

FINANCIAL ECONOMICS AND THE THEORY OF SOLVENCY1. Introduction

The first International Conference on Insurance Solvency was held in June 1986 in Philadelphia, U.S.A. At this Conference, the Solvency Working Party presented a paper which reported their work to date. The Conference was also attended by a collection of individuals from the U.S. who were trained as financial economists and whose approach to the solvency question was entirely different from our own. The second International Conference on Insurance Solvency is now being planned for May 1988. Its stated objective is the integration of the different approaches of actuaries and economists. It was felt that an explanation of the theoretical foundation of financial economics would prove useful to those who are likely to attend the second International Solvency Conference and also that a study of this body of knowledge might provide insights which could enable further progress to be made on the important question of solvency. The purpose of this paper is to provide that explanation.

The development of modern financial economics can be traced back to the early 1950s. Prior to then, the approach to financial theory was normative and prescriptive, that is, it was concerned with what should be done. There was little concern for the empirical validity of the assumptions upon which these normative prescriptions rested. However, a change took place to a focus on positive theories, that is those which are concerned with how the world works. Positive propositions are of the form "if a, then b" and are refutable by normal scientific methods. Normative propositions, in contrast, are concerned with prescription and are of the form "given condition c then alternative d should be chosen". In this form, they are not refutable. However, the logical structure of decision making means that better answers to normative questions occur when a decision maker has a richer set of positive theories available, which provide him with a better understanding of the consequences of his action. Positive theory helps decision makers to decide on action and to achieve their objectives

by providing explanations of how alternative actions affect the desired outcome.

The development of a positive theory of financial economics can be viewed as being comprised of three major building blocks. These are Modern Portfolio Theory (including the Efficient Markets Hypothesis and the Capital Asset Pricing Model), Option Pricing Theory and Agency Theory. This paper, which owes much to Jensen and Smith (1984), attempts to give a broad outline of each of these sets of ideas.

Firstly, however, there is a general point that must be made. A fundamental assumption of financial economics is that firms and individuals interact in open, competitive markets, form rational expectations, seek to increase their expected utilities and are innovative in doing so. An important consequence of this is the Law of One Price, which involves the concept of arbitrage. An arbitrageur is someone who purchases an item (such as a security) in one market and sells it in another at a higher price. The Law of One Price states that, under equilibrium conditions, there exist no arbitrage opportunities.

2. Modern Portfolio Theory

Efficient Market Theory is concerned with an analysis of equilibrium behaviour of price changes through time in speculative markets. The Efficient Market Hypothesis holds that a market is efficient if it is impossible to make economic profits by trading on available information. It postulates that stock price changes behave like a random walk and therefore technical trading rules based on information in past price series cannot expect to produce above normal returns. In a speculative market, unexpected price changes behave as independent random drawings if the market is competitive and economic trading profits are zero. Unexpected price changes reflect new information. Since, by definition, this cannot be deduced from previous information, it must be independent over time and therefore price changes must be similarly independent.

The Efficient Markets Hypothesis is perhaps the most extensively tested hypothesis in the social sciences, due to the widespread availability of security price information and computers. Generally, the evidence is consistent with the hypothesis that it is difficult to earn super-normal profits by trading on publicly available data. However, the evidence is not completely one-sided. Jensen (1978) has reviewed some of the anomalies.

The Efficient Market Hypothesis has several important implications for corporate finance. If Capital Markets are efficient, then the market value of a firm reflects the present value of the firm's expected future cash flows. The firm's objective function is the maximisation of its current market value. There is no benefit in manipulating earnings per share if this does not affect cash flows. Security returns are a meaningful measure of a firm's performance.

Classical Financial theory paid little attention to Portfolio Selection. Security analysis was focused on picking under-valued securities and a portfolio was seen as merely an accumulation of such securities. However, Markowitz (1959) pointed out that accumulating predicted 'winners' was a poor portfolio

selection technique, because it ignores the effect of portfolio diversification on risk.

The question then became how to select portfolios that maximised investors' utility on the basis of expected portfolio returns and portfolio risks. Risk in this context is measured by the variance of expected returns. An efficient portfolio will be one that gives the maximum expected returns for a given variance and the minimum variance for a given return. The Markowitz approach provides a measure on contribution of the co-variance among security returns to the riskiness of a portfolio and defines rules for the construction of an efficient portfolio.

Treynor (1961), Sharpe (1964) and Lintner (1965) apply the analysis of Markowitz to create a positive theory for the determination of asset prices. Given the Markowitz model, they solve for equilibrium security prices, given fixed supplies of assets.

Total risk is measured by the variance of portfolio returns. However, in equilibrium, an individual security is priced to reflect its contribution to total risk, measured by the co-variance of its return with the return on the market portfolio of all assets. This risk measure is usually referred to as 'systematic' risk. This gives rise to the Capital Asset Pricing Model of which the simplest form is as follows:

$$E(R_j) = R_f + [E(R_m) - R_f] \beta_j$$

$E(R_j)$ is expected return on asset j

R_f is the riskless rate of interest

$E(R)$ is the expected return on the market portfolio of all assets

β_j is the measure of systematic risk of asset j .

There have been many empirical tests of the model, with mixed results. Jensen (1972) provides a survey.

Modern Portfolio Theory is not entirely neglected in the actuarial literature. Frost (1982) presented a paper to the Students' Society which discussed its implications for life assurance companies. Clarkson (1982) explored at some length the comparison between his market equilibrium model with Modern Portfolio Theory. Kahane (1978) has demonstrated that the CAPM approach can also be applied to insurance liabilities, and uses it to derive risk loadings for premiums.

3. Option Pricing Theory

The Capital Asset Pricing Model provides a theory for determining expected returns on a security and thereby links the asset price now with the expected future pay-off. Another important class of valuation problems concerns assets where the pay-off is contingent upon the value of another asset. In a seminal paper, Black and Scholes (1973) provided the solution to the valuation of a call option, that is one which gives the right to buy stock at a specific price at any time prior to the exercise date.

Probably no other result in financial economics which was conceived entirely in theory has had as great an impact in practice. This is partly due to the fact that the paper appeared at the same time as the Chicago Options Exchange first started organised trading in options. However, the Black and Scholes analysis contains an insight of possibly even greater practical significance. This is that corporate liabilities in general can be viewed as combinations of option contracts. This insight provides a unified conceptual framework for the whole subject of corporate finance and implies that option pricing models can be used to price corporate securities such as bonds and equities. These generalised option models have given rise to a branch of financial economics known as Contingent Claims Analysis or Option Pricing Theory.

Consider a company which borrows money by means of issuing a bond. If the value of its assets falls to below the face value of the bond, then the company will default on repayment and shareholders will receive nothing. If, however, the value of the assets increases, every additional increment in value goes directly to shareholders. The future value of the firm's assets is uncertain and therefore has a probability distribution associated with it. The market price of the company's shares reflects the market's judgement of the residual value of the company after the bond has been repaid. The share capital of the company can be regarded as an option which the shareholders own on the assets of the company. The price at which the option can be exercised is the face value of

the debt capital.

A call option is defined as one which gives its owner the right to buy stock at a specified price at, or sometimes before, a specified expiration date. The specified price is called the exercise price. A put option is one which gives the right to sell a stock at a specified price at or possibly before a specified expiration date.

The value of a put option immediately before the expiration date will depend upon the price of the underlying stock. If the price of the stock is greater than the exercise price, it will be worthless. If the price of the stock is less, then the price of the option will be difference between the exercise price and the price of the stock. The reverse will be true for a call option. If the stock price is less than the exercise price, the option will be valueless. If the price of the stock is more, the price of the option will be the difference between the stock price and the exercise price.

It is possible to hold call options, put options and shares in different combinations. Consider a portfolio which consists of a stock and a put option on that stock. If the stock price rises above the exercise price, the put option will be worthless and the value of the portfolio will be the value of the share. If the stock price falls below the exercise price, the decline in the value of the stock will be offset by a corresponding increase in the value of the put option. Exactly the same set of pay-offs can be obtained by holding a call option and putting aside sufficient cash to pay the exercise price on the expiration date of the option. By the Law of One Price, the portfolios with identical pay-offs must have identical prices. Thus we have the identity:

$$\text{Value of call option} + \text{Present value of exercise price} = \text{Value of put option} + \text{Stock price} \quad (1)$$

By manipulating this identity, we can show that:

- (a) buying a call option and selling a put option is equivalent to buying the stock and borrowing the present value of the exercise price
- (b) buying a put option is identical to buying a call option, selling the stock and investing the present value of the exercise price.

Thus, it is not necessary to have call options, put options, the stock and borrowing/lending opportunities. Given any three of these, it is always possible to construct the fourth.

Whenever a firm borrows, the lender can be viewed as acquiring the company and the shareholders obtain an option to buy it back by paying off the debt. The shareholders have therefore bought a call option from the lenders. The identity (1) above can therefore be rewritten as:

$$\text{Value of call option} + \text{Present value of exercise price} = \text{Value of put option} + \text{Value of firm's assets} \quad (2)$$

The value of the firm's shares will be equal to the call option on the firm's assets. Rearranging (2) above shows that this is equal to the asset value, less the present value of repayment to bondholders, plus the value of a put option.

What is the significance of the put option? The key to understanding this is the realisation that the firm has limited liability. Thus, if the value of the firm's assets is less than the repayment to lenders, the firm will default. Hence, the set of pay-offs is equivalent to a put option with an exercise price equal to the amount payable to the lenders. Clearly, this option will have a value if the firm is likely to default but will be trivial where default is regarded as highly unlikely.

Owning bonds is equivalent to owning the firm's assets but selling a call option on them. Thus:

$$\begin{aligned}\text{Value of bonds} &= \text{Value of firm's assets} - \text{value of call option.} \\ &= \text{Present value of repayment to lenders} - \text{value of put. (3)}\end{aligned}$$

Hence, the value of a bond is identical to the value of a portfolio constructed by holding the present value of the repayment and selling the put option.

In addition to providing a way of deriving values for shares and bonds, the Option Pricing Model can also give insights into other corporate finance problems, such as underwriting stock issues and issuing warrants. A firm making a rights issue can be regarded as buying a put option from an underwriter, whereby if the stock falls below the rights price the underwriter will take up the stock. The Option Pricing Model provides a methodology for valuing this option. A warrant entitles its holder to purchase a given number of shares at a specified price on or before a given date. It is therefore nothing more than a long-term call option and can be valued as such.

An insurance company presents an interesting special case of the application of the Option Pricing Model. This is because a set of insurance policies can be regarded as the equivalent of a zero coupon bond. Each has an initial payment to the company (premiums/subscription price) followed by at a later date, a payment by the company (claims/redemption). Hence, the general results derived above in relation to the valuation of bonds and shares can be applied to the circumstances of an insurance company. This approach has been developed much further by Doherty and Garven (1986).

What determines the value of an option? Consider a call option. To exercise such an option, the exercise price has to be paid. Other things being equal, the option holder will wish to pay as little for the option as possible. Therefore, the value of an option increases with the ratio of the asset price to

the exercise price.

The exercise price does not have to be paid until the option is exercised. Thus, the option provides its holder with a free loan. The higher the prevailing rates of interest and the time to maturity, the more this loan is worth. Therefore, the value of an option increases with the interest rate per period multiplied by the time to maturity.

If the price of the asset is below the exercise price, the option will not be exercised. The investment in the option will be totally lost, however far the asset price depreciates below the exercise price. If the asset price rises above the exercise price, a profit will be made. The higher the asset price rises, the higher the price will be. Therefore, an option holder does not lose from increased variability if things turn out badly but gains if they go well. The value of an option increases with the variance per period of the stock return, multiplied by the time to maturity.

These imprecise concepts have been extended by Black and Scholes (1973) into their formal option-valuation formula. The key to their argument is that a risk free position can be maintained by a hedge between an option and its related stock, when the hedge can be adjusted continuously through time. By the Law of One Price, this hedge, because it is risk free, will give a return equivalent to the risk free interest rate. It can be shown that there is only one call price formula that meets this requirement and this is as follows:

$$\text{Present value of call option} = PN(d_1) - EXe^{-R_f t} N(d_2)$$

$$\text{where } d_1 = \frac{\log(P/EX) + R_f t + \sigma^2 t/2}{\sigma t}$$

$$d_2 = \frac{\log(P/EX) + R_f t - \sigma^2 t/2}{\sigma t}$$

Where:

$N(d)$ = cumulative normal probability density function
 EX = exercise price of option
 t = time to exercise date
 P = price of stock now
 σ^2 = variance per period of rate of return on the stock
 R_f = risk free rate of interest.

It will be seen that neither the willingness of individuals to bear risk nor the expected return on the stock affect the value of the option. Instead, the value increases with the level of the stock price relative to the exercise price (P/EX), the time to expiration multiplied by the interest rate ($R t$) and the time to expiration times the stock's variability ($\sigma^2 t$).

An increase in the value of a firm's assets will increase the expected pay-offs to its equity capital and also increase debt cover, and therefore increases the current value of both. An increase in the face value of debt increases the claim of bondholders on the assets of the firm, thereby increasing the value of debt and reducing the current value of the equity, because stockholders are residual claimants. An increase in either the time to repayment of debt or in the risk-free interest rate lowers the present value of debt and thereby increases the value of equity. Increases in riskiness or in the time to maturity of debt increases the dispersion of possible values of the firm at the maturity date, which increases the probability of default, lowering the value of debt and increasing the value of equity.

Smith (1979) has drawn attention to the similarity of an insurance contract and an option. The insurance contract calls for the payment of a premium at the outset. If, during the currency of the contract, the value of the asset is reduced by an insured event, the insurance contract will pay the policyholder the difference between the insured value and the current value. If, however, the value is not reduced by an insured event during the currency of the

contract, there is no payment. This contract has a set of pay-offs that are identical to a put option. The price of the option is the premium and the exercise price is the insured value of the asset. Thus, with certain additional assumptions to comply with the Black-Scholes model, the equilibrium price for the insurance contract can be obtained.

Wilkie (1986) points out that the modern theory of option pricing has not received attention in the actuarial literature. His paper, which is concerned with an option pricing approach to bonus policy, describes the mathematical-statistical approach to options, in the hope that it will become more familiar to actuaries.

Cummins (1986) and Doherty and Garven (1986) use Option Pricing Theory to look at the risk loadings appropriate for insurance contracts. Both papers employ contingent claims analysis concepts to provide a setting for the pricing of insurance contracts. Derrig (1986) proposes a method for determining the required solvency margins in terms of Option Pricing concepts. He makes use of the Cummins' model to provide examples based on empirical data from the Massachusetts Rating Bureau.

It has already been noted that the concepts of Option Pricing Theory can be applied to the debit and equity of a firm. The classic problem with which corporate finance is concerned is the optimal capital structure of the firm, i.e. the choice between debt and equity and an insurance company is a particularly interesting example of this problem, as its policyholders' funds can be regarded as a form of debt finance. In Option Pricing Theory terms, the policyholders' funds are an option on the total assets of the insurance firm, where the price of the option is particularly uncertain. The capital structure problem for an insurance company is of course equivalent in one sense to the solvency problem. Doherty (1986) considers the capital structure of insurance firms and produces the interesting result that regulation can convey economic benefits on both policyholders and shareholders.

4. Agency Theory

Agency Theory was originally concerned with contracts whereby a principal engages an agent to perform some service which involves delegating decision making authority. Earlier analyses were concerned with the problems associated with structuring with compensation of the agent to align his incentives with those of the principal. However, Jensen and Meckling (1976) point out that a firm is simply "a legal fiction which serves as a nexus for a set of contracting relationships among individuals." In fact, agency problems are general to all co-operative activity among self-interested individuals. Jensen and Meckling employ this framework to provide a positive analysis of the impact of agency relationships on the investment and financing decisions of the firm.

Agency costs are defined as the sum of three components, monitoring expenditure by the principal, bonding expenditure by the agent and the residual loss. Monitoring expenditure is expenditure by the principal to control the agent's behaviour. Bonding costs are expenditure undertaken by the agent to guarantee that he will not undertake certain agents to harm the principal's interests or, if he does, that he will compensate the principal. The residual loss is the loss suffered by the principal from actions taken by the agent which differ from the actions which would have been taken had the decision been taken by the principal.

Parties to contracts are assumed to make rational forecasts of activities to be accomplished and therefore structure contracts to facilitate those activities. Actions which are motivated by incentives are anticipated and reflected in the contract's prices and terms. Agency costs are therefore borne by the parties to the contracting relationships. The agency framework can be used to analyse the resolution of conflicts between stockholders, managers and bondholders.

Jensen and Meckling show that if financial markets are competitive and efficient, unbiased of agency costs will be reflected in the market price of

securities when they are sold. Therefore, incentives exist to write contracts for monitoring and bonding as long as the marginal benefits of these contracts in reducing the residual loss is greater than the marginal cost of the contract.

One agency problem considered in the financial economics literature concerns the consumption of the non pecuniary benefits (or perks) by owner-managers under partial ownership conditions. An owner manager will seek to maximise his utility from money wages, the market value of the firm and perks. A sole owner bears the full cost of perks but when a fraction of the equity is sold to outsiders, the owner manager still enjoys the full benefit but bears only a proportion of the cost. Rational investors are aware of the incentive to increase perk consumption. They make unbiased estimates of the costs associated with this increased consumption and pass these back in full, in the form of a reduction in the price they are willing to pay for the securities to be sold. As more outside capital is sought, the loss of value due to excessive perk consumption becomes greater. In attempting to finance the firm through outside equity capital, a loss is suffered which is an agency cost.

It has already been seen that equity capital can be regarded as a call option to buy back the firm from bondholders at maturity, at an exercise price equal to the principal amount of the debt. Consider a firm that has to choose between two projects of equal expected value, one of which is high risk, the other low risk. The choice will affect not the total value of the firm but the distribution of the value between bondholders and stockholders. The value of a call option increases with the degree of risk, because the option holder cannot lose from increased variability but can gain. Rational bondholders recognise the risk incentives faced by shareholders and offer a price for debt which reflects the distribution of wealth given by adoption of the high risk project. If the high risk project has a lower net present value, debtholders will assume that the inferior project will be adopted and price the debt accordingly. This will compel the firm to adopt the inferior project, which has a smaller value. This loss represents an agency cost borne by the stockholders.

A firm which is financed only by equity capital will accept any investment for which the net present value is positive. Shareholders in a firm which also has debt capital, however, maximise their wealth by accepting an investment only if its net present value exceeds the associated debt obligation. Otherwise, it is in their best interests to default. Rational bondholders recognise the increased probability of default on their claim on the firm's assets and discount it in the price they are willing to offer for the firm's bonds. This results in a loss, borne by the shareholders, which is an agency cost.

Bankruptcy is not a costless process. It involves legal proceedings which consume part of the remaining value of the firm's assets. It also disrupts the normal trading activities of the firm. The expected value of bankruptcy costs will be allowed for by rational investors when a company raises debt capital. This is another example of an agency cost borne by stockholders.

A further agency problem arises from informational asymmetry. Consider a firm that seeks to finance a project by issuing securities where the returns from the project are unknown to the market but valuable information is possessed by management. The market is unable to distinguish this project from a less profitable project without this information. The asymmetry can be resolved through signalling mechanisms but these are not costless. In the absence of a signal, rational investors will assume that the project is the less profitable one and the price obtained for the securities will reflect this. This loss is yet another agency cost borne by stockholders.

It has already been seen, in discussing the Option Pricing Model, that there is a clear analogy between a set of insurance policies and a bond. Hence, the general conclusions in relation to the incentive conflicts between bondholders and shareholders can be applied to the special case of policyholders and shareholders in an insurance company. Mayers & Smith (1982) discuss the problems of incentives, efficiency and the implications of competition in insurance

markets. The demand for insurance is explained in terms of incentive conflicts, risk shifting and real production economies. Ownership structures and marketing structures of insurance firms are explored in the light of conflicts between owners, managers and policyholders. The impact of incentive conflicts on the asset structure of insurers is also discussed. The structure of the insurance contract is examined and incentive conflicts used to explain the presence of features such as deductibles, upper limits of cover and coinsurance provisions.

Garven (1986) attempts to integrate elements of Agency Theory with the theory of the insurance firm. He identifies the incentive conflicts that occur between policyholders and shareholders. Rational policyholders will recognise the incentives faced by shareholders and the premiums they pay for insurance will reflect their estimate of the shareholders' behaviour. It will therefore be in the interest of shareholders to provide policyholders with guarantees against expropriative behaviour, if such guarantees are less costly than agency problems. Mechanisms for providing such guarantees include the purchase of reinsurance, employing positive levels of equity (ie positive solvency margins) and contractually limiting dividend and investment policy.

5. Conclusions

Cummins (1986) argued that, in spite of its mathematical sophistication, actuarial ruin theory has been of limited practical applicability. He suggested that the principal problems were:

- (1) The mathematical intractability of most of the results,
- (2) The universal tendency to ignore investment risk,
- (3) The failure to recognise that insurance companies operate in a market economy, whereas insurance premiums and asset prices are determined by the interaction of supply and demand.

The Solvency Working Party have included the investment risk in their model right from the outset. The problem of mathematical intractability has been overcome by the use or of simulation methods. Thus, two out of the three criticisms would seem not to apply to our work.

We have to accept, however, that our model as presently constituted does not take account of the interaction of supply and demand. The insights which we can obtain from current developments in modern financial economics hold out the possibility that we may be able to make enhancements on this score also.

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