Age, and ϱ the Remainder of the Elder Age. Then ſhall $R+Y$ be equal to N, $r+y$ to n, and $\varrho+\upsilon$ to ν, and the continual Product of the three Numbers $N n \nu$ ſhall be equal to the continual Product of $R+Y \times r+Y \times \varrho+\upsilon$, which being the whole number of Chances for three Lives is compounded of the eight Products following. (1) $R r \varrho$, which is the number of Chances that all three of the Perſons are living. (2) $r \varrho Y$, which is the number of Chances that the two Elder Perſons are living, and the younger dead. (3) $R \varrho y$ the number of Chances that the middle Age is dead, and the younger and Elder living. (4) $R r \upsilon$ being the Chances that the two younger are living, and the elder dead. (5) $\varrho Y y$ the Chances that the two younger are dead, and the elder living. (6) $r Y \upsilon$ the Chances that the younger and elder are dead, and the middle Age living. (7) $R y \upsilon$, which are the Chances that the younger is living, and the two other dead. And Laſtly and Eightly, $Y y \upsilon$, which are the Chances that all three are dead. Which latter ſubſtracted from the whole number of Chances $N n \nu$, leaves $N n \nu - Y y \upsilon$ the Sum of all the other Seven Products; in all of which one or more of the three Perſons are ſurviving.

To make this yet more evident, I have added *Fig.* 8. wherein theſe Eight ſeveral Products are at one view exhibited. Let the rectangled Parallelepipedon $A B C D E F G H$ be conſtituted of the ſides $A B$, $G H$, *&c.* proportional to N the number of the younger Age; $A C$, $B D$, *&c.* proportional to n; and $A G$, $C E$, *&c.* proportional to the number of the Elder, or ν. And the whole Parallelepipedon ſhall be as the Product $N n \nu$, or our whole number of Chances. Let $R P$ be as R, and $A P$ as Y. let $C L$ be as r, and $L n$ as y; and $G N$ as ϱ, and $N A$ as υ; and let the Plain $P R e a$ be made parallel to the plain

plain *A C G E*; the plain *NVbY* parallel to *A B C D*; and the plain *L X T Q* parallel to the plain *A B G H*. And our firſt Product *R r ϱ* ſhall be as the Solid *S T W I F Z e b*. The Second, or *r ϱ Y* will be as the Solid *E Y Z e Q S M I*. The Third, *R ϱ y*, as the Solid *R H O V W I S T*. And the Fourth, *R r υ*, as the Solid *Z a b D W X I K*. Fifthly, *ϱ Y y*, as the Solid *G Q R S I M N O*. Sixthly, *r Y υ*, as *I K L M G Y Z A*. Seventhly. *R y υ*, as the Solid *I K P O B X V W*. And Laſtly, *A I K L M N O P* will be as the Product of the 3 numbers of perſons dead, or *Y y υ*. I ſhall not apply this in all the caſes thereof for brevity ſake; only to ſhew in one how all the reſt may be performed, let it be demanded what is the value of the Reverſion of the younger Life after the two elder propoſed. The proportion is as the whole number of Chances, or *N n ν* to the Product *R y υ*, ſo is the certain preſent value of the Sum payable after any term propoſed, to the value due to ſuch Chances as the younger perſon has to bury both the elder, by the term propoſed; which therefore he is to pay for. Here it is to be noted, that the firſt term of all theſe Proportions is the ſame throughout, *viz. N n ν*. The Second changing yearly according to the Decreaſe of *R, r, ϱ*, and Encreaſe of *Y, y, υ*. And the third are ſucceſſively the preſent values of Money payable after one, two, three, *&c.* years, according to the rate of Intereſt agreed on. Theſe numbers, which are in all caſes of Annuities of neceſſary uſe, I have put into the following Table, they being the Decimal values of One Pound payable after the number of years in the Margent, at the rate of *6 per Cent.*

Years

(609)

Years.	Present value of 1 *l.*	Years.	Present value of 1 *l.*	Years.	Present value of 1 *l.*
1	0,9434	19	0,3305	37	0,1158
2	0,8900	20	0,3118	38	0,1092
3	0,8396	21	0,2941	39	0,1031
4	0,7921	22	0,2775	40	0,0972
5	0,7473	23	0,2618	45	0,0726
6	0,7050	24	0,2470	50	0,0543
7	0,6650	25	0,2330	55	0,0406
8	0,6274	26	0,2198	60	0,0303
9	0,5919	27	0,2074	65	0,0227
10	0,5584	28	0,1956	70	0,0169
11	0,5268	29	0,1845	75	0,0126
12	0,4970	30	0,1741	80	0,0094
13	0,4688	31	0,1643	85	0,0071
14	0,4423	32	0,1550	90	0,0053
15	0,4173	33	0,1462	95	0,0039
16	0,3936	34	0,1379	100	0,0029
17	0,3714	35	0,1301		
18	0,3503	36	0,1227		

It were needless to advertise, that the great trouble of working so many Proportions will be very much alleviated by using Logarithms; and that instead of using *N n v — Y y υ* for the Second Term of the Proportion in finding the value of Three Lives, it may suffice to use only *Y y υ*, and then deducting the Fourth Term so found out of the Third, the Remainder shall be the present value sought; or all these Fourth Terms being added together, and deducted out of the value of the certain Annuity for so many Years, will leave the value of the contingent Annuity upon the Chance of Mortality of all those three Lives. For Example; Let there be Three Lives of 10, 30, and 40 years of Age proposed, and the Proportions will be thus:

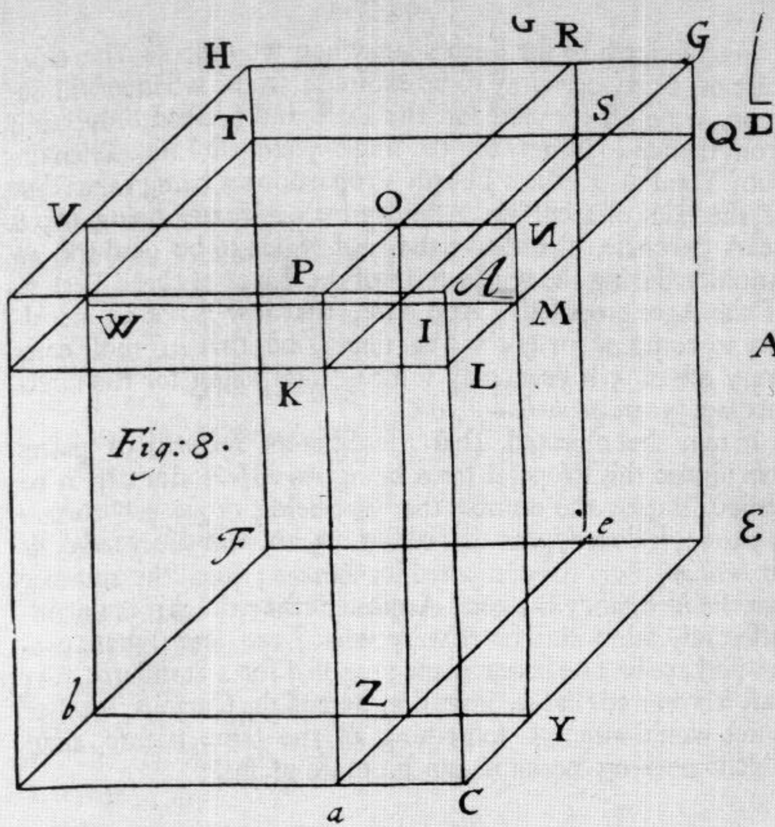
As 661 in 531 in 445 or 156190995, or *N n v*
to 8 in 8 in 9, or 576, or *Y y υ* for the first year, so 0, 9434. to 0,00000348
to 15 in 16 in 18. or 4320, for the second year, so 0, 8900. to 0,00002462
to 21 in 24 in 28, or 14112 for the third year, so 0, 8396 to 0,00008123
to 27 in 32 in 38, for the fourth year, so 0, 7921 to 0,00016650
to 33 in 41 in 48, for the fifth year, so 0, 7473 to 0,00031071
to 39 in 50 in 58, for the sixth year, so 0, 7050 to 0,00051051

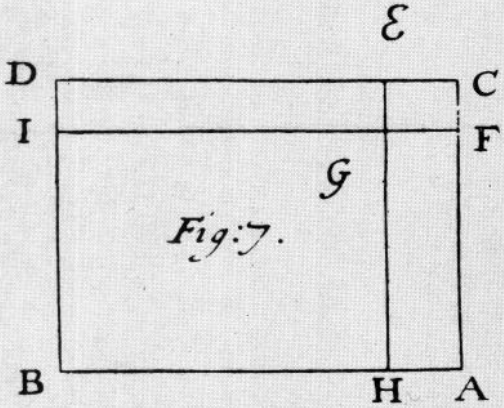
And

And ſo forth to the 60th year, when we ſuppoſe the elder Life of Forty certainly to be expired; from whence till Seventy we muſt compute for the Firſt and Second only, and from thence to Ninety for the ſingle youngeſt Life. Then the Sum Total of all theſe Fourth Proportionals being taken out of the value of a certain Annuity for 90 Years, being 16,58 years Purchaſe, ſhall leave the juſt value to be paid for an Annuity during the whole term of the Lives of three Perſons of the Ages propoſed. And note, that it will not be neceſſary to compute for every year ſingly, but that in moſt caſes every 4th or 5th year may ſuffice, interpoling for the intermediate years *ſecundum artem.*

It may be objected, that the different *Salubrity* of places does hinder this Propoſal from being *univerſal*; nor can it be denied. But by the number that die, being 1174. *per Annum* in 34000, it does appear that about a 30th part die yearly, as Sir *William Petty* has computed for *London*; and the number that die in Infancy, is a good Argument that the Air is but indifferently ſalubrious. So that by what I can learn, there cannot perhaps be one better place propoſed for a Standard. At leaſt 'tis deſired that in imitation hereof the Curious in other Cities would attempt ſomething of the ſame nature, than which nothing perhaps can be more uſeful.

The


R
G
H
S
T
Q
D
V
O
P
A
W
M
I
L
K
Fig: 8.
F
e
b
Z
Y
a
C


D
C
I
F
G
Fig: 7.
B
H
A

I. *Some further Conſiderations on the* Breſlaw *Bills of Mortality. By the ſame Hand*, &c.

SIR,

WHat I gave you in my former Diſcourſe on theſe Bills, was chiefly deſigned for the Computation of the Values of Annuities on Lives, wherein I believe I have performed what the ſhort Period of my Obſervations would permit, in relation to exactneſs, but at the ſame time do earneſtly deſire, that their Learned Author Dr. *Newman* of *Breſlaw* would pleaſe to continue them after the ſame manner for yet ſome years further, that ſo the caſual Irregularities and apparent Diſcordance in the Table, *p.* 599. may by a certain number of Chances be rectified and aſcertain'd.

Were this *Calculus* founded on the Experience of a very great number of Years, it would be very well worth the while to think of Methods for facilitating the Computation of the Value of two, three, or more Lives; which as propoſed in my former, ſeems (as I am inform'd) a Work of too much Difficulty for the ordinary Arithmetician to undertake. I have ſought, if it were poſſible, to find a Theorem that might be more conciſe than the Rules there laid down, but in vain; for all that can be done to expedite it, is by Tables of Logarithms ready computed, to exhibit the *Rationes* of *N* to *Y* in each ſingle Life, for every third, fourth or fifth Year of Age, as occaſion ſhall require; and theſe Logarithms being added to the Logarithms of the preſent Value of Money payable after ſo many Years, will give a Series of Numbers, the Sum of which will ſhew the Value of the Annuity ſought. However for each Number of this Series two Logarithms for a ſingle Life, three for two Lives, and four for three Lives, muſt neceſſarily be be added together. If you think the matter, under the uncertainties I have mentioned, to deserve it, I shall shortly give you such a Table of Logarithms as I speak of, and an Example or two of the use thereof: But by Vulgar Arithmetick the labour of these Numbers were immense; and nothing will more recommend the useful Invention of Logarithms to all Lovers of Numbers, than the advantage of Dispatch in this and such like Computations.

Besides the uses mentioned in my former, it may perhaps not be an unacceptable thing to infer from the same Tables, how unjustly we repine at the shortness of our Lives, and think our selves wronged if we attain not Old Age; whereas it appears hereby, that the one half of those that are born are dead in Seventeen years time, 1238 being in that time reduced to 616. So that instead of murmuring at what we call an untimely Death, we ought with Patience and unconcern to submit to that Dissolution which is the necessary Condition of our perishable Materials, and of our nice and frail Structure and Composition: And to account it as a Blessing that we have survived, perhaps by many Years, that Period of Life, whereat the one half of the whole Race of Mankind does not arrive.

A second Observation I make upon the said Table, is that the Growth and Encrease of Mankind is not so much stinted by any thing in the Nature of the *Species*, as it is from the cautious difficulty most People make to adventure on the state of *Marriage*, from the prospect of the Trouble and Charge of providing for a Family. Nor are the poorer sort of People herein to be blamed, since their difficulty of subsisting is occasion'd by the unequal Distribution of Possessions, all being necessarily fed from the Earth, of which yet so few are Masters. So that besides themselves and Families, they are yet to work for those who own the Ground that feeds them: And of

such

ſuch does by very much the greater part of Mankind conſiſt; otherwiſe it is plain, that there might well be four times as many Births as we now find. For by computation from the Table, I find that there are nearly 15000 Perſons above 16 and under 45, of which at leaſt 7000 are Women capable to bear Children. Of theſe notwithſtanding there are but 1238 born yearly, which is but little more than a ſixth part: So that about one in ſix of theſe Women do breed yearly; whereas were they all married, it would not appear ſtrange or unlikely, that four of ſix ſhould bring a Child every year. The Political Conſequences hereof I ſhall not inſiſt on, only the Strength and Glory of a King being in the multitude of his Subjects, I ſhall only hint, that above all things, Celibacy ought to be diſcouraged, as, by extraordinary Taxing and Military Service: And thoſe who have numerous Families of Children to be countenanced and encouraged by ſuch Laws as the *Jus trium Liberorum* among the *Romans*. But eſpecially, by an effectual Care to provide for the Subſiſtence of the Poor, by finding them Employments, whereby they may earn their Bread, without being chargeable to the Publick.

II. *What*