The Actuarial Profession
making financial sense of the future

Matching Investment Strategies in General Insurance – Is it Worth It?

34TH ANNUAL GIRO CONVENTION

CELTIC MANOR RESORT, NEWPORT, WALES

Aim of Presentation

- To answer a key question:
  " What are the benefit of adopting matching investment strategies for general insurers? "
- We will build up case studies of an insurer
- Demonstrate different approaches to asset liability matching
- Explain how to quantify the benefits and disadvantages of matching

The Actuarial Profession
making financial sense of the future

Background

The Actuarial Profession
making financial sense of the future

What is Matching?

- Matching is the process of constructing an investment portfolio which replicates the timing and amounts of future liability outgo
- If such a portfolio exists, then the insurance company can be certain that their invested assets will be sufficient to meet their obligations
- The key areas of liability outgo uncertainty to consider are
  - timing of payments
  - nature of payments (inflation linked, random nature)
  - currency of payment
- We will only consider timing and uncertainty of amounts

The Actuarial Profession
making financial sense of the future

Why Use Matching?

- Matching is a concept often associated with life or pensions
- It works especially well when the amount and timing of payments is known in advance
- Protects the insurer's solvency position (so is good for policyholders)
- Reduces the level of capital required to support the existing and new business
- Valuable exercise in situations where limited financial backing is available to support a liability (e.g. pension trustees)

The Actuarial Profession
making financial sense of the future

Assessing Benefit of Matching

" Is there any benefit of adopting matching investment strategies for general insurers? "

What does this mean?

- Depends on the goal and targets of the insurer
- We will look at the problem from the perspective of the company's managers. Their main goals are:
  - Maximise economic profit on insurance business
  - Maintain the solvency of the insurer so that commitments to policyholders can be met in most circumstances
- Can matching asset strategies help in these areas?

The Actuarial Profession
making financial sense of the future

Matching Applications and Aims

We will consider whether matching can be applied in two key areas:

- New Business: Can premiums be invested in such a way that will match the liability generated by the new policy?
- Runoff: Can a matching portfolio be found for the runoff of existing business?

Why Does Matching Work For New Business in Life Insurance?

- Payment timing and amounts of annuity type benefits are known in advance when new policies are written
- Premium for life policies covers the actual payments made to the policyholder under the contract
- Above points not normally true for general insurance policies
  - Claims occur with low probability
  - When claims do occur, the individual premium will be significantly less than the claim size (e.g. liability claims)
- This means that it is not possible to create a matching portfolio on an individual policy level

How Can Matching be Applied to New Business in General Insurance?

- If we cannot match on an individual policy level what alternatives are there?

1. Match on a pool of homogenous policies
   - Group by policies with similar claim frequency and severity distributions and loss payment pattern
   - Calculate expected total claim amount and expected payment pattern
   - Set up matching portfolio to these expected amounts
2. Match runoff of liabilities after claim inception
   - Invest premiums in strategic investment fund until claim occurrence
   - Set up matching portfolio
   - (not really solving the problem of matching to new business)
3. Combination of 1 and 2

Measuring Economic Profit

- How do we measure economic profit?
- Premiums – Losses – Cost of Capital
- Therefore assessment of matching depends on regulatory regime
- We will consider the following risk measures:
  - VaR 99.5% (QIS 3)
  - TVaR 99% (QIS 2)
  - Expected Shortfall (EPD)

calculated over a 1 year time horizon

Capital Requirement Calculation

- Following an economic capital approach we assume that in each future year of simulation risk capital will be held on a 1 year time horizon
- In the case studies we assume:
  - 1 year of new business is written and premium is earned over the first year
  - Claims will run-off over a n year payment pattern
- Capital requirement is therefore:

$$C = \sum_{j=1}^{n} C_j \left(1 + i(j)\right)^{-j}$$

where:
C(j) is the capital requirement in year j, calculated on a 1 year time horizon
i(j) is the interest rate applying for a payment in j years time

Can Matching Increase Economic Profit?

- Matching may be able to reduce capital requirements
- Which should then lead to higher economic profits
- If so we have an appealing case for matching:
  - higher economic profit combined with
  - greater policyholder security
- Whether economic profit can be increased will depend on regulatory regime defining risk measure
- Intuitively expect matching to be more effective under VaR / TVaR than EDP
- Need to be aware of the costs involved

Case Study Background

We will compare economic profit and capital requirements under:

a) Non-matching investment strategy of:
   - 80% fixed interest bonds (5 years time to maturity)
   - 20% cash
   - 0% equities (to allow comparison of risk measures)

b) Matching strategy, with any surplus invested into above strategic portfolio

where economic profit is defined as:

Premium – Losses – ROC x Capital Required

What do we Expect?

- Conventionally it is sometimes argued that …
  - Matching not effective for short tailed P&C business
  - Timing and size of payments for longer tailed P&C business make matching ineffective
- We will try to test some of these statements under a simplified framework …
- … gradually removing some of the simplifying restrictions
- This should give some overall indications of when matching is worth considering in practice

Technical Aspects of Matching

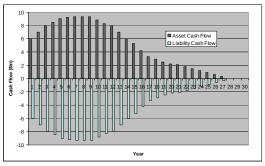
Different Approaches To Matching

There are two main approaches to implementing matching investment strategies

- Pure Matching
- Duration Convexity Matching

Different Approaches to Matching: Pure Matching

- The creation of an asset portfolio which precisely replicates all aspects of the liability outgo
- Normally only possible for fixed liabilities (real or nominal)
- Achieved using a portfolio of fixed / index linked bonds
- Portfolio constructed with aim of holding to redemption …
- … so that all liability payments are met from the proceeds and coupons of the bonds



The Actuarial Profession
making financial sense of the future

How is Pure Matching Implemented?

- We need to create a portfolio of bonds, each of which mature at the same time as each future liability outgo
- Amount of nominal investment in each bond is calculated iteratively…
  … if final liability outgo payment at time t_n is P_n and the bond maturing at time t_n has coupon rate c_n, then invest amount:
  $$NV_n = \frac{P_n}{1+c_n}$$
  into bond n
- Purchase nominal amount:
  $$NV_{n-1} = \frac{P_{n-1} - NV_n \times c_n}{1+c_{n-1}}$$
  into bond n-1
- … continue until reaching first payment

The Actuarial Profession
making financial sense of the future

Limitations of Pure Matching

- Several possible issues affecting implementation:
  - Exact future payments may not be known
  - Timing may be uncertain
  - Matching assets may not exist (either because of long durations or nature of liabilities)
- Costs of implementing matching portfolio
  - Protects from downside risk, but …
  - … removes possibility of benefiting from increasing market values
    (since all assets will be held to redemption to meet liabilities)
  - Analytical costs of implementing strategy
  - Rebalancing costs as new business is written
  - Requires large number of different bonds to be held

Different Approaches to Matching: Duration / Convexity Matching (1)

- In practice it is often not practical to set-up matching portfolios
- Why?
  - Liability profile of large book of business too varied
  - Would require bonds of all durations to be purchased
  - Availability of longer dated bonds limited
- Alternative is classical approach originating from Redington's theory of immunisation
- Protects the insurer from small movements in interest rates causing value of assets and liabilities to move apart
- Requires less complicated asset portfolio that pure matching
- Can outperform pure matching when uncertainty surrounds payment times and amounts

Different Approaches to Matching: Duration / Convexity Matching (2)

- This technique works by developing a dynamic asset portfolio with equal sensitivity to interest rates as the liabilities
- Can be applied to nominal or real liabilities (using ILG's for real liabilities)
- Duration represents the sensitivity of a payment stream to interest rate movements
- Convexity measures the sensitivity of duration to interest rate changes
- Key idea is to match present value, duration and convexity of the assets and liabilities
- Will make surplus process very stable to changes in interest rates
- Analogous to Delta / Gamma hedging in the derivatives market

Different Approaches to Matching: Duration / Convexity Matching (3)

- Duration refers to "Macaulay duration"
  - Let asset or liability have cash-flows C_j at times t_j (j = 1, …, n) and i be the gross redemption yield of the payment stream

$$\text{Duration} = \frac{\sum_{j=1}^{n} t_j C_j (1+i)^{-t_j}}{\sum_{j=1}^{n} C_j (1+i)^{-t_j}}$$

- Convexity is calculated as:

$$\text{Convexity} = \frac{\sum_{j=1}^{n} t_j (t_j + 1) C_j (1+i)^{-(t_j+2)}}{\sum_{j=1}^{n} C_j (1+i)^{-t_j}}$$

The Actuarial Profession
making financial sense of the future

Different Approaches to Matching: Duration / Convexity Matching (4)

- What is "i" in duration / convexity formulae?
- It is the gross redemption yield of the assets / liabilities …
  … that is, the solution of:

$$\sum_{j=1}^{n} C_j (1+i)^{-t_j} = \sum_{j=1}^{n} C_j \left(1 + i(t_j)\right)^{-t_j}$$

where:
  $i(t_j)$ is the spot rate on the yield curve at duration t_j
  C_j is the cash flow of the asset / liability at time t_j

The Actuarial Profession
making financial sense of the future

How is Duration / Convexity Matching Implemented?

1. Select 3 possible bonds for investment (different terms)
2. Calculate present value, duration and convexity of liability (MV_L, Dur_L, Cov_L)
3. Calculate market value of £1 nominal of each bond and the duration / convexity (MV_i, Dur_i, Cov_i for i = 1,2,3)
4. Solve the following system of equations:

$$\begin{aligned} MV_L &= \alpha MV_1 + \beta MV_2 + \gamma MV_3 \\ MV_L \times Dur_L &= \alpha Dur_1 + \beta Dur_2 + \gamma Dur_3 \\ MV_L \times Cov_L &= \alpha Cov_1 + \beta Cov_2 + \gamma Cov_3 \end{aligned}$$

The Actuarial Profession
making financial sense of the future

Limitations of Duration / Convexity Matching

- Often difficult to determine the matching portfolio
  - A solution may not exist for the chosen set of bonds (without allowing short selling)
  - This can be overcome by making the solution space larger by adding more bonds (of different maturities)
  - Requires complicated linear programming techniques
- Will not be immunised against non–parallel shifts in the yield curve
- Requires frequent rebalancing to protect from movements in interest rate
  - N.B. only first two derivatives of yield are matched, so after large interest rate movements, rebalancing is required

Building an Asset Liability Model

The Actuarial Profession
making financial sense of the future

Building Blocks for an Asset Liability Model

- In order to analyse effectiveness of matching strategies, we first need a framework for to build an asset liability model

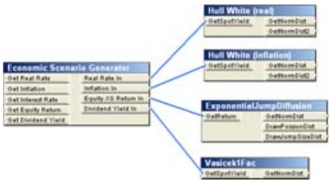
  We require several key components to the DFA model:

  1. Economic scenario generator
  2. Monte Carlo simulation engine
  3. Matching asset portfolio generation

The Actuarial Profession
making financial sense of the future

Economic Scenario Generator

- We implemented an arbitrage free model of the economy based on “A Stochastic Asset Model & Calibration For Long-Term Financial Planning Purposes” by Hibbert, Mowbray & Turnbull (2001)
- This provides a framework for simulating the evolution of key economic variables
- Calibrated to current market conditions in the UK



Monte Carlo Simulation and Asset Matching

- Case studies modelled for 50,000 simulations using Latin Hypercube to accelerate convergence
- Specialised asset allocation components used for setting up the initial matching portfolio and rebalancing the portfolio on annual intervals
  - This should mean that pure matching will outperform duration convexity matching…
  - …since after large movements in interest rates, the portfolio will no longer match the liabilities as closely
  - Whereas pure matching does not require rebalancing (assets held to redemption)

Case Study 1

Matching New Business (Fixed 5 Year Payment Pattern)

Case Study 1: New Business (Fixed 5 Year Payment Pattern)

- We will begin with a simple case study
- Consider insurer writing new business with the following characteristics per policy:
  - Claim Frequency ~ Poisson(0.01)
  - Claim Severity ~ Log Normal (mean = 100, s.d. = 10)
  - Loss payment pattern:

Time	Payment Proportion	Standard Deviation
1	40%	0%
2	30%	0%
3	10%	0%
4	10%	0%
5	10%	0%

- Assume we write 10,000 new policies
- Premium is 1.00 per policy (100% loss ratio)

The Actuarial Profession
making financial sense of the future

Case Study 1: New Business (Fixed 5 Year Payment Pattern)

- We will apply the first matching strategy described for new business: matching pooled liability
- That is we will:
  - Calculate expected total claim amount and expected payment pattern
  - Set up matching portfolio to these expected amounts
- Expected total payout is:

Time	1	2	3	4	5
Payment	4,000	3,000	1,000	1,000	1,000

- Idea is that by investing all premiums matched to this cash-flow schedule then should a claim occur, we will already be approximately matched

The Actuarial Profession
making financial sense of the future

Case Study 1: New Business (Fixed 5 Year Payment Pattern)

- For the non-matching asset portfolio we assume following strategic investment strategy:

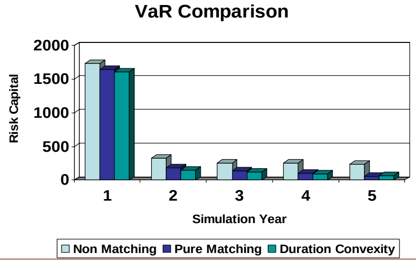
Asset	Equity	Cash	Bonds (5 year)
Allocation	0%	20%	80%

- Under both exact matching and duration / convexity matching strategies, we allocate remaining premiums into the strategic fund
- Remember, we are matching to the expected total loss payments
- Reason for 0% equities: we want to compare matching to non-matched investments (equities will generate higher capital requirements due to larger tails)
- We will run the simulation for 5 years

The Actuarial Profession
making financial sense of the future

Case Study 1: New Business
VaR Under Each Investment Strategy

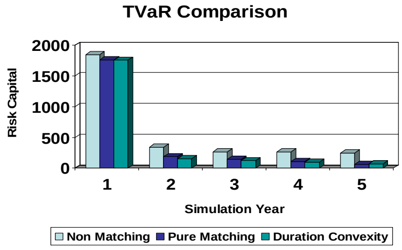
- Significant reduction in risk capital under matching assets for VaR using 1 year time horizon:



The Actuarial Profession
making financial sense of the future

Case Study 1: New Business
TVaR Under Each Investment Strategy

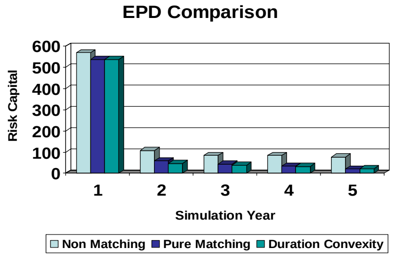
- Significant reduction in risk capital under matching assets for TVaR using 1 year time horizon:



The Actuarial Profession
making financial sense of the future

Case Study 1: New Business
EPD Under Each Investment Strategy

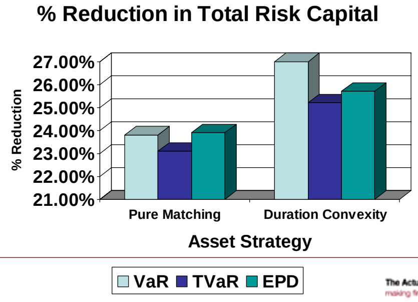
- Less significant reduction in risk capital under matching assets for EPD using 1 year time horizon:



The Actuarial Profession
making financial sense of the future

Case Study 1: New Business
Overall Effectiveness of Matching

- Matching performs best for TVaR and VaR



Case Study 1: New Business
Economic Profit Analysis

- Recall the definition of economic profit:
  Premium – Losses – ROC x Capital Required
- We assume ROC = 10% and calculate the capital required under VaR, TVaR and EPD

		Non Matching	Exact Matching	Dur / Conv Matching
Premium		9,930	9,930	9,930
Initial Reserve		8,937	8,937	8,937
Capital Requirement	VaR	2,722	2,073	1,986
	TVaR	2,866	2,204	2,144
	EPD	888	676	660
Economic Profit	VaR	**721**	**785**	**794**
	TVaR	**706**	**773**	**778**
	EPD	**904**	**925**	**927**

Case Study 1: New Business
(Fixed 5 Year Payment Pattern)

- From the perspective of reducing capital requirements, matching has performed very well: up to 25% reduction in capital
  - This will free up capital for writing new business
- Impact on economic profit is reasonable: around a 8% increase
- Duration convexity matching has performed slightly better than pure matching: likely to be due to pure matching performing less well when amounts and timing differ from expected
- Note that assumptions were not entirely realistic – e.g. fixed payment pattern, timing of payments, no parameter uncertainty
- Next case study will extend this model to allow for stochastically varying payment pattern in terms of amounts and timing

Case Study 2

Matching New Business

(Stochastic 5 Year Payment Pattern)

Case Study 2: New Business (Stochastic 5 Year Payment Pattern)

- We continue the example from case study 1, except now the payment pattern will introduce variability to the payment proportions
- New payment pattern assumption is:

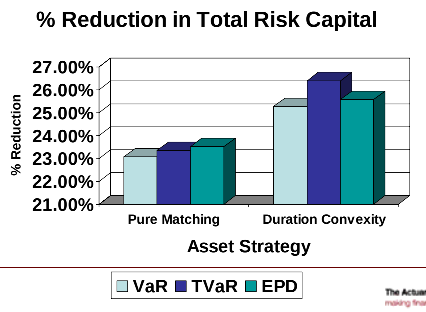
Time	Payment Proportion	Standard Deviation
1	40%	13.33%
2	30%	10%
3	10%	3.33%
4	10%	3.33%
5	10%	3.33%

where each payment proportion follows a Normal distribution with specified mean and standard deviation

- Assume timing of each payment is randomly scaled by Beta distribution with mean 1 and standard deviation 0.1

Case Study 2: New Business Overall Effectiveness of Matching

- Around 25% reduction in capital requirement under matching



Case Study 2: New Business Results

- Very little difference to the fixed payment pattern
- Matching is still very effective on reducing capital requirement and increasing economic profit

		Non Matching	Exact Matching	Dur / Conv Matching
Premium		9,930	9,930	9,930
Initial Reserve		8,937	8,937	8,937
Capital Requirement	VaR	2,731	2,093	2,010
	TVaR	2,883	2,218	2,155
	EPD	891	681	663
Economic Profit	VaR	**720**	**784**	**792**
	TVaR	**705**	**771**	**777**
	EPD	**904**	**925**	**927**

Case Study 2: New Business (Stochastic 5 Year Payment Pattern)

- We have shown that for short tailed business, with reasonably well behaved loss distribution matching can be very beneficial for reducing risk capital
- Important to note we are looking at NPV of capital required in each year of simulation
  - Benefit of matching will be much lower when looking only at the current year
- We will now consider longer tailed business
- This can be intuitively expected to be better suited to matching, due to life insurance analogies

Case Study 3
Matching New Business
(Fixed 15 Year Payment Pattern)

Case Study 3: New Business (Fixed 15 Year Payment Pattern)

- We will begin with a simple case study
- Consider insurer writing new business with the following characteristics per policy:
  - Claim Frequency ~ Poisson(0.01)
  - Claim Severity ~ Log Normal (mean = 100, s.d. = 10)
  - Loss payment pattern:
- Assume we write 10,000 new policies
- Premium is 0.70 per policy (140% loss ratio)

Time	Payment Prop.	Standard Dev.
1	6.67%	2.22%
2	6.67%	2.22%
3	6.67%	2.22%
4	6.67%	2.22%
5	6.67%	2.22%
6	6.67%	2.22%
7	6.67%	2.22%
8	6.67%	2.22%
9	6.67%	2.22%
10	6.67%	2.22%
11	6.67%	2.22%
12	6.67%	2.22%
13	6.67%	2.22%
14	6.67%	2.22%
15	6.67%	2.22%

Case Study 3: New Business Results

- We again apply the matching pooled liability investment strategy
- Matching has performed very well in both reducing capital and increasing economic profit

		Non Matching	Exact Matching	Dur / Conv Matching
Premium		7,093	7,093	7,093
Initial Reserve		6,384	6,384	6,384
Capital Requirement	VaR	3552	2060	2409
	TVaR	3593	2528	2947
	EPD	1085	733	857
Economic Profit	VaR	**394**	**503**	**468**
	TVaR	**340**	**456**	**414**
	EPD	**600**	**636**	**624**

The Actuarial Profession
making financial sense of the future

Case Study 3: New Business Conclusion

- We see a greater reduction in capital requirements with longer tailed business (up to around 40% under exact matching)
- Duration / convexity matching performed less well (due to lower number of bonds, cumulative effect of hedging error)
- Economic profit increases significantly by around 25%
- Overall conclusion: matching very useful for reducing risk capital and increasing economic profit
- Note limitations of case study: deterministic payment pattern, no parameter uncertainty, level payment assumption
- We now will move onto run-off business, which is where we can expect even greater benefits from matching.

The Actuarial Profession
making financial sense of the future

Case Study 4

Matching Runoff

(Variable 15 Year Payment Pattern)

The Actuarial Profession
making financial sense of the future

Case Study 4: Runoff Business (Variable 15 Year Payment Pattern)

- We now look at how matching can be applied to the run-off of existing business
- This is particularly relevant to Loss Portfolio Transfers (LPT)
- Can consider assessing economic profit of accepting company under LPT
- Assumptions:
  - Premium = 79.80
  - Initial Reserve = 63.84
  - Assume timing of each payment is randomly scaled by Beta distribution with mean 1 and standard deviation 0.15
  - No unexpired risk
  - No reserve uncertainty

Time	Payment Prop.	Standard Dev.
1	6.67%	2.22%
2	6.67%	2.22%
3	6.67%	2.22%
4	6.67%	2.22%
5	6.67%	2.22%
6	6.67%	2.22%
7	6.67%	2.22%
8	6.67%	2.22%
9	6.67%	2.22%
10	6.67%	2.22%
11	6.67%	2.22%
12	6.67%	2.22%
13	6.67%	2.22%
14	6.67%	2.22%
15	6.67%	2.22%

The Actuarial Profession
making financial sense of the future

Case Study 4: Runoff Business Results

- We set up matching investment strategy to expected payments under both exact matching and duration / convexity matching

		Non Matching	Exact Matching	Dur / Conv Matching
Premium		79.80	79.80	79.80
Initial Reserve		63.84	63.84	63.84
Capital Requirement	VaR	61.28	0.51	36.50
	TVaR	63.98	0.78	38.50
	EPD	37.54	0	0
Economic Profit	VaR	**9.83**	**15.91**	**12.31**
	TVaR	**9.56**	**15.88**	**12.11**
	EPD	**12.21**	**15.96**	**15.96**

The Actuarial Profession
making financial sense of the future

Case Study 4: Runoff Business (Variable 15 Year Payment Pattern)

- Exact matching has performed extremely well
  - Around 60% increase in profits for VaR / TVaR
  - Removed most of capital requirement under all risk measures
- Duration / convexity matching also showed strong improvements over non-matching strategy
  (difference due to cumulative hedging error)
- Variability in timing and amounts had no significant impact on matching ability
  - Payment times scaling factor has 95% confidence interval [0.7, 1.3]
  - Payment proportions have 33% standard deviation
- Case study excluded reserve uncertainty. This will reduce the effectiveness of matching.

Conclusion

- Matching investment strategies can have valuable applications in general insurance and should not be ignored
- Capital requirements for new business in reasonably stable classes can be significantly reduced
- Economic profit has been shown to increase considerably
- Matching extremely successful for run-off business due to expected payments being known with greater certainty
- Many issues discussed here are worth further investigation with more realistic assumptions

Questions?