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Steven Morrison and Alex McNeil

Application of Extreme Value Theory to Risk Capital Estimation

7-9 November 2010

Application of EVT to risk capital estimation: Agenda

- Motivation
- Background theory
- VaR case study
- Summary
- Questions or comments?

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Motivation

Motivation

- Measures of risk capital are based on the (extreme) tail of a distribution
  - Value at Risk (VaR)
  - Conditional Value at Risk (CVaR) / Expected Shortfall

- In particular, Solvency II SCR is defined as a 99.5% VaR over a one year horizon

- Generally needs to be estimated using simulation
  1. Generate real-world economic scenarios for all risk drivers affecting the balance sheet over one year
  2. Revalue the balance sheet under each real-world scenario
     - e.g. Monte Carlo (‘nested stochastic), Replicating Formula, Replicating Portfolio
  3. Estimate the statistics of interest

Motivation

- An insurer who has gone through such a simulation exercise states
  - “Our Solvency Capital Requirement is £77.5m”
- How confident can we be in this number?
- Many sources of uncertainty
  - Choice of economic scenario generator (ESG) models and their calibration
  - Liability model assumptions e.g. dynamic lapse rules
  - *Choice of scenarios sampled i.e. choice of real-world ESG random number seed*

Motivation

- The same insurer re-runs their internal model using a different random number seed (but all other assumptions are unchanged)
  - “Our Solvency Capital Requirement is now £82.8m”

- So, simulation-based capital estimates are subject to statistical uncertainty
  - Can we estimate this statistical uncertainty?
  - How can we reduce the amount of statistical uncertainty?

- In this presentation, we will address these questions using a statistical technique known as Extreme Value Theory (EVT)

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Background theory

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VaR case study

VaR Case study

Liability book

- UK-style with profits
  - Management actions, dynamic EBR, dynamic bonus rates, regular premiums

Valuation methodology

- Nested stochastic
  - 1,000 real-world outer scenarios
  - 1,000 risk-neutral inner scenarios per outer scenario

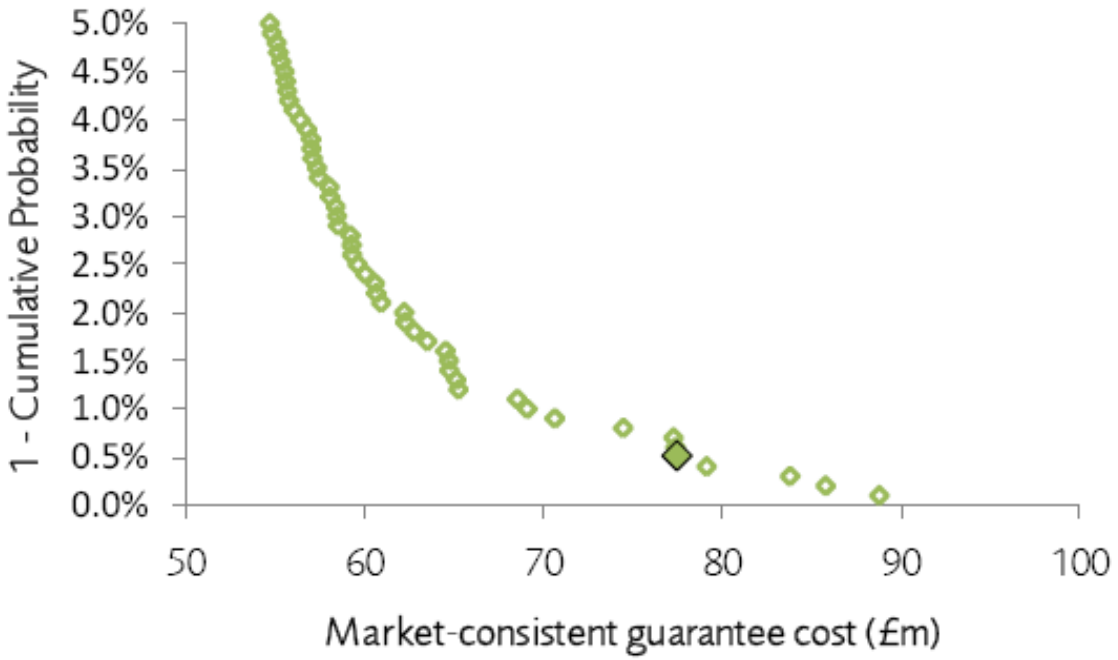
Estimated distribution of liability value at end of year (empirical quantile method)

- Estimate ‘empirical quantiles’ by ranking 1,000 scenarios

- Estimated 99.5% VaR = £77.5m

  (995th worst-case scenario)

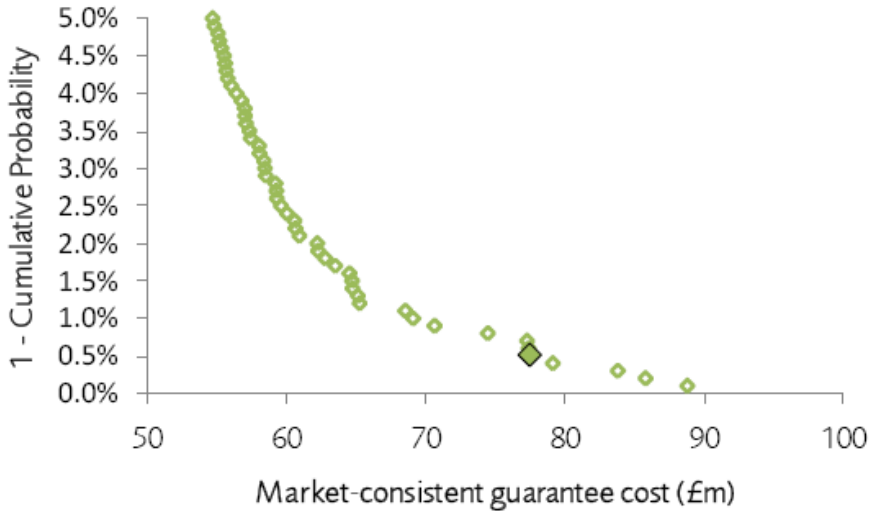
- Note that estimated distribution is ‘lumpy’, particularly as we go further out in the tail



Estimated distributions using different scenario sets (empirical quantile method)

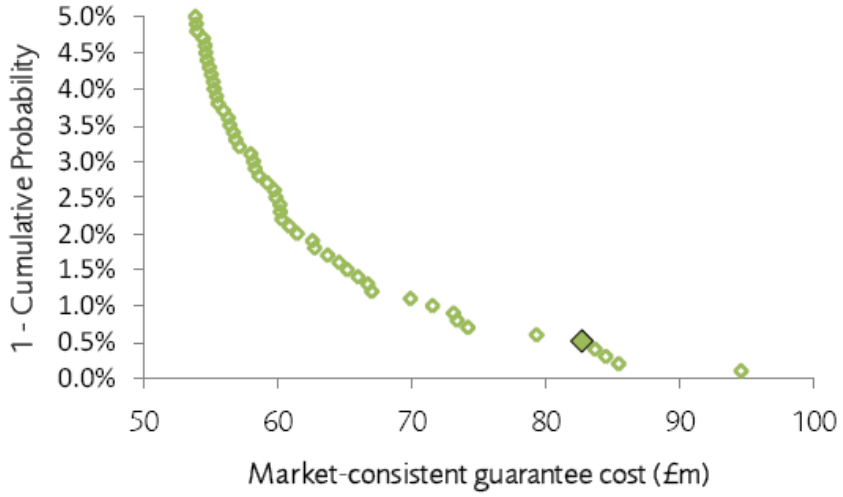
Initial set of 1,000 real-world scenarios

- 99.5% VaR = £77.5m



Second set of 1,000 real-world scenarios

- 99.5% VaR = £82.8m



What is the ‘true’ VaR?

- Two different *estimates* for VaR
  - £77.5m
  - £82.8m
  - Which is ‘correct’?

- Both use same (subjective) modelling assumptions
  - Same economic scenario generator and calibration
  - Same liability model assumptions e.g. dynamic lapse rules

- Difference is purely due to different random number streams used to generate the economic scenarios

Two important questions

1. Can we reduce the sensitivity of the estimate to the choice of random numbers?
   - Run more scenarios
     - May not be feasible because of model run-time
   - Find a ‘better’ estimator than the empirical quantile

2. Given a particular estimate of the 99.5% VaR, can we estimate the uncertainty around this?

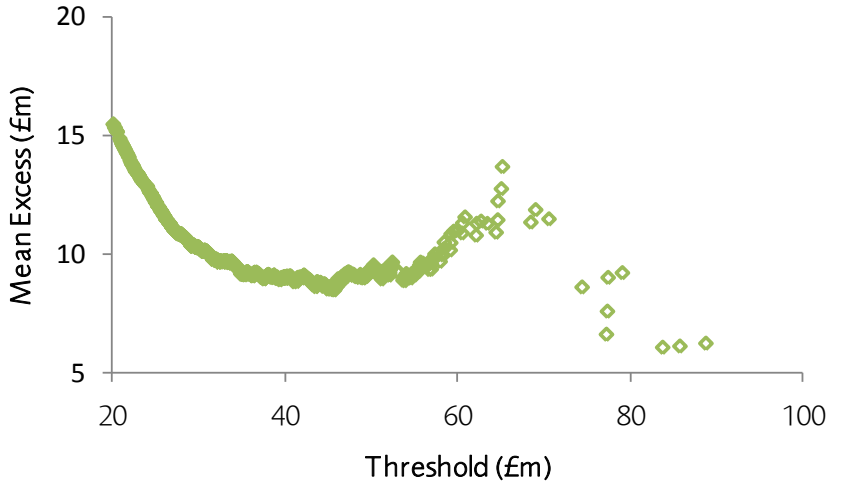
Application of Extreme Value Theory

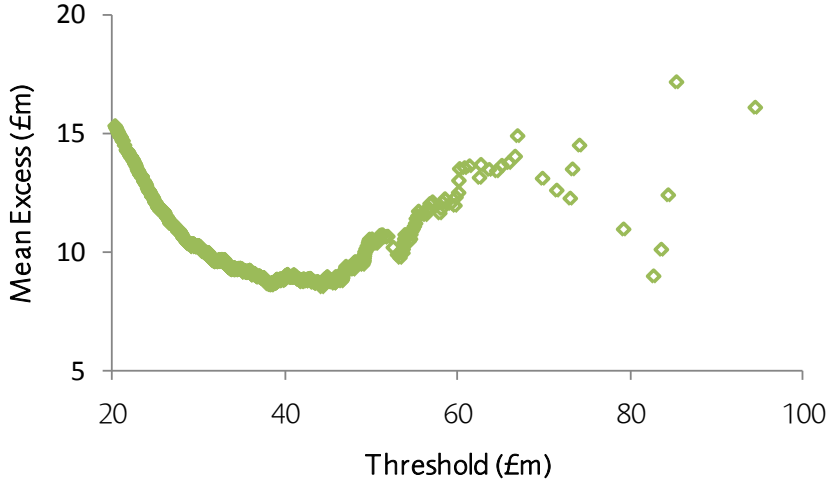
- Recall that Extreme Value Theory tells us something about the shape of the distribution in the tail
  - Distribution of liability value beyond some threshold is (approximately) Generalised Pareto
  - Parameterised by 2 parameters

- Estimate the tail of the distribution by:
  1. Picking a threshold
  2. Fitting the 2 parameters of the Generalised Pareto Distribution to values in excess of the threshold

Choice of threshold

- Choice of threshold is subjective
  - But examination of ‘mean excess function’ *helps* identify a suitable choice
- We have judged that a threshold of £40m is suitable for this particular case study
  - Approximately 26% of scenarios exceed the threshold

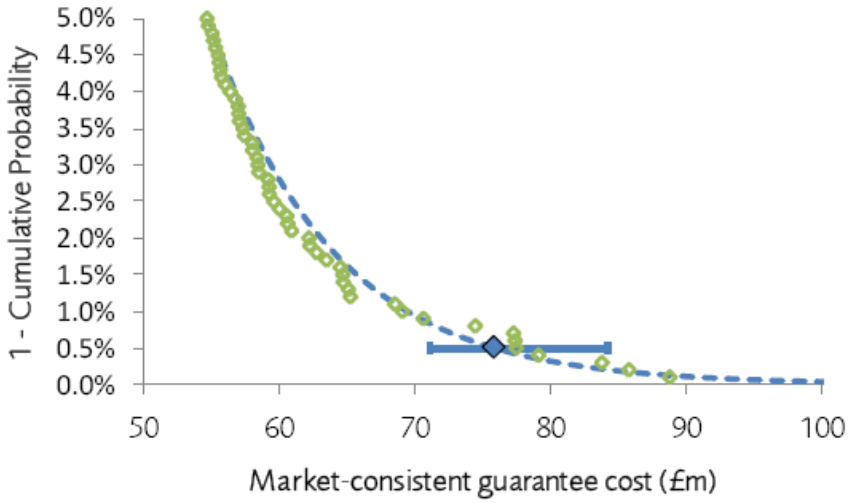




Estimated distributions using different scenario sets (Extreme Value Theory method)

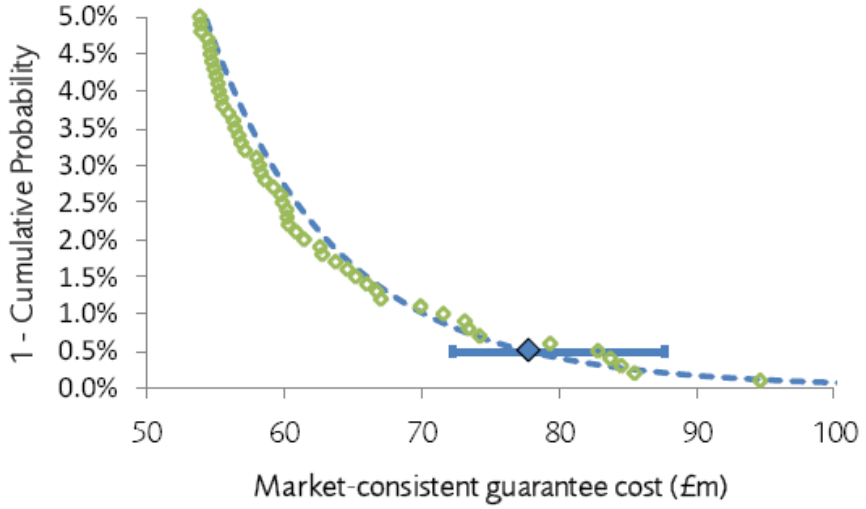
Initial set of 1,000 real-world scenarios

- 99.5% VaR = £75.9m
- 95% confidence interval = [71.1m, 84.3m]



Second set of 1,000 real-world scenarios

- 99.5% VaR = £77.8m
- 95% confidence interval = [72.2m, 87.7m]



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Summary

Summary

- Simulation-based measures of risk capital, e.g. VaR, are subject to statistical uncertainty

- Extreme Value Theory provides a robust method for estimating VaR
  - Allows statistical uncertainty to be estimated
  - Statistical uncertainty lower than ‘naïve’ quantile estimation
  - Provides an estimate of entire tail of distribution, allowing estimate of more extreme VaR, CVaR etc.

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.