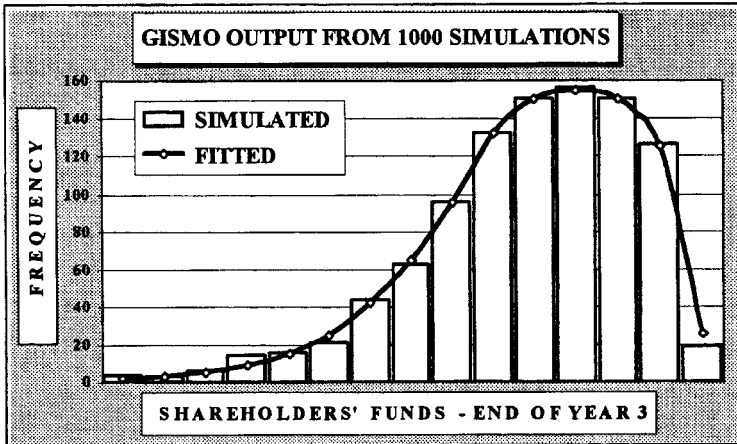


G I S M O

GENERAL INSURANCE STOCHASTIC MODEL OFFICE

'SHORT-TERM MODELLING FOR MANAGEMENT DECISIONS'



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SUMMARY

GENERAL INSURANCE STOCHASTIC MODEL OFFICE

GISMO is a stochastic insurance cash flow model implemented as an Excel 5 spreadsheet. It is intended to provide managers with useful information on the quality, or likely variability, of the key financial figures in their plans and enable them to gauge the impact on these figures from changes in, say, asset mix or catastrophe reinsurance protection.

GISMO incorporates an economic model which allows for 'jumps' in the values of the underlying indices. This is a pre-requisite, in our view, in any attempt to model the financial strength of a general insurer. It provides a consistent economic framework for valuing assets and liabilities. It has an option to model the impact of catastrophe type losses both gross and net of a specified amount of traditional catastrophe excess of loss reinsurance.

An assumed asset mix is input together with simple parameters that broadly define the insurance characteristics of the company. The model can then produce a probability distribution of any item from a simplified Balance Sheet and Revenue Account which stretches quarterly over the short to medium term.

It is, for example, possible to try a new asset mix assumption and see what happens to any of the balance sheet or revenue account figures at any point in the projection period. The model could be used in investigations of capital requirements, dividend strategies, catastrophe reinsurance requirements, as well as to consider the balance between asset and insurance risk.

There is no optimisation capability built into the model, although it should be possible, with a lot of trial and error, to use the model in an optimising process, as long as the identified objectives are based on one or more of the values included in the output.

The latest version of GISMO will be demonstrated at the GISG Conference at Stratford-Upon-Avon on the 2nd of October, 1996. It is our intention to make GISMO widely available via the Internet.

GISMO

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1. INTRODUCTION

Terms Of Reference:

“To consider the financial risks faced by general insurers and investigate how these may be classified, quantified and combined in a technically sound and dynamic manner so as to help insurance managers ensure that their day-to-day and strategic decision making processes are consistent with their objectives.”

The year on year results of General Insurance and Reinsurance Companies (Non-Life, Property & Casualty) show significant variation arising from a variety of sources, some of which are systematic, such as pricing changes, and some random, such as catastrophe or weather losses and movements in the market values of assets.

The performance of these companies is generally measured by their pre-tax profit figure, primarily the underwriting result plus investment income on all assets. Their financial strength is measured by their solvency ratio, that is the ratio of assets less liabilities (the solvency margin) to the premium income for the year.

Attempts to ‘model’ a general insurance operation have tended to focus on solvency from a regulatory perspective. As a result the models developed (for example, by the Finnish Working Party and the various GISG Solvency Working Parties) have projected results over the medium to long term (10 to 30 years) and have generally used autoregressive asset models, such as the Wilkie Model. The models have made allowance for the underwriting cycle, catastrophe losses and the demand elasticity of the market.

These solvency orientated models appear to have fallen by the wayside, with attention in the nineties turning towards Risk Based Capital, Dynamic Solvency and Asset-Liability Modelling, whatever these terms may mean. It is not surprising, in the circumstances, to find that insurance managers are somewhat confused by all this technical jargon. These managers have many pressing financial questions that they would like to ask, if only they could find a means to answer them.

The Working Party has attempted to provide a framework for answering these questions by developing a ‘General Insurance Stochastic Model Office’, or GISMO. This is a short-term model, concentrating on values around the means rather than the extremes of the underlying distributions.

In other words, the GISMO is intended to provide measures of the quality of financial estimates or plan figures, and the likely impact on these from different asset or reinsurance strategies. For example, the plan for Year 2 will have a figure for the expected underwriting profit and an estimate of the solvency ratio at the end of this year. We should like to be able to comment on the probability of the actual values (profit or solvency ratio) turning out to be above or below the Plan figures by some amount or percentage, given our asset or business mix.

We suggest that this 'focused' approach is needed in order to make any real progress with modelling the financial dynamics of a general insurer and produce results that are of use to the managers of these companies. Perhaps the easiest way to illustrate this point is to consider whether the insurance manager is more interested in estimating his 'probability of ruin' or the probability that his solvency margin falls by some amount or his solvency ratio falls by some percentage over the plan period.

These measures should assist the manager to determine whether he needs to raise more capital and, perhaps, how much he needs, or to amend his plans by, for example, reducing the volume of business he will write. Some estimate of a ruin probability is not of immediate use to him, especially if he has only five years to retirement. We should also exercise care in interpreting low probability estimates from models which do not take account of parameter uncertainty.

Insolvencies of insurance companies are rarely due to stochastic variation and tend to involve some element of management failure, often as the main contributory factor. They could have the wrong model altogether. However, the development of stochastic insurance models will not stop insurance managers making costly mistakes which may, in the extreme, lead to insolvency.

The development and proper understanding and use of such models should improve management's appreciation of the issues involved and, hopefully, help them to manage better. If this happens, then we may see fewer companies getting into financial difficulties than might otherwise have been the case!

Work on the computer model is continuing. A working version exists at the time of writing (early July 1996). The model will be refined further in time for the 1996 GISG Conference in October. The latest version of the model will be demonstrated 'live' at the Conference and we intend to then make it available through the Internet.

2. RISK AND MANAGEMENT OBJECTIVES

2.1 WHAT IS RISK?

Although insurance is often described as a 'risk business', 'risk' is usually not well defined. The term is used to describe any number of things: the probability of some event occurring, an individual policy (for example 'this motor risk'), an amount we insure (such as the Sum Insured), or some upper bound on the amount we stand to lose on a claim (such as the PML or Probable Maximum Loss - whatever this means!).

To add further confusion, the mathematical theory that attempts to describe the insurance process is known as 'Risk Theory'. In simple terms, this theory combines loss frequency and loss severity distributions to derive the aggregate loss distribution of a given portfolio of policies. The elegant results can then be used to consider questions such as the probability of ruin and safety loading in premiums.

It is helpful to associate the term 'risk' with the existence of a model of the underlying process that enables us to estimate the distribution of possible outcomes.

2.2 WHAT IS UNCERTAINTY?

There is, however, a further problem with the risk theory description of the insurance process.

The results derived from this theory often show significantly less variation than that seen in practice. The main reason for this would appear to be that model error and parameter error are not considered, and that these may be much more significant than the stochastic variability inherent in the model with the chosen parameters.

The insurance process is a complex one and any attempt to model it is bound to have limitations. How does one allow for new types of latent claim, for example, or for the effect of future legal judgements on one's frequency or severity parameters?

We describe situations where we do not have an adequate model as cases of 'uncertainty' rather than 'risk', and accept that in such cases we are simply not able to estimate the distribution of likely outcomes. We may have to rely on gut-feel or guess work.

We have attempted to differentiate between risk and uncertainty in order to ensure that our readers are aware that any model will have limitations and that any answers derived from a model have to be interpreted with care. More particularly, the sensitivity of the results to the chosen parameters should be investigated at every stage.

It is not unrealistic, however, to assume that more detailed analysis of the insurance process and the development and use of stochastic insurance models could turn some of these 'uncertainties' into 'risks'.

Any model will inevitably fail to capture some features of the real world. The level of modelling will depend on the budget and time scale for model development, as well as the amount of detail required of the outputs. Features of the real world may be deliberately omitted because they are not believed to be of primary importance, or because they are conceptually difficult and therefore possibly costly to implement.

Other features get missed out because we don't think of them. It is easy to get carried on a wave of euphoria and simply extrapolate past favourable trends. We don't consider what might happen if new competitors appear, and we may tend to forget latent claims because none have arisen recently.

More sinister are the corporate factors which lead to certain features being taboo and therefore outside the scope of our models. Modelling may be carried out as a matter of routine by staff who are not privy to strategic level decisions. In several recent cases of corporate mergers or acquisitions, financial modellers were not advised of the impending restructuring, which limited the usefulness of their efforts.

By the same token, technical staff may well be capable of estimating the probability of a stupid and costly management error, and even the likely costs of various suboptimal courses of action. However, managers can be sensitive about such matters - if a prophetic subordinate forecasts managerial ineptitude, the first concrete evidence of such ineptitude may be the termination of the subordinate's employment!

Even given infinite resources, free information and a favourable management climate, no stochastic model captures all the items of risk. In particular, there are significant risks that the model being employed is wrong. From a statistical perspective, this may be measured by computing parameter standard errors. However, this also fails to capture some of the risks. There is a tendency for the real world to change periodically from one regime to another, usually at rather short notice. The problem this causes is that, within a particular regime, a certain statistical model may appear to be a good fit; the parameter standard errors may give no warning of any catastrophic shift which may be around the corner.

Dealing with parameter and model uncertainty is still an unsolved problem from a practical point of view. As far as we are aware, current practice is to simulate based on 'central estimates' of parameters. Parameter standard errors may be used in the model selection process, but having decided on a model they are rarely incorporated into simulations in any way. Instead, experienced model users tend to 'take uncertainty into account' in a subjective fashion by suitably interpreting the model output. To some extent, such manual amendments defeat the object of having a quantitative model.

2.3 THE EXPOSURE CONCEPT

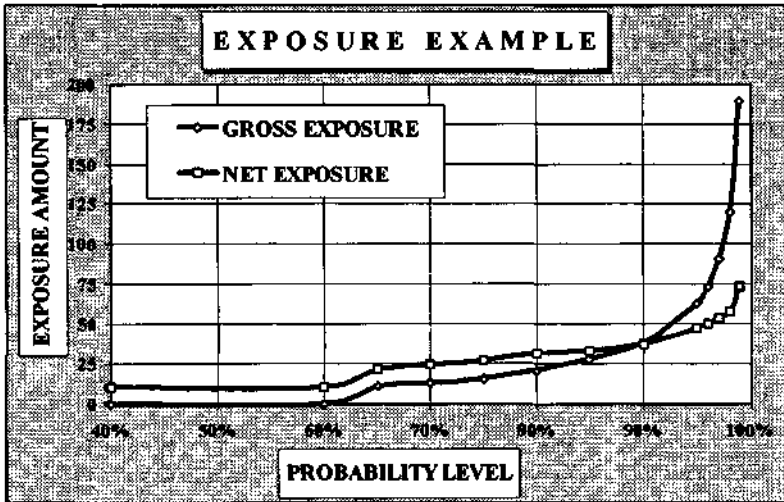
We have discussed the use of the terms 'risk' and 'uncertainty' in insurance and tried to draw a distinction between them. Insurance is full of terminology that defies consistent definition. Probable Maximum Loss (PML), Estimated Maximum Loss (EML) and Exposure are such examples. At least with these three examples, we know that they are measured in money terms, even if we are not too sure of their exact probabilistic definition.

We believe that this situation can be improved and that it is possible to define 'exposure' and apply this definition to estimate usable values under certain circumstances.

We propose that **exposure** be defined as the amount that can be lost at some given probability or risk level. In other words, we are interested in the complete probability distribution of the outcome and not some value from this distribution.

Clearly our definition requires that we be able to derive the aggregate loss distribution. For the purposes of the GISMO, we are dealing with this exposure at company level rather than at class of business or policy level. It is worth recalling that adding 'exposures' involves correlations and convolutions of probability distributions which is a lot more complicated than adding a series of 'exposure' values.

The following chart shows an example in the case of a particular company's catastrophe exposure, before and after reinsurance. It is intended to illustrate the concept of 'exposure' to loss as a function of a risk or probability variable, and indicate how this approach can provide management with useful information on the effects of a management action, which in this case is the purchase of some catastrophe reinsurance.



In the example, a loss classified as a catastrophe event occurs with an annual probability of around 60%. The gross exposure is then calculated using a simple catastrophe model with the Poisson distribution for the event frequency and the Pareto distribution for the event severity. For more details on this approach please refer to Appendix 2.

The chart also illustrates how reinsurance may 'reduce' this exposure. Clearly, there is a cost associated with this reduction in exposure, as the chart shows. In this example, the insurer incurs a higher cost with a probability of 90%, primarily as the cost of the reinsurance and any reinstatement premium exceeds the amount recovered from the reinsurer, but benefits with probability of 10% by recovering more from the reinsurer than he has paid in total.

2.4 RISK, EXPOSURE AND MANAGEMENT OBJECTIVES

The main 'risk' faced by any manager is that of failing to meet his objectives. We have set out to develop a model that can assist insurance managers to get a better feel of the size of this risk. In other words we should like to provide them with a model of their 'exposure' to this 'risk' that puts probability distributions around these objectives.

Pre-tax profits for a period and accounted net assets at the end of a period are **examples of objectives** for which we would like our model to provide probability distributions.

2.5 SOURCES OF RISK TO MANAGEMENT OBJECTIVES

The main sources of 'risk' to these objectives are those resulting from the insurance pricing cycle, exposure to catastrophe losses and variations in asset values.

To make any progress we need an appropriate model of these three key aspects of the insurance process that captures enough of the complexities and interactions and is capable of outputting the distributions we are interested to see.

Ideally, we should like to comment on the likelihood (risk) of failing to achieve these objectives by a given amount (the exposure). The manager can then attempt to limit the 'risk of not meeting his objectives' by investigating the effects of changing his insurance or asset portfolios and his catastrophe reinsurance cover.

There are clearly going to be limitations in the sources of 'risk' that can be incorporated into any insurance model. For example, shortfalls in claims reserves have caused difficulties to insurers from time to time and, in extreme cases, have led to some spectacular insolvencies. Should our model attempt to allow for such a possibility? Our answer, at this stage, is no. Companies vary significantly in their management of this risk and this is likely to be a feature more dependent on management competence than random or stochastic variation that can be incorporated in such a model.

In other words, we are not attempting to model 'uncertainty'. This does not mean that it is not a threat, rather that it is additional to what we can measure and has to be considered as such.

3. GISMO - GENERAL INSURANCE STOCHASTIC MODEL OFFICE

3.1 THE CONCEPT OF A MODEL OFFICE

A model office is a piece of software which projects cash flows and capital items for an insurance enterprise. These items may be based on stated assumptions regarding the insurance and capital markets. Model offices can be used to assess likely consequences of various management strategies, and can therefore act as an aid to financial decision-making.

The model office can also be used to examine the consequences of changing the stated assumptions - in particular, how far can the assumptions be flexed without the outcome becoming uncomfortable? This is the classical way of measuring risk. An office is following a *'high risk'* strategy if it is not resilient to changes in underlying parameters.

3.2 THE STEP TO STOCHASTICS

The introduction of stochastic models is, conceptually, a moderately small step. Broadly speaking, the idea is to prescribe probability distributions for the various cash flows and capital items. These distributions may be based on empirical data, or theoretical descriptions of the underlying generating process.

Potentially, there are many ways of combining distributions of underlying elements to obtain distributions of output quantities. For example, one could attempt to perform all the necessary operations algebraically or by storing densities numerically. However, it is now almost universally assumed that calculations will be carried out by simulations. The user generates the driving variables according to the specified distributions, each set of values representing one 'scenario'. Each scenario is then pushed through a standard model office to obtain specified performance indicators. Finally, the distribution of any required output variable is estimated by the sampling distribution from the scenarios tested.

One significant drawback of the pure Monte-Carlo approach is the difficulty of calculating conditional distributions. For example, in an ideal world, a reserve might be the conditional mean of a suitable loss distribution, given information obtained to date. This reserve figure may be an important consideration in the decisions made for the following year. However, calculation of this conditional mean at, say, time 2, would appear to require the simulation of a large number of scenarios starting from the current situation at time 2, which has itself been generated stochastically.

This leads to a 'scenarios within scenarios' situation - the computational burden grows exponentially over time and soon becomes unmanageable. The only practical solution to this problem that we are aware of is to keep the loss generation process to a relatively simple algebraic form so that the conditional expectations can be calculated analytically. This is the solution that we have adopted. It would appear that previous work in this area has not advanced far enough to discuss the problem, let alone suggest a solution. However, our approach immediately rules out from practical consideration many models which might be perfectly plausible descriptions of reality.

3.3 MODELLING THE MARKET PLACE

Some of the items in a model office may be supposed to derive from natural processes which may be modelled in a physical fashion, such as hurricanes, storms, floods and so on. A wider class of risks may be taken as essentially natural in the sense of being random and, to some degree, at least in the large, beyond human control. Motor accidents, burglaries or accidents may come into this category.

Other aspects of an insurer's operations will be determined by various forms of human negotiation. Premiums come into this category - there is a trade-off between price and volume which is not determined by any physical law but by the interaction between customers, competing insurers, and possibly brokers or other third parties.

Often, macro-economic aspects are also determined by human negotiation, but more indirectly. For example, market values of assets are determined by trades in a market. Our insurer may not be actively trading, but traded prices of investments will still impact the solvency calculation. To a lesser degree, expenses, taxes or even the supervisory environment are the outcome of lobbying and political negotiation.

There is an essential difference between natural processes and negotiated ones. If an insurer takes his eye off the negotiation process, all future negotiations are likely to go against him. This is most noticeable in the phenomenon of 'winner's curse' in premium rating - if you underprice you get loads of business and lose money, while if you overprice you get no business and are crippled by overheads. A deep understanding of competitor positions and the negotiating process is essential to sound management and the avoidance of such mishaps. On the other hand, insurers cannot by negotiation influence the incidence of natural disasters - or at least, few have adopted daily prayers or animal sacrifices on a corporate scale to appease the Gods!

3.4 THE EQUILIBRIUM CONSTRUCTION

The existence of a great deal of micro-level detail is possibly the largest obstacle to building a coherent higher level stochastic model office. How is this detail to be subsumed into a simple form in such a way that the essential macro features are preserved? This difficult problem is still very far from having been solved. Most of the solutions currently in place seem pretty crude.

However, the pitfalls of not thinking about this issue properly are severe. Let us suppose that we had obtained a highly simplified and stylised model of the premium negotiation process. As a result of the simplification, the model may overlook constraints or possible competitor actions which may not, at first sight, appear important. When we come to take decisions for our own insurer, we will be attempting to solve an optimisation problem to maximise some generic form of utility. The relaxation of constraints or failure to consider competitor action will generally have the effect of increasing the apparent maximum utility achievable. In effect, by modelling competitor actions in a superficial fashion and then optimising our own behaviour in a mathematical sense, the dice will appear to be loaded in our favour. In reality, competitors may also be optimising and we ignore such a possibility at our peril!

Economists have approached this on the asset side with the equilibrium approach. This is one way of investigating investment prices without having to model the behaviour of every market maker, investor and capital raiser. Instead, one assumes that each of these agents is doing something broadly optimal, that is, each agent is negotiating to the best of his ability, usually using a very simple utility function, such as a power law. This kind of equilibrium model is consistent with efficient financial markets, where each investment earns an expected return consistent with the level of risk; risk needs to be rather carefully defined to make this work. Such models constrain the extent to which an insurer can exploit competitor irrationality and therefore overcome the over-optimistic forecasting problem mentioned above.

The same principle can be applied to insurance contracts, at least in the aggregate. The assumption would then lead to changes in insurance pricing forced on the markets by the rational behaviour of the players in this market. This would lead to insurance contracts being priced by discounting using an appropriate risk-adjusted discount rate according to the level of 'systematic risk'. If this were actually the case, then insurance markets would be efficient; nothing could be gained by surfing the insurance cycle to maximise volumes when premiums are high.

In reality, there are various constraints which prevent insurance markets from being quite as efficient as one might hope. The most significant feature is the barrier to new entrants - setting up an insurance company and generating business is expensive and involves a substantial gestation period, during which any perceived opportunity has potentially diminished.

There may be capitalists sitting on the sidelines with large cheque books, but they only start writing when premiums are very dear relative to where they 'should' be, and are likely to remain so for some period of time. By the same token, having committed capital and resources to developing an area of business, an existing insurer may not be prepared to let go of a community of customers just because rates are low for one year. Instead, a substantial burning of fingers is required to force an exit. Thus, there are bands within which market premiums may fluctuate before management action has any remedial effect.

Despite the inefficiencies of insurance pricing, we believe that the theoretical 'equilibrium' premium is a useful concept. We have modelled premium rates relative to equilibrium rates so that we can at least quantify (and put limits on) how inefficient the market is. The way in which this is modelled is highly influential on the model outcome, since the major area in which shareholder value is created is by writing profitable insurance business.

The use of equilibrium models gives rise to a phenomenon known as 'risk aliasing'. This occurs because equilibrium models attempt to explain current investor behaviour in terms of optimisation in the face of uncertainty.

Suppose the model omits some risk which investors are, in fact, taking into account. The equilibrium procedure will ascribe that investor behaviour, wrongly, to some other feature which is captured by the model. For example, if a model does not capture the possibility of corporate bonds going into default, their high yield may wrongly be ascribed to an extreme intolerance of interest rate risk on the part of investors. This is particularly problematic in the insurance world, where, of necessity there are numerous latent risks not captured by our models.

The *ad hoc* management approach to containing such risks is to apply exposure limits to geographic areas, lines of business, particular reinsurers and so on. However, an equilibrium model which does not measure these latent risks is unlikely to provide useful guidance as to where these exposure limits ought to be placed, and may even seem to imply that such limits are unnecessary. Once again, we see that considerable skill is required in interpreting these models in order to avoid being misled by their weaknesses.

3.5 THE ECONOMIC MODEL

There are three aspects to an economic model:

1. generation of cash flows as statistical time-series;
2. modelling the rate at which uncertainty is resolved as information reaches the market;
3. modelling how a rational market would price these uncertain cash flows, making an appropriate and consistent allowance for risk.

There is a parallel between changes in claims reserves, which are the result of updated expectations of claims cash flows, and capital gains on investments, which are the result of updated expectations of asset cash flows. We propose in this paper a single framework for valuing asset and liability cash flows, which provides consistency between the two and constrains the complexity of the relationships modelled. Without such a framework the valuation process becomes unmanageable: on the liability side, for example, we would need to consider three-dimensional arrays of expected cash flows (expectation at time t of cash flows during quarter u arising from business written in quarter s). This would require a six-dimensional array of correlations!

The economic model is described briefly below. The interested reader is referred to Smith (1995,1996) for fuller descriptions.

The model is based around five indices $P_i(t)$,

$$P_i(t) = V_i(0, t) \exp \left[-\psi_i(t) + \sum_j \left\{ \beta_{ij} G_j(t) - \gamma_{ij} H_j(t) \right\} \right],$$

where

$i = 1 \Rightarrow$ one pound sterling

$i = 2 \Rightarrow$ retail prices

$i = 3 \Rightarrow$ total return on equities

$i = 4 \Rightarrow$ total return on property

$i \geq 5 \Rightarrow$ claims per unit exposure inflation index (with $P_i(0)$ set to 1)

and

$$\psi_i(t) = \sum_{j: \gamma_{ij} \neq 0} \alpha_j \left[t + \frac{(1 - \beta_{ij}) \log(1 - \beta_{ij}) - (1 - \beta_{ij} + \gamma_{ij} t) \log(1 - \beta_{ij} + \gamma_{ij} t)}{\gamma_{ij}} \right] - \sum_{j: \gamma_{ij} = 0} \alpha_j t \log(1 - \beta_{ij}).$$

The $P_i(t)$ describe the 'spot' prices of the underlying quantities, denominated in some notional currency. To observe the prices in sterling terms, for example, they must be divided by the price of sterling in the notional currency, $P_1(t)$. Thus, $P_2(t)/P_1(t)$ represents the RPI index, $P_3(t)/P_1(t)$ represents the total return on equities, in sterling.

The price indices are driven by a series of compound Poisson 'jump' processes $G_i(t)$, starting from zero. $G_i(t) - G_i(s) \sim \Gamma[\alpha_i(t-s)]$, where Γ denotes the Gamma distribution. These represent fundamental variables underlying the economy, on which the values of all investments depend.

The $H_i(t) = \int_0^t G_i(s) ds$ represent the cumulative effect of the jump processes $G_i(t)$.

The α_j , β_{ij} and γ_{ij} are constants, such that $\alpha_j > 0$, $\beta_{ij} < 1$ and $\gamma_{ij} \geq 0$. The β_{ij} measure the sensitivity of the price indices to the processes $G_i(t)$, which generate jumps in the price, γ_{ij} measure the sensitivity to the cumulative effect of the $G_i(t)$.

The quantity $V_i(s,t)$ is defined to be the value at time s , in units of $P_i(s)$ of a bond delivering one unit of $P_i(t)$ at time t . In other words, it represents a term-dependent factor at time s for discounting quantities due at time t . The $V_i(s,t)$ are given by the formula

$$V_i(s,t) = \frac{V_i(0,t)}{V_i(0,s)} \exp \left[-\psi_i(s) + \psi_i(t-s) - \psi_i(t) - (t-s) \sum_j \gamma_{ij} G_j(s) \right].$$

The $V_i(0,t)$ therefore describe the initial term structure of the five investment classes.

Bond prices (in units of the underlying asset) are given by:

$$V_i(s,t)P_i(s) = V_i(0,t) \exp \left[\begin{array}{l} -\psi_i(t-s) - \psi_i(t) + \sum_j \beta_{ij} G_j(s) \\ -\sum_j \gamma_{ij} [H_j(s) + (t-s)G_j(s)] \end{array} \right].$$

Total return indices, obtained from a portfolio of zero-coupon bonds linked to the underlying asset i , constantly rebalanced to retain maturity τ_i , are given by:

$$RP_i(t, \tau_i) = \prod_j (1 - \beta_{ij} + \gamma_{ij} \tau_i)^{\alpha_j} \exp \left(\sum_j (\beta_{ij} - \gamma_{ij} \tau_i) G_j(t) \right).$$

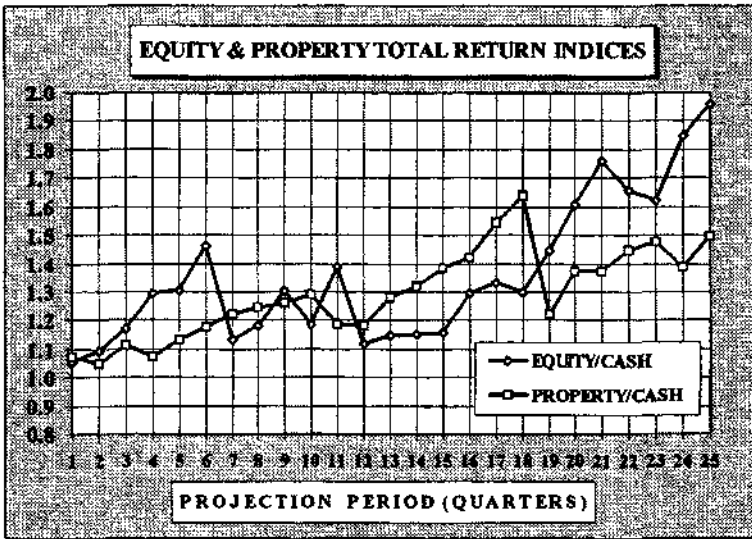
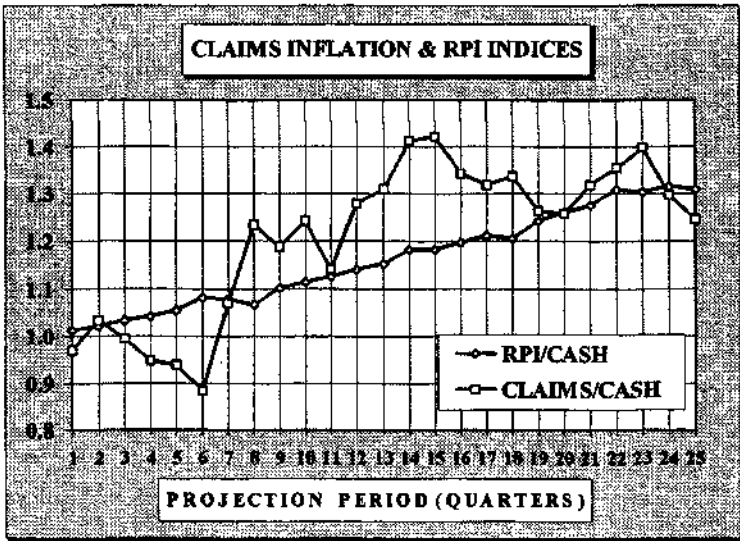
3.5.1 Calibration of the Economic Model

1. The model is calibrated to a set of expected returns and variance-covariances for the different investment classes. The variances/covariances for the asset classes have been obtained from historical data. The expected returns are derived by assuming a utility function and optimal portfolio for an investor and 'fitting' the expected returns so that the implied optimal portfolio for the investor, given those expected returns, equals the optimal portfolio assumed.
2. The variances/covariances for the claims inflation index are largely arbitrary - the parameters are round multiples of the other model parameters to reflect, for example, 50% equity exposure. The mean returns and term structure have been calibrated for the other classes so that the representative investor holds 0% (as opposed to a long or short position) in claim-index-linked bonds.

3.5.2 Statistical Features of the Economic Model

1. The error terms used are more highly skewed than the normal error terms commonly used; this has the advantage, in particular for modelling events over the short term, that significant jumps in investment prices will occur.
2. The model is essentially short-term - the long-term asymptotic properties are not reliable.
3. The rates of inflation in effect perform random walks. There may be scenarios, therefore, in which these rates become negative. The expected shape of future inflation is determined by the shape of the initial yield curves used - they have been chosen, fairly arbitrarily, to be flat here.
4. The parameters of the RPI model have been chosen to produce jumps in the gradient of the RPI index but not in its level. The claims inflation index incorporates jumps in the level to reflect shocks arising from court settlements, foreign exchange rates for parts, and so on.

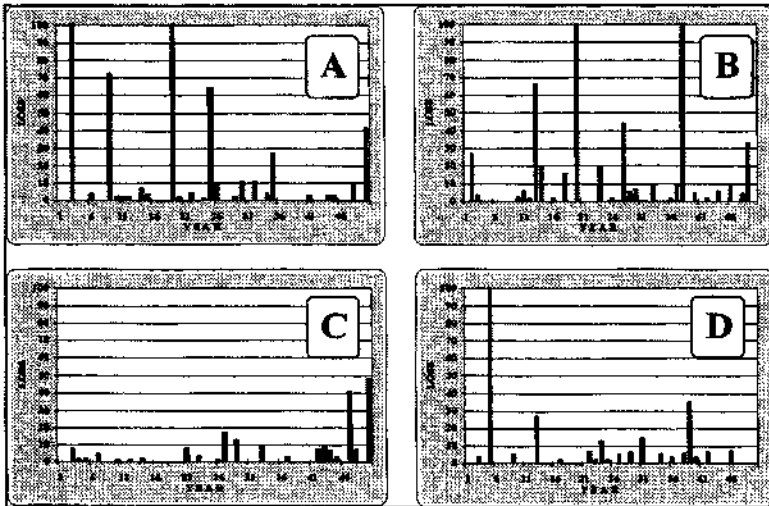
Example Simulations over 25 Quarterly Periods



3.6 MODELLING CATASTROPHE LOSSES

From time to time catastrophe events result in huge losses for insurers and reinsurers. Such events, which include hurricanes, earthquakes and floods, may be regarded as being the result of natural phenomena which are not directly related to investment markets or pricing cycles (although reinsurance costs for these exposures may vary within a pricing cycle).

An insurance model that did not include a catastrophe loss element would be incomplete and so GISMO has a basic facility for modelling such losses before and after traditional excess of loss reinsurance. The number of such events in a period are assumed to follow a Poisson distribution and the cost of these events a Pareto distribution whose scale is dependent on the claims index at the time. Once an event has occurred and its cost has been simulated, the cumulative payments are assumed to follow a Craighead curve. The following charts show three typical outcomes from such a model, together with an actual series of catastrophe losses. The reader is invited to identify the real data. The answer can be found in Appendix 2.



A novel feature of the GISMO is that it 'estimates' the market cost of a reinsurance programme utilising a 'cat reinsurance' pricing indicator - (soft, medium, hard) and is then able to calculate the net costs of these events resulting from the simulated gross amounts. Further details can be found in Appendix 2.

3.7 THE CASH FLOW MODEL

The elements of the cash flow model, as at July 1996, are described below. The description is deliberately superficial, since work is continuing and the final version will differ in some respects from that discussed here.

Written Premium & Exposure

The user inputs target written premiums over the plan period. The premium target represents planned changes in business volume (or exposure), rather than the actual level of written premiums (changes in which combine, of course, both changes in volume and changes in the average premium). The 'actual' written premium projected in each projection period therefore varies (stochastically, to an extent) as the average written premium varies.

The average written premium is expressed as a multiple of the theoretical, 'equilibrium' premium:

actual premium = equilibrium premium * premium adjustment

The equilibrium premium per unit exposure is calculated by discounting claims payments arising from that exposure according to the market (claims-inflation-linked) bond prices. The premium adjustment is a factor which captures the state of the market at that point. The user inputs the premium level and the position in the cycle at the start of the projection period, and the premium adjustment then varies through the projection period according to the initial conditions and the loss experience in prior periods. A price 'elasticity' term has been included.

written premium = premium target * premium adjustment^{elasticity}.

The 'elasticity' term is a very crude device to enable the volume of business (in exposure terms) to vary with price levels.

The unit of exposure is defined to be the amount of risk which gives rise to total expected claims, as at the inception date, of one claims inflation unit.

$$\begin{aligned}
 \text{written exposure} &= \frac{\text{written premium}}{\text{actual premium rate}} \\
 &= \frac{\text{premium target} * \text{premium adjustment}^{-\text{elasticity}}}{\text{equilibrium premium} * \text{premium adjustment}} \\
 &= \frac{\text{premium target} * P_i * \text{premium adjustment}^{-\text{elasticity}-1}}{\text{equilibrium premium}}
 \end{aligned}$$

A price elasticity of -1 may be described as a 'neutral' assumption. When this is the case, the written exposure remains unchanged as the market price level varies relative to the equilibrium level; in effect, the insurer is maintaining market share by pricing at the 'market' level. A price elasticity < -1 corresponds to "surfing the cycle" - more business is written when price levels are high.

Claims

The expected claims payments arising from a period of exposure are assumed to follow a Craighead curve, with different parameters for the catastrophe events. The actual non-catastrophe claims payments in a period are then simulated around this expected value by a Gamma variable whose variance is an assumed multiple of its mean. For both types of claims cost, expected claims rise in line with the claim inflation index produced by the economic model.

The claims reserves are a prospective assessment of future claims on an undiscounted basis.

3.7.1 Assumptions

A number of simplifying assumptions have been made in the current implementation of the cash flow model, at least some of which will have been modified by the time of the conference. They include:

1. Claims expenses are assumed to be contained within the cost of claims. Written premiums are effectively assumed to be net of other costs.

2. All the company's insurance business is modelled as a single class in the current version of the model. By October 1996 we anticipate that a number of classes will be modelled.
3. The impact of exchange rates has not been modelled explicitly.
4. Cash flows are assumed to occur at the end of each quarter.
5. Premiums and non-catastrophe claims are assumed to be net of reinsurance.
6. The premiums written in a quarter are assumed to result in 'claim reserves' based on the best estimate of the losses relating to the whole of this premium. In other words the model does not calculate explicit UPR's which are implicitly included in the outstanding claims.
7. No allowance is made for the possibility of bringing forward tax losses from prior years.
8. No distinction is drawn between investment income and capital gains.
9. The total (non-catastrophe) claims in each period have been modelled by a Gamma variable.
10. The portfolio of assets is re-balanced to maintain a constant split between the different asset classes. There are alternatives - an obvious one would be to re-balance the portfolio in proportions which vary according to the level of shareholders' funds as a proportion of total assets, say, on either a discounted or an undiscounted basis.
11. The expected claims in each period vary to the extent that claims inflation varies stochastically. The actual payments are then subsequently simulated as they fall due. This feature of incorporating stochastic parameters in the model should reduce, possibly significantly, the amount of parameter error left out of the GISMO estimates albeit that there will still be an element of model error which has not been included in these estimates.
12. The thorny question of how best to model changes in premium levels has not yet been fully addressed, with various alternatives being considered. There will always be a difficulty in modelling a market-wide phenomenon in the context of a single company, which may or may not be 'typical' of that market. An individual company faces a demand curve which shifts as the cycle passes - modelling the impact of varying the prices charged by a company relative to those charged by its competitors merits a paper on its own.

3.8 USER INPUTS

The initial assumptions are as follows:

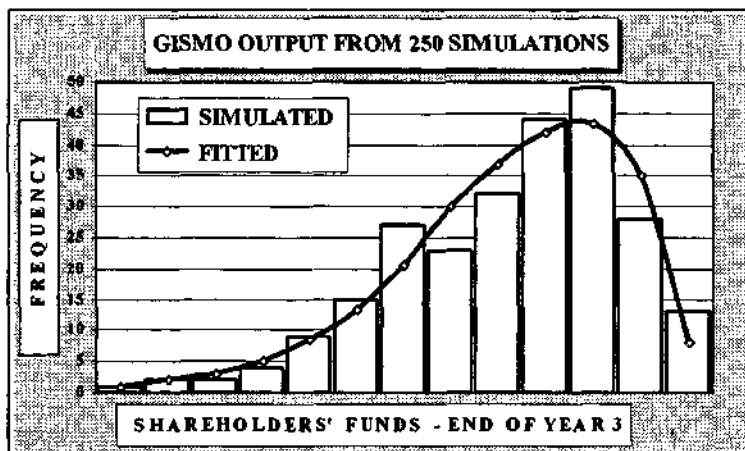
1. target written premiums over the projection period,
2. the total market value of assets at the start of the projection period,
3. the split of assets between gilts, index-linked gilts, equities and property at the start of the projection period,
4. the total technical reserves at the start of the projection period,
5. the tax rate applicable,
6. the 'target' solvency margin %; the solvency margin is defined as the ratio of shareholders' funds to written premium over the past four quarters,
7. the ratio of the variance of non-catastrophe claims to their mean,
8. parameters defining the development of claims payments (according to a Craighead curve),
9. parameters describing the initial level of current market premium rates relative to the 'equilibrium' level and the recent history of premium levels relative to the equilibrium level,
10. a measure of premium elasticity to the cycle,
12. parameters for the stochastic investment model,
13. parameters describing catastrophe losses and the state of reinsurance market

3.9 OUTPUT FROM THE MODEL

A simulation produces output of the following form, on a quarterly basis over a time horizon, which is currently ten years. This horizon may be reduced to five years in the version of the model to be released in order to reduce run times.

Time	0	¼	½	¾	1	1¼	1½	10
Balance Sheet									
Securities	320	426	478	565	387	454	504	1078
Outstanding Claims	58	118	180	245	258	388	358	802
Tax Provision	0	15	12	19	0	0	6	0
Shareholders' Funds	262	293	287	301	129	65	140	274
Revenue Account									
Written Premium		88	90	92	93	89	91	180
Investment Income		24	4	5	(7)	12	24	32
Paid Claims		(6)	(41)	(11)	(23)	(34)	(65)	(176)
Transfer from Reserves		(61)	(62)	(65)	(13)	(130)	31	58
Tax		0	0	0	(36)	0	0	(8)
Dividend		0	0	0	(206)	0	0	0

The user can select any cell in the output table and, by pressing a button, produce the graduated density function of the quantity of that cell over, say, 250 simulations. In addition, the model will display a bar chart of the actual simulation results with an overlay of the fitted density function as shown in the chart below.



For each projection period i , the economic and cash flow models produce the following quantities:

1. written premium(i)
2. paid claims(i)
3. outstanding claims(i)
4. investment return index(i).

The balance sheet and revenue account items for each projection period are derived from these according to the formulae below.

For each projection period i ,

$$1. \text{ securities}(i) = \text{securities}(i-1) + \text{written premium}(i) + \text{investment income}(i) + \text{paid claims}(i) + \text{tax}(i) + \text{dividend}(i)$$

(Paid claims, tax and dividends are assumed by convention to be negative.)

$$2. \text{ investment income}(i) = \text{securities}(i-1) * \left[\frac{\text{investment return index}(i)}{\text{investment return index}(i-1)} - 1 \right]$$

$$3. \text{ transfer from reserves}(i) = \text{outstanding claims}(i-1) - \text{outstanding claims}(i)$$

$$4. \text{ tax provision}(i)$$

$$= \begin{cases} \text{tax rate} * \text{cumulative pre-tax profit in year to date} & (i = 1,2,3) \\ 0 & (i = 4) \end{cases}$$

(subject to a minimum of 0)

$$5. \text{ tax}(i) = \begin{cases} 0 & (i = 1,2,3) \\ \text{tax rate} * \text{cumulative pre-tax profit in year to date} & (i = 4) \end{cases}$$

(subject to a minimum of 0)

6. Dividends are set (subject to a maximum of zero) to achieve a pre-defined solvency margin at the year-end. The solvency margin is defined as shareholders' funds at the year end as a proportion of written premiums over the year. Thus, at the year-end,

$$\begin{aligned} \text{solvency margin \%} * \sum_{k=0}^3 \text{written premium}(i-k) = \\ \text{securities}(i-1) + \text{written premium}(i) + \text{investment income}(i) + \\ \text{paid claims}(i) + \text{tax}(i) + \text{dividend}(i) - \text{outstanding claims}(i) \end{aligned}$$

or, equivalently,

$$\begin{aligned} \text{dividend}(i) = \text{solvency margin \%} * \sum_{k=0}^3 \text{written premium}(i-k) \\ - \text{securities}(i-1) - \text{written premium}(i) - \text{investment income}(i) - \\ \text{paid claims}(i) - \text{tax}(i) + \text{outstanding claims}(i). \end{aligned}$$

4. GISMO APPLICATIONS

4.1 APPLICATIONS IN GENERAL

All companies have plans extending a number of years into the future. These plans are used as a basis for making decisions. However, the one sure thing we know about plans is that the actual results will turn out to be different.

Looking at general insurance companies, any plan projecting the state of the company forward will be wrong for a number of reasons. The company will have to make some sort of average assumption about capital gains and investment income, which will turn out to be wrong. They will have to make some assumption about the number of policies written in future, which they will get wrong. Finally, they will also have to make assumptions about the non-catastrophe related premium adequacy, and the cost of catastrophes, which, along with almost every other aspect of the financial position of the company, will be wrong.

This makes planning sound like a pretty futile exercise. This seems rather harsh, as there is obviously still a great deal of merit in having expected values of the future state of a company. However, one would like to have more information than this for two broad reasons.

Firstly, given that the various statistics of interest have quite a wide range (the likely results are not usually tightly bunched about the mean), it would be useful for managers to have some idea of the various percentiles of the distribution, rather than just the mean. It is not terribly helpful to know that, on average, the company will remain solvent, when there is 1 in 5 chance that the company will be bust in a year's time!!

Secondly, when, in a year or two's time, one looks back at what has happened compared to what was planned, one has no way of assessing whether the difference is "reasonable", or should lead to changes in the assumptions going forwards. Having some gradation (25th and 75th percentiles for example) of the outcomes of various statistics of interest (profit, solvency and so on) allows one more meaningfully to refine existing plans in the light of experience.

When making any decision, one looks at the impact of that decision on some financial measure, be it a balance sheet measure such as solvency, or a revenue account figure such as underwriting result, or some combination of such measures.

Obvious candidates where a GISMO can provide useful input are when considering:

- asset strategies
- reinsurance strategies
- the mix of insurance business
- the balance of business between different territories within a Group
- the balance between asset and insurance risk
- how much capital one might want to raise
- future dividend strategies, given likely levels of distributable profit
- local objectives that are sufficient to meet Group needs

and so on. For any of these possible applications, the GISMO will give a manager more than just a point estimate of the financial measures of interest, and some objective way of then, in future, assessing how good, bad or indifferent his decisions were.

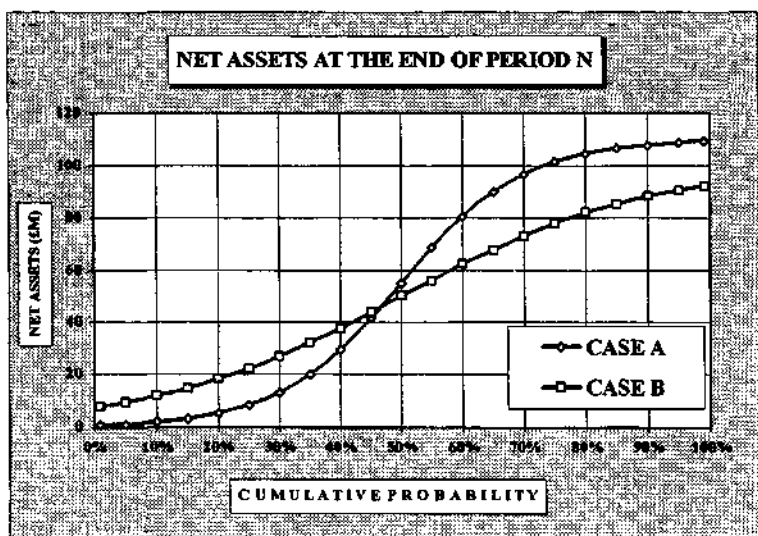
Some simple examples are given below.

4.2 ASSET STRATEGY APPLICATIONS

Say for example, one wanted to take look at the implications of investing more of one's assets in equities. Looking at this over a plan period of say 3-5 years, one expects to achieve a greater return on average, but this is achieved at the expense of greater volatility in those returns. One expects higher profits, due to increased investment performance, but potentially sharp drops in solvency, should the value of equities drop substantially. The GISMO lets one look at this balance between risk and return.

One could just look at the assets side of a companies balance sheet in isolation, when considering different strategies, but we may expect that over a period of time there will be a link between a period of high price inflation / equity increases and increases in the cost of claims. A companys' revenue account and balance sheet may also be affected by catastrophe claims. For example, if one assumes that catastrophic asset movements and catastrophic insurance claims are independent then the combined exposure, under our earlier definition, will be less than the sum of the exposures, at a given risk level. One can only assess the benefit of this diversification by modelling the two components together within a framework such as the GISMO which allows for some correlation through the link provided by the use of the inflation indices.

The following chart shows an example of the GISMO output for the level of net assets at the end of a Plan period when considering two different asset strategies:



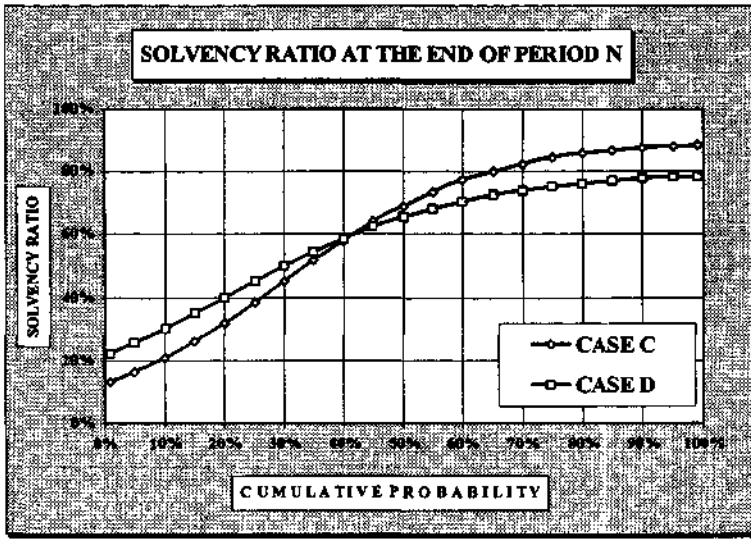
Case B is the base case, Case A is the case where one has increased equity investments. Case A has expected Net Assets at the end of the period of £60m, Case B has expected Net Assets at the end of the period of £50m.

Looking at expected values alone one might automatically choose Case A. However, we can observe that Case A leaves the company with a 30% chance that net assets will be less than £10m at the end of the period, whereas Case B, with a limited equity exposure, has a very small, less than 5% chance, that net assets will fall below £10m.

4.3 THE BALANCE BETWEEN ASSET AND INSURANCE RISK

Similar considerations apply when considering the balance between asset and insurance risk. For example, one might want to have either a small amount of catastrophe XL reinsurance cover, or have a higher proportion of one's assets invested in equities, but not necessarily both. In both cases, we expect to achieve higher overall returns, but the deciding factor may be the extent to which either strategy leaves one exposed to the risk of failing to meet any target levels of solvency, for example, that a company may have.

The following chart shows an example of the GISMO output for solvency margins at the end of a plan period when comparing relative levels of equity investment and reinsurance:



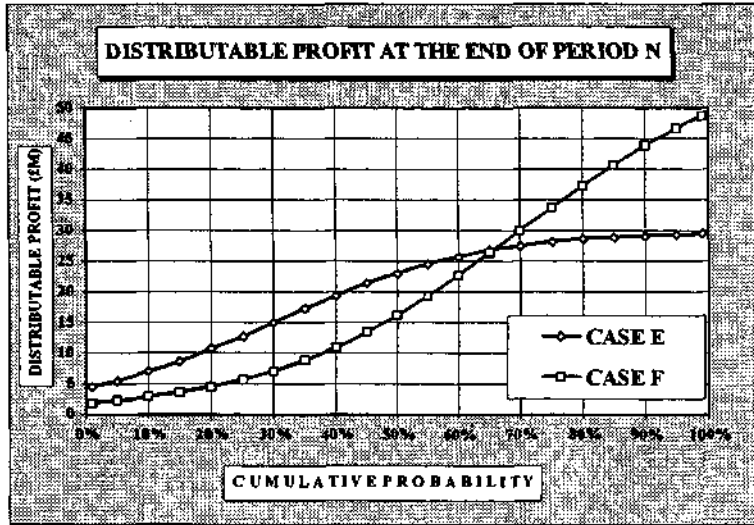
Case C maintains current reinsurance levels, but invests more heavily in equities, Case D has the current level of equity investment but with a much reduced level of reinsurance cover.

There is not too much between the two different strategies looking just at expected values. Case C has an expected solvency at the end of the period of 42%, the expected solvency for Case D is 45%. However, Case C has a 1 in 5 chance of solvency falling below 30%, Case D has only a 1 in 10 chance of falling below this level. This indicates that whilst the expected benefit of the two alternative strategies is similar, Case C, with a heavier equity investment, exposes the company to a greater risk of solvency falling below certain target levels when compared to the particular reduction in reinsurance.

4.4 FINANCIAL STRATEGY APPLICATIONS

A company's ability to pay dividends depends on the amount of distributable profit. Typically companies like to indicate to analysts and outside observers what their plan for dividends is going to be. Clearly it would therefore be useful to have some measure of the leeway a company has to increase its dividends over a period of time.

The following chart shows an example of the GISMO output for distributable profit. The two competing options are two different mixes of business:



Case F has much higher Liability content in the mix of business than Case E. Case F has a lower expected profit, but offers the chance of high profits (a 25% chance of making more than £35m), but with a 40% chance of only making £10m or less. Clearly if the company needs to satisfy analyst's expectations by delivering a distributable profit of £15m-£25m, whilst Case F may have a higher expected profit, the company may feel that the high chance of having a relatively low level of profits does not compensate for the possibility of very high profits.

5. CONCLUSION

Stochastic models are simplified representations of very complex processes. A good model will capture some of the main features of the processes it is intended to represent and thus produce some output that helps us develop a better understanding of the underlying process.

The financial dynamics of a general insurer are particularly complex, involving both volatility in the value of the assets as well as in the insurance outcomes. Additionally, insurance companies face statutory regulations requiring them to demonstrate solvency or balance sheet (financial) strength on at least an annual basis. The companies' ability to maintain market share may depend on their perceived financial strength.

Developing a model that can be useful to both the managers of these companies and their external observers, such as regulators, has proved to be a very demanding task. Most of the published attempts so far appear, for example, to use asset models that do not exhibit the jumps in values that occur in practice, from time to time, and which can play havoc with a company's perceived financial strength or solvency.

GISMO is intended to be, primarily, a management tool rather than a 'Solvency Model' designed to consider regulatory type questions. The output is designed to provide information useful to the managers of these companies rather than to be restricted to the sort of information that may be of interest to regulators. Apart from the ability to produce probability distributions of any balance sheet or revenue account item, the model can also be used to consider financial strength through a plan period and could be used for Dynamic Solvency Testing, which involves more than just mandated changes in some of the valuation parameters.

In determining our Terms Of Reference we recognised that the initial focus for the GISMO had to be clear and precise in order to ensure that at least some progress could be achieved in the time available. We believe that we have met our objectives and that the GISMO will prove to be a real step forward in the attempts to model the financial dynamics of general insurers.

We hope that GISMO will turn out to be useful to a wide audience. It is our intention to release a version through the Internet and we look forward to receiving constructive comments and suggestions from other testers.

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7. APPENDICES

7.1 APPENDIX 1 : SUMMARY OF RELEVANT PAST PUBLICATIONS

7.1.1 The ASIR System (1981).....M. Brown & L.C. Galitz

The insurance model described in the manual has 2 main aims:-

1. the modelling system should be sufficiently powerful and flexible to allow investigations into 'areas of topical interest to insurance and reinsurance'
2. the model should be accessible to a wide range of users wishing to apply the model to areas of theoretical and practical interest.

The design of the ASIR model is such that it is considered as a collection of sub-models, some of which are deterministic, some of which are stochastic. Stochastic models are used to generate future claim numbers and claim amounts, whilst all other cash flows (including investment returns) are generated deterministically. For a given risk type claim amounts are divided into normal claims and 'very large' claims in the ASIR model. Distributions can be specified for each of these two claim types and these are then combined to form a mixture distribution for claim amounts. The weight, reflecting the average proportion of all claims which are 'very large', used to determine the mixture distribution is an input parameter

Three modes of use of the model are identified:-

1. deterministic in the sense that the model is run only once (although the model as a whole is actually stochastic in nature).
2. sensitivity analyses i.e. the model is run several times with varying input parameters to test the sensitivity of the results to the input data. What are described as the 'uncertain elements' (e.g. future claims) are held constant between each successive run.
3. stochastic in the sense that the model is run many times, with the 'uncertain elements' being allowed to vary with each run. 1000, say, runs are performed indicating the extent of risk or uncertainty attaching to the business.

The ASIR model deals with a simulated market of up to 10 companies linked by a network of insurance and reinsurance treaties, 40 risk classes, reinsurance (quota share, surplus, excess of loss and stop loss), and also 20 countries and currencies.

The intended applications of the ASIR model are given as:-

1. to be of use in both insurance research and insurance practice
2. to answer "What-if" questions and in so doing explore the effects of different management strategies (rating, investment policy, growth, reinsurance policy) and different market environments (inflation, interest rates, exchange rates, tax)
3. corporate planning

7.1.2 Solvency (1986).....W.M. Kastelijn & J.C.M. Remmerswaal

This publication provides a summary of a number of methods for determining a company's solvency. The emphasis is generally on methods for statutory purposes. Many of the limitations of the various methods are attributed to practical factors, e.g. time, money and complexity (lack of?). Comparison of the various methods of calculating solvency requires consideration of 3 particular aspects:-

1. Time-frame i.e. static vs. dynamic models
2. Which risks are to be considered: on the assets side these include overall investment values (macro risk), fluctuations in the individual portfolio (micro risk) and other risks such as valuation method; on the liabilities side, these include claim fluctuations, nature of the portfolio, inflation, reinsurance and catastrophes.
3. Different methods will give rise to different measures of (in)solvency e.g. a one year probability of insolvency of x ; a probability of insolvency over 10 years of y ; a probability of ruin of z over an infinite time period. The question of which is more acceptable is not straightforward.

Three types of methods have been distinguished:-

1. Methods based on ratios
2. Methods based on risk theory i.e. considers the risk of variations in the aggregate claims amount;
3. 'Comprehensive' methods i.e. models which consider both liabilities and assets

A summary of all the methods reviewed by the publication is attached.

The 'comprehensive models' reviewed are as follows:-

The Finnish solvency study

This study considered solvency as needing to achieve the following objectives:-

- to safeguard the interests of policyholders
- to ensure the long-term continuation of the operation of the insurance company

The solvency margin is defined to be the sum of:-

- free reserves
- underestimation of reserves (hidden reserves)
- overestimation of liabilities (equalisation reserves) - although not considered a 'normal' part of the solvency margin

Aggregate claim amounts are subject to 4 types of fluctuation:-

- random fluctuations (according to the Poisson Law)
- short-term variations in the basic parameters
- cyclical variation
- trends

The study gives approximations for determining the probability of ruin at given points during the period under consideration.

1983 GISG Solvency Study - Daykin et al

- No general model was constructed.
- Solvency margin is defined by summing the reserves for different types of risks i.e. an RBC-type approach.
- Emphasis is put on consistent valuation of assets and liabilities.

A number of simulation methods are described:-

Dickinson & Roberts (1983)

- Performs a deterministic simulation for a group of insurance companies between 1960-80.
- Aim was to investigate the link between volatility of assets and rate of return and external capital requirements.

Ryan (1980 and 1984)

- 3 types of risk are considered: insurance, investment and inflation.
- Insurance risk - claim cost is determined by reference to the Central Limit Theorem and therefore requires a large portfolio.
- Investment risk - includes equities assumed to follow a random walk model.
- Inflation - assumed to perform a random walk around a mean of 8%.

Berkouwer et al (1983)

- Considered a range of risks: stochastic (fluctuations in claim payments), investment risks, expense risk, and others. Quantitative investigations were carried out for the stochastic risks only, by simulation techniques. The results showed this particular risk to be of relatively minor significance.

Coutts, Devitt & Ross (1983, 1984)

The 'micro' and 'macro' aspects of 3 risks are considered:-

- Liability risk: Macro- fluctuations in basic claim probabilities
- Micro - particular mix of business
- Asset risk: Macro - fluctuations in investment values
- Micro - particular investment portfolio
- Economic risk (e.g. inflation): Macro - economic situation, as affects all companies
- Micro - features of the company and structure of its assets/liabilities
- The authors believe that a simple general solvency model is not possible due to the micro aspects. Hence simulation is used.
- Assets were modelled as per Wilkie.
- Assumes company is in run-off.

GISG Working Party - Daykin & Bernstein (1985)

As with Coutts (above), this report considers solvency on a break-up basis and simulates the run-off of the existing liabilities. Three main types of uncertainty are identified:-

- random (e.g. claim amounts, fluctuations in asset values)
- less random, but outside company's control (inflation)
- within company's control (e.g. investment strategy)

Claim amounts and inflation were determined stochastically. For most of the assumptions used were sensitivity was tested. Asset variability was found to be most significant.

7.1.3 Managing Uncertainty in a General Insurance Company (1990) **C. Daykin & G. Hey**

A cash flow model is proposed as a way of analysing uncertainty in the future development of a general insurance company. Simulation techniques are used to explore the consequences of uncertainty. The company is modelled alongside the market in aggregate to assess the impact of changes in premium rates relative to the market. A computer model was developed for the practical application of the model.

The report recognises that accounting implications cannot be ignored, and hence the model operates at two levels: the level of actual transactions and the level at which the transactions will be reported using accounting conventions.

Whilst stochastic models may not be available for all types of uncertainty, scenario testing allows a range of scenarios to be explored to test the resilience to extreme assumptions or to interactions between assumptions.

Design aspects of the model:

- Up to 6 types of liability can be allowed for.
- Each class is characterised by a number of parameters e.g. claim settlement pattern, growth rate
- Company is initially assumed to have premium rates at the market rate and is then assumed to deviate from the market in one of several ways.
- Non-claim settlement expenses are modelled explicitly.
- Provision is made for claim ratios to vary on a cyclical basis
- Claim ratios are assumed to vary according to the Normal distribution.
- Seven types of investments are allowed for
- Wilkie's model is used.
- Additional allowance is made for tax and dividends.

7.1.4 Practical Risk Theory for Actuaries (1994).....Daykin et al

This is split into two parts. The first part is on Risk Theory. Broadly, this looks at various models for the frequency and severity of claims, and approaches to producing compound distributions of the two.

The second part analyses the variability of the assets and liabilities of an insurance business. Although it says it does this over the short and long term, the emphasis seems to be on the long term (20 years+). Variability of inflation and investment returns are considered. Various inflation models are considered, such as auto-regressive models in general, the Wilkie model in particular and the Clarkson model with periodic and sporadic random shocks. Investment models are also described, from the MGWP on to the Wilkie model again. The authors don't actually use the Wilkie model, but a simpler approach Modelling the value and income of each investment category.

The claims process is looked at over a long timescale into the future. The number of claims is deemed to be subject to: long term trends, irregular cyclical effects (in response to, say, economic cycles), and short-term random variation. The reserving process is modelled, as to given levels of payments and case estimates, say, what level of reserves would be set up. Various types of catastrophe are considered: single object (Jumbo crash, or oil rig sinking), multi-object (windstorm) or multi-channel (reinsurance spiral). A further compounding effect is reinsurance failure.

Premiums are considered and some simple models of price elasticity looked at (for example exponential pricing elasticity).

There are various tedious assumptions about tax, expenses and dividends.

The authors look at insurance cycles around the world. These have typically been of the order of 10-20% of the combined ratio, with a length of 4-8 years+. Some simple mechanisms for generating cycles are considered, for example just lagging premium rates in accordance with solvency.

The paper considers various ways one can model the impact of competitors - in relation to either a well populated, diversified market, or a market with a small number of major competitors.

The authors finally consider briefly Life and Pensions business.

Most of the focus of the paper is related to solvency considerations, usually over time periods measured in decade rather than years.

7.2 APPENDIX 2 : THE GISMO CATASTROPHE MODEL

7.2.1 Introduction

In a well-managed and well-diversified insurance organisation, catastrophe losses tend to be the main contributors of insurance-related stochastic variation. Any model of the general insurance process would therefore be incomplete without considering the impact of catastrophes on cash flow and solvency, the associated implications for the assets, and the impact on reinsurance coverage and costs.

The frequencies and severities of a given type of catastrophe are modelled separately using appropriate distributions. The Poisson and Pareto distributions are used in the GISMO as this combination produces loss patterns that appear similar to those observed in practice. Other distributions, such as the Binomial, Negative Binomial, Lognormal or Gamma, could equally have been used. The gross losses generated by these modelling processes are then netted down in accordance with the reinsurance assumptions and projected in line with an appropriate payment pattern.

The following sections describe the modelling process which generates the number of catastrophe events and the aggregate loss arising from each of these events in the period under consideration. Also discussed are the required user inputs and the assumptions made. The treatment of reinsurance is then considered, and finally, other possible refinements to the Modelling process are discussed.

7.2.2 Frequency Distribution

The outcome of the frequency analysis is more sensitive to the selected parameters than to the selected model, and therefore any of the standard discrete distributions would have been appropriate for estimating the catastrophe frequencies. The Poisson distribution was selected in this case. A single parameter only is required as an input - the estimated frequency of the lowest cost event that could be classified as a catastrophe event. These two parameters (frequency and minimum event cost) are inter-dependant. Increasing the cost of this benchmark event reduces the probability of a loss exceeding this amount. We would expect that an appropriate Poisson parameter to be around 0.5, on an annual basis, for GISMO applications.

7.2.3 The Severity Distribution

As for the frequency distribution, there is more than one possible distribution which could be used to simulate the cost of a catastrophe event. Any of the standard skew distributions could be appropriate in some situations, such as Pareto, Log-Normal or Gamma. The Pareto distribution was selected for the GISMO as it has the fatter tail and produces estimates of risk premiums that are comparable with market premiums across a wide range of values. In other words it has been chosen as it seems to be the distribution that the reinsurance market appears to be basing its premiums on.

Two parameters are required for the Pareto distribution, the scale and the shape. The density function is given below:-

$$f(x) = c \frac{a^c}{x^{c+1}}$$

where c = shape
 a = scale
and $a \leq x < \infty$

The scale is simply the lower base cost of the catastrophe event and is an input required from the user. It will be consistent with the Poisson frequency as discussed in the previous section.

The shape can be an input parameter, or can be derived using two other inputs in addition to the information already held:-

- the expected upper limit cost of a benchmark catastrophe event (eg £200m storm)
- the expected frequency of this benchmark catastrophe (eg one in 40 years, or 0.025)

Clearly, a good understanding of the catastrophe risks and exposure to these risks is required to provide these inputs, but it is felt that, where such understanding exists, the nature of the required inputs should in many instances be fairly readily obtainable. The return periods of benchmark losses of certain risks are often widely discussed, for example the return period of a 1990A UK storm is thought to be in the region of 30 to 40 years.

The Pareto values, that is, the gross cost of each of the events generated by the Poisson process are derived by using the following formula which incorporates a unit random variable:-

$$\text{Pareto value} = \frac{a}{R^{1/c}}$$

where R = a random number between 0 and 1

The distribution and nature of these costs are such that the amounts can be expected to vary within a wide range of values and this may require some truncation of this distribution. This can be done simply by setting the infimum of this range to something other than zero.

7.2.4 Projected Cash Flows

The Poisson/Pareto process described above produces the aggregate cost of any catastrophes for each period under analysis. This aggregate cost must be spread over future periods in accordance with some payment pattern. In line with the non-catastrophe payments in the model, a Craighead curve is used to fit these projected payments. This payment pattern can be expected to be relatively short-tail and will be based on market information, or the insurance organisation's own experience. Analysis of such information has suggested that a Craighead curve with parameters $b = 0.55$ and $c = 1.2$ produces a reasonable fit to gross payments for UK storm losses.

The particular version of the Craighead curve used for these purposes is given by the following formula:-

$$P(t) = 1 - e^{-1/2t^c}$$

where $P(t)$ = the percentage paid at time t .

An inflation index is applied to the projected catastrophe payments in line with the non-catastrophe payments and this is described in Section 3.5.

It is assumed that for a claim occurring in a given period, the first claim payment takes place at the end of the next period.

7.2.5 Reinsurance

An entirely separate model could have been devoted to the reinsurance aspects of the catastrophe claims and, indeed, all other claims. It was felt, however, that the extra complexity of such a reinsurance model was not warranted within GISMO, and that this extra complexity would not be offset by any additional benefit. A simpler and more pragmatic approach was therefore adopted, with no loss of usefulness. The model assumes a simple excess of loss reinsurance programme with four layers, the uppermost and lower limits being defined by the user. A maximum of one reinstatement is assumed for all layers.

The required additional user inputs are limited to just three items of information:-

- lower limit of first layer
- upper limit of highest layer
- current reinsurance pricing conditions (soft, medium or hard)

The model determines upper and lower bounds for each of the four layers, within the limits entered by the user. This is done simply by means of geometric averaging, the end results not being sensitive to the choice of layer limits. The risk premium in each layer is calculated by reference to the Pareto and Poisson assumptions and allowing for up to one reinstatement. This risk premium is converted to a market premium by applying multiples which are dependent on reinsurance market conditions at the outset of the period under analysis. These factors are naturally subjective, but are not felt to be out of line with the actual relativities between reinsurance costs and the calculated risk premium in recent years.

The net reinsurance recoveries (total recovery less the reinsurance premium and any reinstatement premium) are then deducted from the gross catastrophe costs to obtain the net costs.

7.2.6 Possible future refinements:

- Dynamic adjustment of reinsurance market conditions dependent on (simulated) occurrence of catastrophe losses in earlier periods;
- Min/Max Rates On Line;
- Linking exposure to cats to overall portfolio exposure changes;
- Allowing for the partial placement of the reinsurance cover.

(The answer to the question in section 3.6 is Chart A)