

MODELLING OF LONGEVITY TRENDS

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AGENDA

Historical overview of mortality trends

Modelling issues

Morbidity trends

Subjective responses

Implications for pension programs

Concluding comments

FIRST, THE GOOD NEWS

- Life expectancy at birth is increasing – we are all living longer.
- Life expectancy at birth increased by about 25 years in 10,000 years up to 1800.
- Life expectancy at birth increased by about 25 years in 200 years from 1800-2000.

MORE GOOD NEWS

MALES AGE 65

MORTALITY TABLE

LIFE EXPECTANCY (YEARS)

Ulpianus (230)

5.3

Breslau-Halley (1693)

9.6

Karlsruhe (1864)

10.3

England and Wales (2016)

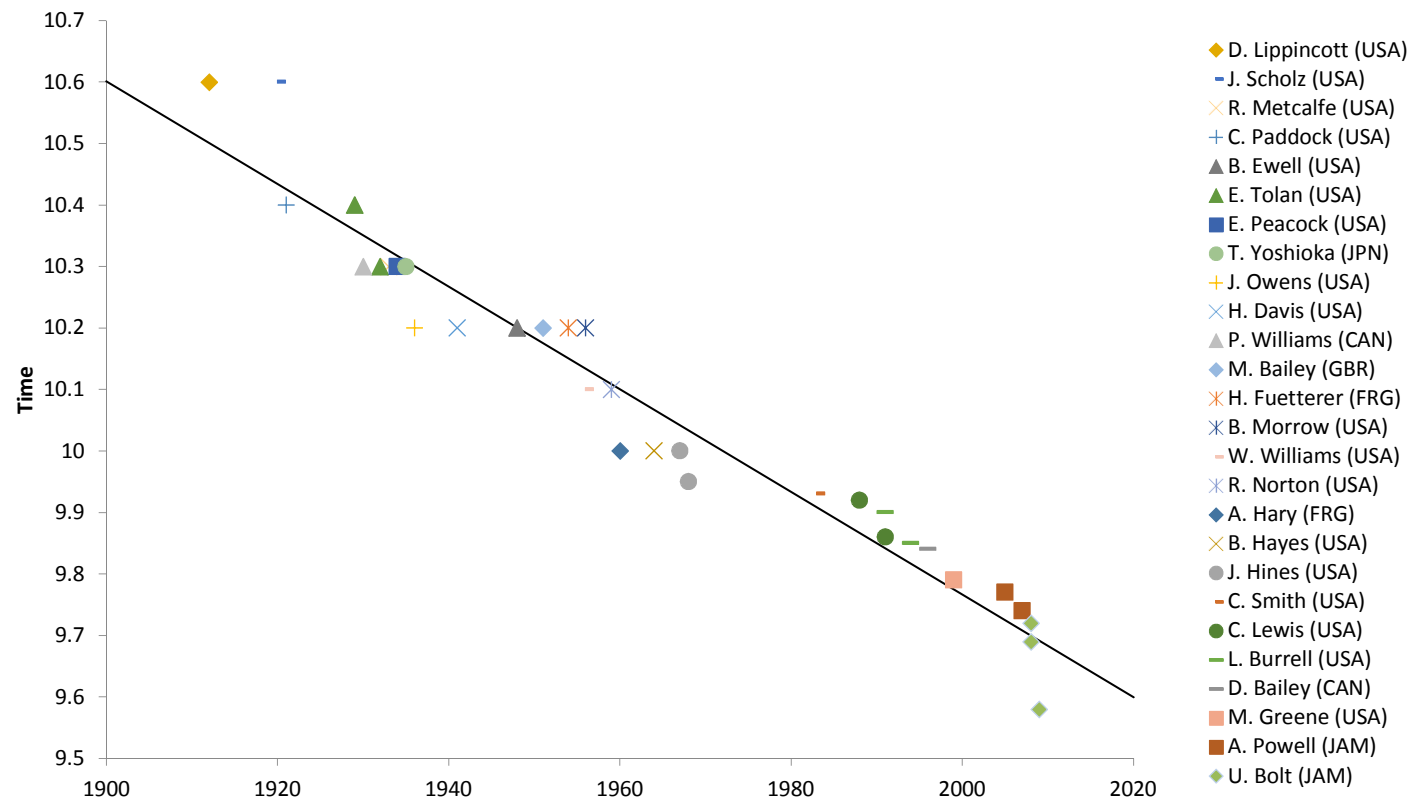
18.7

NOW, THE BAD NEWS

Life expectancy is increasing – we are all living longer.

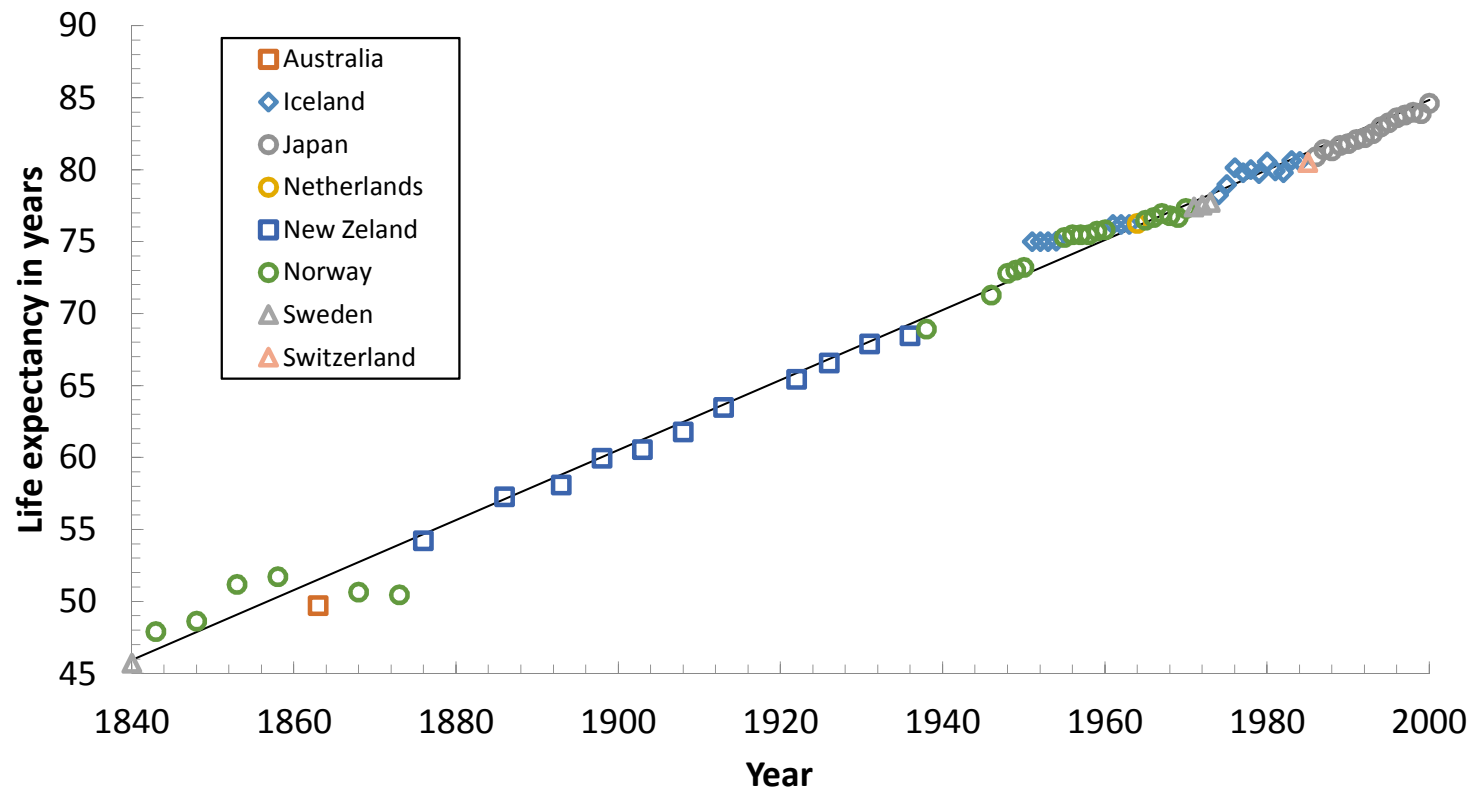
This is a systematic effect leading to longevity risk. See later for implications.

Male 100 Metres World Record



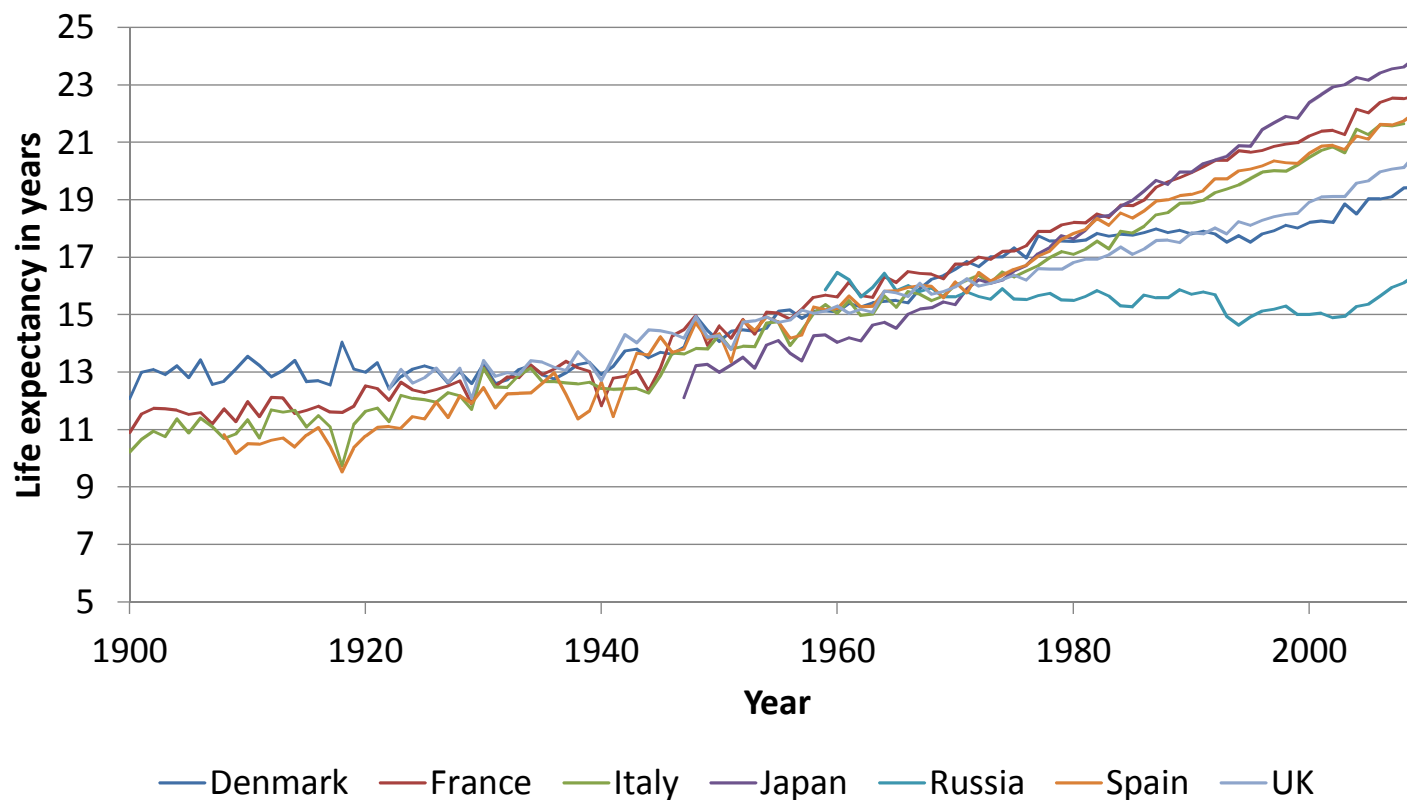
Source: IAAF

Record (“best practice”) female life expectancy



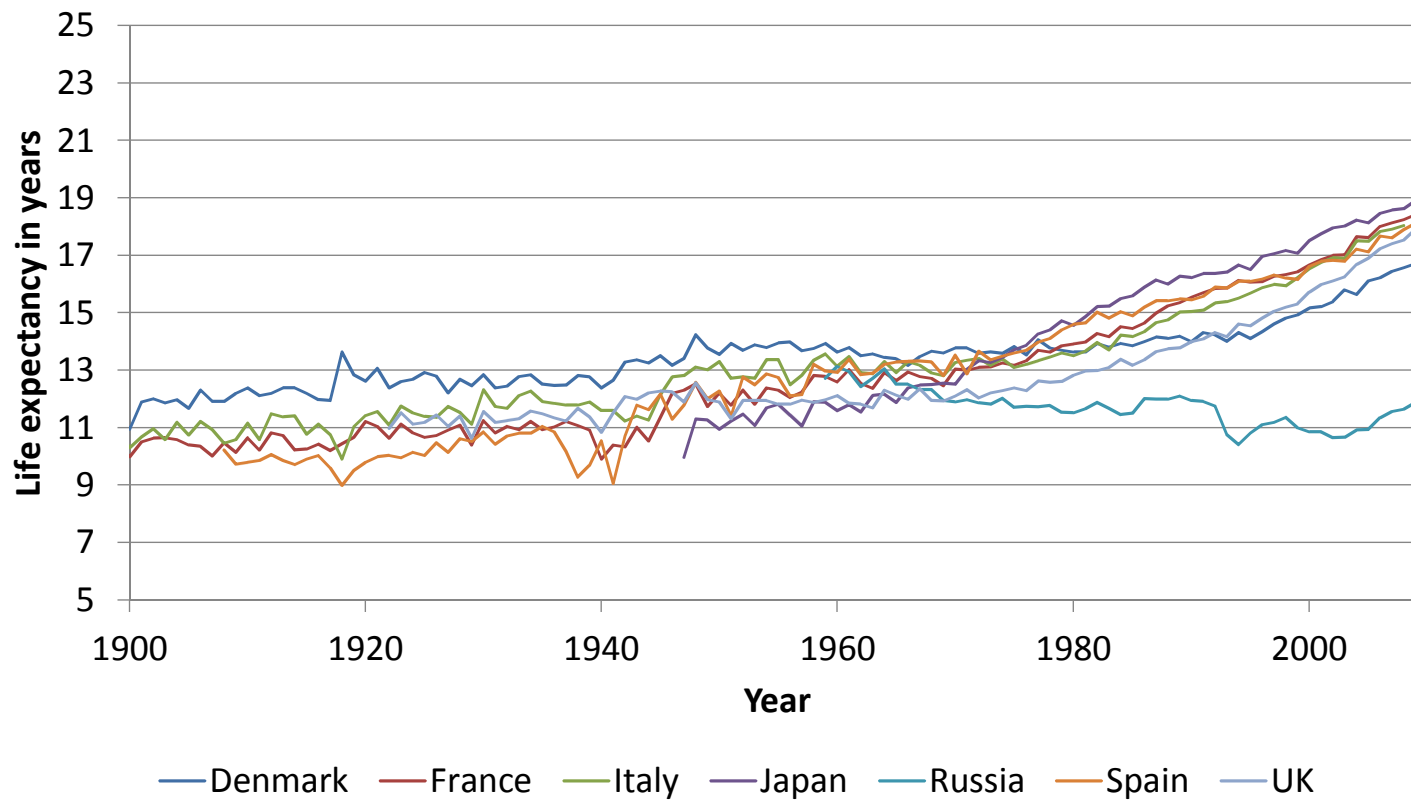
Source: Supplementary material Oeppen and Vaupel (2002)

Female period life expectancy at age 65



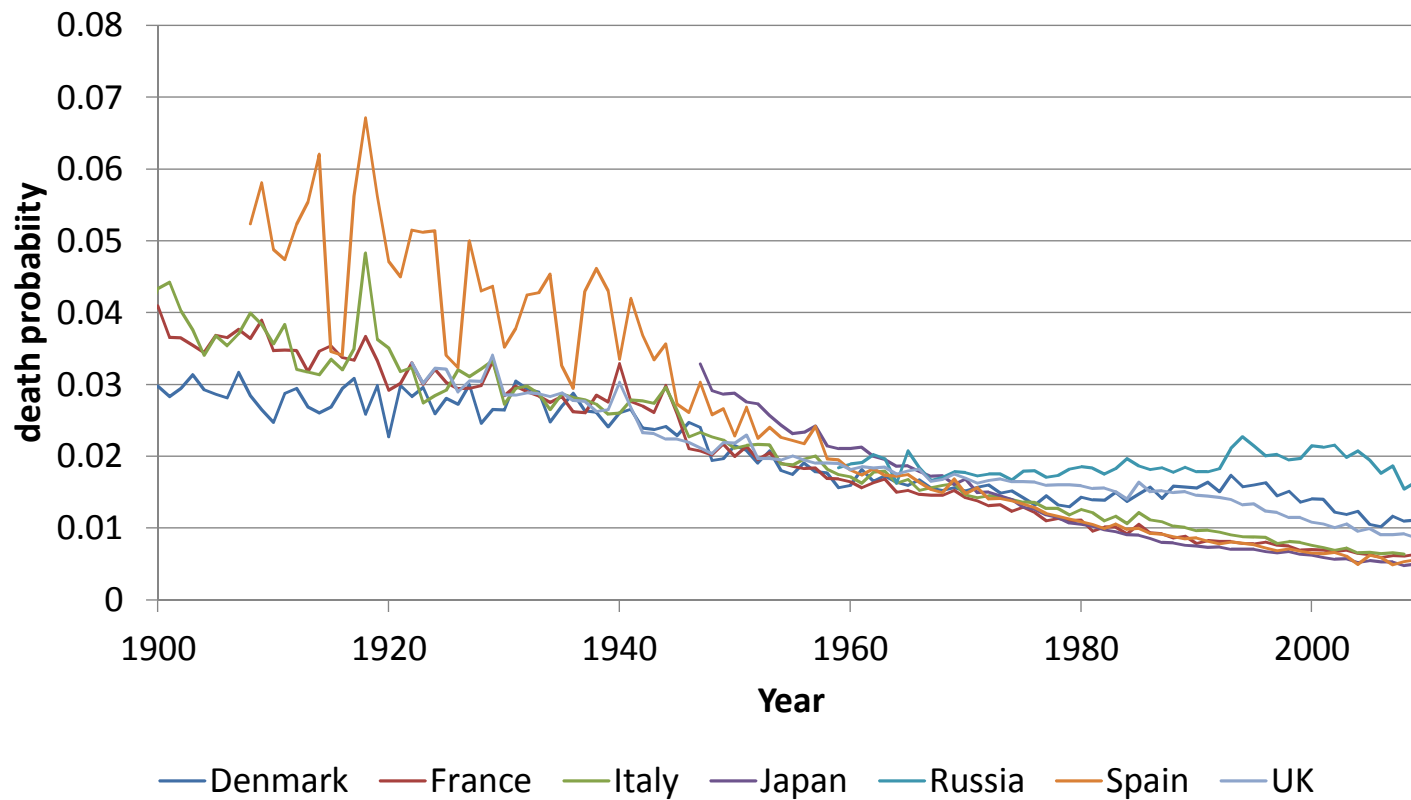
Source: Human Mortality Database

Male period life expectancy at age 65



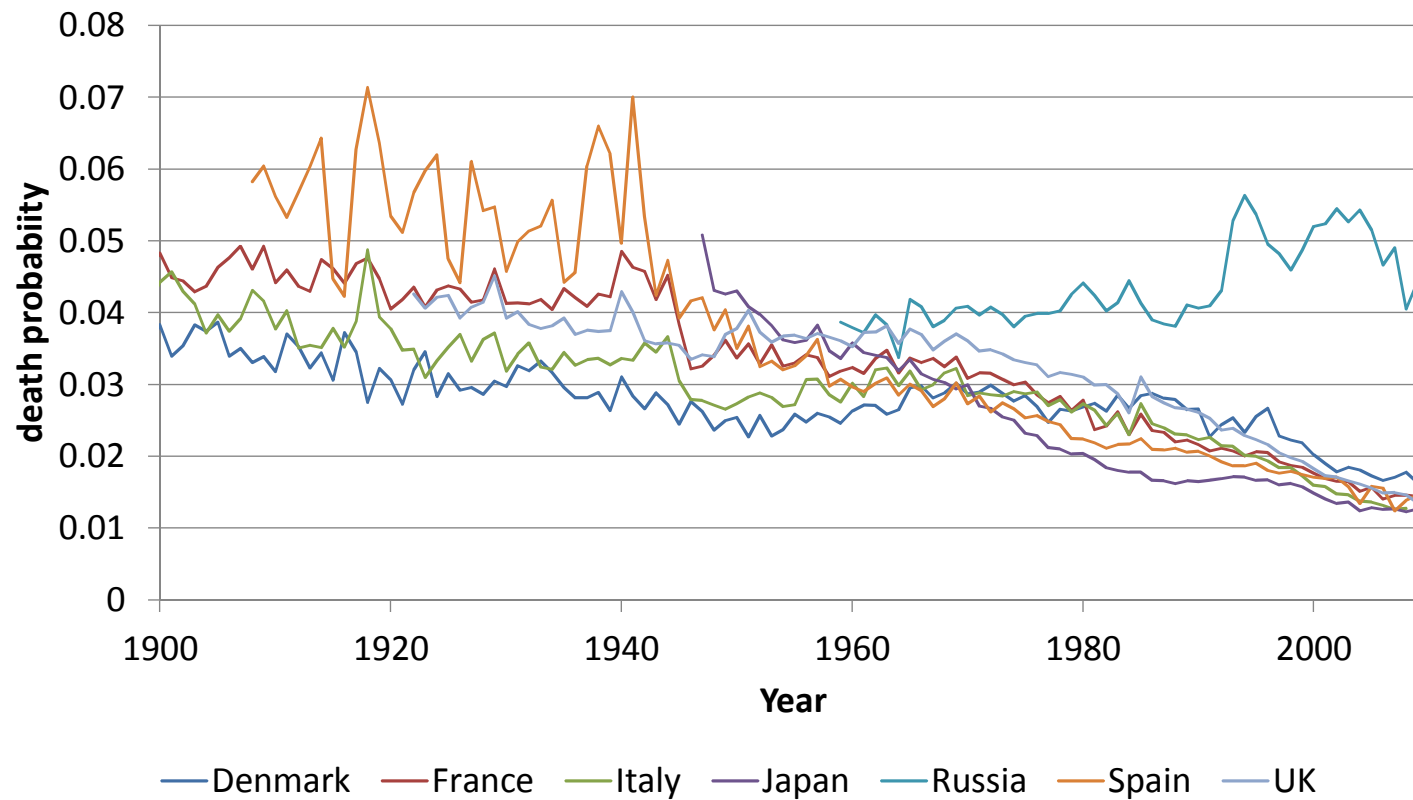
Source: Human Mortality Database

Female death probability at age 65



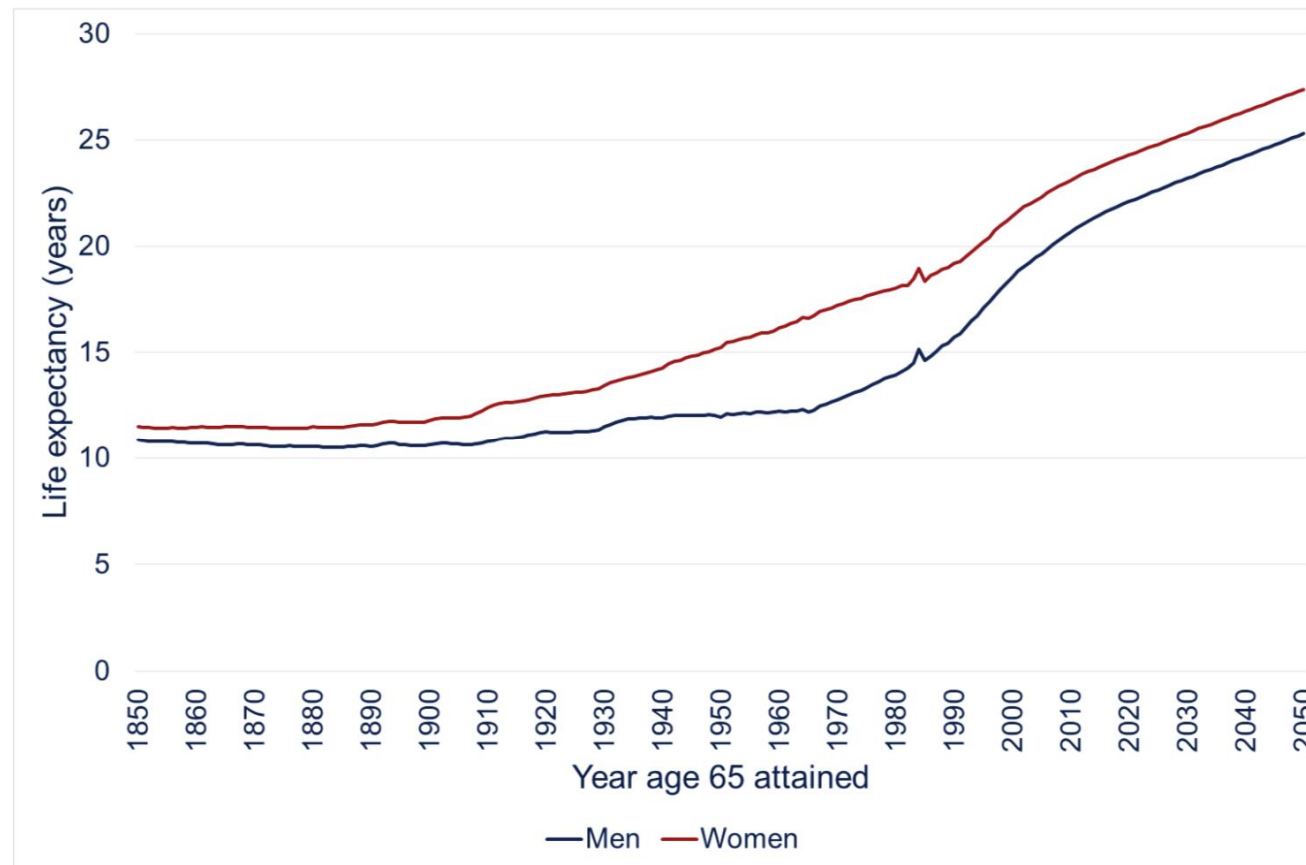
Source: Human Mortality Database

Male death probability at age 65



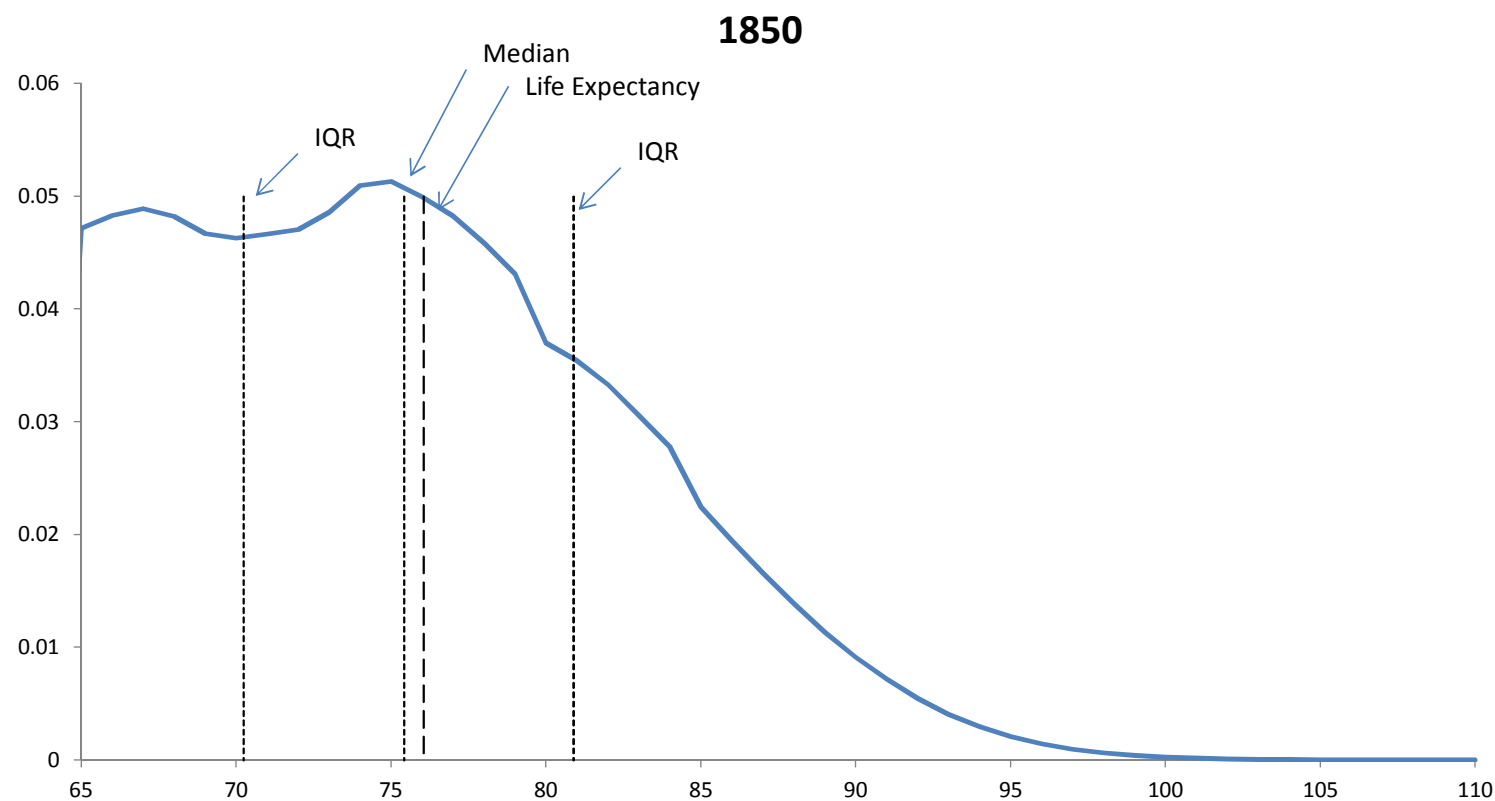
Source: Human Mortality Database

PROJECTED COHORT LIFE EXPECTANCY AT AGE 65, 1950 TO 2050



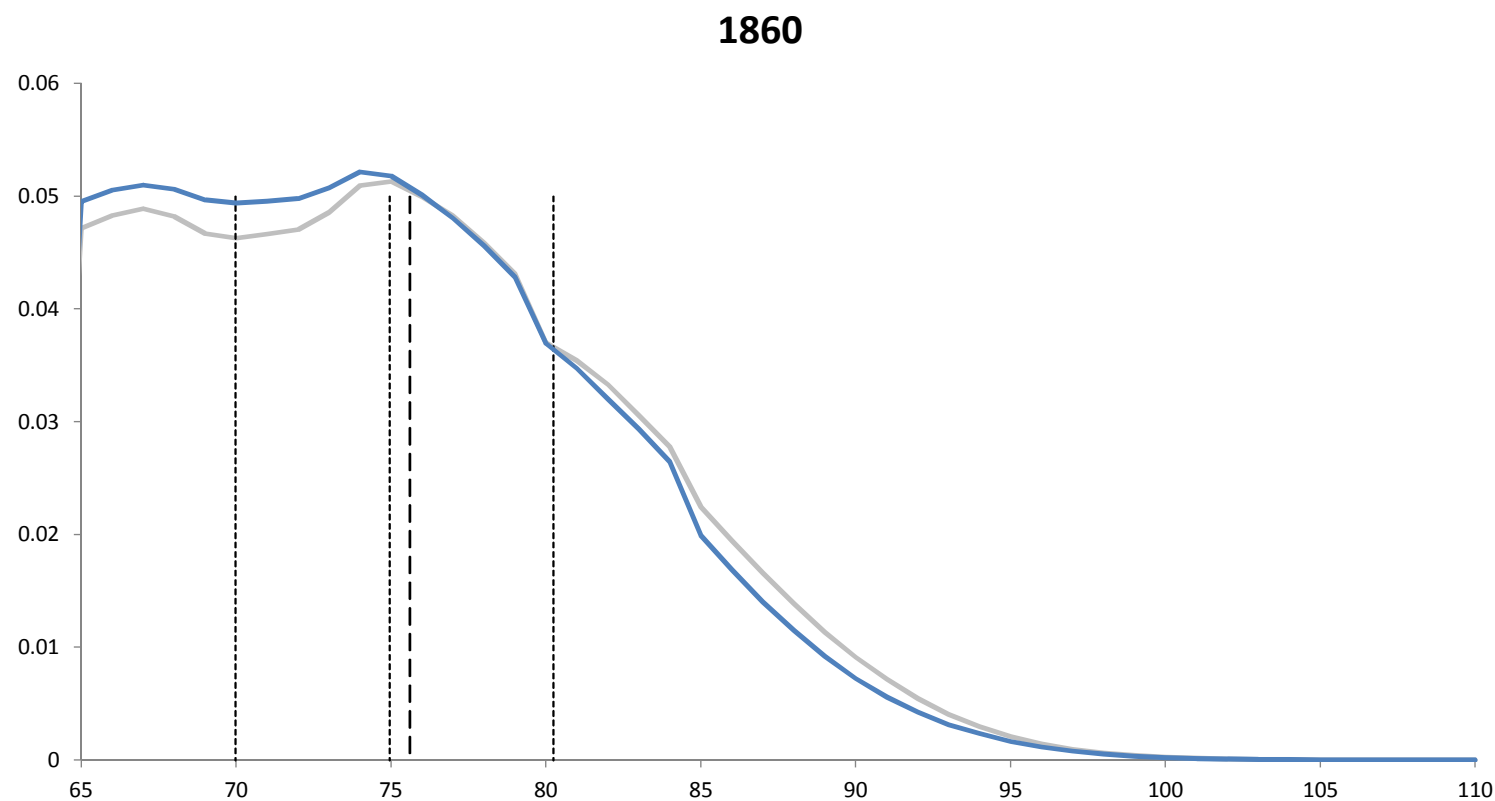
Source: ONS. ONS have noted that the 'blip' in the trend line in 1984 relates to the birth cohorts of 1918 to 1920, where the births were not evenly distributed throughout the year.

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



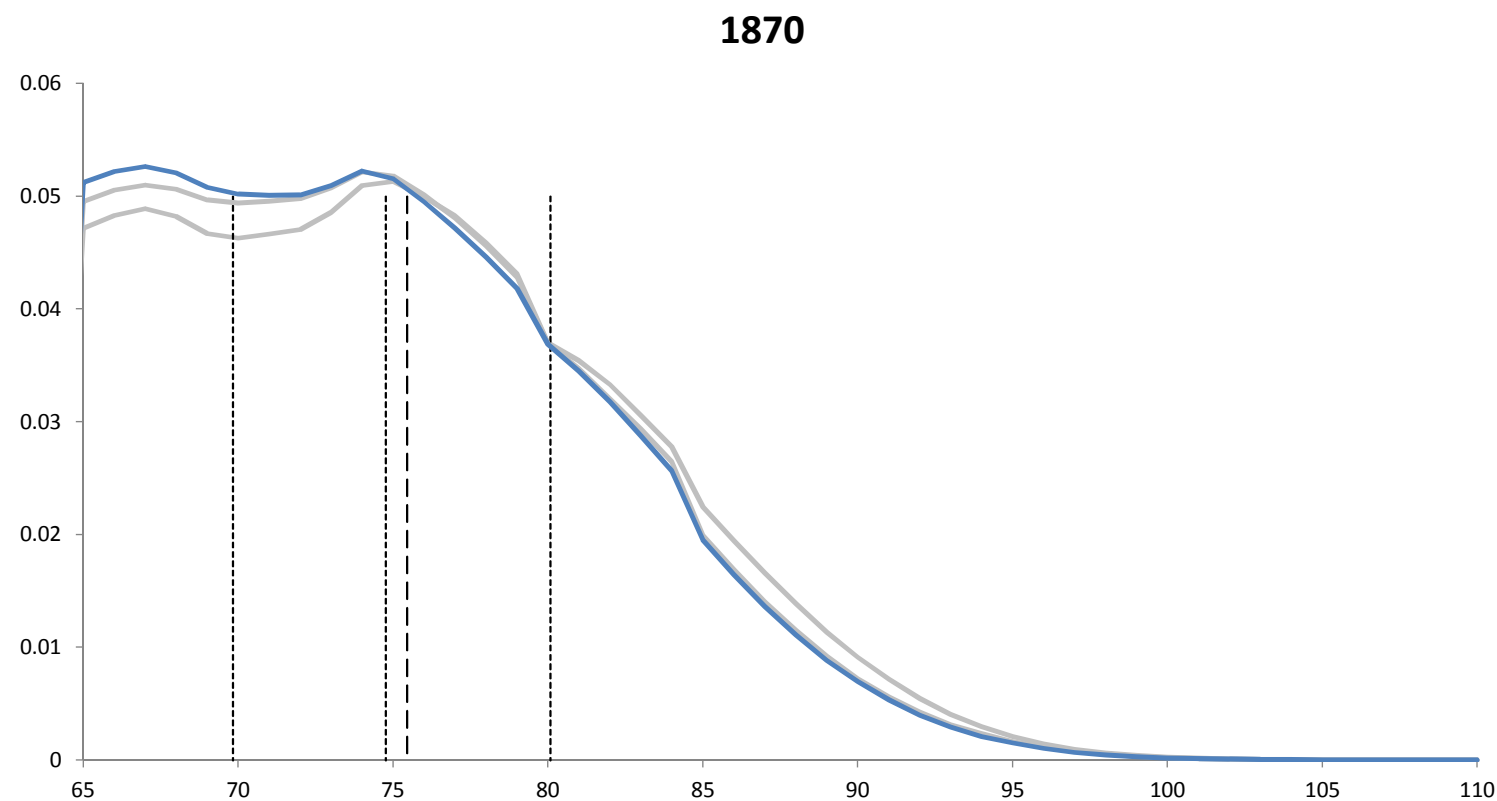
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



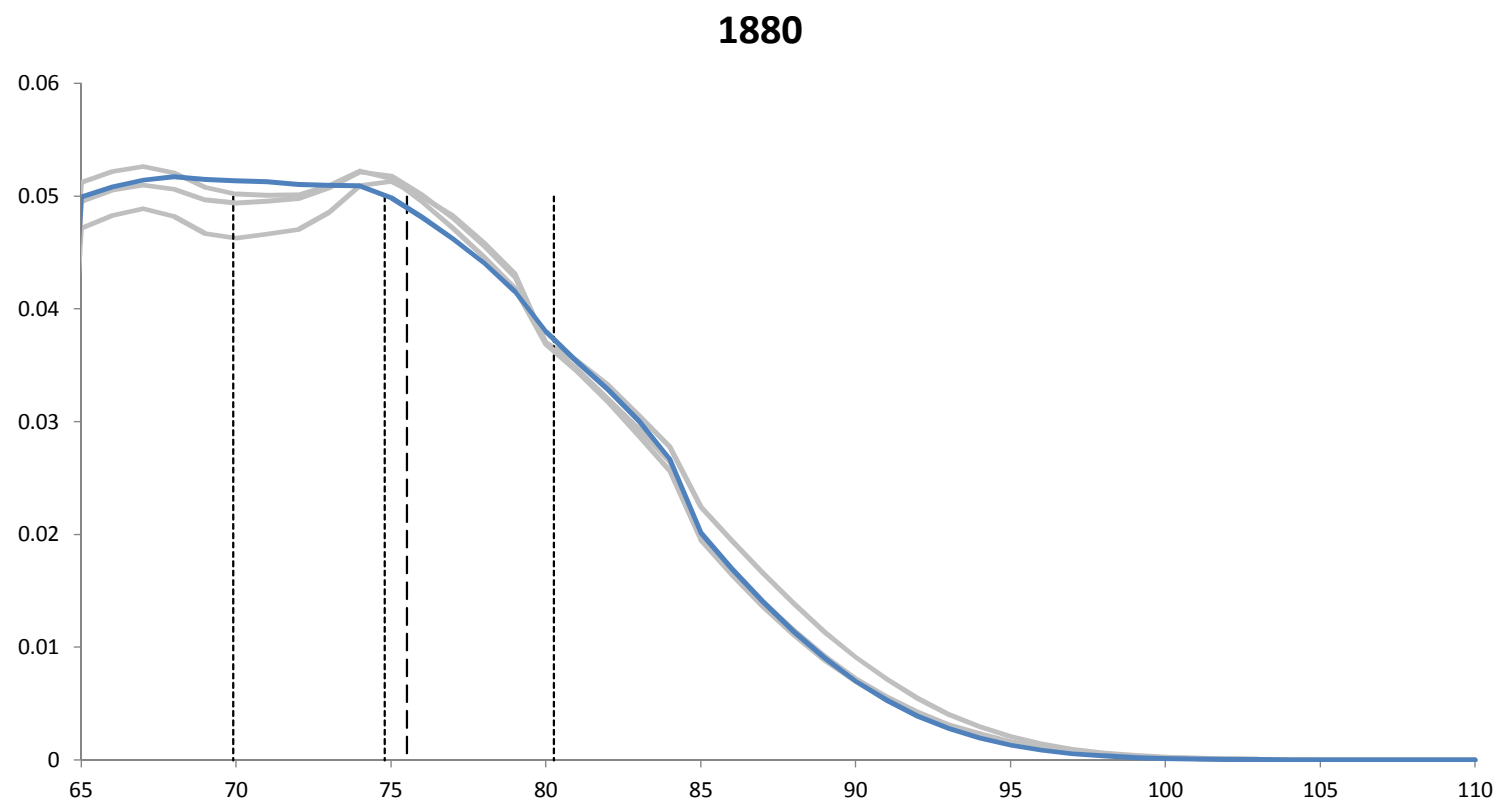
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



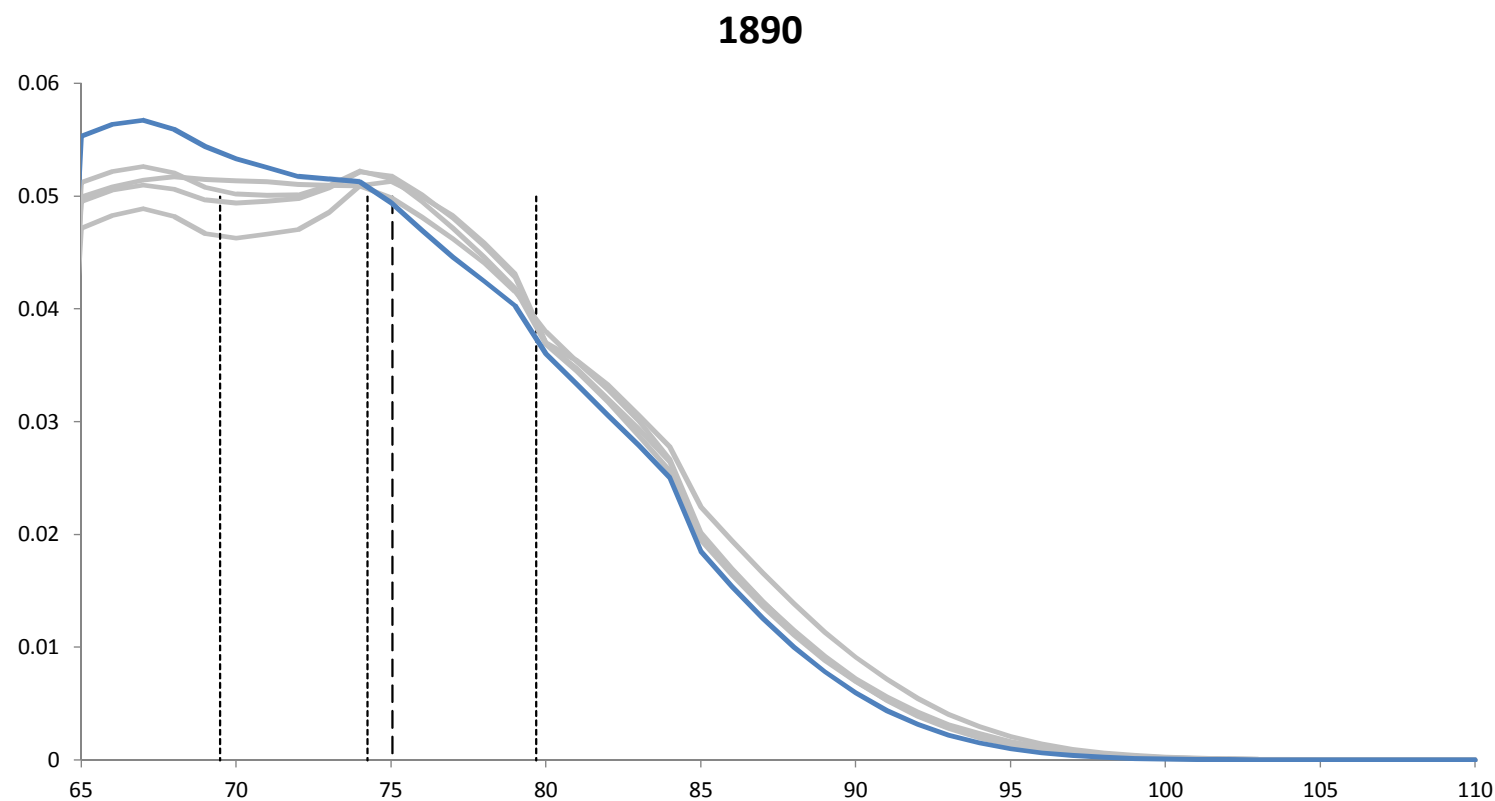
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



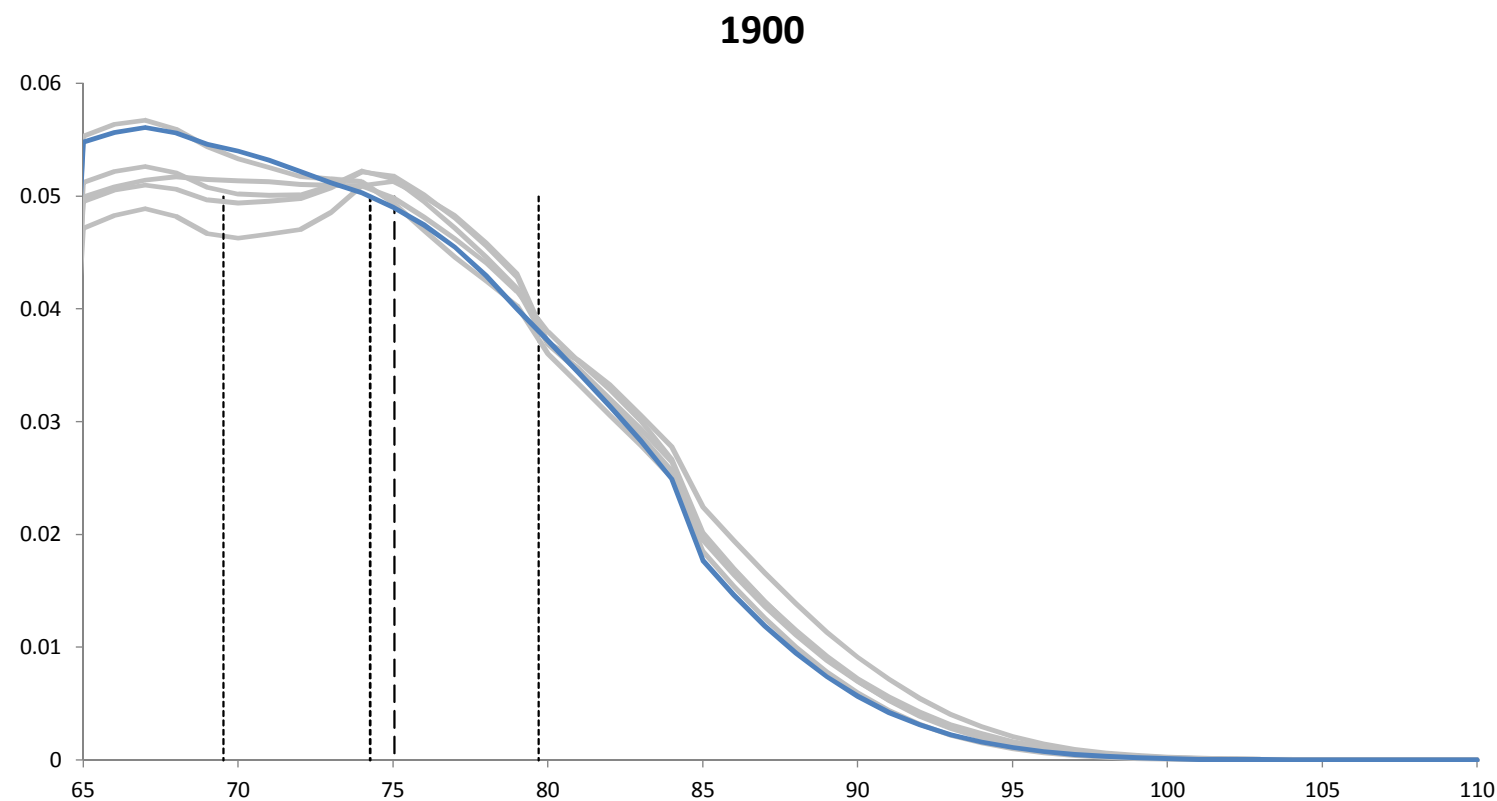
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



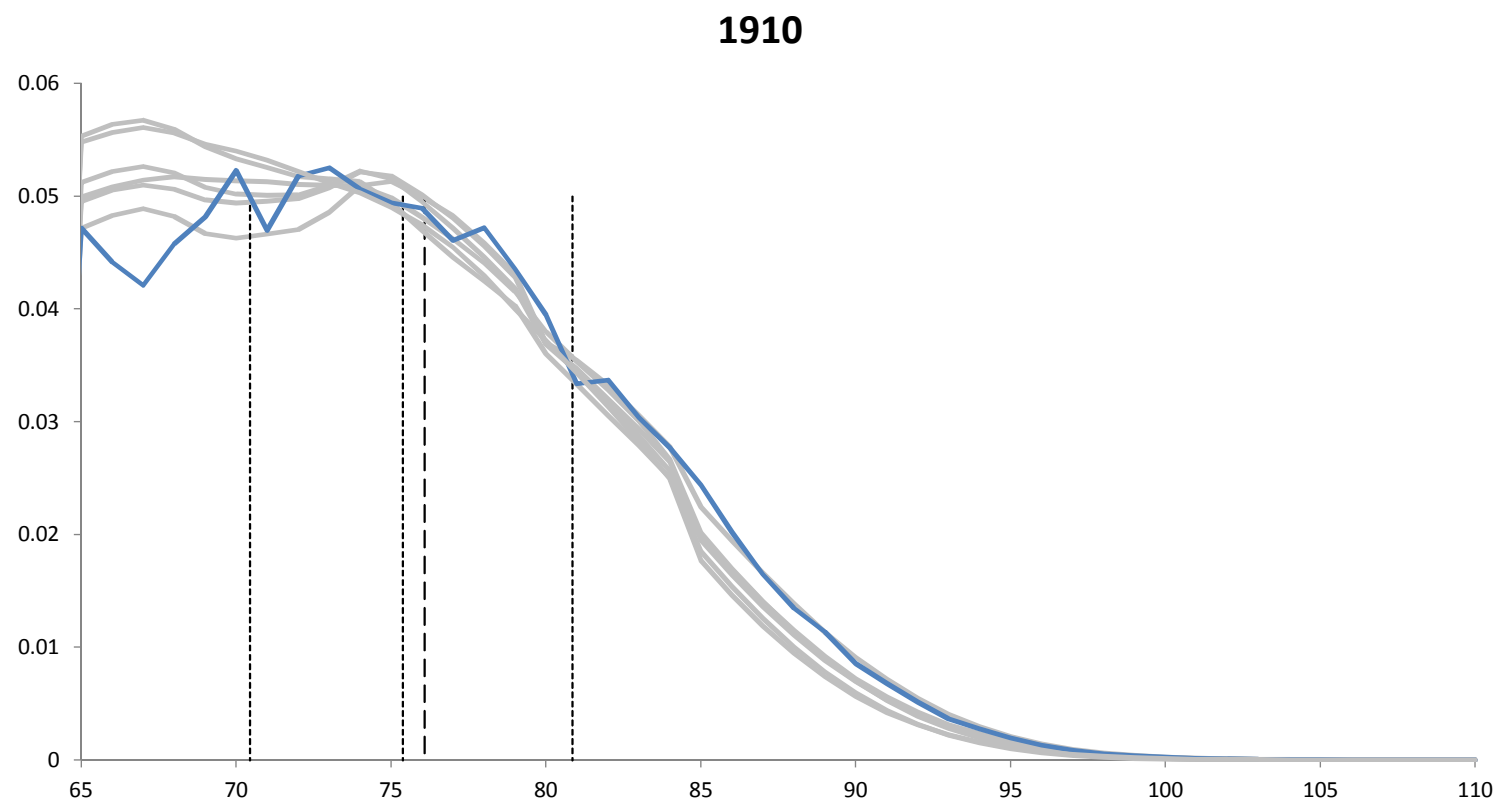
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Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



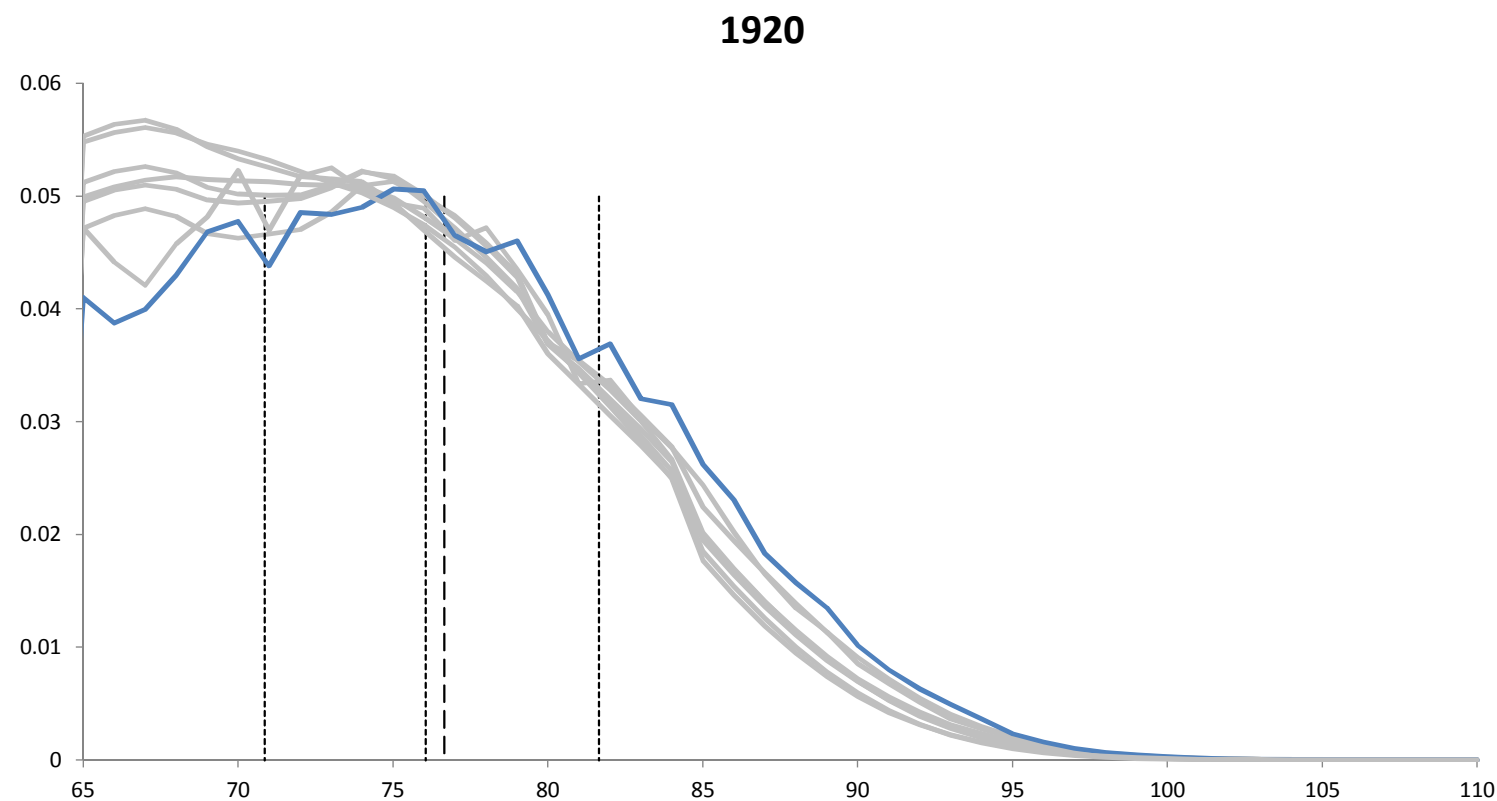
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



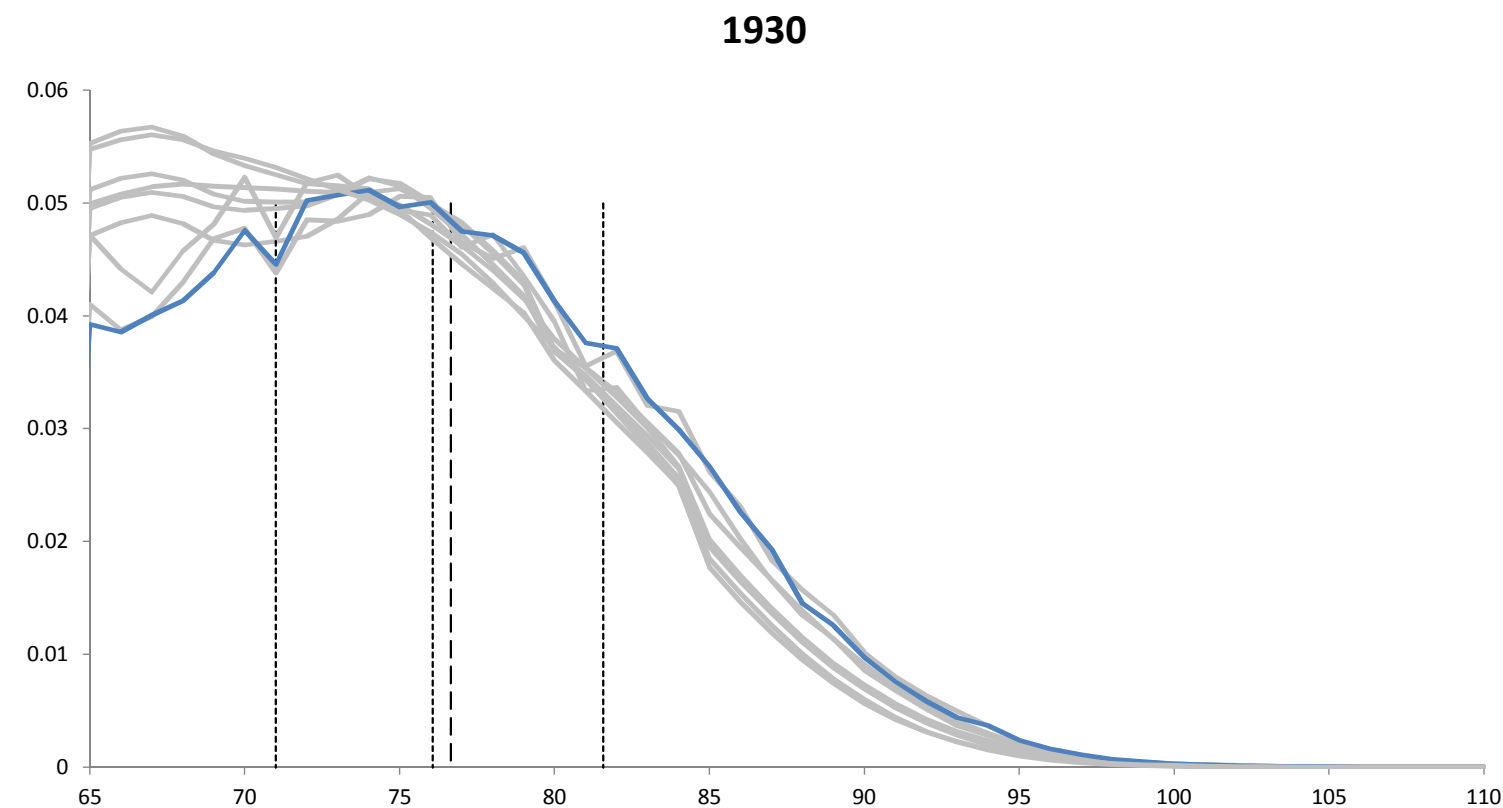
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



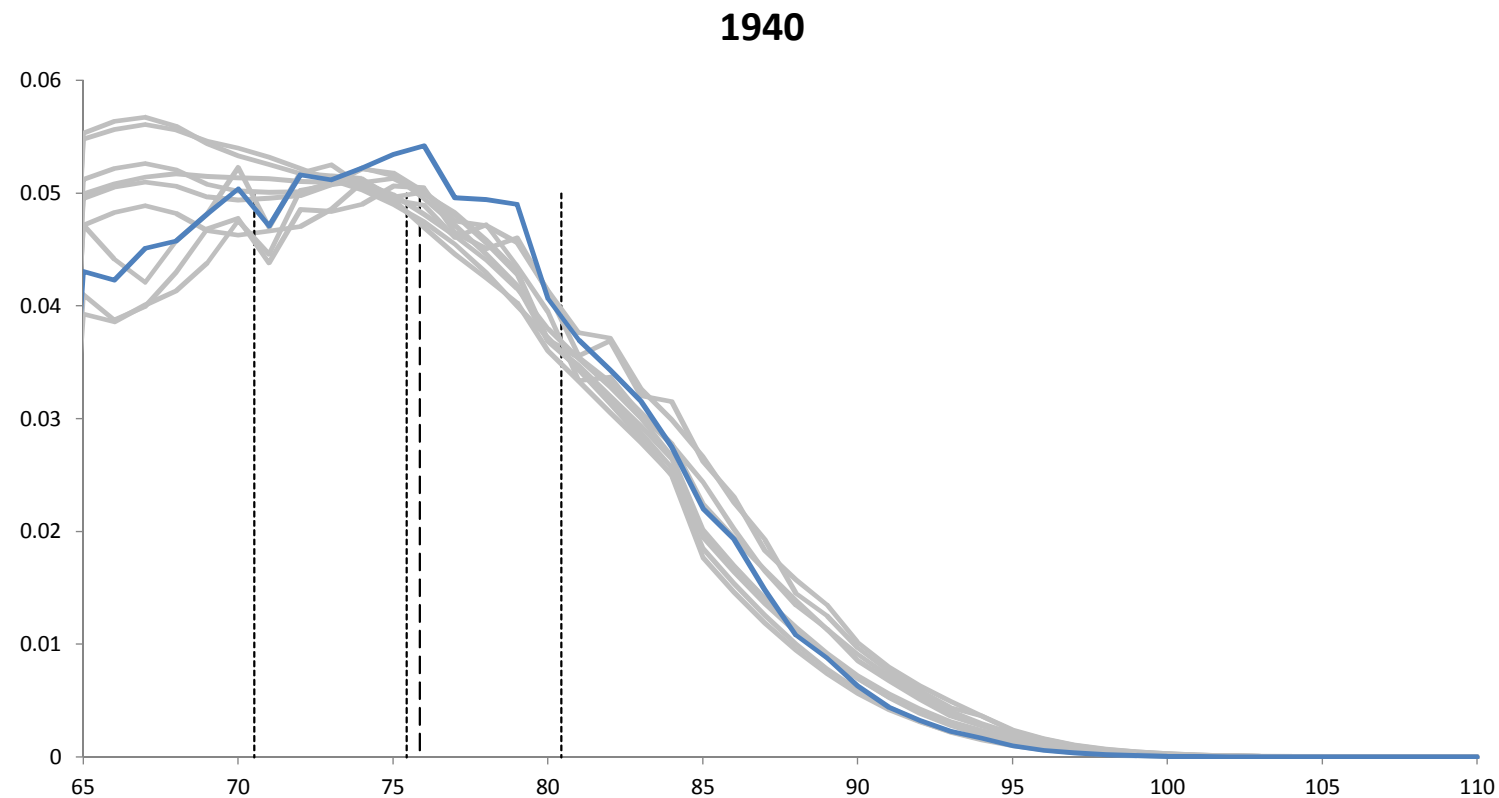
Source: Human Mortality Database

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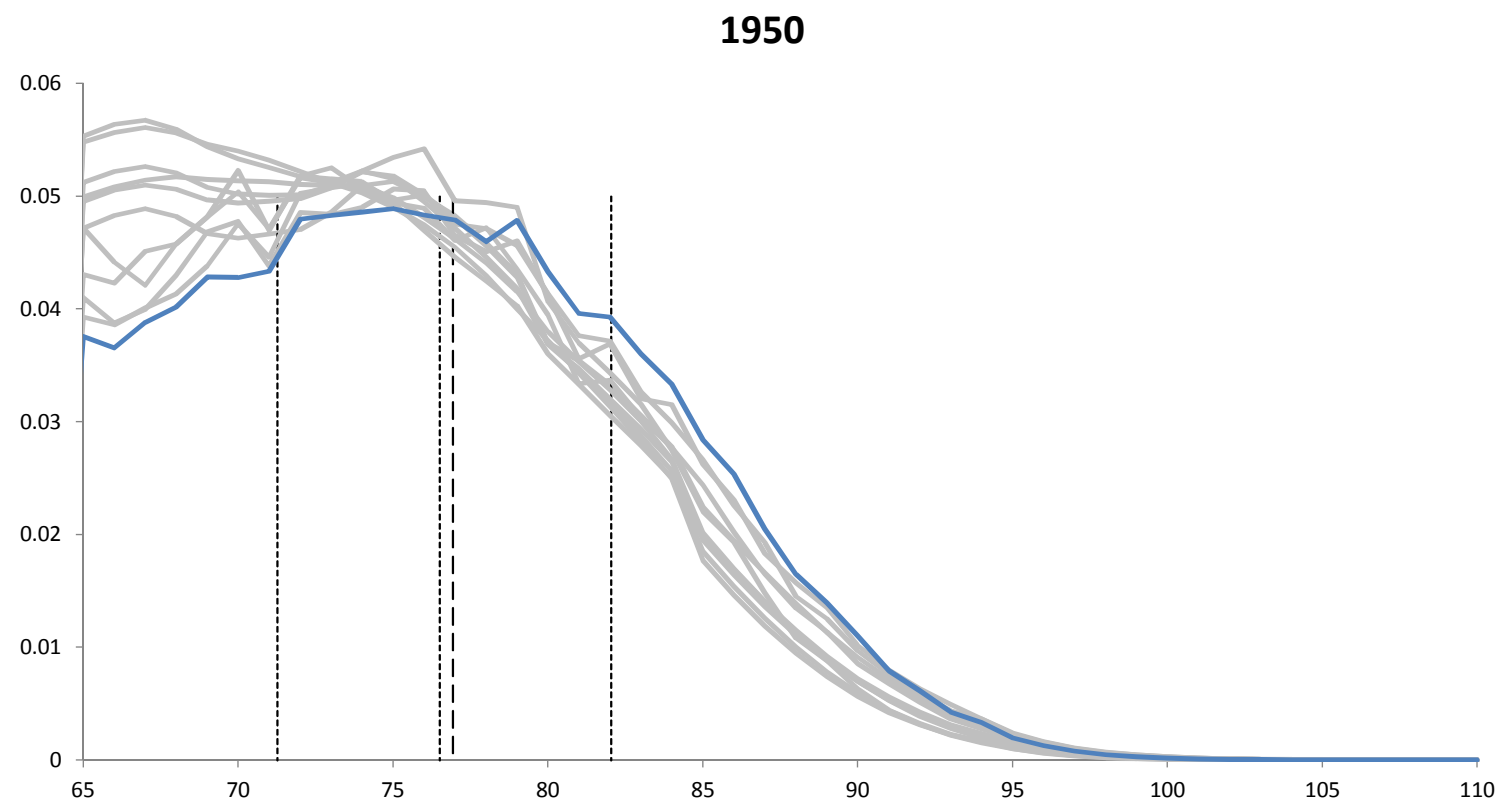
Source: Human Mortality Database

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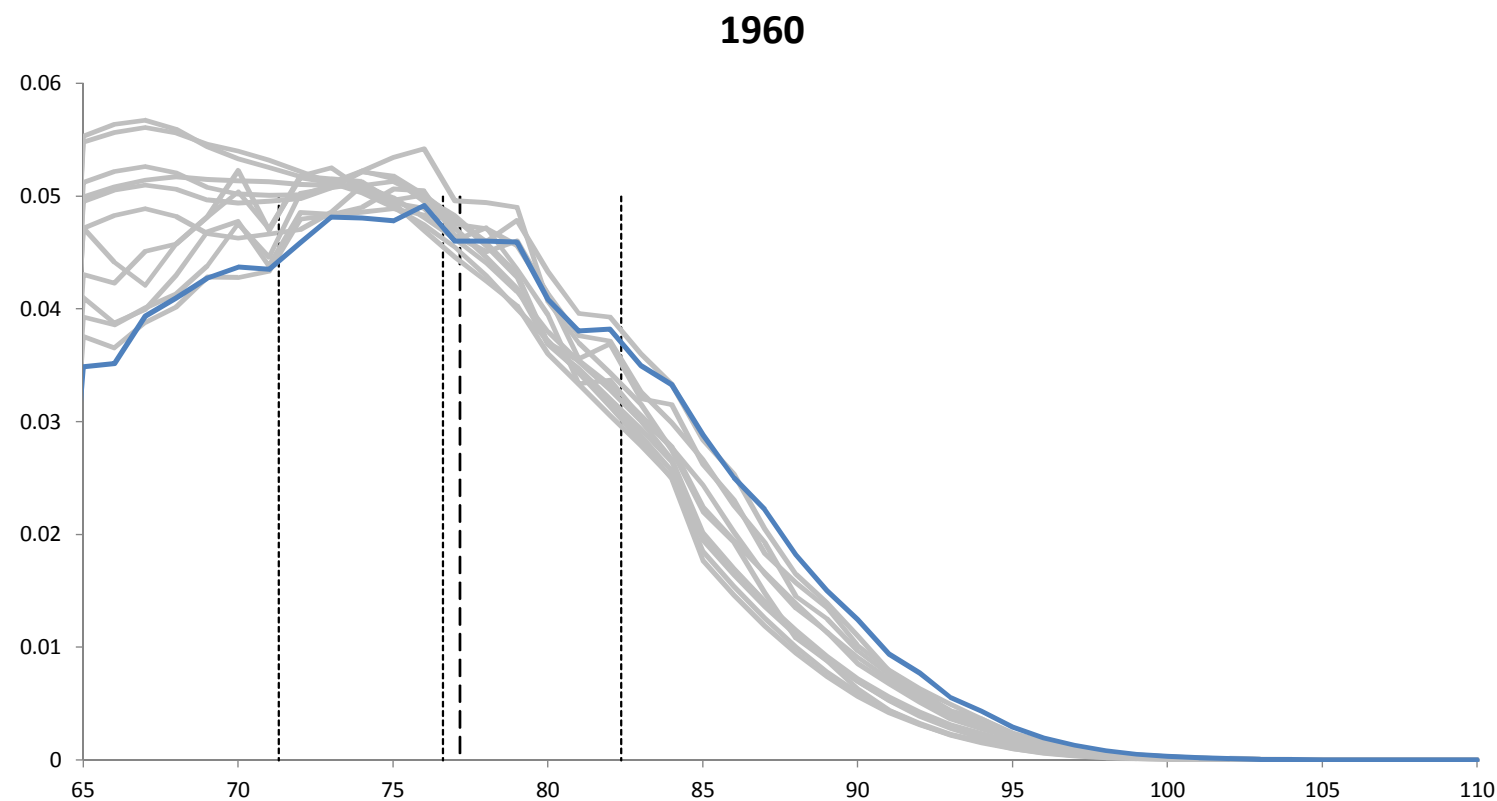
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



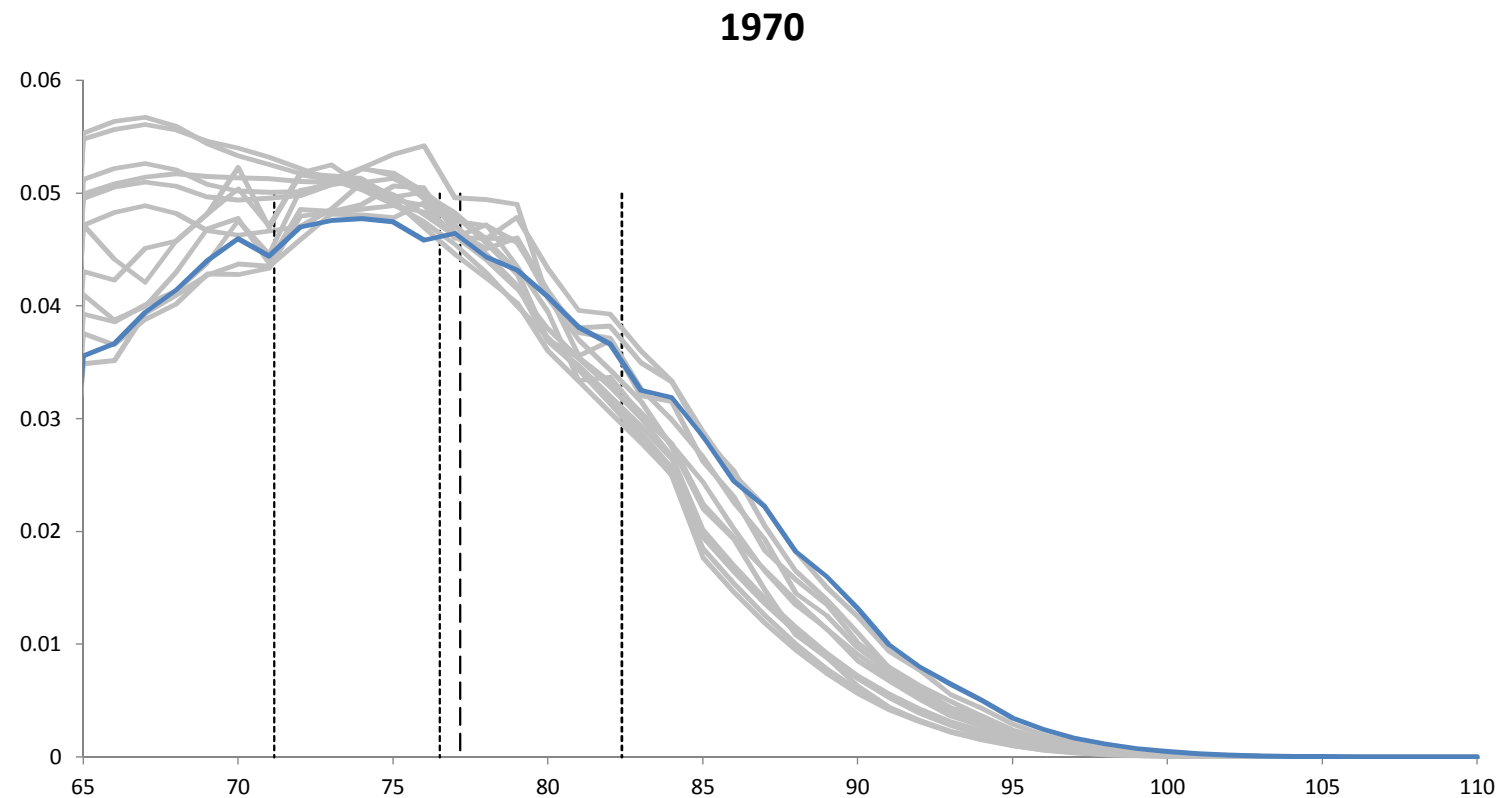
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



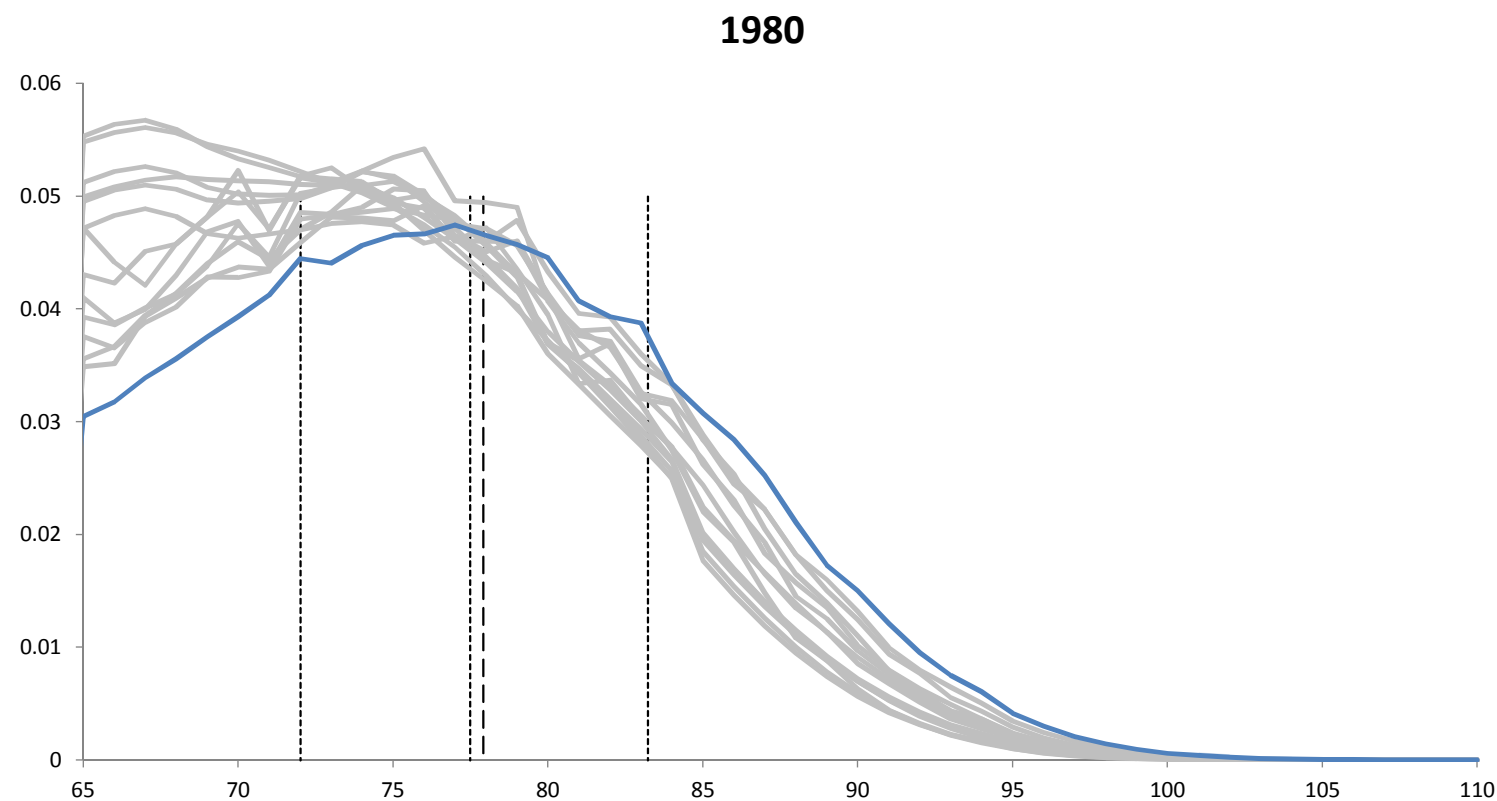
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Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



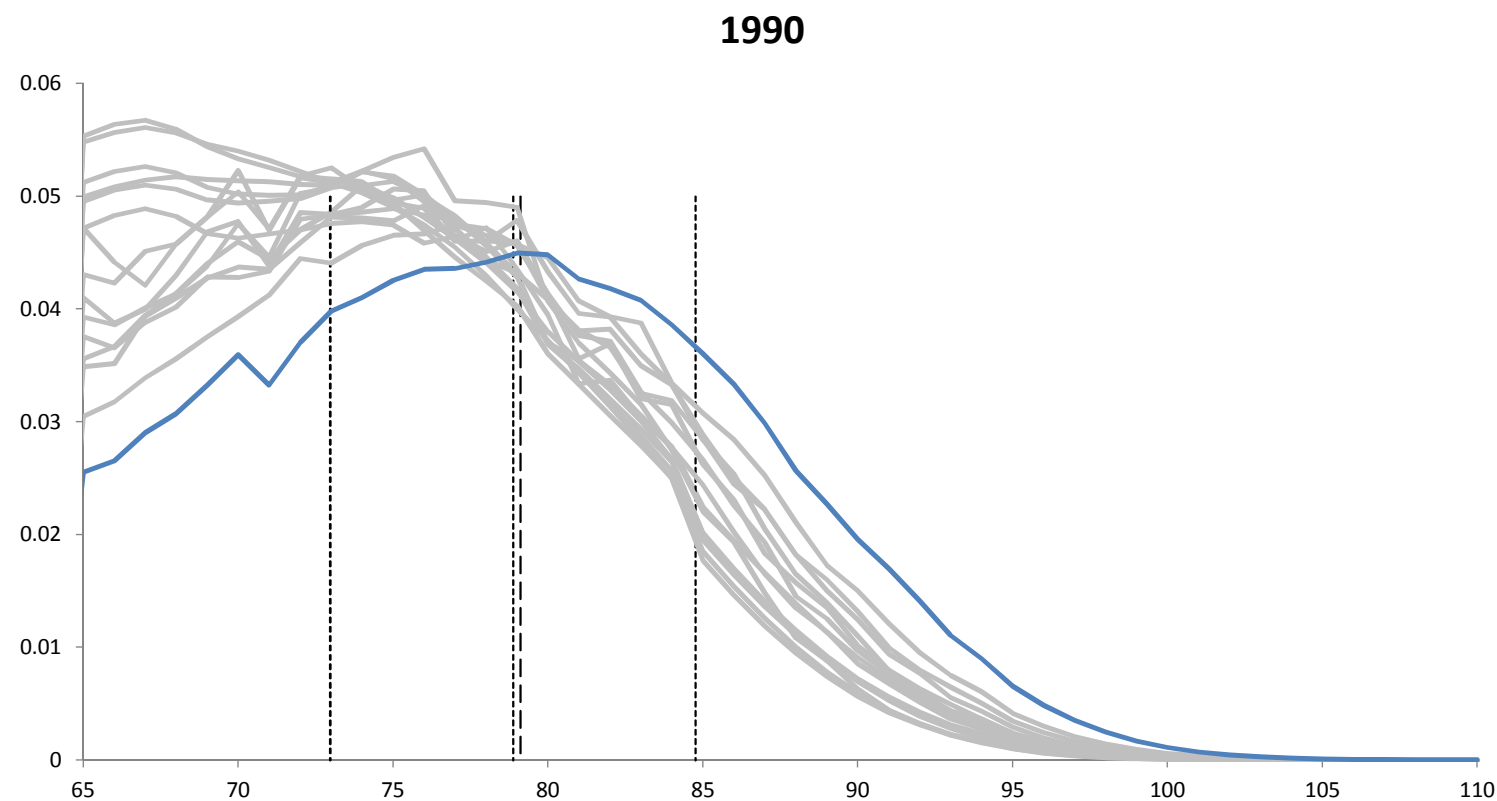
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Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



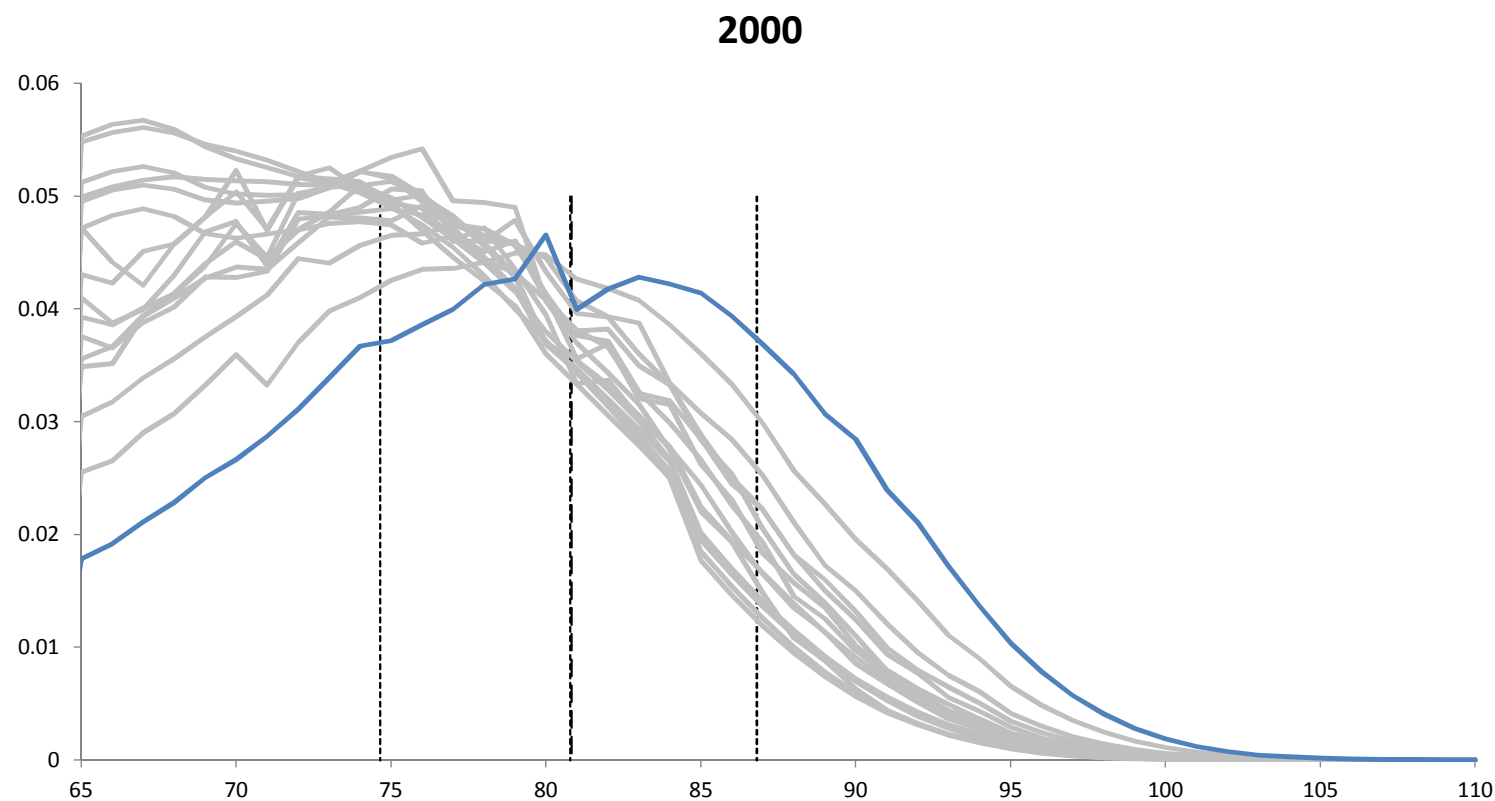
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Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



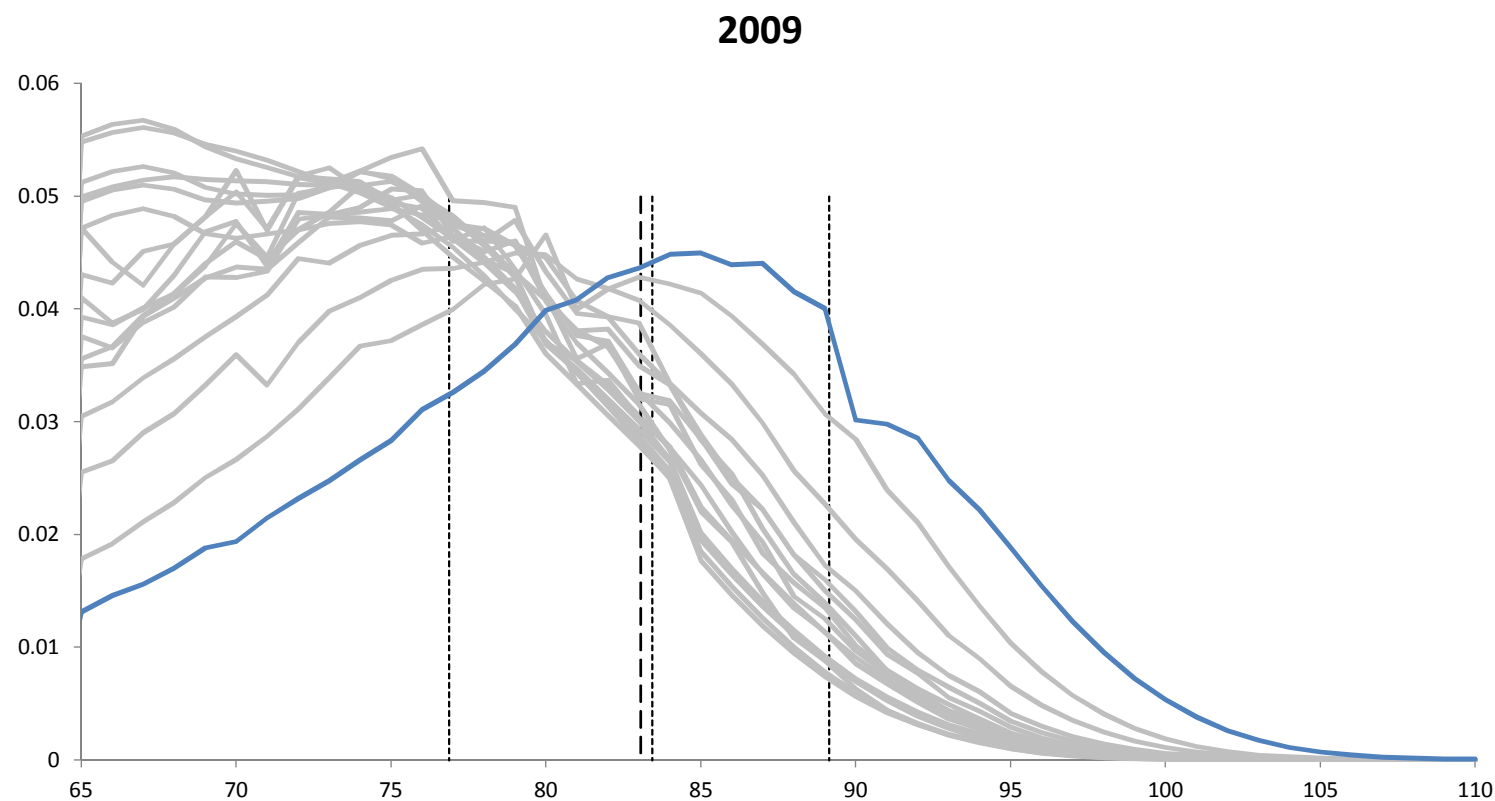
Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



Source: Human Mortality Database

Male life Table distribution of deaths conditional on reaching age 65, England and Wales 1850-2009



Source: Human Mortality Database

DISTRIBUTION OF AGES AT DEATH CONDITIONAL ON REACHING AGE 65 (ENGLAND AND WALES)

	Males		Females	
Year	Median	IQR	Median	IQR
1851	75.0	70.1 – 80.1	75.8	70.6 – 81.5
1871	74.8	70.0 – 80.3	75.7	70.6 – 81.4
1891	74.6	69.8 – 80.0	75.6	70.5 – 81.2
1911	75.3	70.4 – 80.8	76.9	71.5 – 82.4
1931	75.8	70.8 – 81.1	77.8	72.4 – 83.3
1951	76.3	71.1 – 81.7	79.3	73.7 – 84.7
1971	76.6	71.2 – 82.4	81.2	75.1 – 86.9
1991	79.0	72.0 – 85.0	83.5	76.7 – 89.5
2001	81.1	72.9 – 87.0	84.7	78.1 – 90.4
2011	83.9	77.3 – 89.5	86.7	80.2 – 92.1

WHAT IS HAPPENING?

- Reductions in probabilities of dying at each age → increases in life expectancy, increases in median age at death.
- Reductions in probabilities of dying at “younger ages” → reductions in life span inequality (as measured by IQR, for example).
- Now, we have reductions at “older ages” → increases in life span inequality. This has important implications and makes modelling more difficult.

WHY ARE WE LIVING LONGER?

- Epidemiologic transition: pattern of disease has changed from high mortality among infants and children and episodes of famine and epidemics affecting all ages to a pattern of degenerative and man-made diseases (e.g smoking) affecting mainly the elderly.
- What has happened – improved public health and hygiene; control of infectious diseases and epidemics; safer environment; new medical treatments preventing onset of disease and delaying death etc.
- In developing countries, the transition has happened more quickly than developed countries like EW.

CAUSES OF PREMATURE DEATH

UK 2010

Rank	Disorder	% of total YLL
1	Ischemic heart disease	15.9
2	Lung cancer	7.2
3	Stroke	6.8
4	COPD	4.5
5	Lower respiratory infections	4.4
6	Colorectal cancer	3.7
7	Breast cancer	3.2
8	Self harm	2.6
9	Cirrhosis	2.6
10	Alzheimer's disease	2.6
11	Other cardio and circulatory	2.3
12	Road injury	1.8
13	Pancreatic cancer	1.7
14	Oesophageal cancer	1.5
15	Prostate cancer	1.5

(Source: Global Burden of Disease Study, 2010).

(YLL – years of life lost)

BURDEN OF DISEASE ATTRIBUTABLE TO 15 LEADING RISK FACTORS (2010)

Rank	Risk Factor
1	Dietary risks
2	Smoking
3	High blood pressure
4	High body-mass index
5	Physical inactivity
6	Alcohol use
7	High total cholesterol
8	High fasting plasma glucose
9	Drug use
10	Occupational risks
11	Ambient PM pollution
12	Lead
13	Low bone mineral density
14	Childhood sexual abuse
15	Intimate partner abuse

(Ranked by % of total DALY: Global Burden of Disease Study 2010).

(DALY – disability adjusted life years)

POTENTIAL DRIVERS FOR FUTURE MORTALITY CHANGES

- Changes in bio-medical technology
- Effectiveness of health care systems
- Behavioural changes related to health
- Smoking prevalence
- Lifestyles
- Obesity
- Emergence of new diseases (eg HIV, SARS)
- Antibiotic resistance
- Re-emergence of old diseases (eg TB)
- Environmental change, disasters, wars
- Changes in population composition: cohort effects, migrants

NEED FOR MODELS OF LONGEVITY

- Understand better past trends
- Reduce the dimension of the problem
- Help with forecasting the future
- Quantify risk – for example, for insurance companies and pension schemes.

MODELS REQUIRE CARE

“A model should be as simple as possible, but no simpler”
(Einstein).

“The truth . . . is much too complicated to allow anything but approximations”.
(Von Neumann).

“All models are wrong but some are useful”.
(Box).

ALTERNATIVE EXPERT VIEWS ON LONGEVITY: THE EXTREMES

- ‘Pessimists’ suggest that life expectancy might level off or decline (Olshansky).
 - Impact of obesity, poor diet, sedentary lifestyles etc.
- ‘Optimists’ suggest no natural limit to human life (Vaupel).
 - Supported by extrapolative methods.
 - Future scientific advances?

MORTALITY FORECASTING METHODOLOGIES

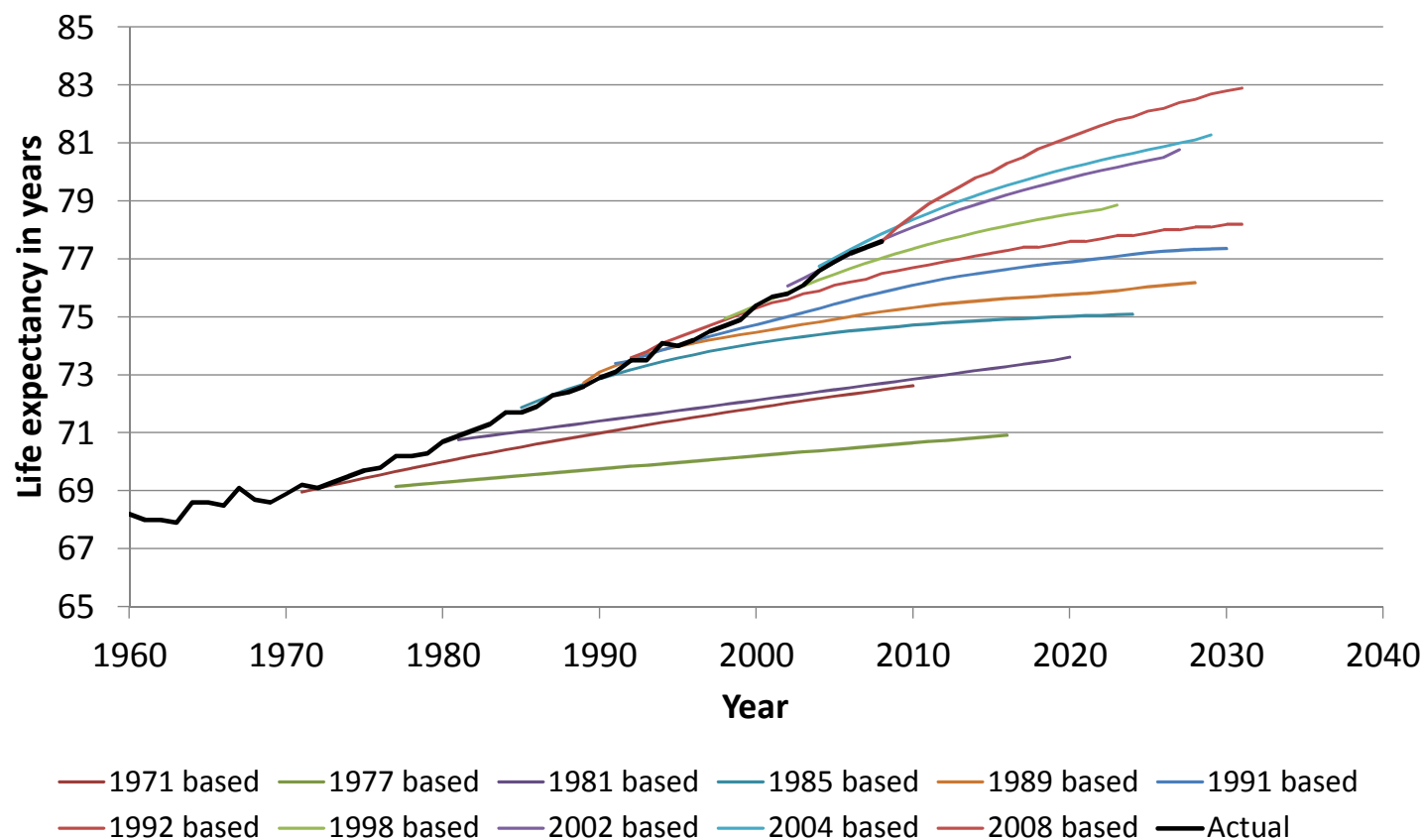
(Booth and Tickle, 2008)

- Expert based.
- Structural Modelling (Explanatory or Econometric).
- Decomposition.
- Trend Modelling (Extrapolation).

REFLECTIONS

- Extrapolation methods fail to account for future structural change.
- Expert opinion has been conservative e.g. choice of target, target date, interpolation path.
- Theoretical advantages of structural models not matched by forecasting performance.
- Decomposition by cause of death has led to conservative forecasts.

Accuracy of Office of National Statistics Mortality assumption (Actual and projected UK male period life expectancy at birth)



Source: Government Actuary's Department

JUSTIFICATION FOR EXTRAPOLATION METHODS

- Complexity and stability of historical trends.
- Extrapolation may be the most reliable approach in terms of forecast accuracy.
- “...we cannot afford to be ashamed of extrapolating the observed regularities of the past” (Keyfitz, 1982).
- Most statistical offices in Europe now use extrapolative methods for mortality forecasting (Janssen, 2018).
- But explaining (i.e. fitting) the past and forecasting the future are difficult.

GENERAL EXTRAPOLATION MODEL STRUCTURE

$$\eta_{x,t} = \alpha_x + \sum_{i=1}^N \beta_x^{(i)} k_t^{(i)} + \beta_x^{(o)} \gamma_{t-x}$$

x = age, t = time, $t - x$ = year of birth (cohort).

Choice of predictor: $\ln \mu_{x,t}$, logit $q_{x,t}$

FAMILY 1: LEE-CARTER (1992)

$$\ln \mu_{x,t} = \alpha_x + \beta_x k_t$$

Not a regression model

FAMILY 1: LEE-CARTER (continued)

Non Linear structure and non-parametric $\beta_x^{(i)}$

$N=1$ and $\beta_x^{(0)} = 0$: Lee and Carter (1992).

$N=2$ and $\beta_x^{(0)} = 0$: Booth et al (2002), Haberman and Renshaw (2003).

$\beta_x^{(0)} \neq 0$: Haberman and Renshaw (2006); cohort term involves estimation problems.

FAMILY 1: LEE-CARTER (continued)

Parameter constraints to ensure identifiability

Interpretation of parameters

Methods of fitting: SVD/PC, WLS, GLM

Smoothing of $\beta_x^{(i)}$

Forecasting: based on time series models of $k_t^{(i)}$

FAMILY 2: CAIRNS, BLAKE, DOWD (2008, 2009)

Linear structure, with pre-determined parametric choices for $\beta_x^{(i)}$

$$\beta_x^{(1)} = 1$$

$$\beta_x^{(2)} = x - \bar{x}$$

$$\beta_x^{(3)} = (x - \bar{x})^2 - \sigma^2$$

FAMILY 3: MORTALITY IMPROVEMENTS RATES

CMI 1924: $\frac{q_{x,t}}{q_{x,t-1}} = r(< 1)$

A constant reduction in mortality rates over time.

FAMILY 3: MORTALITY IMPROVEMENT RATES (continued)

PERIOD

$$\eta_{x,t} = 1 - \frac{\mu_{x,t}}{\mu_{x,t-1}}$$

COHORT

$$\eta_{x,t} = 1 - \frac{\mu_{x,t}}{\mu_{x-1,t-1}}$$

Haberman and Renshaw (2012, 2013)

WHAT IS A GOOD MODEL?

Desirable properties include

- Parsimony
- Transparency
- Straightforward to implement
- Goodness of fit to historical data
- Robustness
- Forecasting success and plausible levels of uncertainty
- Biologically reasonable forecasts

CASE STUDY: 13 COMMONLY USED EXTRAPOLATION MODELS - FORECASTING PERFORMANCE

- Use England and Wales male experience for 1961-1982, ages 55-89 to fit models and then calculate life expectancies (and annuity values) in 1982 by cohort method.
- Calculate values for same indices using raw mortality rates for 1983-2007, and consider relative errors: $(\text{predicted} - \text{actual}) / (\text{actual})$.

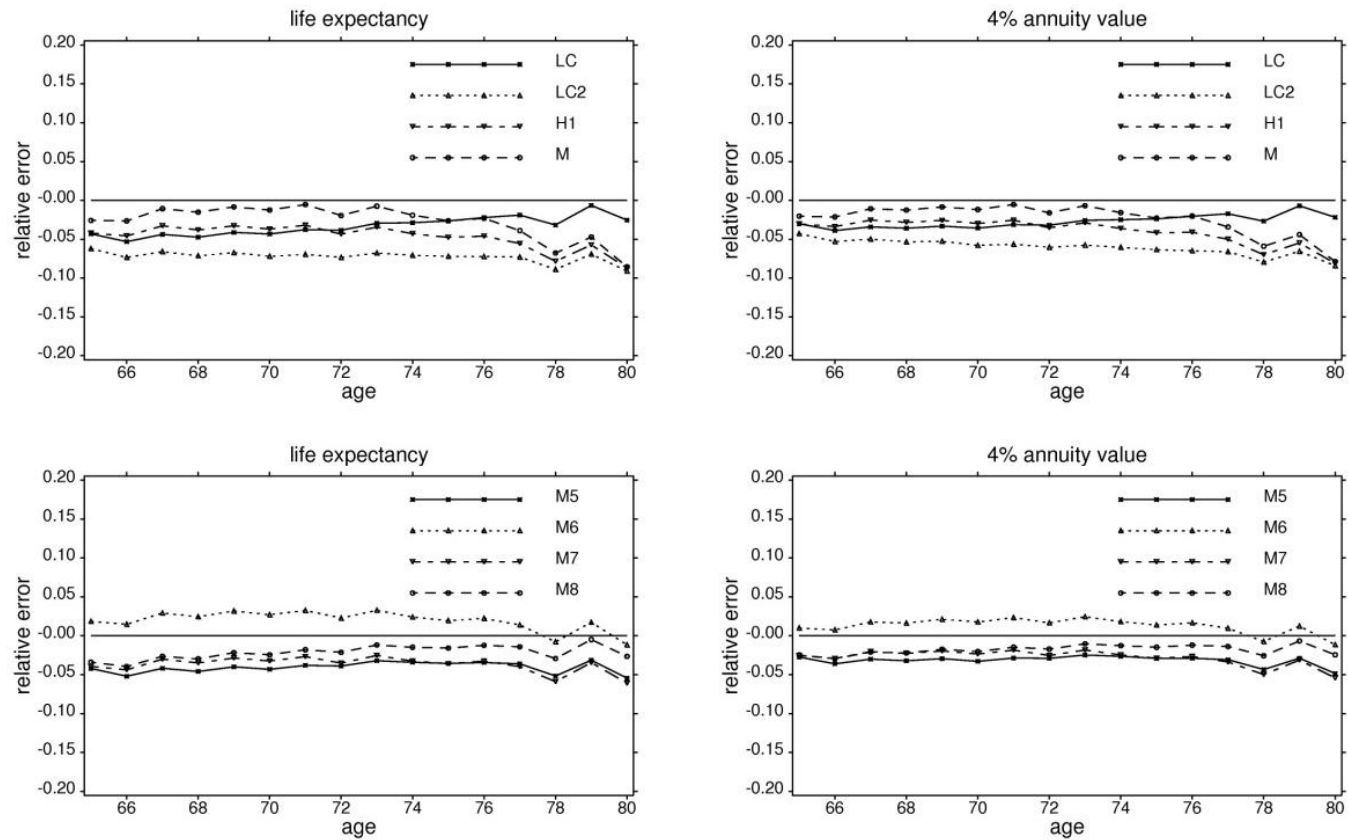
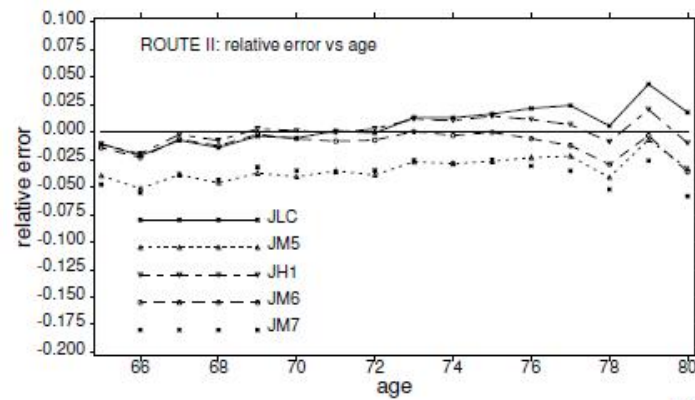
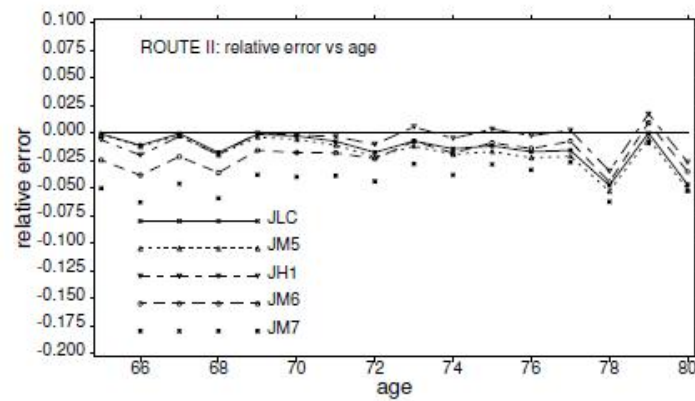
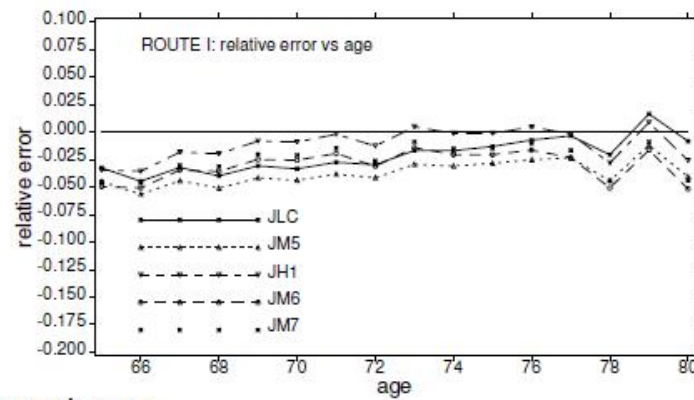


Fig 13a. Retrospective relative error in 1982 predicted life expectancies (LH panels) and annuity values (RH panels): ages 65-80.



male experience



female experience

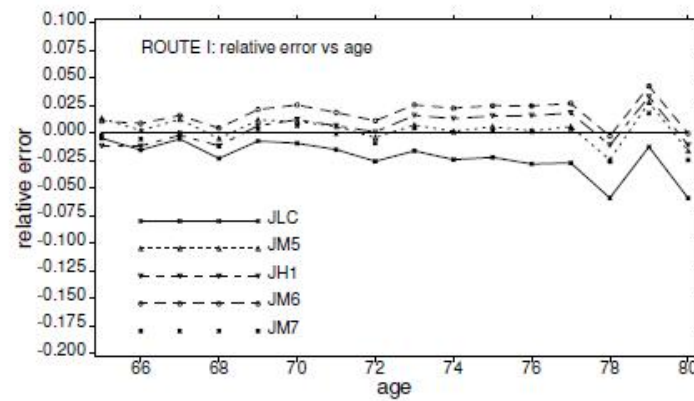
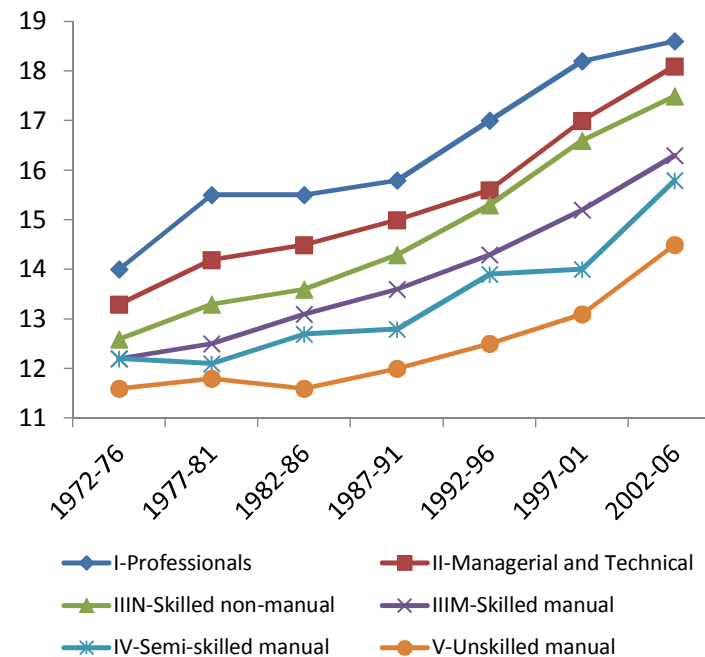


Fig 15a. E&W male and female mortality experiences. Retrospective relative error in 1982 predicted life expectancies.(Route II LH panels; Route I RH panels): ages 65-80.

SOCIO-ECONOMIC DIFFERENCES IN MORTALITY

- Well-documented relationship between mortality and socioeconomic variables
 - Education
 - Income
 - Occupation
- Important implications on social and financial planning
 - Public policy for tackling inequalities
 - Social security design
 - Annuity reserving and pricing
 - Longevity risk management

Male life expectancy at age 65 by social class -England and Wales



Source: ONS Longitudinal Study

RELATIVE MODELLING APPROACH

National mortality trends

$$\ln \mu_{xt} = \alpha_x + \beta_x k_t + \gamma_{t-x}$$

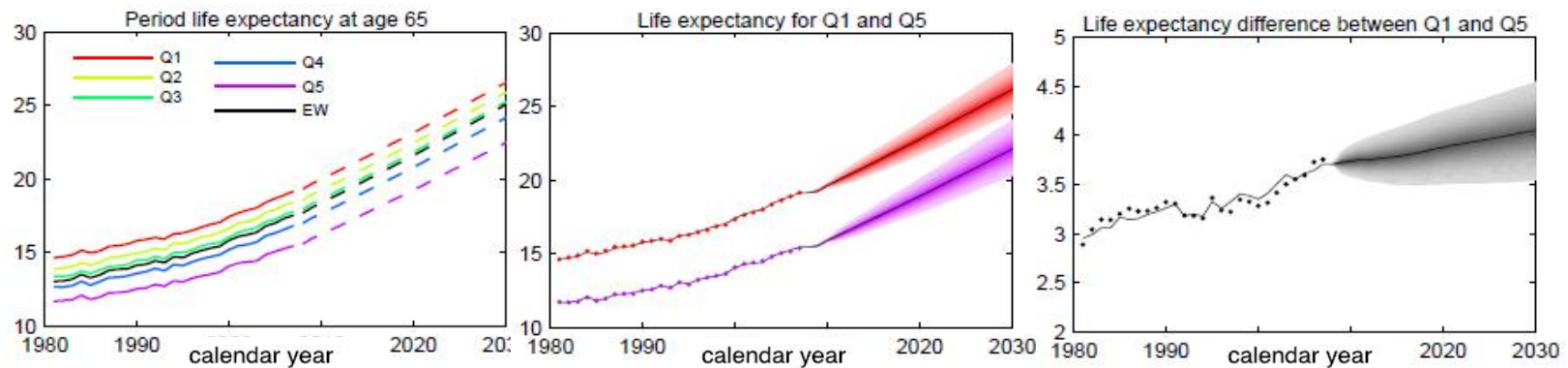
Sub population trends

$$\ln \mu_{xtg} = \ln \mu_{xt} + \alpha_{xg} + \beta'_x k_{tg} \quad \text{for subpopulation } g$$

(Villegas and Haberman, 2014)

Case study: Mortality by deprivation in England

Period life expectancies males aged 65

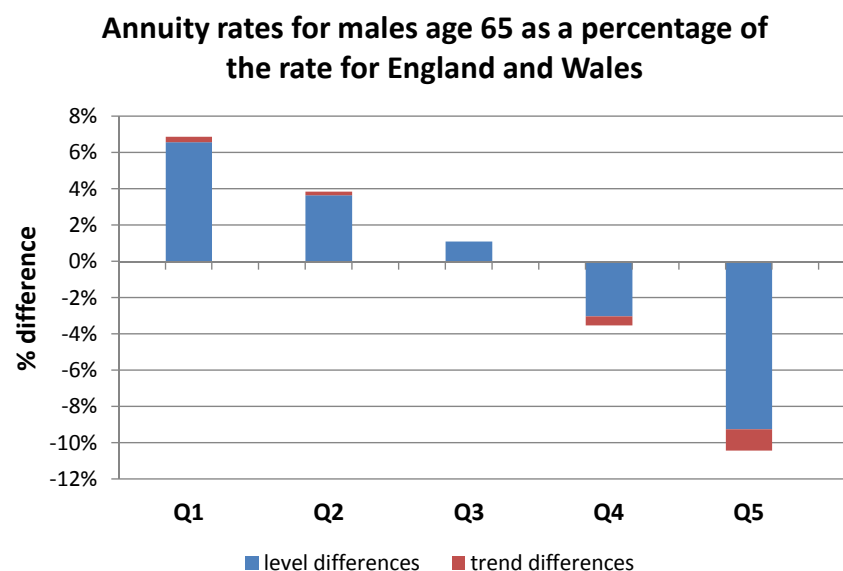


Male period life expectancy at age 65					
	1981	1995	2007	2020	2030
EW	13.1	14.6	17.54	21.6	25.0
Q1	14.6	16.3	19.1	23.1	26.5
Q2	13.9	15.6	18.4	22.4	25.9
Q3	13.4	15.0	17.8	21.8	25.2
Q4	12.7	14.1	16.8	20.8	24.2
Q5	11.7	12.9	15.4	19.3	22.5
Q1-Q5	2.9	3.4	3.8	3.9	4.1

Case study: Mortality by deprivation in England

Implications for life annuities

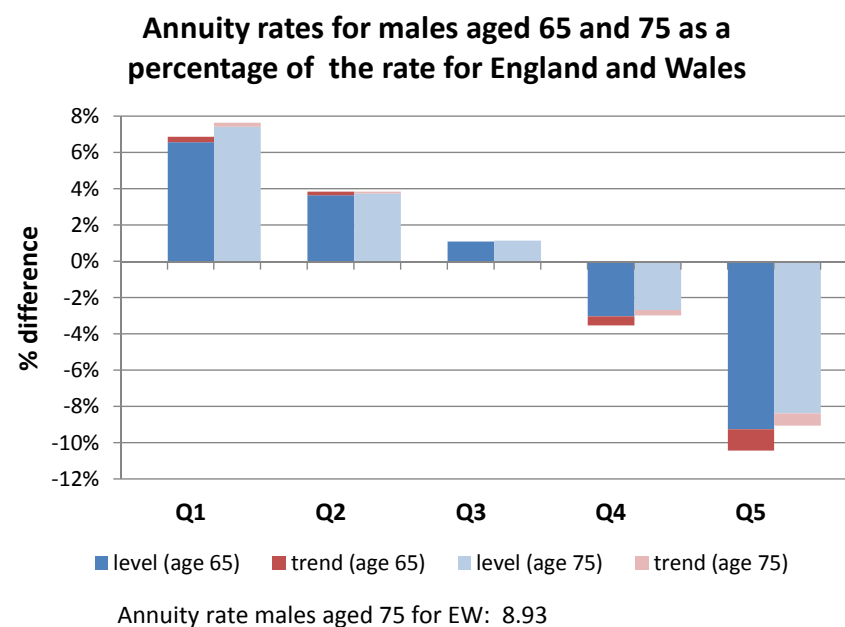
- Discount rate: 4%
- Calculations under cohort trajectory for men age x in 2007
- Annuity rate males aged 65 for EW: 13.53
- Significant variation of annuity rates with the level of deprivation
- Baseline mortality differentials have a very significant impact
- The impact of improvement differences is of second order



Case study: Mortality by deprivation in England

Implications for life annuities

- Discount rate: 4%
- Calculations under cohort trajectory for men age x in 2007
- Annuity rate males aged 65 for EW: 13.53
- Significant variation of annuity rates with the level of deprivation
- Baseline mortality differentials have a very significant impact
- The impact of improvement differences is of second order
- Although mortality differentials decrease with rising age their impact on annuity rates does not



SENSITIVITY OF LONGEVITY MEASURES AND ANNUITY VALUES TO LONGEVITY TRENDS

- Effect of changing assumptions and different models
- Rates of change in response to underlying trend

SENSITIVITY OF LIFE EXPECTANCY AND DISCOUNTED ANNUITY VALUES TO LONGEVITY ASSUMPTIONS (STATIC – EW MALES 2011)

	Discount Rate			
	0%	1%	3%	5%
<u>Annuity issued in 2011</u>				
Immediate – age 65	18.15	16.33	13.46	11.34
10 year deferred – age 55	16.73	13.57	9.09	6.21
20 year deferred – age 45	16.19	11.84	6.42	3.49
<u>Effect of 25% reduction in 2012</u>	Percentage increases			
Immediate – age 65	11	10	8	7
10 year deferred – age 55	14	13	11	10
20 year deferred – age 45	15	14	12	11
<u>Effect of 50% reduction in 2012</u>				
Immediate – age 65	28	25	20	16
10 year deferred – age 55	33	31	26	22
20 year deferred – age 45	36	33	29	26

SENSITIVITY OF LIFE EXPECTANCY AND DISCOUNTED ANNUITY VALUES TO STOCHASTIC MORTALITY MODELS

Ranges of percentage increases relative to static EW2011 males				
	Discount Rate			
<u>Annuity – issued in 2011</u>	0%	1%	3%	5%
Immediate – age 65	7-17	6-15	5-11	4-8
10 year deferred – age 55	5-25	5-23	4-18	3-15
20 year deferred – age 45	6-38	5-35	3-29	3-26

(5 models fitted to EW males data for 1960-2011.

Model: LC, APC, CBD-M5, CBD-M7, Plat) Central estimates only.

SIMPLE GOMPERTZ MODEL AND EXPONENTIAL DECLINE

$$\mu(x, t) = \alpha e^{\beta x} e^{-\rho t}$$

As $t \rightarrow \infty$

$$\frac{\delta}{\delta t} e^p(x, t) \rightarrow \frac{\rho}{\beta} = \frac{.02}{.12} = 17\% \text{ pa}$$

$$\frac{\delta}{\delta t} e^c(x, t) \rightarrow \frac{\rho}{\beta - \rho} = \frac{.02}{.10} = 20\% \text{ pa}$$

Range of ρ : 0.5% to 3%
 β : .08 to .16

(Missov and Lenart, 2011)

MORBIDITY TRENDS: ARE WE LIVING LONGER AND HEALTHIER?

Table 2 from Jagger (2015) Foresight Report

		LE	DFLE (95% CI)	HLE* (95% CI)
Women				
At birth	2000-2002	80.4	62.8 (62.5 – 63.1)	62.4 (62.1 - 62.7)
	2009 - 2011	82.4	64.7 (64.4 – 65.1)	66.1 (65.8 – 66.5)
	Difference	2.0	1.9	3.7
Age 65	2000 - 2002	19.0	10.2 (10.0 – 10.4)	10.8 (10.6 – 11.0)
	2009 - 2011	20.7	11.0 (10.7 – 11.2)	12.1 (11.8 – 12.3)
	Difference	1.7	0.8	1.3
Age 85	2000 - 2002	6.2	2.1 (2.0 – 2.4)	2.9 (2.7 – 3.1)
	2009 – 2011	6.8	2.2 (2.0 – 2.4)	3.5 (3.3 – 3.7)
	Difference	0.6	0.1	0.6

LIVING HEALTHIER: RESULTS FROM A DYNAMIC MICROSIMULATION MODEL

“In the next 20 years, the English population aged 65 and over will see increases in the number of individuals who are independent but also in those with complex care needs. The increase is due to more individuals reaching age 85 years or older who have higher levels of dependency, dementia and co morbidity”.

(Kingston et al, 2018)

PROJECTED YEARS LIVED FROM AGE 65 FOR FEMALES BY DEPENDENCY AND YEAR

	Years lived from age 65		
	2015	2025	2035
Total life expectancy	21.1	22.7	24.1
Independent years	10.7	11.4	11.6
Proportion	50.6%	49.9%	48.0%
Low dependence years	7.2	7.7	8.5
Proportion	34.0%	33.9%	35.4%
Medium dependency years	1.3	1.3	1.3
Proportion	6.0%	5.5%	5.4%
High dependence years	2.0	2.4	2.7
Proportion	9.5%	10.7%	11.2%

(Kingston et al, 2018)

SUBJECTIVE VIEW OF LONGEVITY

Systematic underestimation of how long we are going to live

- “individuals underestimate their chances of survival to ages 75, 80 and 85 on average
- Individuals in their late 70s and 80s are, on average, mildly optimistic about surviving to ages 90, 95 and above”

(O’Dea and Sturrock, 2018)

PERCEPTIONS BY AGE: DIFFERENCES BETWEEN OBJECTIVE AND SUBJECTIVE LIFE EXPECTANCY

	Average self-perception	Average GAD forecast figure	Self-estimate minus GAD forecast figure
Males			
16-19	75.47	82.41	-6.94
20-29	75.83	82.34	-6.51
30-39	75.90	82.20	-6.30
40-49	76.84	82.09	-5.24
50-59	78.54	82.34	-3.79
60-69	80.61	83.45	-2.83
70-79	83.76	85.42	-1.66
80-89	89.97	89.19	0.77
90-99	98.75	96.45	2.30
Females			
16-19	78.35	86.42	-8.07
20-29	77.40	86.22	-8.82
30-39	79.34	85.88	-6.53
40-49	79.85	85.66	-5.80
50-59	80.67	85.79	-5.12
60-69	81.82	86.44	-4.62
70-79	84.57	87.51	-2.94
80-89	89.44	90.46	-1.01
90-99	96.00	95.55	0.45

Source: O'Brien et al (2005)

SUBJECTIVE VIEWS OF RETIREMENT AND RETIREMENT PLANNING

- Underestimation of life expectancy
- Misunderstanding of longevity risk and risk of outliving assets
- Shift from DB to DC plans and inadequate contribution levels
- Underestimation of post retirement medical expenses and costs of long term care
- Average levels of DC funds at retirement are inadequate
- Impact of health shocks on finances
- Reluctance to purchase an annuity voluntarily (“annuity puzzle” but annuity is optimal choice – improves welfare in face of uncertain future lifetime – Yaari 1965)
- Early claiming of social security benefits rather than deferring
- Mismatch between numbers who think that risk management and insurance products are a good idea and the numbers who actually buy them.

(SOA, 2019)₆₈

IMPLICATIONS OF DEMOGRAPHIC TRENDS FOR PENSION PROGRAMS

- PAYG Government backed pensions and social security programs
- Funded DB pension programs
- Funded DC pension programs

Mortality and morbidity trends imply increased costs of benefits

RESPONSE TO INCREASED COSTS TO MAINTAIN FINANCIAL SUSTAINABILITY

- Increase contributions or premiums
- Reduce benefits in payment e.g. lower levels of indexation, initial pension level based on forecast life expectancy
- Change benefit eligibility conditions e.g. age at retirement
- New contract designs to share costs

But variability of lifetimes within the population.

(UK Pensions Commission, 2003)

COMMENT ON MORTALITY TRENDS

Downward trend in mortality rates (and corresponding increases in lifetimes) is a systematic effect. It affects everyone and undermines the pooling of risk which is an important element of insurance (and pension systems).

IMPACT OF DEMOGRAPHIC TRENDS ON FUNDED PENSION PROGRAMS

- DB Pension plans guarantee retirement income for life – however long members live
 - Sponsoring employers face extra costs – may need to divert resources away from investment and dividend programs (UK - £2T liabilities; OECD - \$29T)
- DC pension plans involve more risk transfer to plan members: financial competence problems and cognitive decline in later life.
- DC pension plans – members face tail risk of out living their savings; or underspend savings leading to an unintentional bequest on death.
- Insurance companies selling annuities (important DC delivery mechanism) face adverse selection and longevity risk – have inadequate reserves (exacerbated by regulatory rules e.g. Solvency II)

VOLTAIRE ON ANNUITIES

- “I advise you to go on living solely to enrage those who are paying your annuities. It is the only pleasure I have left.”

JANE AUSTEN ON ADVERSE SELECTION

“If you observe, people always live forever when there is any annuity to be paid them. An annuity is a very serious business; it comes over and over every year, and there is no getting rid of it.”

(Fanny Dashwood: *Sense and Sensibility*)

BEHAVIOURAL ASPECTS OF ANNUITIES

Insights from behavioural finance and work of Kahneman and Tversky

- Cumulative prospect theory
 - Hyperbolic discounting
- } point to a role for deferred annuities (Chen et al, 2019)
- Framing effects: contrast annuity as “investment vehicle” and annuity as “product allowing consumption”.

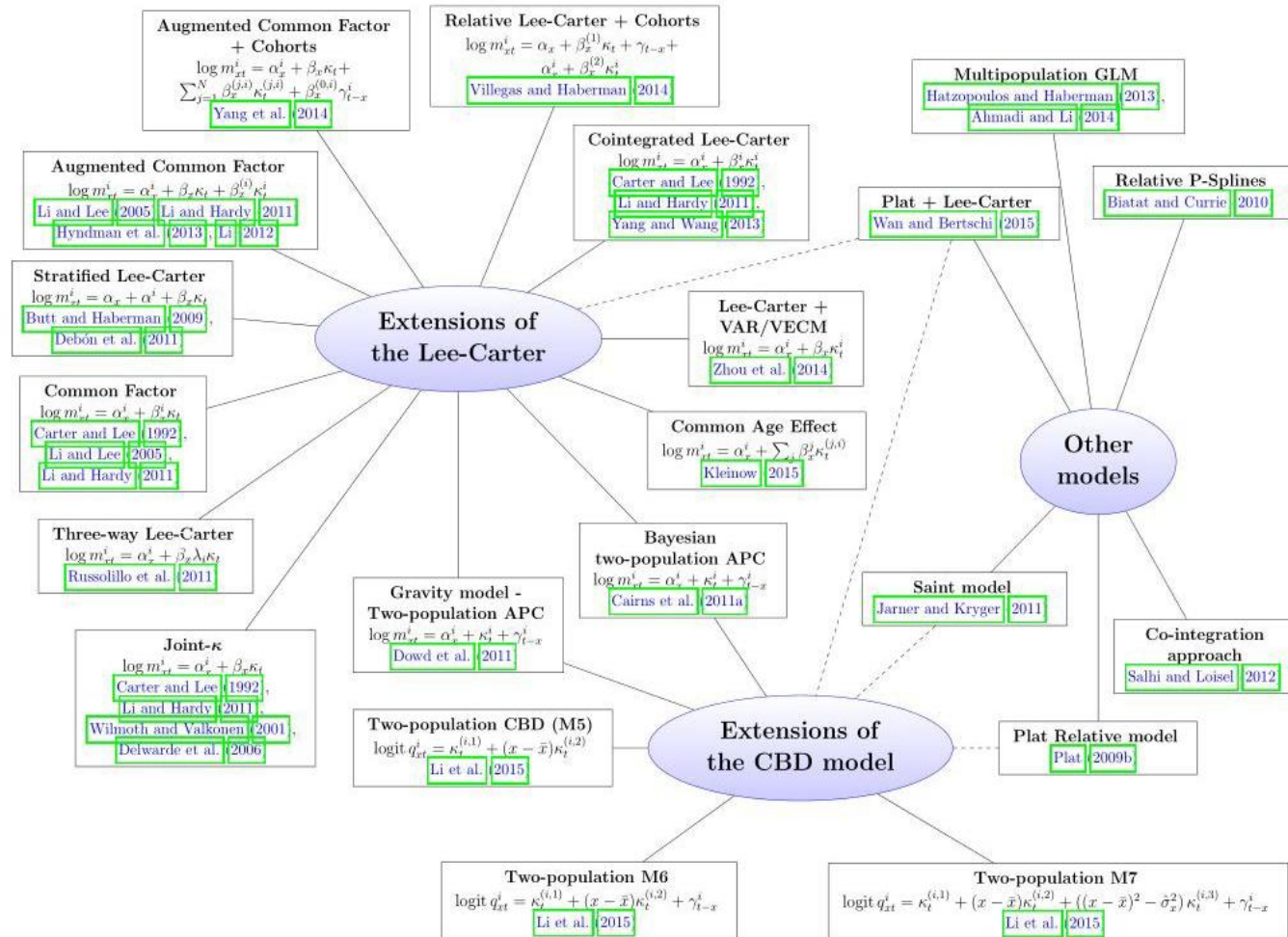
POTENTIAL SOLUTIONS FOR FUNDED PENSION PROGRAMS

- Transfer of risk: buyin and buyout, partial and complete (UK market – about £100B of £2T liabilities have been transferred)
- Flexible product design with sharing of risk e.g. annuities with benefits that depend on actual mortality experience of risk pool; annuities with longevity-linked deferred periods; with profits product design (Richter and Weber, 2011; Denuit et al, 2011, 2015).
- New products like GSA, modern versions of tontines (Piggott et al, 2005; Donnelly et al, 2014; Milevsky and Salisbury, 2016; Bernhardt and Donnelly, 2019).
- Use of housing wealth to provide retirement income e.g. reverse mortgages,
- Flexible product design to meet future LTC costs (Murtagh et al, 2001),
- Adverse selection problems: mandatory elements to reduce selection; enhanced annuities and improved risk classification.

NEW FRONTIERS FOR RESEARCH

- Use of functional time series models.
- Allowing for jumps, regime switches.
- Joint modelling of populations
 - different countries, different regions, different subgroups
 - coherence, cointegration, panel data methods
 - longevity basis risk between two populations (Cass – Hymans)
- Joint modelling and allowing for coherent aggregation, across sub-populations
- Joint modelling of cause of death data and allowing for classification changes .
- Allowing for additional covariables - epidemiological and macroeconomic factors.
- Bayesian modelling.
- Improved methods of bootstrapping
 - including allowing for dependence.
- Modelling of healthy life expectancy.
- Addition of machine learning techniques.

Universe of Multipopulation Models



MODELLING: CONCLUDING COMMENTS

- Trade-off between goodness of fit and forecasting accuracy.
- Time series methods and their application to long forecasting periods.
- Appropriateness of data sources for particular applications e.g. hedging: adverse selection and “basis risk”.
- Model error – essential to investigate more than one modelling framework.
- Not all sources of uncertainty can be quantified.

OVERALL CONCLUSIONS

- We are all living longer, but not necessarily healthier
- Measures of inequality and diversity within populations are worsening – mortality and morbidity
- Pensions programs need to be adapted to respond to mortality and morbidity trends
- Effect of low fertility adds to mortality and morbidity trends – demographic ageing and wider implications
 - One of 4 global megatrends (McKinsey, 2014).
- Longevity is an important societal problem – many unanswered questions and a need for further research contributions.