The Impact of Longevity Risk Hedging on Economic Capital

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ARC Research Programmes

Actuarial Research Centre (ARC):
funded research arm of the Institute and Faculty of Actuaries

Three major programmes started in 2016, including

Modelling, Measurement and Management of Longevity and Morbidity Risk

- New/improved models for modelling longevity
- Management of longevity risk
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance

Outline

- Introduction and motivation
- Hedging longevity risk with an index-based call-spread option contract
- Anatomy of a hedging calculation in 22 easy steps!
- Numerical example
- Discussion

Motivation

- Longevity risk
- Measurement
  - e.g. Capital Requirement
  - Best estimate + extra for risk
- Longevity risk management
  - customised hedges
  - index-based hedges

Motivation

- Why use General Population Longevity Index based risk transfer instruments?
  $\rightarrow$ Capacity and **Price**
- Pros/cons
  - Transferred risk is efficiently priced
  - But hedger left with basis risk
- Thus we need
  - a clear and rigorous approach to quantify basis risk
  - hedger and regulator agreement on approach
  - to quantify properly the Capital Relief

Introduction

- Underlying problem:
  - Life insurer
  - Aim 1: measure mortality/longevity risk
  - Aim 2: manage mortality/longevity risk
  - e.g. to *reduce* regulatory capital

    $\Longleftrightarrow$ regulatory engagement/acceptance
  - e.g. to *reduce* economic capital
  - e.g. to *increase* economic value
- Further aim:

  to bridge the Academic/Practitioner gap

Regulatory Capital Requirements: Annuity Portfolio

- Solvency II options:
  - Solvency Capital Requirement,
    SCR= difference between
    Best estimate of annuity liabilities (BE) and
    Annuity liabilities following an immediate 20% reduction in mortality
  - or SCR= extra capital required at time 0 to ensure solvency at time 1 with 99.5% probability
  - or *SCR= extra capital at time 0 to ensure solvency at time T with x% probability*

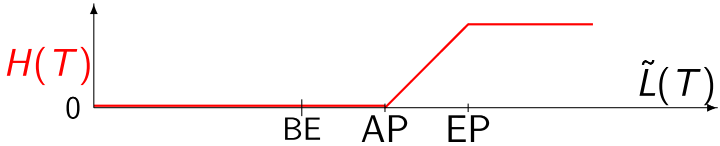
Liability to be Hedged

- L = random PV at time 0 of liabilities
- $L(0)$ = point estimate of L based on time 0 info
- $L(T)$ = point estimate of L based on info at T
  = PV of actual cashflows up to T
  + PV of estimated cashflows after T
- Risk $\Rightarrow$ capital requirements

What type of hedge to modify capital requirements and manage risk?

- Index-based hedge
  - Synthetic $\tilde{L}(T) \approx$ true $L(T)$
  - Call spread derived from underlying $\tilde{L}(T)$
    Payoff at T, *per unit*

$$H(T) = \begin{cases} 0 & \text{if } \tilde{L}(T) < AP \text{ (Attachment Point)} \\ \tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP \text{ (Exhaustion Point)} \\ EP - AP & \text{if } EP \leq \tilde{L}(T) \end{cases}$$



The Synthetic $\tilde{L}(T)$

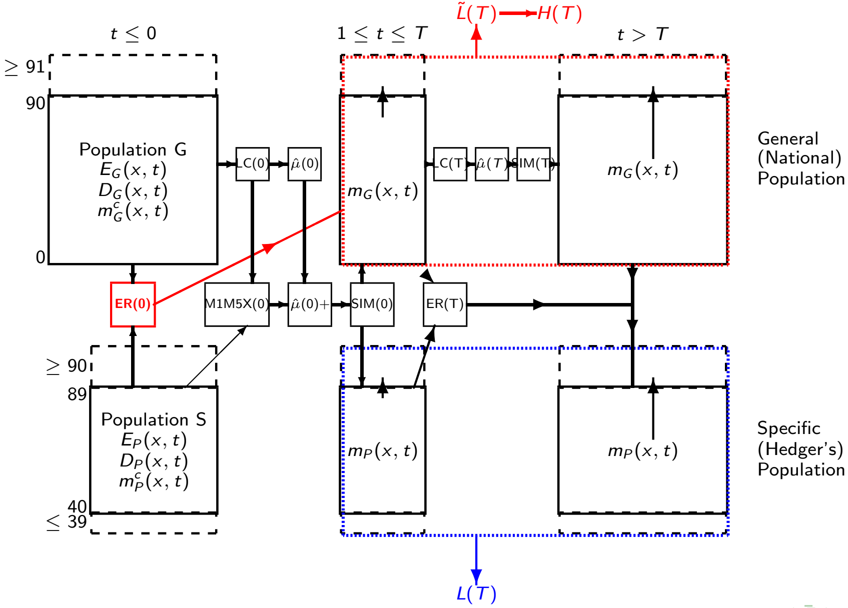
- $\tilde{L}$ = random PV at time 0 of a portfolio of synthetic liabilities
- Synthetic mortality experience
  - based on general population mortality
  - adjusted using experience ratios

- $\tilde{L}(T)$ = point estimate of $\tilde{L}$ based on info at T
  = PV of actual *synthetic* cashflows up to T
  + PV of estimated *synthetic* cashflows after T

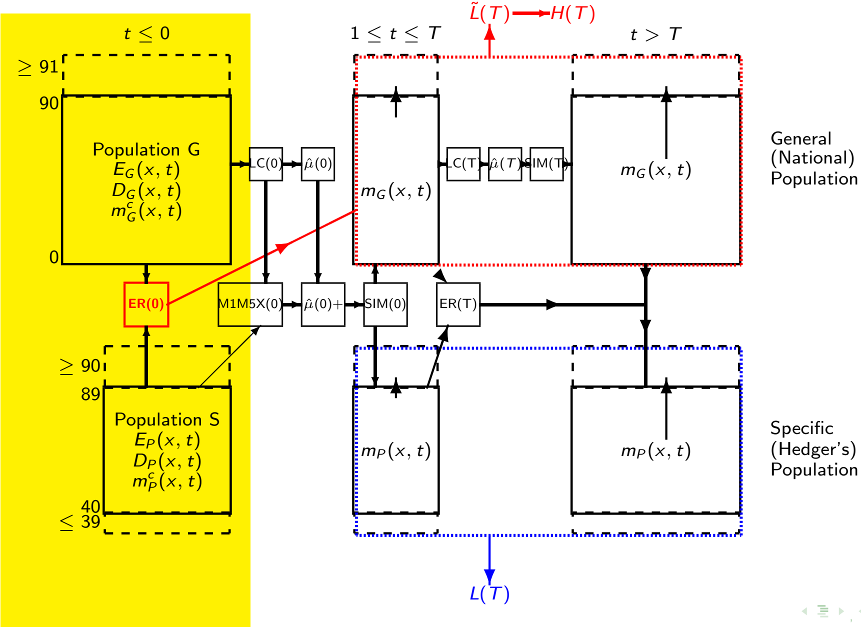
Questions and Observations

- What impact $L(T) \longrightarrow L(T) - H(T)$?
- Need a two population mortality model
- Practical reality: calculation is more complex than academic 'ideal world'
- What are good choices of AP, EP, T?

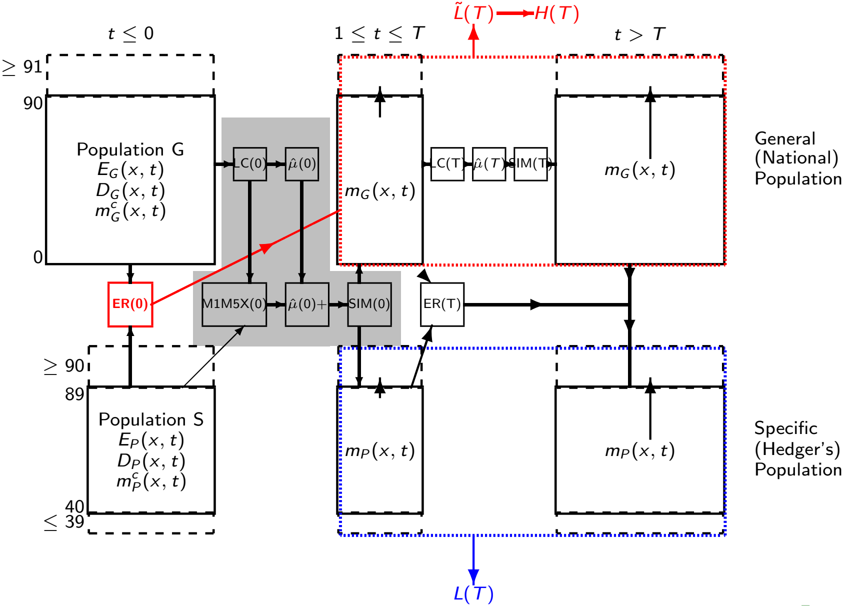
Anatomy of a Hedging Calculation in 22 Easy Steps!



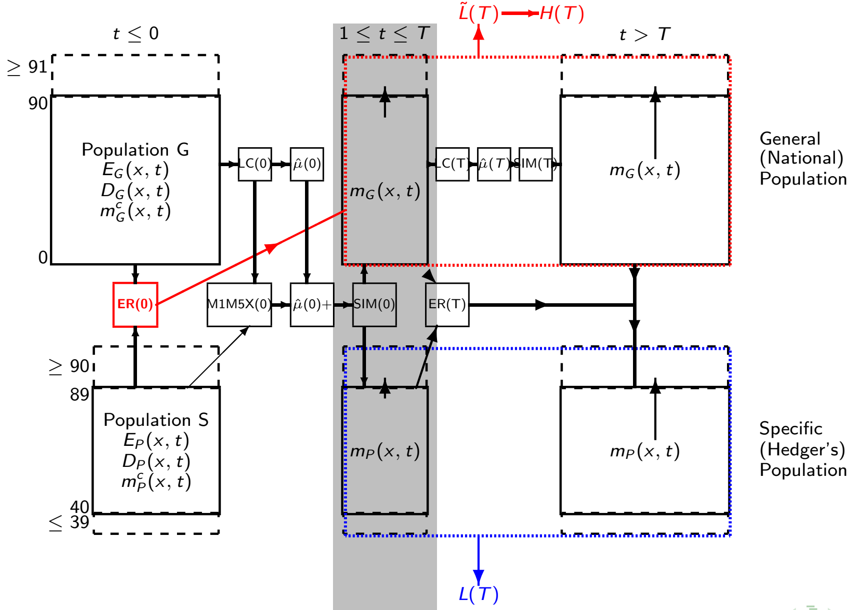
Anatomy of a Hedging Calculation: Steps 1, 2



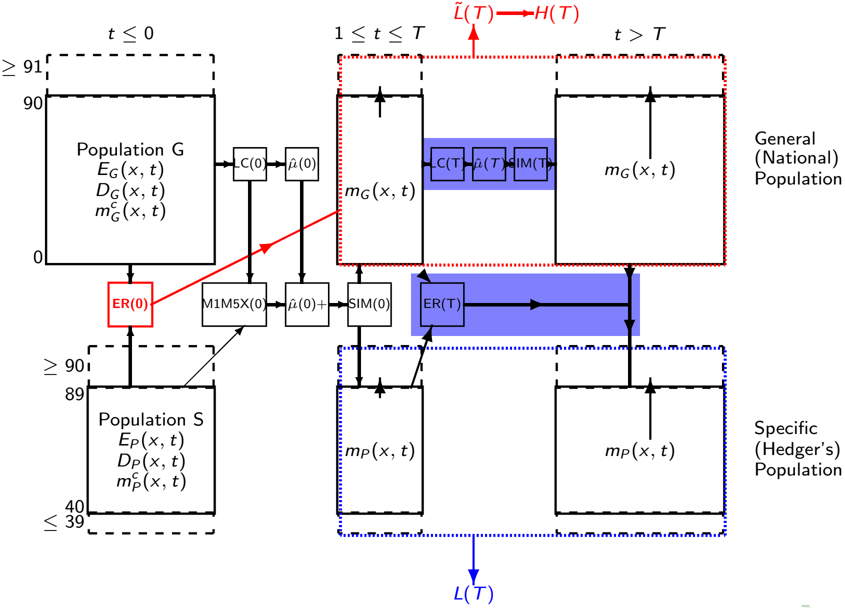
Anatomy of a Hedging Calculation: Steps 3-5

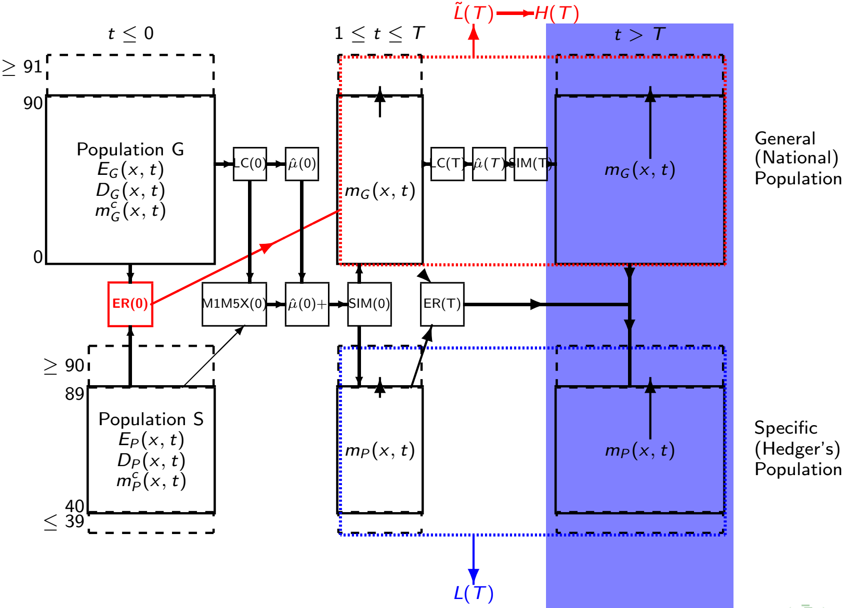


Anatomy of a Hedging Calculation: Steps 6, 7, 14, 15, 17



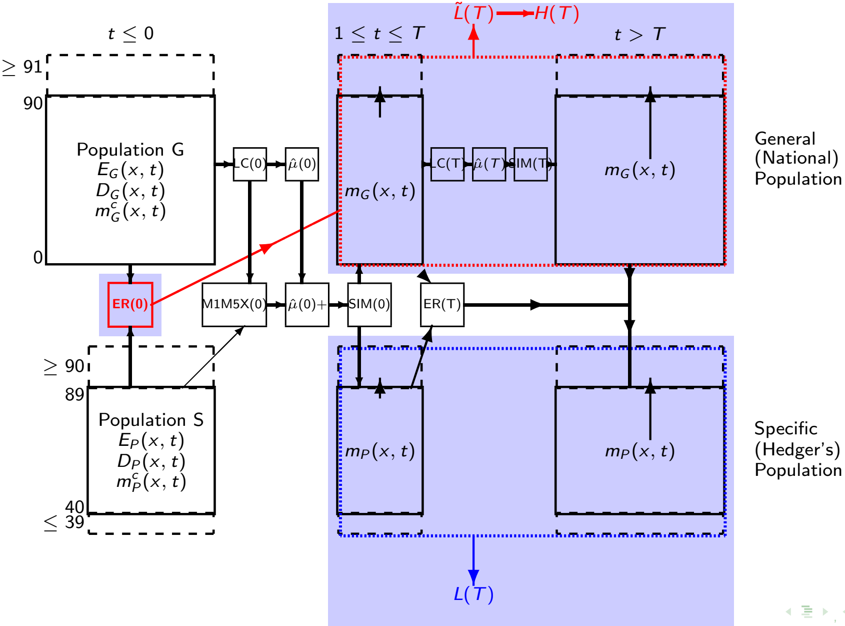
Anatomy of a Hedging Calculation: Steps 8, 9, 12





$\tilde{L}(T)$
$H(T)$
$t \le 0$
$1 \le t \le T$
$t > T$
≥ 91
90
0
Population G
$E_G(x,t)$
$D_G(x,t)$
$m^c_G(x,t)$
LC(0)
$\hat{\mu}(0)$
$m_G(x,t)$
LC(T)
$\hat{\mu}(T)$
SIM(T)
$m_G(x,t)$
General
(National)
Population
ER(0)
M1M5X(0)
$\hat{\mu}(0)+$
SIM(0)
ER(T)
≥ 90
89
Population S
$E_P(x,t)$
$D_P(x,t)$
$m^c_P(x,t)$
40
≤ 39
$m_P(x,t)$
$m_P(x,t)$
Specific
(Hedger's)
Population
$L(T)$

Anatomy of a Hedging Calculation: Steps 19-22



How many models do you need?

Academic 'ideal': One model

In practice:

- Time 0:
  - Liability valuation model (BE + SCR)
  - Simulation model ($0 \to T$)
- Time T:
  - Hedge instrument valuation model
  - Liability valuation model
- 'Models' for extrapolating to high (and low) ages

Time 0 Models

- Unhedged Liabilitiies:
  Deterministic BE + 20% stress
- Simulation: (by way of example)
  - General population: (Lee-Carter/M1)

$$\ln m_{gen}(x,t) = A(x) + B(x)K(t) \quad \text{(Lee-Carter/M1)}$$

  - Hedger's own population: (M1-M5X)

$$\ln m_{pop}(x,t) = \ln m_{gen}(x,t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})$$

Time T models

- Hedge instrument:
  - Lee-Carter (M1) for general population
  - Recalibration: *on basis specified at time 0*

$$q^H_{pop}(x,t) = q^H_{gen}(x,t) \times ER(x,0) \rightarrow \tilde{L}(T) \rightarrow H(T)$$

- Liability: specific (hedger's) population
  - Lee-Carter (M1) for general population
  - Possibly different calibration from the hedge instrument
  - $q^L_{pop}(x,t) = q^L_{gen}(x,t) \times ER(x,T) \rightarrow L(T)$
  - Approach must mimic local practice

Hedging Example

- Data: Netherlands
  - CBS national data
  - CVS insurance data (Dutch aggregated industry experience data)

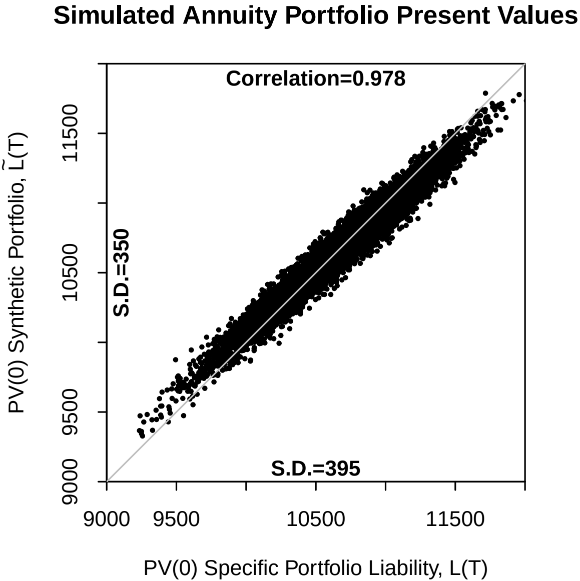
- Hedge instrument maturity: $T = 10$
- Attachment and exhaustion points at 60% and 95% quantiles of $\tilde{L}(T)$
- Key point: EP "<<" 99.5% quantile of $\tilde{L}(T)$

Hedging Example

- Portfolio of deferred and immediate annuities
- Current ages 40 to 89
- Weights ($\equiv$ pension amounts):

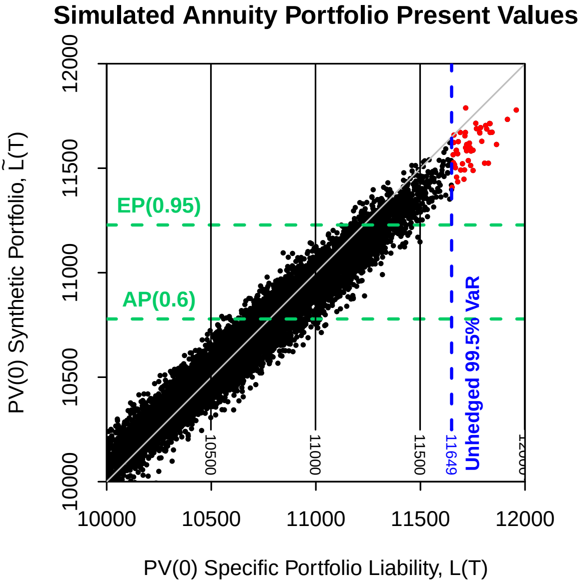
$$w_x = \begin{cases} x - 25 & \text{for } 40 \leq x < 50 \\ 25 & \text{for } 50 \leq x < 65 \\ 90 - x & \text{for } x \geq 65 \end{cases}$$

- Deferred to age 65
- Before and after: Compare $L(T)$ with $L(T) - H(T)$
- SCR = 99.5% quantile − mean

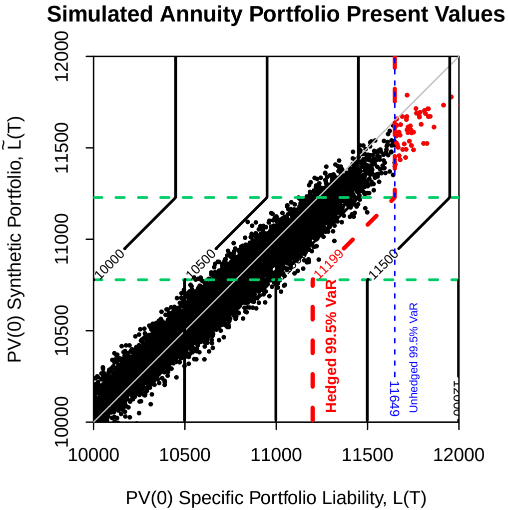


Simulated Annuity Portfolio Present Values
Correlation=0.978
S.D.=350
S.D.=395
PV(0) Synthetic Portfolio, L̃(T)
PV(0) Specific Portfolio Liability, L(T)
9000
9500
10500
11500
9000
9500
10500
11500

Hedging Example: Unhedged VaR $= 11,649$

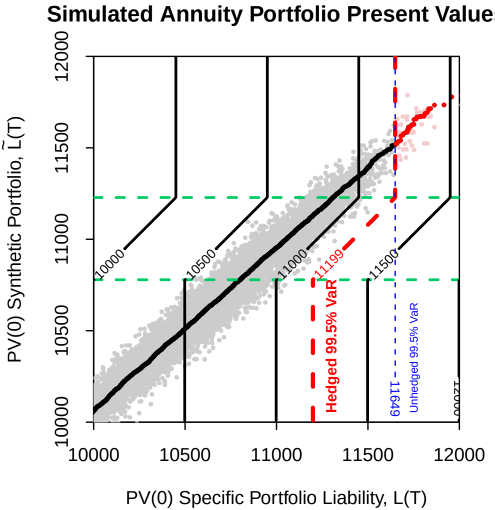


Hedging Example: Hedged VaR $= 11,199$



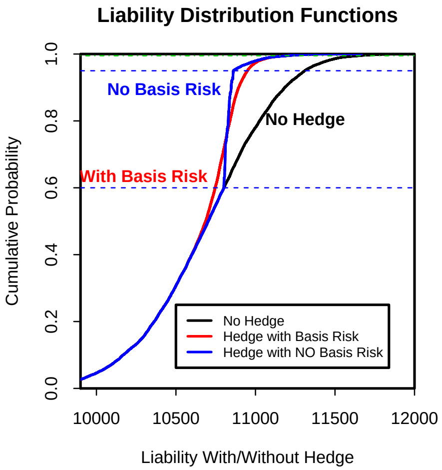
Plot shows kinked contours of $L(T) - H(T)$.

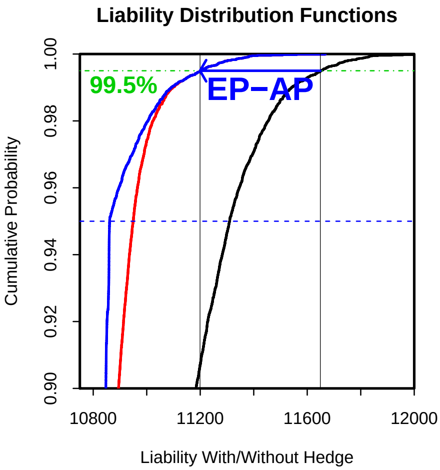
Hedged VaR $= 11,119$ with no Pop. Basis Risk



Plot shows kinked contours of $L(T) - H(T)$.

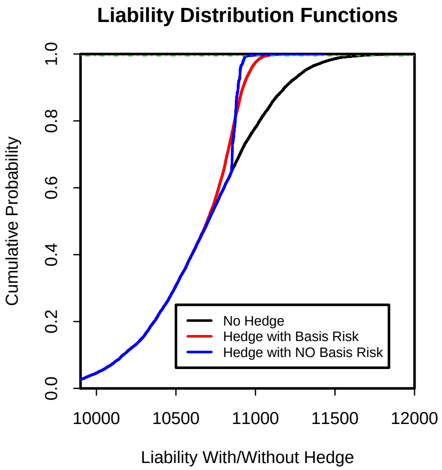
Hedging Example: VaR Calculations

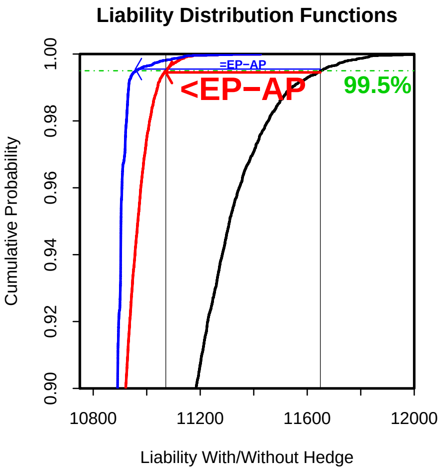




Note: CDF makes no allowance for the price of the hedge.

Hedging Example: Higher AP (0.65) and EP (0.995)





Numerical Example: AP, EP $=$ 60% and 95% quantiles

$L(0)$:	$SCR_{20\%stress}$	840	
$\tilde{L}(T)$:	SCR_{10}	840	(Pop 1; no hedge)
$\tilde{L}(T) - H(T)$:	SCR_{11}	478	(Pop 1; with $\tilde{L}(T)$ hedge)
$L(T)$:	SCR_{20}	960	(Pop 2; no hedge)
$L(T) - H(T)$:	SCR_{21}	598	(Pop 2; with $\tilde{L}(T)$ hedge)

Table: SCR values in excess of the mean liability. For the hedging instrument $AP = 10779$ (60% quantile) and $EP = 11228$ (95% quantile). Pop 1: synthetic $\tilde{L}(T)$. Pop 2: true $L(T)$.

How good is the hedge?

- "Good" $\Rightarrow$ price and risk reduction
- "Good" $\leftrightarrow$ Types of basis risk
  - Structural (e.g. non-linear payoff)
  - Population basis risk
    - Within population (e.g.linkage to different cohort)
    - Different population
- Hedge effectiveness $\Rightarrow$ % reduction in required capital
- Haircut $\Rightarrow$ impact on capital relief as a result of population basis risk
- EIOPA Solvency II guidelines $\Rightarrow$
  regulatory approval should focus on the haircut

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What is the impact of Population basis risk on hedge effectiveness?

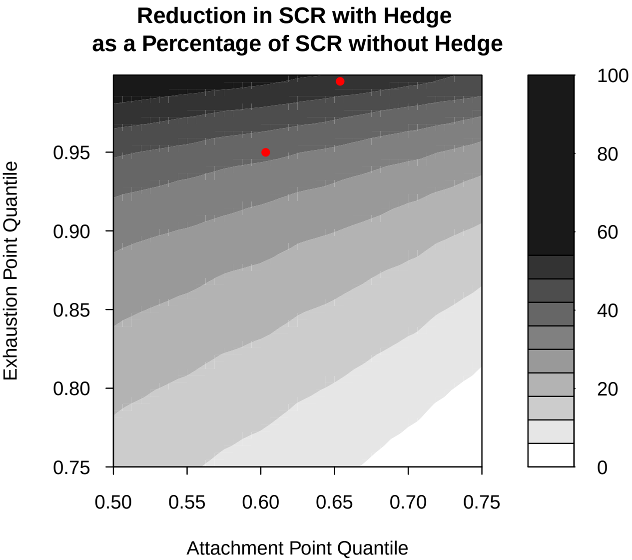
$$\text{Haircut} \quad HC = 1 - \frac{SCR_{20} - SCR_{21}}{SCR_{10} - SCR_{11}} = 0.000.$$

Haircut $\approx$ 0: Interpretation

- Here *EP* "$<<$" 99.5% quantile
- Above the 99.5% quantile the call spread (almost) always pays off in full
- So population basis risk $\Rightarrow$ little impact
- Structural basis risk prevails

- More detailed analysis $\Rightarrow$
  Haircut is *worst* (highest) when EP is close to the 99.5% quantile.

Reduction in SCR: Dependence on AP and EP



Sensitivity to Hedge Maturity, T

- e.g. $T = 20$

- % reduction in SCR is *slightly* higher
- Haircut is *slightly* worse
- Haircut is still ≈ 0 for $EP \leq 99.5\%$ quantile

- The longer the maturity:
  - less liquid market
  - less confidence in future reserving method
  - more future capital relief (everything else held constant)

Summary

- Bridging the gap:
  Academics $\leftrightarrow$ Insurance practitioners $\leftrightarrow$ Regulators
- Academics:
  practice is more messy than you would like!
- Practitioners: insightful exercise, ultimately allows for flexible longevity risk management.

Thank You!

Questions?

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