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# The Modelling of Reinsurance Credit Risk

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## Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues

## Reinsurance Credit Risk
### What is Reinsurance Credit Risk

- Definition:
  "The risk of loss if another party fails to perform its obligations or fails to perform them in a timely manner."
    - Key counterparties include reinsurers, brokers, insureds, and reinsureds
- Examples of Risk Factors:
  - Reinsurance Failure (of individual reinsurers)
  - Credit Deterioration (of individual reinsurers)
  - Bad Debt provision inadequacy
  - Reinsurance Recovery exposure
  - Correlation in extreme loss scenarios
  - Credit Concentration
  - Duration of Recoveries
  - Willingness to Pay / Dispute Risk
  - Treatment of intra-group reinsurance
  - Non-reinsurance related credit risk



## Reinsurance Credit Risk
### Why it is important to Understand

- Regulatory Capital Requirements
  - ICA Capital – VaR (@99.5%) over 12-months
  - SCR (Solvency II) Capital – same risk measure and probability
- Economic Capital Modelling
  - As above but reflecting say assumptions for a desired credit rating
- Minimising the risk of insolvency
  - Related to the above
- Risk Management Best Practices
  - An understanding of risks and issues might translate into better practices
  - e.g. Regular aged debt analysis → highlight future potential issues with certain reinsurers ('Willingness to Pay')
- Capital Markets Solutions
  - Securitisation and risk transfer products
  - e.g. Aspen Re Credit Wrap and Merlin (Hannover Re) transactions (2007)

## Reinsurance Credit Risk
### Why it is important to Understand

- Reinsurance Purchasing decision making:
  - Can play a part in determining the optimal reinsurance structure
  - Modification in the NPV of the net loss and underwriting profit distributions
    - Impact greatest at the highest loss percentiles
  - More relevant for longer-tail lines:
    - Reserves take a few years to run-off (albeit declining exposure)
    - Not a big number in year 1 – highly rated companies
    - Yesterday's 'A' rated companies suffer downgrades over time
  - In addition at the extreme loss percentiles
    - Very Large Property Cat Loss → increase in reinsurance default rates

- Reinsurance Panel Evaluation:
  - Given a new reinsurance program how should it be placed
    - 100% with one reinsurer
    - Smaller shares with others (Rating ?)
  - Benefits of Diversification → Credit Risk
  - Similar considerations when making reinsurance purchasing decisions



## Reinsurance Credit Risk
### Managing Reinsurance Counterparty Risk

- Risk Management Practices of ways to manage the Risk :
  - Greater risk retention – i.e. reinsure less
  - Establishment of an established credit risk committee, which reviews the credit ratings of reinsurers, brokers and coverholders on a regular basis.
  - Focus on reinsurer's 'Willingness to Pay' and not just credit rating
  - The instigation of formal procedures for reinsurance purchasing
  - Having a formal policy and procedures for the evaluation, usage and monitoring of new and existing reinsurance security.
  - As above but the process to embrace new and existing brokers.
  - Regular review of concentrations within individual custodians, group companies, or geographic locations.
  - The monitoring and reporting of historical accumulated exposures
  - Regular aged debt analysis and reporting
  - Regular internal audit reviews of controls over third party credit risk
  - Downgrade clauses in reinsurance treaties.

## Topics

- Reinsurance Credit Risk
- **The Loss Process**
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues



## The Loss Process
### Expected Loss ("EL") and Unexpected Loss ("UL")

- Let $Y_i$ be a binary variable for obligor i at time 1 year.
  - $Y_i$ takes values - 1 (Default) or 0 (No Default) given non-default state at t= 0.

- $EL_i = PD_i \times EAD_i \times LGD_i$
- $UL_i = [PD_i \times (1 - PD_i)]^{1/2} \times EAD_i \times LGD_i$ *($EAD_i$ and $LGD_i$ constant)*
  - EAD = Exposure at Default
  - LGD = Loss Given Default (i.e. severity per unit of exposure)
  - PD = Probability of Default
- Otherwise:

$$UL_i = [\, PD^2_i \times EAD^2_i \times \sigma^2_{LGDi} + EAD^2_i \,.\, LGD^2_i \,.\, \sigma^2_{PDi} + LGD^2_i \,.\, PD^2_i \times \sigma^2_{EADi} +$$

$$+ PD^2_i \times \sigma^2_{EADi} \times \sigma^2_{LGDi} + EAD^2_i \times \sigma^2_{LGDi} \times \sigma^2_{PDi} + LGD^2_i \times \sigma^2_{PDi} \times \sigma^2_{EADi}$$

$$+ \sigma^2_{PDi} \times \sigma^2_{EADi} \times \sigma^2_{LGDi} \,]^{0.5}$$

  - This further assumes that $PD_i$, $EAD_i$ and $LGD_i$ are independent

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# The Loss Process
## Expected Loss ("EL") and Unexpected Loss ("UL")

| Obligor | PD | LGD | EAD | EL | UL |
|---|---|---|---|---|---|
| Obligor 1 | 2.0% | 40% | 2,000 | 16.0 | 131.7 |
| Obligor 2 | 5.0% | 60% | 2,000 | 60.0 | 283.5 |
| Portfolio | 3.80% | 50% | 4,000 | 76.0 | 319.8 |

| | |
|---|---|
| Asset Correlation | 25% |
| Joint Default Prob | 0.28% |
| Default Correlation | 6.03% |

| | |
|---|---|
| Diversification Benefit | 95.5 |
| as % of ($UL_1 + UL_2$) | 23.0% |

PD = Probability of Deafult
LGD = Loss Given Default (%)
EAD = Exposure at Default
EL = Expected Loss
UL = Unexpected Loss

$$\mathbf{UL_i = EAD_i \times [LGD^2_i \times PD_i \times (1 - PD_i) + PD_i \times LGD_i \times (1 - LGD_i) / 4]^{0.5}}$$

$$\mathbf{UL_T = (UL^2_1 + UL^2_2 + 2 \times \rho_d \times UL_1 \times UL_2)^2}$$

$\rho_d$ = *Default correlation between obligor 1 and obligor 2*

$\sigma^2_{PDi} = PD_i \times (1 - PD_i)$

$\sigma^2_{LGDi} \sim LGD_i \times (1 - LGD_i) / 4$ *(and assuming a Beta Distribution)*

$EAD_i$ = *constant*



# The Loss Process
## Probability of Default

- Actuarial Model
  - Based on historical default probabilities over time (e.g. rating agency studies)
  - Do not infer an underlying causal or default process
  - Default probabilities assigned to each rating class
- Merton Model ('Structural Model')
  - Based on the firm's capital structure and asset return volatility
  - Firm defaults when value of assets < value of liabilities at maturity
  - Equity is a call option on the asset of firm – Black-Scholes framework
- Conditioning on the State of the Economy
  - Default probabilities based on an econometric model
    - Conditional on the state of the economy
  - Similar to the actuarial model
- Market Prices of Traded Debt ('Reduced Form Models')
  - Default probabilities and Loss amount derived from traded debt
  - If constructed properly can be used to extract implied parameters from
    - Debt prices, Subordinated prices and Credit Derivative prices

## The Loss Process
### Loss Severity

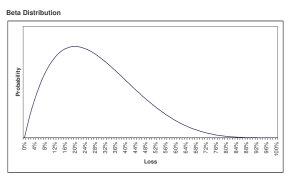
- Two ways of modelling loss severity
  - Recovery % amount is known with certainty
  - Recovery % amount is uncertain
- Recovery % amount is uncertain
  - Beta Distribution is often used to model Loss Severity

$$f(x) = x^{(\alpha - 1)} \times (1 - x)^{(\beta - 1)} \times [\Gamma(\alpha + \beta) / \Gamma(\alpha) \times \Gamma(\beta)] \; \ldots\ldots \text{ for } 0 < x < 1$$

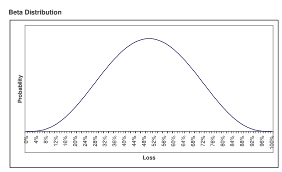
$$f(x) = 0 \; \ldots\ldots\ldots \text{ for } x < 0 \text{ and } x > 1$$

$$\mu = \alpha / (\alpha + \beta)$$

$$\sigma^2 = (\alpha \times \beta) / [(\alpha + \beta)^2 \times (\alpha + \beta + 1)]$$




α 2.0 E(X) 28.6%
β 5.0 σ(X) 16.0%




α 4.0 E(X) 50.0%
β 4.0 σ(X) 16.7%

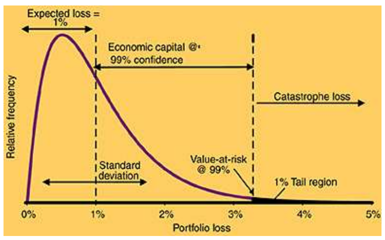


## The Loss Process
### Credit Exposure

- Banking – financial assets e.g. fixed-income, equities, derivatives
  - Crucial assumptions for volatilities, dependencies and correlation
- Reinsurance Exposures are Stochastic
  - NPV of Reinsurance Recoveries – Amount (~ Gross) and Payment patterns
  - Interest rates – could be stochastic (NPV - Economic Value)
  - Prior year and Current year – different loss dynamics
- Reinsurance – Current Year Exposure
  - More accurate modelling of Stochastic Gross → Net process
    - Gross – Attritional and Large (Frequency / Severity)
    - Detailed knowledge of current reinsurance structures
  - Sampling error could be an issue
    - High minimum rating criteria (say ‘A-’ and above) – very low default rates
- Reinsurance – Prior Year Exposure
  - Mix of reinsurers different to Current year
  - Average credit rating likely to be lower (rating downgrades)
  - Gross to Net Process – less accuracy
    - Typical ‘Actuarial’ Reserving techniques (approx methods)
    - Typical Reserve Volatility techniques (e.g. Bootstrap)

## The Loss Process
### Loss Paradigms and Economic Capital

- Default Loss Paradigm
  - A loss is only recognised on default
- Mark-to-Market Loss Paradigm
  - A loss (or gain) also occurs if there is a change in the credit quality
  - Values being determined by the discounting of cash flows using credit curve
- Mark-to-Model Loss Paradigm
  - A slight variation on the Mark-to-Market paradigm
  - None or limited secondary market – Value estimated by model
- Economic Capital






## The Loss Process
### Credit Risk Modelling Challenges (vs Market Risk)

- The lack of a liquid market
  - Makes it difficult to price products
  - Time horizon tends to be longer than for market risk
  - Requirement for more refined simulation techniques (evolution of exposures)
- “True” probabilities cannot be observed - need to be estimated
  - Historical experience of credit ratings
  - Market Prices
  - Subjective assessment criteria
- Default Correlation are difficult to measure (Risk Aggregation)
  - Sparse data
- Capital Adequacy calculations
  - Tails of asymmetric fat-tailed distributions
- Reinsurance Credit Risk Modelling
  - As above but additional issues

## Topics

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- Diversification and Correlation
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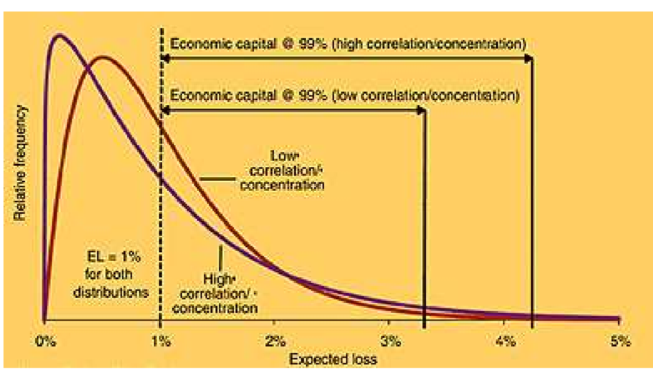
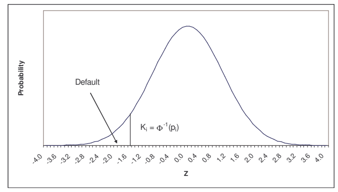
## Diversification and Correlation
### Asset Return vs Default Correlation




- Higher default correlation will significantly increase the probability of abnormally large losses due to multiple "bad" credit events
- Correlations mostly influenced by macroeconomic factors - state of the economy.

## Diversification and Correlation
### Asset Return and Default Correlation relationship




**$Y_i = 1 \Leftrightarrow X_i \leq D_i \Leftrightarrow AR_i \leq K_i$**

Where:

$X_i$ = Value of the Assets for obligor i at the end of time t.

$D_i$ = Value of the Asset Threshold (or cut-off level) for obligor i at the end of time t.

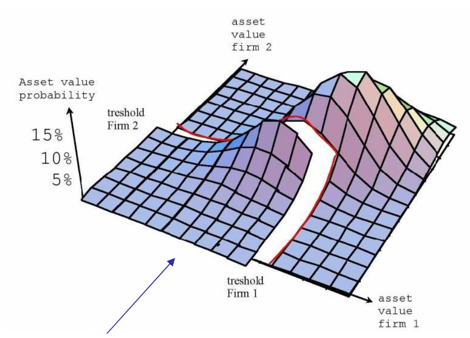
$AR_i$ = Asset Return for obligor i over time t.

$K_i$ = Asset Return threshold for obligor i over time t

**Number of defaults within a portfolio of M obligors = $\sum_{i=1}^{M} Y_i$**



## Diversification and Correlation
### Asset Return and Default Correlation relationship



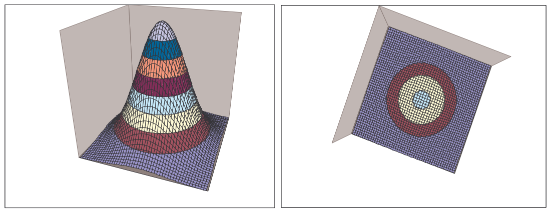

Assume that the joint asset return distribution is bi-variate normal

- Joint Default Probability = Probability that value of their assets jointly falls below their respective thresholds at the same time
  - Bottom left corner of the bi-variate normal distribution

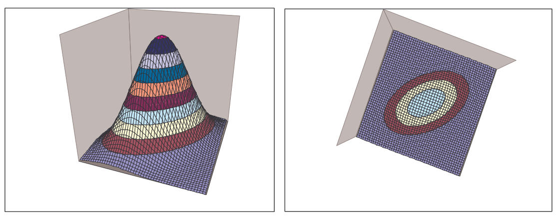
$$PD_{12} = \int_{-\infty}^{K_1}\int_{-\infty}^{K_2} (1/(2\pi(1-\rho_A^2)^{0.5}) \exp(-(x_1^2 + x_2^2 - 2 x_1 x_2 \rho_A) / (2(1-\rho_A^2))) dx_1 dx_2$$

## Diversification and Correlation
### Asset Return and Default Correlation relationship

**Joint Default Probability Distribution for $\rho_A = 0\%$**



**Joint Default Probability Distribution for $\rho_A = 50\%$**





## Diversification and Correlation
### Asset Return and Default Correlation relationship

$$\rho_d = (PD_{12} - PD_1 \times PD_2) / (PD_1 \times (1 - PD_1) \times PD_2 \times (1 - PD_2))^{0.5}$$

*Where:*

$PD_1 = P(Y_1 = 1) = P(X_1 \leq D_1)$ *and*

$PD_{12} = P(Y_1 = 1, Y_2 = 1) = P(X_1 \leq D_1, X_2 \leq D_2)$

## Diversification and Correlation
### Asset Return and Default Correlation relationship

| $PD_1$ and $PD_2$ | Asset Corr | Joint Def Prob | Default Corr |
|---|---|---|---|
| 0.2% | 10.0% | 0.00% | 0.31% |
| 0.2% | 30.0% | 0.00% | 2.05% |
| 0.2% | 50.0% | 0.01% | 6.93% |
| 0.2% | 70.0% | 0.04% | 18.61% |
| | | | |
| 1.0% | 10.0% | 0.02% | 0.95% |
| 1.0% | 30.0% | 0.06% | 4.64% |
| 1.0% | 50.0% | 0.13% | 12.12% |
| 1.0% | 70.0% | 0.27% | 26.06% |
| | | | |
| 10.0% | 10.0% | 1.32% | 3.54% |
| 10.0% | 30.0% | 2.14% | 12.67% |
| 10.0% | 50.0% | 3.21% | 24.58% |
| 10.0% | 70.0% | 4.64% | 40.47% |

- Implied default correlation is much lower than the asset correlation
  - Values - VBA routine for the numerical approximation to the integral



## Diversification and Correlation
### One-Factor Modelling alternative

$$AR_i = [R^2_i]^{0.5} \times X + [1 - R^2_i]^{0.5} \times \varepsilon_i$$

Where:

$\varepsilon_i$ = Obligor Specific (Non-Systematic) component

X = State of the Economy

$R^2_i$ = Obligor asset return correlation with the Economy

$\rho_A = \text{Corr}(AR_1, AR_2) = [R^2_1]^{0.5} \times [R^2_2]^{0.5}$

*Example:*

$R^2_1 = 50\%$ and $R^2_2 = 25\%$ then $\rho_A = 35.4\%$

- Values of $R^2$ can vary from 15% or so for (SME) up to 60% for large multinationals
- Can also consider multi-factor models – country, industry indices etc.
- Large Portfolio - Obligor-specific part can be diversified away

## Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues



## Rating Agency Studies
### Cumulative Probability of Default

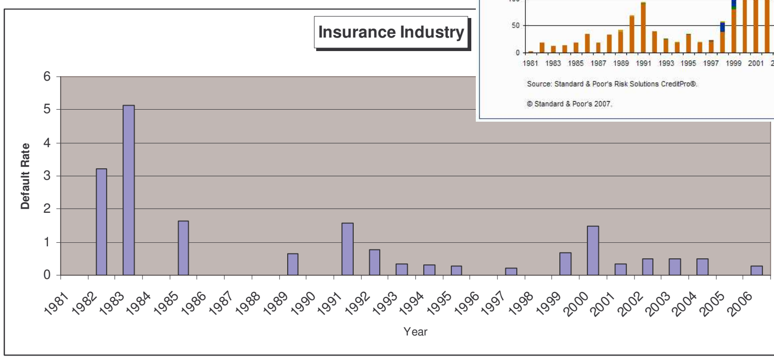
| Cumulative Average Default Rates By Rating (1981-2006) (%) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Time Horizon (Years) | | | | | | | | | |
| Rating | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| AAA | 0.00% | 0.00% | 0.09% | 0.19% | 0.29% | 0.43% | 0.50% | 0.62% | 0.66% | 0.70% |
| AA+ | 0.00% | 0.07% | 0.07% | 0.14% | 0.21% | 0.29% | 0.37% | 0.37% | 0.37% | 0.37% |
| AA | 0.00% | 0.00% | 0.00% | 0.09% | 0.21% | 0.29% | 0.39% | 0.53% | 0.65% | 0.78% |
| AA- | 0.02% | 0.09% | 0.21% | 0.34% | 0.48% | 0.65% | 0.81% | 0.95% | 1.07% | 1.20% |
| A+ | 0.05% | 0.10% | 0.26% | 0.47% | 0.63% | 0.80% | 1.02% | 1.18% | 1.38% | 1.57% |
| A | 0.07% | 0.19% | 0.32% | 0.44% | 0.63% | 0.85% | 1.06% | 1.29% | 1.52% | 1.85% |
| A- | 0.06% | 0.22% | 0.35% | 0.53% | 0.79% | 1.11% | 1.57% | 1.87% | 2.14% | 2.33% |
| BBB+ | 0.16% | 0.50% | 1.00% | 1.43% | 1.92% | 2.46% | 2.86% | 3.23% | 3.74% | 4.14% |
| BBB | 0.25% | 0.59% | 0.93% | 1.52% | 2.14% | 2.72% | 3.25% | 3.84% | 4.34% | 4.90% |
| BBB- | 0.33% | 1.11% | 1.94% | 3.04% | 4.07% | 5.04% | 5.77% | 6.47% | 7.00% | 7.67% |
| BB+ | 0.57% | 1.54% | 3.12% | 4.62% | 5.94% | 7.36% | 8.65% | 9.25% | 10.32% | 11.18% |
| BB | 0.86% | 2.67% | 4.92% | 6.99% | 9.02% | 10.92% | 12.36% | 13.73% | 14.81% | 15.70% |
| BB- | 1.54% | 4.47% | 7.62% | 10.72% | 13.39% | 15.86% | 17.76% | 19.68% | 21.34% | 22.57% |
| B+ | 2.70% | 7.46% | 12.04% | 15.91% | 18.75% | 20.87% | 22.86% | 24.53% | 25.95% | 27.41% |
| B | 7.10% | 14.23% | 19.47% | 23.21% | 25.77% | 28.03% | 29.45% | 30.56% | 31.48% | 32.48% |
| B- | 10.11% | 18.61% | 24.89% | 29.10% | 32.20% | 34.48% | 36.44% | 37.67% | 38.44% | 38.94% |
| CCC/C | 26.29% | 34.73% | 39.96% | 43.19% | 46.22% | 47.49% | 48.61% | 49.23% | 50.95% | 51.83% |

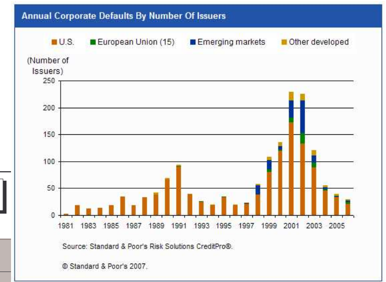
Sources: Standard & Poor's Global Fixed Income Research & Standard & Poor's CreditPro

- There are some inconsistencies by rating within term
  - Top-left: Higher rating, shorter time horizon
  - There are also some zero entries
  - Function of the methodology - Static Pool Methodology
- Default Rates need to be smoothed (See later)
- Corporate Debt – Adaptability for reinsurance default process ?

## Rating Agency Studies
### Annual Corporate Default Rates

- Default Rates are very cyclical
- There is no obvious relationship between the pattern of Insurance industry defaults and those of other industry groupings.

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## Rating Agency Studies
### Transition Matrices

| Global Average Transition Rates (1981-2006) (%) - 1 Year | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| From/To | AAA | AA | A | BBB | BB | B | CCC/C | D | NR |
| AAA | 88.34 | 7.84 | 0.47 | 0.09 | 0.09 | - | - | - | 3.17 |
| AA | 0.59 | 87.31 | 7.54 | 0.57 | 0.06 | 0.10 | 0.02 | 0.01 | 3.79 |
| A | 0.05 | 2.00 | 87.39 | 5.47 | 0.40 | 0.15 | 0.02 | 0.06 | 4.46 |
| BBB | 0.01 | 0.15 | 3.98 | 84.17 | 4.14 | 0.73 | 0.16 | 0.24 | 6.42 |
| BB | 0.03 | 0.06 | 0.22 | 5.18 | 75.71 | 7.20 | 0.84 | 1.07 | 9.69 |
| B | - | 0.05 | 0.18 | 0.30 | 5.78 | 72.77 | 4.10 | 4.99 | 11.83 |
| CCC/C | - | - | 0.26 | 0.39 | 1.10 | 11.15 | 47.49 | 26.29 | 13.34 |

| Global Average Transition Rates (1981-2006) (%) - 3 Years | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| From/To | AAA | AA | A | BBB | BB | B | CCC/C | D | NR |
| AAA | 68.39 | 18.98 | 2.55 | 0.40 | 0.12 | 0.03 | 0.03 | 0.09 | 9.41 |
| AA | 1.41 | 66.46 | 18.06 | 2.38 | 0.41 | 0.25 | 0.02 | 0.10 | 10.92 |
| A | 0.10 | 4.57 | 67.34 | 12.21 | 1.52 | 0.62 | 0.11 | 0.32 | 13.21 |
| BBB | 0.04 | 0.47 | 9.09 | 60.55 | 8.06 | 2.33 | 0.43 | 1.32 | 17.72 |
| BB | 0.05 | 0.10 | 0.81 | 11.33 | 43.82 | 11.87 | 1.54 | 5.92 | 24.57 |
| B | 0.01 | 0.07 | 0.45 | 1.30 | 11.01 | 37.08 | 4.34 | 17.04 | 28.71 |
| CCC/C | - | - | 0.30 | 1.06 | 2.43 | 14.25 | 13.57 | 42.61 | 25.78 |

- Largest values are along the diagonal
  - Values fall off very quickly moving off the diagonal
- Investment Grade companies tend to exhibit lower ratings volatility
- Transition matrices are based on historical rating changes
  - There is volatility in transition rates from year to year – macroeconomic etc.

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## Rating Agency Studies
### Transition Matrices – Conditional vs Unconditional

**Comparison Of Conditional Versus Unconditional Transition Matrices—One Year (1981-2006) (%)**

| From / To | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- | BB+ | BB | BB- | B+ | B | B- | CCC/C | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AA+ | 0.30 | 0.91 | 1.58 | 1.15 | 2.59 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | N.A. | N.A. | N.A. | 0.00 | N.A. | N.A. | N.A. | N.A. |
| AA | 0.00 | 0.36 | 0.94 | 1.34 | 2.31 | 1.49 | 4.34 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | N.A. | N.A. | 0.00 | 0.00 | N.A. |
| AA- | 0.00 | 1.30 | 0.51 | 0.89 | 1.63 | 2.25 | 1.30 | 0.75 | 1.30 | 0.00 | 0.00 | N.A. | N.A. | 0.00 | 5.22 | 0.00 | N.A. | 0.00 |
| A+ | N.A. | 0.00 | 0.29 | 0.37 | 0.92 | 1.59 | 1.74 | 3.04 | 1.52 | 4.45 | 0.00 | 0.00 | 0.00 | 2.23 | 0.00 | 0.00 | N.A. | 3.71 |
| A | 0.00 | 0.00 | 0.80 | 1.34 | 0.56 | 0.92 | 1.84 | 2.18 | 0.87 | 2.15 | 2.41 | 0.80 | 1.21 | 1.10 | 0.00 | 0.00 | 0.00 | 2.01 |
| A- | 0.00 | 0.00 | 0.00 | 1.61 | 0.37 | 0.68 | 0.93 | 1.38 | 1.94 | 2.15 | 1.43 | 2.64 | 5.36 | 0.00 | 4.29 | 0.00 | 0.00 | 2.15 |
| BBB+ | N.A. | 0.00 | 0.00 | 1.25 | 2.63 | 0.30 | 0.38 | 0.91 | 1.61 | 2.61 | 3.39 | 3.25 | 1.25 | 0.36 | 1.36 | 0.00 | 1.25 | 1.50 |
| BBB | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.89 | 0.95 | 0.32 | 0.95 | 2.09 | 1.57 | 2.44 | 3.33 | 1.17 | 0.67 | 0.00 | 5.59 | 4.14 |
| BBB- | 0.00 | N.A. | 0.00 | 0.00 | 0.00 | 1.17 | 1.41 | 0.63 | 0.45 | 0.89 | 2.12 | 2.59 | 2.03 | 2.33 | 2.35 | 0.73 | 1.85 | 1.55 |
| BB+ | 1.75 | 7.02 | N.A. | 0.00 | 0.00 | 0.00 | 2.81 | 0.00 | 0.44 | 0.48 | 0.99 | 1.59 | 1.72 | 1.87 | 1.53 | 4.21 | 1.65 | 3.90 |
| BB | N.A. | N.A. | 3.12 | N.A. | N.A. | 0.00 | 2.34 | 1.04 | 1.20 | 0.17 | 0.51 | 0.93 | 1.49 | 1.22 | 3.74 | 4.21 | 2.27 | 3.21 |
| BB- | N.A. | N.A. | N.A. | 22.14 | 0.00 | 0.00 | 0.00 | 1.23 | 2.60 | 0.82 | 0.74 | 0.91 | 0.84 | 1.53 | 2.20 | 1.67 | 2.82 | 3.01 |
| B+ | N.A. | 0.00 | N.A. | 0.00 | N.A. | 2.77 | 1.39 | 2.22 | 4.44 | 0.00 | 0.85 | 0.74 | 0.60 | 0.91 | 1.57 | 1.79 | 2.06 | 1.91 |
| B | N.A. | N.A. | 0.00 | 5.26 | N.A. | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 1.32 | 0.29 | 0.50 | 0.46 | 0.90 | 1.67 | 2.05 | 1.92 |
| B- | N.A. | N.A. | N.A. | N.A. | 0.00 | 2.01 | N.A. | 1.34 | 2.01 | 0.00 | 0.00 | 3.22 | 0.31 | 0.67 | 0.68 | 0.81 | 1.76 | 2.25 |
| CCC/C | N.A. | N.A. | N.A. | N.A. | 2.24 | N.A. | 2.24 | 0.00 | 1.12 | 0.00 | 2.24 | 2.24 | 0.93 | 0.71 | 0.78 | 0.56 | 0.79 | 1.59 |

Conditional transition matrix generated using entities that experienced downgrade in prior year. N.A.—Not available. Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

- Conditional – Experienced a ratings downgrade in prior period
  - Value = 1.0: Transitions conditioned on prior downgrade are no different
  - Value > 1.0: Future ratings depends on Current AND Prior ratings



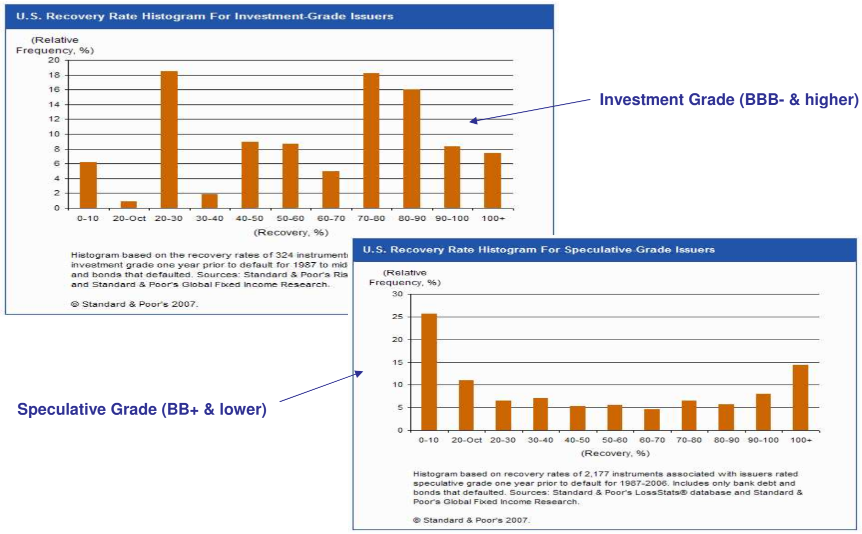
## Rating Agency Studies
### Recovery Rates

**Discounted Recovery Rate By Emergence Year**


*Data through the third quarter of 2006. Recoveries are discounted at each instrument's pre-default interest rate. Sources: Standard & Poor's Risk Solutions LossStats® Database and Standard & Poor's Global Fixed Income Research.

# Rating Agency Studies

## Recovery Rates





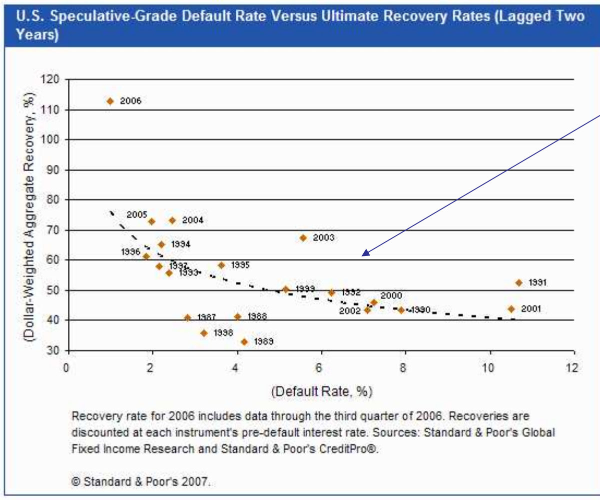

# Rating Agency Studies

## Recovery Rates

| Ultimate Recovery Rates | | | |
|---|---|---|---|
| **Original Rating** | **Recovery** | **Standard Deviation** | **Observations** |
| Bank Debt | 77.5 | 30.9 | 1,204 |
| Senior Secured Bonds | 62.0 | 33.3 | 301 |
| Senior Unsecured Bonds | 42.6 | 34.8 | 769 |
| Senior Subordinated Bonds | 30.3 | 33.3 | 469 |
| Subordinated Bonds | 29.2 | 34.2 | 394 |
| Junior Subordinated Bonds | 19.1 | 30.6 | 49 |
| Standard & Poor's Global Fixed Income Research & Standard & Poor's CreditPro | | | |

- Recovery rates are conditional on the level of debt seniority
- Higher security → greater expected recovery
- Standard deviation High
  - Measurement does not 'neutralise' impact of economic cycle

## Rating Agency Studies
### Default Rate vs Recovery Rate




Inverse relationship between Probability of Default and Recovery Rate



## Rating Agency Studies
### Impairment Rates – A.M. Best Studies

- A.M. Best rated U.S. domiciled insurance companies
- General Corporate Bond Default Rates are inappropriate for insurance:
  - Unique regulatory and accounting environments
  - Relatively few insurers issue public debt
- Impairment is a wider category of financial duress than default
  - Impairment often occurs when insurer able to meet policyholder obligations
    - Regulators sufficiently concerned about future solvency to intervene
  - → Impairment rates > Default rates for a given rating
- Definition of Impairment
  - Financially Impaired Company ("FIC") - First official regulatory action taken
    - Ability to conduct normal insurance operations is adversely affected
    - Capital and Surplus inadequate to meet legal requirements
    - General financial condition has triggered regulatory concern
  - State Actions include:
    - Regulatory Supervision, Rehabilitation, Liquidation, Receivership etc.
    - and any other action that restricts a company's freedom to conduct its insurance business as normal

## Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues



## Modelling Reinsurance Credit Risk Loss
### Assumptions – For the numerical examples

- Loss Process
  - Loss only due to default
- Time Horizon – 12-months
  - Modelling of losses arising in a 12-month calendar year period
  - Same principles for 12-month intervals multi-year modelling (rating migration)
- Monte Carlo Simulation
  - Not an analytical solution (as per Solvency II, Vasicek)
- Probability of Default
  - Rating Agency (“S&P”) default rates
  - Two durations:
    - 12-months ( per Solvency II and multi-year modelling using 12-month intervals)
    - Mean-term of liabilities (as per some ICA submissions)
  - ‘Stressed’ Default rates – Adjustments to base rates :
    - Allowance for Impairment / Dispute / Willingness to Pay risk
    - Non-linear loadings - Allowance for ‘critical’ ratings, say ‘A-’ below which premium volumes and earnings fall → momentum leading to further downgrades
    - Allowance for position in economic / insurance cycle

    - ***However for modelling purposes have assumed zero loadings***

## Modelling Reinsurance Credit Risk Loss

### Assumptions – For the numerical examples

- Loss Given Default
  - Not easy to determine – see sample from Final Dividend % (London Market)
  - Use of E(LGD) values (by Rating) in “GDV Solvency II paper (Dec 05)”
  - Assumed to be variable with a Beta Distribution ( Standard Deviation 15%)

**Examples of Paid Recoveries - Finalised Settlements**

| Name | Final Dividend |
|---|---|
| Andrew Weir | 49.7% |
| Anglo American | 100% + |
| BNIB | 100.0% |
| Fremont (UK) | 38.3% |
| Hawk | 23.0% |
| ICS Re | 88.8% |
| Pine Top | 24.9% |
| RMCA Re | 93.0% |
| Scan Re | 80.5% |
| Stockholm Re | 36.4% |
| NEMGIA | 37.6% |

*Source:*
Bulmer R. et al; Reinsurance Bad Debt Provisions for GI companiesParty - Supplementary Advisory Note of Oct 2005 (Appendix 3); GIRO WP, (Jan 2000)

- Probability of Default and Loss Given Default – Independent
- Reinsurer Asset Returns are Multi-variate Normal



## Modelling Reinsurance Credit Risk Loss

### Data Inputs – Information at individual Reinsurer level

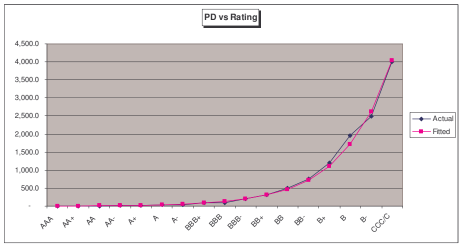
| | |
|---|---|
| No. of Reinsurers | 16 |
| Recoveries | 10,000,000 |
| Expected Loss | 158,027 |

| INPUT DATA | | Years | Prior | Severity | | Variable | Yes |
|---|---|---|---|---|---|---|---|
| **Reinsurer** | **Recoveries** | **Rating** | **PD** | **E(Loss)** | **SD(Loss)** | **Alpha (α)** | **Beta (β)** |
| Reinsurer A | 100,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer B | 200,000 | BBB | 1.520% | 58.0% | 15.0% | 5.70 | 4.13 |
| Reinsurer C | 300,000 | BB | 6.990% | 60.0% | 15.0% | 5.80 | 3.87 |
| Reinsurer D | 400,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer E | 200,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer F | 400,000 | BBB | 1.520% | 58.0% | 15.0% | 5.70 | 4.13 |
| Reinsurer G | 600,000 | BB | 6.990% | 60.0% | 15.0% | 5.80 | 3.87 |
| Reinsurer H | 800,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer I | 300,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer J | 600,000 | BBB | 1.520% | 58.0% | 15.0% | 5.70 | 4.13 |
| Reinsurer K | 900,000 | BB | 6.990% | 60.0% | 15.0% | 5.80 | 3.87 |
| Reinsurer L | 1,200,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer M | 400,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |
| Reinsurer N | 800,000 | BBB | 1.520% | 58.0% | 15.0% | 5.70 | 4.13 |
| Reinsurer O | 1,200,000 | BB | 6.990% | 60.0% | 15.0% | 5.80 | 3.87 |
| Reinsurer P | 1,600,000 | A- | 0.530% | 55.0% | 15.0% | 5.50 | 4.50 |

- Exposure (assumed to be Constant) – Separate for Prior and Current Year
- Credit Rating
  - Probability of Default (duration)
  - Loss Given Default
    - Variable (No) – LGD Fixed %
    - Variable (Yes) – LGD Beta Distribution( $\alpha,\beta$)

# Modelling Reinsurance Credit Risk Loss

## Data Inputs – Default Probabilities (after Smoothing)




- Curve y = exp (a + b.RC) – Rating given values from 1 to 17. (Similar to Solvency II calibration)
- Curve Fitting only for shorter durations. Often omitting highest ratings → use implied 'smoothed' values.
- Adjusted R ~ 98%. t-distribution statistics OK. Standardised Residuals ?

**Term** 3 **Curve** y = exp(a+b.RC)

| Rating | RC | Actual | Fitted | Log-Actual | Log-Fitted | Weights |
|---|---|---|---|---|---|---|
| AAA | 1 | 9.0 | 4.1 | 2.1972 | 1.4186 | 0 |
| AA+ | 2 | 7.0 | 6.4 | 1.9459 | 1.8489 | 0 |
| AA | 3 | 0.1 | 9.8 | -2.3026 | 2.2791 | 0 |
| AA- | 4 | 21.0 | 15.0 | 3.0445 | 2.7094 | 1 |
| A+ | 5 | 26.0 | 23.1 | 3.2581 | 3.1397 | 1 |
| A | 6 | 32.0 | 35.5 | 3.4657 | 3.5700 | 1 |
| A- | 7 | 35.0 | 54.6 | 3.5553 | 4.0002 | 1 |
| BBB+ | 8 | 100.0 | 84.0 | 4.6052 | 4.4305 | 1 |
| BBB | 9 | 93.0 | 129.1 | 4.5326 | 4.8608 | 1 |
| BBB- | 10 | 194.0 | 198.6 | 5.2679 | 5.2911 | 1 |
| BB+ | 11 | 312.0 | 305.3 | 5.7430 | 5.7213 | 1 |
| BB | 12 | 492.0 | 469.5 | 6.1985 | 6.1516 | 1 |
| BB- | 13 | 762.0 | 721.9 | 6.6359 | 6.5819 | 1 |
| B+ | 14 | 1,204.0 | 1,110.1 | 7.0934 | 7.0122 | 1 |
| B | 15 | 1,947.0 | 1,706.9 | 7.5740 | 7.4424 | 1 |
| B- | 16 | 2,489.0 | 2,624.7 | 7.8196 | 7.8727 | 1 |
| CCC/C | 17 | 3,996.0 | 4,036.0 | 8.2930 | 8.3030 | 1 |
| **Units** | 10,000 | | | | | |

**SUMMARY OUTPUT**

| *Regression Statistics* | |
|---|---|
| R Square | **98.812%** |
| Adjusted R Sq | **98.713%** |
| SE | **20.540%** |
| Observations | **14** |

**ANOVA**

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 42.1188 | 42.1188 | 998.3019 | 6.36671E-13 |
| Residual | 12 | 0.5063 | 0.0422 | | |
| Total | 13 | 42.6251 | | | |

| | ***Coefficients*** | ***SE*** | ***t Stat*** | ***P-value*** | ***2.5%*** | ***97.5%*** |
|---|---|---|---|---|---|---|
| Intercept | 0.9883 | 15.317% | 6.4525 | 3.14858E-05 | 0.6546 | 1.3220 |
| X Variable 1 | 0.4303 | 1.362% | 31.5959 | 6.36671E-13 | 0.4006 | 0.4599 |



# Modelling Reinsurance Credit Risk Loss

## Correlation – Cholesky Matrix decomposition

**CORRELATION MATRIX**

| | **No.** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Reinsurer A | 1 | 1.00 | 0.50 | 0.50 | 0.25 | 0.25 | 0.25 |
| Reinsurer B | 2 | | 1.00 | 0.50 | 0.25 | 0.25 | 0.25 |
| Reinsurer C | 3 | | | 1.00 | 0.25 | 0.25 | 0.25 |
| Reinsurer D | 4 | | | | 1.00 | 0.25 | 0.25 |
| Reinsurer E | 5 | | | | | 1.00 | 0.25 |
| Reinsurer F | 6 | | | | | | 1.00 |

The pair-wise correlations between 1,2 and 3 are higher (50%) than the others (25%)

**CHOLESKY MATRIX**

| **No.** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.50 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.50 | 0.29 | 0.82 | 0.00 | 0.00 | 0.00 |
| 4 | 0.25 | 0.14 | 0.10 | 0.95 | 0.00 | 0.00 |
| 5 | 0.25 | 0.14 | 0.10 | 0.16 | 0.94 | 0.00 |
| 6 | 0.25 | 0.14 | 0.10 | 0.16 | 0.14 | 0.93 |

**TRANSPOSE CHOLESKY MATRIX**

| **No.** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 0.50 | 0.50 | 0.25 | 0.25 | 0.25 |
| 2 | 0.00 | 0.87 | 0.29 | 0.14 | 0.14 | 0.14 |
| 3 | 0.00 | 0.00 | 0.82 | 0.10 | 0.10 | 0.10 |
| 4 | 0.00 | 0.00 | 0.00 | 0.95 | 0.16 | 0.16 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.94 | 0.14 |
| 6 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.93 |

**ORIGINAL MATRIX - CHECK**

| **No.** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 0.50 | 0.50 | 0.25 | 0.25 | 0.25 |
| 2 | 0.50 | 1.00 | 0.50 | 0.25 | 0.25 | 0.25 |
| 3 | 0.50 | 0.50 | 1.00 | 0.25 | 0.25 | 0.25 |
| 4 | 0.25 | 0.25 | 0.25 | 1.00 | 0.25 | 0.25 |
| 5 | 0.25 | 0.25 | 0.25 | 0.25 | 1.00 | 0.25 |
| 6 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 1.00 |

- Cholesky Matrix is used to generate 'correlated' standard normals from 'independent' standard normals
- Original Matrix needs to be 'Positive Definite' – not all matrices work
- Product of the Cholesky Matrix and its Transpose equals the Original Matrix

## Modelling Reinsurance Credit Risk Loss
### Multi-year Modelling considerations

- Default Probabilities vary over time – ‘Probability Drift’
  - Good credit rating → higher probability of downgrade than upgrade
    - Over time higher rated companies have larger probability of transitioning to lower ratings than is the case over 12-months only.
  - Mean reversion in credit ratings

| Global Average Transition Rates (1981-2006) (%) - 1 Year | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| From/To | AAA | AA | A | BBB | BB | B | CCC/C | D | NR |
| AAA | 88.34 | 7.84 | 0.47 | 0.09 | 0.09 | - | - | - | 3.17 |
| AA | 0.59 | 87.31 | 7.54 | 0.57 | 0.06 | 0.10 | 0.02 | 0.01 | 3.79 |
| A | 0.05 | 2.00 | 87.39 | 5.47 | 0.40 | 0.15 | 0.02 | 0.06 | 4.46 |
| BBB | 0.01 | 0.15 | 3.98 | 84.17 | 4.14 | 0.73 | 0.16 | 0.24 | 6.42 |
| BB | 0.03 | 0.06 | 0.22 | 5.18 | 75.71 | 7.20 | 0.84 | 1.07 | 9.69 |
| B | - | 0.05 | 0.18 | 0.30 | 5.78 | 72.77 | 4.10 | 4.99 | 11.83 |
| CCC/C | - | - | 0.26 | 0.39 | 1.10 | 11.15 | 47.49 | 26.29 | 13.34 |

| Global Average Transition Rates (1981-2006) (%) - 3 Years | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| From/To | AAA | AA | A | BBB | BB | B | CCC/C | D | NR |
| AAA | 68.39 | 18.98 | 2.55 | 0.40 | 0.12 | 0.03 | 0.03 | 0.09 | 9.41 |
| AA | 1.41 | 66.46 | 18.06 | 2.38 | 0.41 | 0.25 | 0.02 | 0.10 | 10.92 |
| A | 0.10 | 4.57 | 67.34 | 12.21 | 1.52 | 0.62 | 0.11 | 0.32 | 13.21 |
| BBB | 0.04 | 0.47 | 9.09 | 60.55 | 8.06 | 2.33 | 0.43 | 1.32 | 17.72 |
| BB | 0.05 | 0.10 | 0.81 | 11.33 | 43.82 | 11.87 | 1.54 | 5.92 | 24.57 |
| B | 0.01 | 0.07 | 0.45 | 1.30 | 11.01 | 37.08 | 4.34 | 17.04 | 28.71 |
| CCC/C | - | - | 0.30 | 1.06 | 2.43 | 14.25 | 13.57 | 42.61 | 25.78 |



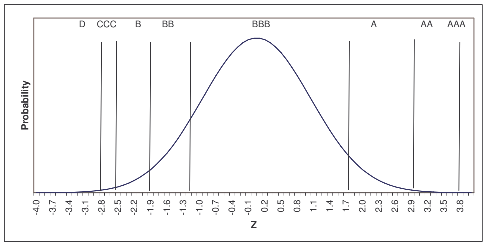
## Modelling Reinsurance Credit Risk Loss
### Multi-year Modelling considerations

- Stochastic exposures becomes more important
  - Variance of the underlying variables increase over time
- Ratings Momentum exist
  - Markov Process for transition rates – a convenient modelling approach
    - i.e. conditional probability distribution of future states depends only on the current state and not prior states.
  - Often used for multi-year modelling of future states - $M_T = (M_1)^T$
    - Where $M_T$ = T-year transition matrix
  - Empirical evidence suggests otherwise
- Correlated Credit migration
  - Can use Asset Return correlation framework to determine future ratings of a company
    - i.e. the rating changes of any two reinsurers are more likely to move together either upwards or downwards - rather than being independent processes
  - A convenient mathematical representation of economic or insurance cycle impacts on reinsurers
  - Consistent with the loss default process (assuming asset correlation)

# Modelling Reinsurance Credit Risk Loss
## Multi-year Modelling considerations

- Correlated Credit migration

**Credit Migration Thresholds**




| Current | BBB | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| t=1 | D | CCC | B | BB | BBB | A | AA | AAA |
| Z | -2.82 | -2.51 | -2.00 | -1.19 | 1.73 | 2.95 | 3.72 | + Infinity |
| Φ(.) | 0.24% | 0.60% | 2.27% | 11.69% | 95.86% | 99.84% | 99.99% | 100.00% |

**Adjusted Transition Matrix (re-spreading of NR)**

1 Year

| From/To | AAA | AA | A | BBB | BB | B | CCC/C | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 88.34% | 7.84% | 2.76% | 0.53% | 0.53% | 0.00% | 0.00% | 0.00% |
| AA | 0.59% | 87.31% | 11.00% | 0.83% | 0.09% | 0.15% | 0.03% | 0.01% |
| A | 0.05% | 2.00% | 87.39% | 9.51% | 0.70% | 0.26% | 0.03% | 0.06% |
| BBB | 0.01% | 0.15% | 3.98% | 84.17% | 9.42% | 1.66% | 0.36% | 0.24% |
| BB | 0.03% | 0.06% | 0.22% | 5.18% | 75.71% | 15.88% | 1.85% | 1.07% |
| B | 0.00% | 0.05% | 0.18% | 0.30% | 5.78% | 72.77% | 15.93% | 4.99% |
| CCC/C | 0.00% | 0.00% | 0.26% | 0.39% | 1.10% | 13.68% | 58.28% | 26.29% |



# Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues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# Numerical Examples

## Some Results – 16 Reinsurers as previously described

**OUTPUTS**

| | | | | | | |
|---|---|---|---|---|---|---|
| Exposure | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 |
| No. Reinsurers | 16 | 16 | 16 | 16 | 16 | 16 |
| Average Rating | As Given | As Given | As Given | As Given | As Given | As Given |
| Correlation | 0% | 25% | 50% | 0% | 25% | 50% |
| Mean Term | 4 | 4 | 4 | 1 | 1 | 1 |
| No. Simulations | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 |
| Stress Load | 0% | 0% | 0% | 0% | 0% | 0% |

| | CreditLoss | CreditLoss | CreditLoss | CreditLoss | CreditLoss | CreditLoss |
|---|---|---|---|---|---|---|
| **EC(VaR 99.5%)** | **1,218,583** | **1,675,993** | **2,329,250** | **742,709** | **839,581** | **963,792** |
| ***as % Exposure*** | **12.2%** | **16.8%** | **23.3%** | **7.4%** | **8.4%** | **9.6%** |
| **EC(TVaR 99.5%)** | **1,395,832** | **2,045,597** | **3,130,242** | **910,275** | **1,093,465** | **1,453,602** |
| ***as % Exposure*** | **14.0%** | **20.5%** | **31.3%** | **9.1%** | **10.9%** | **14.5%** |
| Minimum | 0 | 0 | 0 | 0 | 0 | 0 |
| Maximum | 2,255,585 | 3,152,566 | 4,798,956 | 1,706,195 | 1,912,611 | 3,578,554 |
| **Expected** | **157,230** | **156,978** | **156,381** | **21,107** | **22,135** | **23,110** |
| Std Dev | 287,674 | 340,268 | 422,968 | 107,820 | 119,783 | 143,783 |
| 10.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 20.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 30.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 40.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 50.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 60.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 70.0 %ile | 134,494 | 0 | 0 | 0 | 0 | 0 |
| 80.0 %ile | 338,950 | 252,720 | 124,527 | 0 | 0 | 0 |
| 90.0 %ile | 604,829 | 640,051 | 608,555 | 0 | 0 | 0 |
| **95.0 %ile** | **786,723** | **886,391** | **990,414** | **51,578** | **0** | **0** |
| 99.0 %ile | 1,192,029 | 1,530,445 | 2,036,799 | 637,558 | 676,659 | 728,219 |
| **99.5 %ile** | **1,375,812** | **1,832,972** | **2,485,631** | **763,816** | **861,717** | **986,902** |
| 99.9 %ile | 1,644,649 | 2,494,736 | 3,968,448 | 994,438 | 1,281,578 | 1,737,091 |
| 100.0 %ile | 2,255,585 | 3,152,566 | 4,798,956 | 1,706,195 | 1,912,611 | 3,578,554 |

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# Numerical Examples

## Some Results – 16 Reinsurers 'A' rated – 1 year and 4 years

**OUTPUTS**

| | | | | | | |
|---|---|---|---|---|---|---|
| Exposure | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 | 10,000,000 |
| No. Reinsurers | 16 | 16 | 16 | 16 | 16 | 16 |
| Average Rating | A | A | A | A | A | A |
| Correlation | 0% | 25% | 50% | 0% | 25% | 50% |
| Mean Term | 4 | 4 | 4 | 1 | 1 | 1 |
| No. Simulations | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 |
| Stress Load | 0% | 0% | 0% | 0% | 0% | 0% |

| | CreditLoss | CreditLoss | CreditLoss | CreditLoss | CreditLoss | CreditLoss |
|---|---|---|---|---|---|---|
| **EC(VaR 99.5%)** | **766,342** | **858,361** | **1,091,418** | **189,134** | **191,624** | **122,406** |
| ***as % Exposure*** | **7.7%** | **8.6%** | **10.9%** | **1.9%** | **1.9%** | **1.2%** |
| **EC(TVaR 99.5%)** | **955,221** | **1,143,465** | **1,596,704** | **442,287** | **478,171** | **435,931** |
| ***as % Exposure*** | **9.6%** | **11.4%** | **16.0%** | **4.4%** | **4.8%** | **4.4%** |
| Minimum | 0 | 0 | 0 | 0 | 0 | 0 |
| Maximum | 1,338,558 | 1,976,112 | 3,021,969 | 1,180,514 | 1,016,423 | 1,281,835 |
| **Expected** | **22,977** | **24,764** | **24,881** | **2,490** | **2,617** | **2,273** |
| Std Dev | 109,581 | 125,224 | 153,297 | 35,241 | 38,463 | 36,404 |
| 10.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 20.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 30.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 40.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 50.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 60.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 70.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 80.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| 90.0 %ile | 0 | 0 | 0 | 0 | 0 | 0 |
| **95.0 %ile** | **142,524** | **128,863** | **0** | **0** | **0** | **0** |
| 99.0 %ile | 627,190 | 700,237 | 799,973 | 0 | 0 | 0 |
| **99.5 %ile** | **789,319** | **883,125** | **1,116,299** | **191,624** | **194,242** | **124,679** |
| 99.9 %ile | 1,113,006 | 1,340,567 | 2,002,843 | 560,834 | 663,513 | 637,175 |
| 100.0 %ile | 1,338,558 | 1,976,112 | 3,021,969 | 1,180,514 | 1,016,423 | 1,281,835 |

**The Actuarial Profession**
making financial sense of the future

## Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- **Modelling Issues**



## Modelling Issues
### Issues (and Solutions)

- Assumptions for:
  - Probability of Default (setting "Stressed levels")
  - Loss Given Default
  - Asset (or Default Correlation)
  - Dependencies
    - Amongst the above e.g. PD and LGD; or Value of Asset Return and LGD
    - Other variables – insurance loss and default rate
- Monte Carlo Sampling error:
  - Problem for highly rated portfolios and for high loss percentiles (~Capital)
    - e.g. probability of default = 0.05% → average one default per 2,000 simulations
  - Especially for very lumpy exposures
  - Error term decreases as $N^{-0.5}$ (N – No. of simulations)
  - Need to either:
    - Run a very large number of simulations
    - Use Monte Carlo acceleration methods (i.e. 'variance reduction techniques')
  - Methods:
    - Stratified Sampling, Low-Discrepancy sequences, Control Variates etc.

## Modelling Issues
### Issues (and Solutions)

- Multi-variate Normal distribution:
  - May be reasonable for non-financial corporate sector
  - Could be issue for the insurance sector:
    - Correlation between lines of business
    - Interdependence within the industry - reinsurance
    - Shared exposures to aggregate industry losses (Large Cats, Systemic issues)
  - Multi-variate t-distribution → 'Fatter' Tails (perhaps more realistic)
- VaR as a Risk Measure
  - An issue – linked to the Monte-Carlo sampling error
  - Especially lumpy exposures
  - TVaR a better risk measure