This short paper refers to a fuller, 80 page, paper being available.
Due to work commitments the author has in fact not been able to complete the longer paper

The Modelling of Reinsurance Credit Risk

GIRO 2007 Working Party

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Summary

This paper addresses the modelling of reinsurance credit risk and covers a number of different topics. Given the advanced credit risk modelling approaches developed within the banking industry since the mid-1990's there are ideas and approaches that can be usefully applied within the insurance industry. Economic capital modelling requires the consideration of adverse credit loss scenarios, a key to which is the treatment of correlation. The stochastic model that is at the core of this paper relies on the idea of a***sset return correlation*** as a way of 'triggering' multiple reinsurance defaults. It overcomes some of the issues of working directly with default correlation parameters and as is shown by way of numerical examples the default correlation parameters are very different to the corresponding asset correlations.

Keywords

Asset Return Correlation, CreditMetrics, CreditRisk+, CreditPortfolioView, Default Correlation, Economic Capital, Exposure at Default, Loss Given Default, Probability of Default, Markov Process, Merton Model, Multivariate Normal Distribution, Portfolio Manager, Transition Matrix, Willingness to Pay

This note provides an overview of the content of the GIRO 2007 paper which is well in excess of 80 pages. The paper will also form the basis of the workshop on this topic. It will be available for download from the Faculty and Institute of Actuaries website.

The views expressed in this paper are those of the working party and do not necessarily represent the views of every member or of any organisation with which any member of the group is, or has been, associated.

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2 Introduction and Background

2.1 Introduction

This paper addresses the modelling of reinsurance credit risk, namely the risk associated with reinsurance receivables, recoveries and other reinsurance related assets, such as broker balances sitting on the asset side of the balance sheet.

For insurance companies insurance risk will always remain the largest risk exposure, be it the Non-Cat and Cat components of underwriting risk or the reserve run-off risk which has particular significance for long-tail business lines. Credit risk capital under the FSA'S Individual Capital Assessment ("ICA") or the proposed Solvency II SCR framework is not going to be as significant as insurance risk. However, trying to model it in a rationale and scientific way poses some interesting challenges caused not only by the intracies of the credit risk loss process itself but by the paucity of credible data and the complex interaction with the insurance loss process.

Given the advanced credit risk modelling approaches developed within the banking industry since the mid-1990's there are a lot of ideas and approaches that can be usefully applied within the insurance industry either with or without modification. Well known banking models include CreditMetrics (JP Morgan), Portfolio Manager (Moody' KMV), CreditRisk+ (CSFB) and CreditPortfolioView (McKinsey's).

Some of these ideas are being usefully employed in insurance companies and Lloyd's syndicates within their ICA stochastic models and not in others. To the extent that they are not then this paper may serve as a useful reference point in that it covers a number of topics and issues. Moreover, it describes a stochastic modelling approach based on ***asset return correlation*** that borrows simple ideas from the banking world and applies them within a reinsurance framework. It should be stressed that there are other possible modelling approaches to reinsurance credit risk that are equally valid.

The paper is very practical and has many numerical examples. It has not been written solely with an actuarial audience in mind.

2.2 Credit Risk

Credit risk within the banking industry covers two distinct risk types, (i) counterparty risk for loan and derivative portfolios and (ii) issuer risk for corporate bonds. The credit risk arises from potential changes in the credit quality of counterparty in a transaction. There are in principle two parts (i) Default risk and (ii) Credit Spread risk.

Default Risk

Default risk is driven by the potential failure of a counterparty to make promised payments, either wholly or in part.

Credit Spread Risk

If a counterparty does not default there is still a risk due to the possible widening of the credit spread or worsening in credit quality. There are two quite distinct components of credit spread risk:

Jumps in the credit spread:

These may arise from a rating change and will usually be something company specific that reflects either ‘good’ or ‘bad’ information.

Credit spread volatility:

This is likely to be driven by the market’s appetite for certain levels of risk. For example the spreads on bonds may widen or narrow.

Unless a portfolio is marked-to-market only default risk is important. However, if a portfolio is marked-to-market then a transaction with a counterparty whose credit quality has got worse will result in an economic loss. This is because the future cash-flows with that counterparty are now more ‘risky’ and should be discounted at a higher discount rate.

Risk Management

A few years ago traditional approaches managing credit risk involved the use of credit limits, netting agreements and collateral. These traditional techniques, whilst useful at the time, have proved inadequate for the range of capital market products that are now traded. Stochastic internal capital models have been developed in recent years by major financial institutions that involve new mathematical and statistical techniques to deal with (i) the ever increasing needs for the quantification of traded instruments, and (ii) the management of risk and capital under the Basel II regime.

Credit Risk vs Market Risk

Credit Risk Modelling poses a number of challenges and is more difficult to model than Market Risk (i.e. investment assets):

- The lack of a Liquid Market makes it difficult to price products for specific entities or time periods. The time horizon tends to be longer than for market risk and there is a requirement for more refined simulation techniques for the evolution of the exposures.
- “True” probabilities of default within the market cannot be observed but need to be calculated based on either historical experience of credit ratings, deduced from a process involving some form of market prices; or on some subjective credit assessment criteria.
- Default Correlations are difficult to measure, an issue for risk aggregation.
- Capital requirement calculations at the extreme loss percentiles involve examination of the tails of asymmetric fat-tailed loss distributions.

Summary of Paper

In what follows extracts are either taken from the paper or summaries given of the relevant sections. The practical concerns like data, parameterisation and modelling issues for the modelling approaches put forward are addressed in full in the paper and not in this overview.

3. Reinsurance Credit Risk

This section begins with a definition of reinsurance credit risk and then goes on to discuss in detail twelve different risk factors be it reinsurance default, credit migration, credit concentration, willingness to pay to issues such as model and parameter risk.

Why it is important to understand the reinsurance credit risk loss dynamics

The potential for uncollectible reinsurance has always been a major concern for both insurers and reinsurers. For some companies, reinsurance recoveries represent one of the largest assets on their balance sheet. Some of the more obvious reasons are listed:

- Regulatory Economic Capital Requirements
- Economic Capital Modelling
- Minimising the risk of insolvency
- Risk Management Best Practices

However there are wider applications involving:

- Reinsurance Structuring
- Reinsurance Placement Evaluation
- Capital Markets Solutions

The paper discusses each of these seven topics and in particular for (i) reinsurance structuring and (ii) reinsurance placement evaluation describes how reinsurance credit risk modelling, especially for longer-tail business could play a useful role in the decision making process.

Finally there is a listing of practical steps a company can take through its risk management practices to manage reinsurance counterparty risk in an efficient way.

4. The Loss Process

Binary Loss

We begin by thinking in terms that a loss occurs only in the event of the default of an obligor. This can be modelled as a binary event.

Mathematically speaking let Y_i be a binary variable for obligor i at some fixed time horizon T, e.g. one year. Y_i can either take the value 1 (Default) or 0 (No Default) given a non-default state at t= 0.

Probability of Default

There are at least three main approaches to the estimation of default rates:

- *Actuarial Model* - based on probabilities alone that do not infer an underlying causal or default process unlike some of the alternative model types.
- *Merton model* – an application of Merton's firm model to calculate default probabilities based on the firm's capital structure and asset return volatility.
- *Conditioning on the State of the Economy* – an econometric model that incorporates default rates that are conditional on the current state of the economy

Loss Severity

There are in principle two ways to model loss severity:

- Recovery amount is known with certainty
- Recovery amount is uncertain

The Beta Distribution is often used to model the uncertainty in the recovery value where the severity per unit of exposure can vary between 0% and 100%.

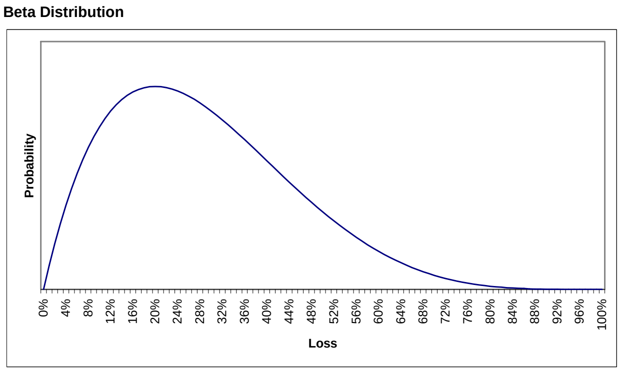
$$f(x) = \begin{cases} x^{(\alpha - 1)} \text{ x } (1 - x)^{(\beta - 1)} \text{ x } [\Gamma(\alpha + \beta) / (\Gamma(\alpha) \text{ x } \Gamma(\beta))] & \ldots\ldots\text{ for } 0 < x < 1 \\ 0 & \ldots\ldots\text{ for } x < 0 \text{ and } x > 1 \end{cases}$$

The values of μ and σ are given by:

$$\mu = \alpha / (\alpha + \beta)$$

$$\sigma^2 = (\alpha \text{ x } \beta) / [(\alpha + \beta)^2 \text{ x } (\alpha + \beta + 1)]$$

α and β can be estimated from historical data, using MLE or the method of moments. The distribution can assume a wide range of shapes.



α	2.0	E(X)	28.6%
β	5.0	σ(X)	16.0%

Loss Paradigm

There are fundamentally two principle definitions of loss, as described earlier:

- Default Mode paradigm – i.e. default only
- Marked-to-Market (or Model) paradigm – i.e. allowing for change in credit spread

This paper is only concerned with the first loss paradigm.

5. Diversification and Correlation

Introduction

One of the major challenges within credit risk management is the modelling of the correlation between default events. Higher positive default correlation will significantly increase the probability of abnormally large losses due to multiple "bad" credit events within a portfolio of many assets. These correlations are mostly influenced by factors that are by and large dependent on the state of the economy.

Asset Return Correlation vs Default Correlation

The question arises on how to correlate two or more obligors. There are issues in trying to model default correlations directly (i) be it the non-triviality of trying to simulate correlated binary variables or the (ii) lack of credible historical data in trying to estimate default correlation directly.

The way multiple defaults are generated will have a large influence on the level of economic capital. Various models will give different probabilities for multiple defaults and economic capital.

Asset Return Correlation

Two of the most important industry models such as Moody's KMV and CreditMetrics rely on the idea of 'asset return correlation' or simply asset correlation. The underlying premise here is that default occurs if the value of the obligor's assets at some time horizon, say 12 months, falls below some 'asset threshold', often interpreted as the value of the obligor's liabilities i.e. when the net asset value becomes negative.

An alternative way of thinking of this is that default occurs if the obligor's so-called asset return at time t is lower than an 'asset return threshold', at which point default occurs. Mathematically we have:

$Y_i = 1 \Leftrightarrow X_i \leq D_i \Leftrightarrow AR_i \leq K_i$
X_i = *Value of the Assets for obligor i at the end of time t.*
D_i = *Value of the Asset Threshold (or cut-off level) for obligor i at the end of time t.*
AR_i = *Asset Return for obligor i over time t.*
K_i = *Asset Return threshold for obligor i over time t, i.e. an asset return below this level will lead to default.*

A core assumption of the Moody's KMV and CreditMetrics models are that asset returns are multivariate normal which makes it easier to handle simulation processes involving correlated random variables.

The Asset Return and Default Correlation relationship

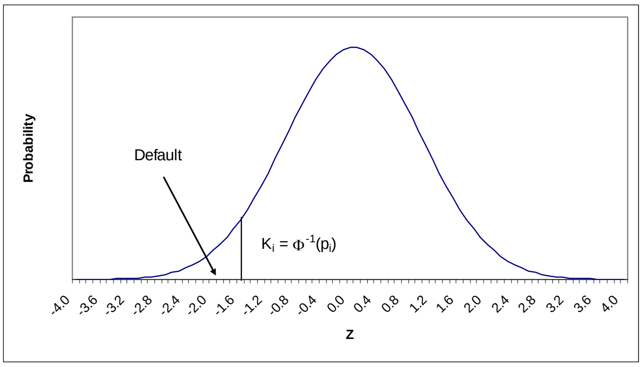
There is a direct relationship between the asset return correlation ("asset correlation") and default correlation which will be explored in the case of two obligors 1 and 2. Let us assume that (X_1, X_2) are bivariate normal.

The Default Correlation ρ_d is related to the marginal probabilities of default PD_1 and PD_2 for obligors 1 and 2 respectively (assuming a binary process $Y_i = 1$ or 0), the asset correlation ρ_A and the Joint Default Probability ("JDP") distribution PD_{12} as follows:

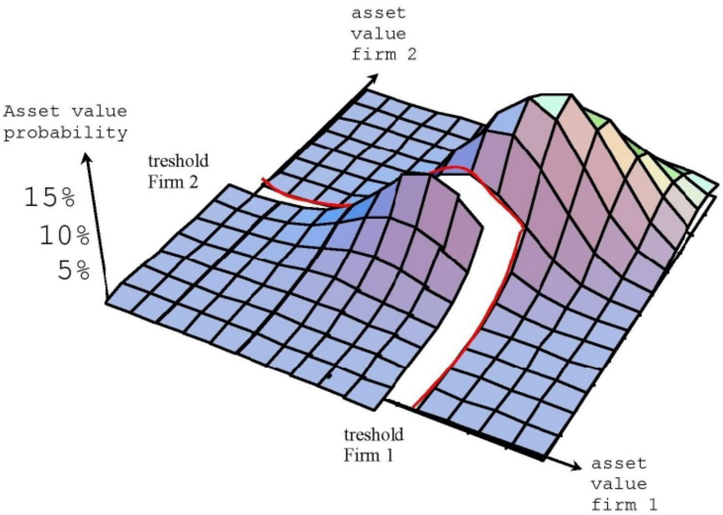
$$\rho_d = (PD_{12} - PD_1 \text{ x } PD_2) / (PD_1 \text{ x } (1 - PD_1) \text{ x } PD_2 \text{ x } (1 - PD_2))^{0.5}$$

$$PD_1 = P(Y_1 = 1) = P(X_1 \leq D_1) \text{ and } PD_{12} = P(Y_1 = 1, Y_2 = 1) = P(X_1 \leq D_1, X_2 \leq D_2)$$

Without loss of generality we can assume that the marginal asset return distributions are standard normals. The asset return threshold K_i at which default is assumed to take place is given by $K_i = \Phi^{-1}(p_i)$ where p_i is the probability of default and $\Phi(.)$ is the cumulative standard normal distribution.



The JDP of two obligors is the probability that value of their assets jointly falls below their respective thresholds at the same time i.e. the area in the bottom left corner of the bivariate normal distribution in the diagram below.



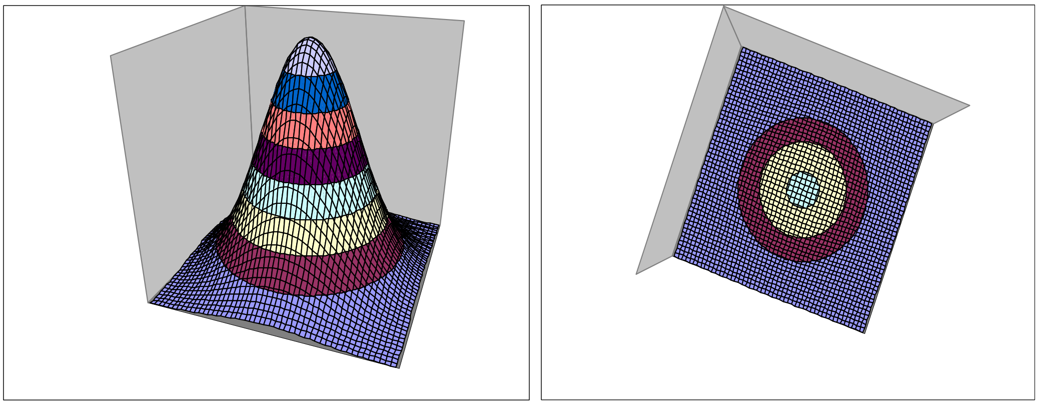
Source: GARP Sep / Oct 2006 – Economic Capital, Peter-Paul Hoogbruin

This area can be mathematically solved via the double integral calculation:

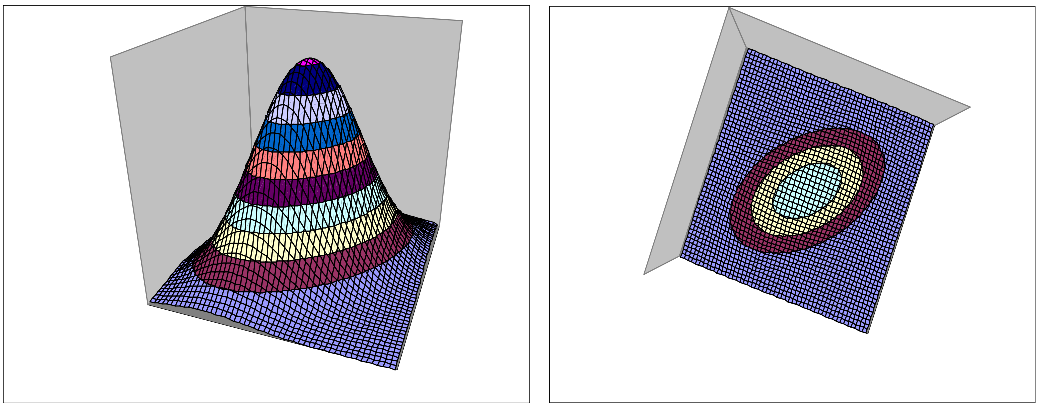
$$PD_{12} = \int_{-\infty}^{K_1}\int_{-\infty}^{K_2} (1/(2\pi(1- \rho_A^2)^{0.5}) \exp(- (x_1^2 + x_2^2 - 2\, x_1\, x_2\, \rho_A) / (2(1- \rho_A^2))) \, dx_1\, dx_2$$

Examples of the joint probability distribution for different asset correlations of 0% and 50% are shown in the following diagrams.

Joint Default Probability Distribution for ρ_A = 0%



Joint Default Probability Distribution for □A = 50%



The following table shows that the default correlation is often much lower than the assumed asset return correlation assumption. The JDP's are derived using a VBA routine that provides a numerical approximation to the double integral.

PD_1 and PD_2	**Asset Corr**	**Joint Def Prob**	**Default Corr**
0.2%	**10.0%**	0.00%	**0.31%**
0.2%	**30.0%**	0.00%	**2.05%**
0.2%	**50.0%**	0.01%	**6.93%**
0.2%	**70.0%**	0.04%	**18.61%**
1.0%	**10.0%**	0.02%	**0.95%**
1.0%	**30.0%**	0.06%	**4.64%**
1.0%	**50.0%**	0.13%	**12.12%**
1.0%	**70.0%**	0.27%	**26.06%**
10.0%	**10.0%**	1.32%	**3.54%**
10.0%	**30.0%**	2.14%	**12.67%**
10.0%	**50.0%**	3.21%	**24.58%**
10.0%	**70.0%**	4.64%	**40.47%**

6. Some important Credit Risk Models

During recent years a number of financial institutions have developed proprietary or commercially available models. A review is made of the models that have attracted a lot of interest, namely:

- Portfolio Manager (Moody's KMV) - 1993
- CreditMetrics (JP Morgan) - 1997
- CreditRisk+ (Credit Suisse Financial Products (CSFP)) – 1998
- CreditPortfolioView (McKinsey's) – 1998

The review of each model covers the common themes of:

- Model Description
- Probability of Default
- Loss Severity
- Loss Paradigm
- Correlation
- Suitability for reinsurance credit risk modelling

7. Making Use of Rating Agency Studies

A review is made of some of the rating agency outputs that often form the basis of modeling assumptions. The strengths and weaknesses of the data are discussed.

Key assumptions where use is often made of rating agency data are:

- Probability of default
- Loss given default (and variance)
- Correlation
- Transition matrices (or probabilities)

Of particular interest is the cyclical nature of both default rates and recovery rates and the repeated academic studies the show a negative correlation between default rate and recovery rate which can be overlooked in modelling. One possible explanation being that economic conditions that cause defaults to rise may cause recovery rates to decline and vice-versa.

8. Willingness to Pay

Willingness to Pay addresses the risk that even though a reinsurer is able to pay claims as reflected in their claims paying ability rating they may not be willing to pay or to do so in a timely manner.

Willingness to pay and dispute risk are important consideration within the modelling process and to this extent the topic is discussed in some detail.

9. Modelling the Reinsurance Credit Risk Loss Distribution

Monte Carlo Simulation

Monte Carlo simulation is often required to derive the probability distribution of losses. For a medium to large size portfolio the number of possible default combinations is extremely large.

Monte Carlo simulation will converge to the ‘real’ result as the number of simulations increase. But, the convergence can be extremely slow especially if (i) we require losses at the 99.5% loss level over 12-months to determine the economic capital and (ii) we have a very high grade portfolio with very low probabilities of default.

Stochastic Exposure

One of the interesting challenges in modeling reinsurance credit risk is the stochastic nature of the exposures which consist of two quite distinct components, namely (i) exposure arising from writing new business and (ii) prior year reinsurance recoveries.

Current Year Exposure:

Most insurance companies and Lloyd’s syndicates are able to model the direct ceded loss impact from gross loss scenarios under their current reinsurance programmes thus introducing one source of direct dependency in the modeling process.

Prior Year Exposure:

Credit risk modeling of prior year exposures has characteristics that are different to new business. Reserve run-off credit risk is on a known asset at the beginning of the year, however there will be volatility arising in each future year from the run-off of the reserves. Furthermore, there are additional issues such as the level of granularity of data in modelling net from gross losses and the robustness or otherwise of techniques used to model loss reserve volatility; not forgetting credit migration.

Probability of Default Assumptions

The selection of ‘stressed’ default rates is discussed, covering topics such as appropriate loadings to corporate bond default rates, impairment rates, duration, adjustments for the economic cycle and ‘critical’ reinsurance ratings.

Loss Given Default Assumptions

There are two options here, to use either a fixed loss percentage or one that can vary about a fixed amount. If the latter then further assumptions are needed for the loss distribution and appropriate parameters.

Use is made of rating agency data, insurance run-off data and work under Solvency II to determine expected LGD and standard deviations) that vary by rating.

Correlation Assumptions

There are two selection issues:

- Form of the Multivariate asset distribution
- Correlation assumptions between reinsurer pairs

Use of the multivariate normal distribution to deal with asset correlation may be reasonable for some corporate sectors but this assumption but may be viewed as weak when considering the insurance industry where there is a lot of interdependence through the use of reinsurance which shares aggregate loss exposure arising from catastrophes or other forms of loss. A more conservative assumption would be to use a 'fatter-tailed' distribution. The multivariate t-distribution is one possible choice and this is discussed in the paper.

Determining asset correlation assumptions between reinsurer pairs is a challenge. One could use techniques that are described in section 5 of the paper (but not in this summary) or something more pragmatic involving a classification process.

For example one could classify each reinsurer by one of six classes A to F which could be based on (i) Geographical location (e.g. Bermuda, US, Europe), (ii) Size (Small, Large), (iii) Level of Diversification or some combination of the three. The resulting 6 x 6 correlation matrix could then be populated by one of four correlation rankings of 0, Low (L), Medium (M) or High (H). Correlation parameters being determined from a broader study of data together with say expert opinion.

Finally one could also use simple economic factor models to determine dependent asset returns directly for each reinsurer as a weighted average of index values which are in themselves correlated. This is explored further in the paper.

Economic Capital

Monte Carlo simulation is used to determine the loss distribution. The simulation algorithm is consistent with the asset return correlation framework.

Mathematically, within each simulation i of say N, 'dependent asset returns' for each reinsurer are generated from a vector of 'independent asset returns' using a cholesky decomposition of the asset return correlation matrix. For each reinsurer, default occurs if their 'dependent asset return' falls below their 'asset return threshold' which is the default trigger and is derived from the probability of default. The aggregate loss for each simulation being equal to the sum of reinsurer losses for those that default.

10. Numerical Examples

This section focuses on numerical examples derived from a VBA model that incorporates the modelling ideas presented.

Modelling Assumptions

The probability of default and loss given default are assumed to be independent processes and in addition in the event of default the LGD is assumed to be stochastic.

Results

Various results have been produced using the ‘asset return’ modelling algorithm. Sample output is shown in the following table with key variations for:

- Average Rating (for simplicity all reinsurers are assumed to have the same rating)
- Asset Return Correlation (same for all reinsurer pairs)
- Mean term of liabilities (for PD)
- Stress PD – i.e. with and without % loading.

The modelling time horizon is 12-months.

OUTPUTS

Exposure	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000
No. Reinsurers	25	25	25	25	25	25	25	25
Average Rating	A	A	BBB	BBB	BB	A	BBB	BB
Correlation	25%	50%	25%	50%	25%	25%	25%	25%
Mean Term	1	1	1	1	1	4	4	4
No. Simulations	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
Stress Load	0%	0%	0%	0%	0%	0%	0%	0%

	CreditLoss	**CreditLoss**	**CreditLoss**	**CreditLoss**	**CreditLoss**	**CreditLoss**	**CreditLoss**	**CreditLoss**
EC(VaR 99.5%)	**69,095**	**70,348**	**111,264**	**143,561**	**204,773**	**138,212**	**205,251**	**618,459**
as % Exposure	**2.8%**	**2.8%**	**4.5%**	**5.7%**	**8.2%**	**5.5%**	**8.2%**	**24.7%**
EC(TVaR 99.5%)	**89,309**	**132,802**	**155,005**	**243,776**	**269,435**	**202,758**	**292,184**	**744,142**
as % Exposure	**3.6%**	**5.3%**	**6.2%**	**9.8%**	**10.8%**	**8.1%**	**11.7%**	**29.8%**

Minimum	0	0	0	0	0	0	0	0
Maximum	218,182	637,705	268,541	721,894	466,408	652,700	728,001	1,340,380
Expected	**1,007**	**1,027**	**3,594**	**3,292**	**13,001**	**6,155**	**21,357**	**105,316**
Std Dev	8,413	12,072	17,676	22,686	37,090	24,167	49,818	140,228

10.0 %ile	0	0	0	0	0	0	0	0
20.0 %ile	0	0	0	0	0	0	0	0
30.0 %ile	0	0	0	0	0	0	0	0
40.0 %ile	0	0	0	0	0	0	0	32,463
50.0 %ile	0	0	0	0	0	0	0	62,501
60.0 %ile	0	0	0	0	0	0	0	84,884
70.0 %ile	0	0	0	0	0	0	0	129,204
80.0 %ile	0	0	0	0	0	0	42,268	184,250
90.0 %ile	0	0	0	0	60,464	0	78,310	285,420
95.0 %ile	**0**	**0**	**17,410**	**0**	**84,299**	**56,482**	**120,931**	**394,330**
99.0 %ile	50,972	41,278	86,928	89,577	175,737	106,138	226,607	629,756
99.5 %ile	**70,102**	**71,375**	**114,858**	**146,852**	**217,775**	**144,367**	**295,401**	**723,775**
99.9 %ile	95,217	155,805	193,529	298,483	323,851	240,820	403,766	925,657
100.0 %ile	218,182	637,705	268,541	721,894	466,408	652,700	728,001	1,340,380

11. Extension to Multi-year modelling

When consideration is given to an extension of the 12-month time-frame to a multi-year framework there are additional aspects that need to be considered. There are in particular two important effects:

- Default probabilities vary over time – i.e. there is probability drift.
- Stochastic exposures become even more important as the variance of the underlying variables increase over time

Drift in Default Probabilities

The default probability is not constant but can change significantly over time. A company with a good credit rating has a higher probability of downgrade than of upgrade and vice versa. We have mean reversion in credit ratings. The default rate of investment grade companies will increase over time as the calculations will reflect the increased likelihood of downgrade to a lower rating with a higher default rate.

Credit Migration

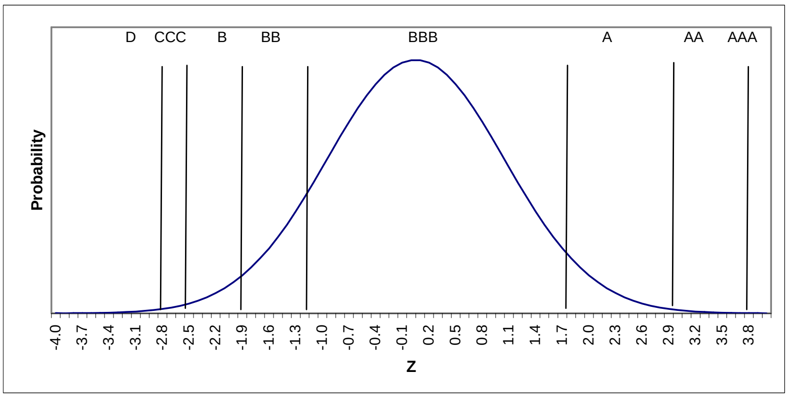
If one is modelling over a multi-year time frame and for each future 12-month period there is a need to simulate what rating-state a company might be in there is a very appealing method that takes advantage of the ‘asset return’ framework. The rating being determined by the simulated value of the ‘dependent asset return’.

Correlated Credit Migration:

The intuition behind this approach is that a large asset return will lead to an upgrade, whereas a small asset return could result in a downgrade or default. Also, the correlation behind default events will apply to credit migrations as well. Moreover, there is an increased likelihood that the ratings of two reinsurers will move together either upwards or downwards and thus is a convenient mathematical representation of economic (or underwriting cycle) impacts.

Below are the asset return thresholds for a currently ‘BBB’ rated company. The calculations involved using an S&P transition matrix are shown in the paper.

Credit Migration Thresholds



Current	**BBB**

t=1	**D**	**CCC**	**B**	**BB**	**BBB**	**A**	**AA**	**AAA**
Z	-2.82	-2.51	-2.00	-1.19	1.73	2.95	3.72	+ Infinity
Φ(.)	0.24%	0.60%	2.27%	11.69%	95.86%	99.84%	99.99%	100.00%

12. Monte Carlo Acceleration Methods

The problem with Standard Monte Carlo simulation is that the error term decreases as $N^{-0.5}$ where N is the number of simulations, i.e. an increase of 100x the number of simulations is needed to increase the accuracy by 10x.

There are various techniques that can be used to overcome some of these issues. These are discussed in the paper:

- Stratified Sampling
- Stratified Sampling – Latin Hypercube
- Low-Discrepancy Sequences
- Importance Sampling
- Control Variates