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making financial sense of the future

The Modelling of Reinsurance Credit Risk in Non-Life Insurance

RMSIG: Credit Risk Evening

Richard Shaw
Guy Carpenter

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Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Issues
- Conclusions

Reinsurance Credit Risk
What is Reinsurance Credit Risk

- Definition:
  "The risk of loss if another party fails to perform its obligations or fails to perform them in a timely manner."
  - Key counterparties include reinsurers, brokers, insureds, and reinsureds
- Examples of Risk Factors:
  - Reinsurance Failure (of individual reinsurers)
  - Credit Deterioration (of individual reinsurers)
  - Bad Debt provision inadequacy
  - Correlation in extreme loss scenarios
  - Credit Concentration
  - Duration of Recoveries
  - Willingness to Pay / Dispute Risk
  - Non-reinsurance related credit risk

Reinsurance Credit Risk
Why it is important to Understand

- Regulatory Capital Requirements
  - ICA Capital – VaR (@99.5%) over 12-months
  - SCR (Solvency II) Capital – same risk measure and probability
- Economic Capital Modelling
  - As above but different assumptions e.g. percentile and time horizon
- Risk Management Best Practices
  - Regular aged debt analysis → highlight future potential issues with certain reinsurers ('Willingness to Pay' / Dispute Risk)
  - Setting Reinsurer Counterparty Limits
- Capital Markets Solutions
  - Risk transfer solutions and mechanisms
  - e.g. Aspen Re Credit Wrap and Merlin (Hannover Re) transactions (2007)

Reinsurance Credit Risk
Why it is important to Understand

- Reinsurance Purchasing decision making:
  - Can play a part in determining the optimal reinsurance structure
  - Modification in the NPV of the net loss and underwriting profit distributions
    - Impact greatest at the highest loss percentiles
  - Longer-tail lines (more relevant):
    - Reserves take a few years to run-off - declining exposure
    - Not a big number in year 1- highly rated companies
    - Yesterday's 'A' rated companies suffer downgrades over time
  - Loss Dependency at the extreme loss percentiles
    - Very Large Property Cat Loss → increase in reinsurance default rates
- Reinsurance Panel Evaluation:
  - Given a new reinsurance program how should it be placed
    - 100% with one reinsurer
    - Smaller shares with others (Rating ?)
  - Benefits of Diversification → Credit Risk
  - Plus Reinsurance Purchasing Criteria considerations as above

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The Loss Process
Expected Loss (“EL”) and Unexpected Loss (“UL”)

- Let Y_i be a binary variable for obligor i at the end of year 1.
  - Y_i = 1 (Default) or 0 (No Default) given non-default state at t= 0.
- $EL_i = PD_i \times EAD_i \times LGD_i$
- $UL_i = [PD_i \times (1 - PD_i)]^{1/2} \times EAD_i \times LGD_i$ *(EAD_i and LGD_i constant)*
  - EAD = Exposure at Default
  - LGD = Loss Given Default (i.e. severity per unit of exposure)
  - PD = Probability of Default
- Otherwise:

$$UL_i = [\, PD^2_i \times EAD^2_i \times \sigma^2_{LGDi} + EAD^2_i \,.\, LGD^2_i \,.\, \sigma^2_{PDi} + LGD^2_i \,.\, PD^2_i \times \sigma^2_{EADi} + PD^2_i \times \sigma^2_{EADi} \times \sigma^2_{LGDi} + EAD^2_i \times \sigma^2_{LGDi} \times \sigma^2_{PDi} + LGD^2_i \times \sigma^2_{PDi} \times \sigma^2_{EADi} + \sigma^2_{PDi} \times \sigma^2_{EADi} \times \sigma^2_{LGDi} \,]^{0.5}$$

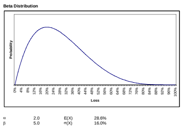
  - This further assumes that PD_i ,EAD_i and LGD_i are independent

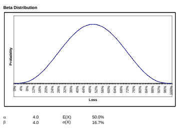
The Loss Process
Loss Severity

- Two ways of modelling loss severity
  - Recovery % amount is known with certainty
  - Recovery % amount is uncertain
- Recovery % amount is uncertain
  - Beta Distribution is often used to model Loss Severity

$$f(x) = x^{(\alpha - 1)} \times (1 - x)^{(\beta - 1)} \times [\Gamma(\alpha + \beta) / \Gamma(\alpha) \times \Gamma(\beta)] \;\ldots\ldots\; \text{for } 0 < x < 1$$
$$0 \;\ldots\ldots\; \text{for } x < 0 \text{ and } x > 1$$

$$\mu = \alpha / (\alpha + \beta)$$
$$\sigma^2 = (\alpha \times \beta) / [(\alpha + \beta)^2 \times (\alpha + \beta + 1)]$$





The Loss Process
Reinsurance Credit Exposure

- Economic Capital - Reinsurance Exposures are Stochastic
  - Variation in the underlying Gross loss process
  - Variation in Interest rates (NPV Calculations)
  - Variation in Payment patterns (NPV Calculations)
- Current Year Reinsurance Exposure
  - More accurate modelling of Stochastic Gross → Net process
    - Gross Distributions
    - Current Reinsurance Structures
  - Sampling error could be an issue
    - High minimum rating criteria (say ‘A-’ and above) – very low default rates
- Prior Year Reinsurance Exposure
  - Mix of reinsurers different to Current year
  - Average credit rating likely to be lower (rating downgrades)
  - Gross → Net Process – less accuracy
    - Actuarial Reserving techniques (approx methods)
    - Reserve Volatility techniques (e.g. Bootstrap)

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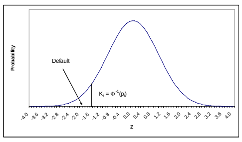
Diversification and Correlation
Asset Return and Default Correlation relationship



$\mathbf{Y_i = 1 \Leftrightarrow X_i \le D_i \Leftrightarrow AR_i \le K_i}$

Where:

X_i = Value of the Assets for obligor i at the end of time t.

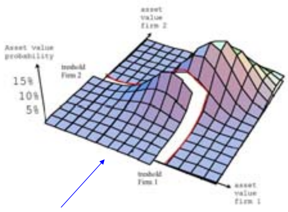
D_i = Value of the Asset Threshold (or cut-off level) for obligor i at the end of time t.

AR_i = Asset Return for obligor i over time t.

K_i = Asset Return threshold for obligor i over time t

Number of defaults within a portfolio of M obligors = $\sum_{i=1}^{M} \mathbf{Y_i}$

Diversification and Correlation
Asset Return and Default Correlation relationship



Assume that the joint asset return distribution is bi-variate normal

- Joint Default Probability
  - Probability that the value of assets jointly falls below the respective thresholds
  - Bottom left corner of the bi-variate normal distribution

$$PD_{12} = \int_{-\infty}^{K_1}\int_{-\infty}^{K_2} (1/(2\pi(1-\rho_A^2)^{0.5}) \exp(-(x_1^2 + x_2^2 - 2 x_1 x_2 \rho_A) / (2(1-\rho_A^2))) dx_1 dx_2$$

Diversification and Correlation
Asset Return Correlation and Default Correlation relationship

PD_1 and PD_2	Asset Corr	Joint Def Prob	Default Corr
0.2%	10.0%	0.00%	0.31%
0.2%	30.0%	0.00%	2.05%
0.2%	50.0%	0.01%	6.93%
0.2%	70.0%	0.04%	18.61%
1.0%	10.0%	0.02%	0.95%
1.0%	30.0%	0.06%	4.64%
1.0%	50.0%	0.13%	12.12%
1.0%	70.0%	0.27%	26.06%
10.0%	10.0%	1.32%	3.54%
10.0%	30.0%	2.14%	12.67%
10.0%	50.0%	3.21%	24.58%
10.0%	70.0%	4.64%	40.47%

$\rho_d = (PD_{12} - PD_1 \times PD_2) / (PD_1 \times (1 - PD_1) \times PD_2 \times (1 - PD_2))^{0.5}$

Where:

$PD_1 = P(Y_1 = 1) = P(X_1 \leq D_1)$ and

$PD_{12} = P(Y_1 = 1, Y_2 = 1) = P(X_1 \leq D_1, X_2 \leq D_2)$

Diversification and Correlation
Correlation – Cholesky Matrix decomposition

CORRELATION MATRIX

	No.	1	2	3	4	5	6
Reinsurer A	1	1.00	0.50	0.50	0.25	0.25	0.25
Reinsurer B	2		1.00	0.50	0.25	0.25	0.25
Reinsurer C	3			1.00	0.25	0.25	0.25
Reinsurer D	4				1.00	0.25	0.25
Reinsurer E	5					1.00	0.25
Reinsurer F	6						1.00

CHOLESKY MATRIX

No.	1	2	3	4	5	6
1	1.00	0.00	0.00	0.00	0.00	0.00
2	0.50	0.87	0.00	0.00	0.00	0.00
3	0.50	0.29	0.82	0.00	0.00	0.00
4	0.25	0.14	0.10	0.95	0.00	0.00
5	0.25	0.14	0.10	0.16	0.94	0.00
6	0.25	0.14	0.10	0.16	0.14	0.93

TRANSPOSE CHOLESKY MATRIX

No.	1	2	3	4	5	6
1	1.00	0.50	0.50	0.25	0.25	0.25
2	0.00	0.87	0.29	0.14	0.14	0.14
3	0.00	0.00	0.82	0.10	0.10	0.10
4	0.00	0.00	0.00	0.95	0.16	0.16
5	0.00	0.00	0.00	0.00	0.94	0.14
6	0.00	0.00	0.00	0.00	0.00	0.93

ORIGINAL MATRIX - CHECK

No.	1	2	3	4	5	6
1	1.00	0.50	0.50	0.25	0.25	0.25
2	0.50	1.00	0.50	0.25	0.25	0.25
3	0.50	0.50	1.00	0.25	0.25	0.25
4	0.25	0.25	0.25	1.00	0.25	0.25
5	0.25	0.25	0.25	0.25	1.00	0.25
6	0.25	0.25	0.25	0.25	0.25	1.00

The pair-wise correlations between 1,2 and 3 are higher (50%) than the others (25%)

- Cholesky Matrix is used to generate 'correlated' standard normals from 'independent' standard normals
- Original Matrix needs to be 'Positive Definite' – not all matrices work
- Product of the Cholesky Matrix and its Transpose equals the Original Matrix

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Modelling Reinsurance Credit Risk Loss
Data Inputs – Information at Reinsurer level

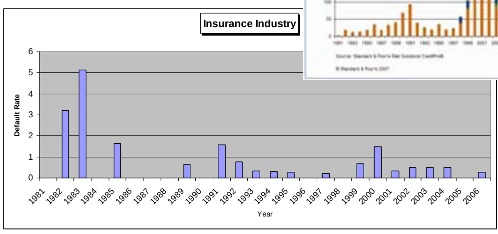
No. of Reinsurers	16
Recoveries	10,000,000
Expected Loss	20,637

INPUT DATA		Years	Prior	Severity		Variable	Yes
Reinsurer	Recoveries	Rating	PD	E(Loss)	SD(Loss)	Alpha (α)	Beta (β)
Reinsurer A	100,000	A	0.053%	55.0%	20.0%	2.85	2.33
Reinsurer B	200,000	BBB	0.250%	58.0%	20.0%	2.95	2.14
Reinsurer C	300,000	BB	0.860%	60.0%	20.0%	3.00	2.00
Reinsurer D	400,000	A-	0.089%	55.0%	20.0%	2.85	2.33
Reinsurer E	200,000	A	0.053%	55.0%	20.0%	2.85	2.33
Reinsurer F	400,000	BBB	0.250%	58.0%	20.0%	2.95	2.14
Reinsurer G	600,000	BB	0.860%	60.0%	20.0%	3.00	2.00
Reinsurer H	800,000	A-	0.089%	55.0%	20.0%	2.85	2.33
Reinsurer I	300,000	A	0.053%	55.0%	20.0%	2.85	2.33
Reinsurer J	600,000	BBB	0.250%	58.0%	20.0%	2.95	2.14
Reinsurer K	900,000	BB	0.860%	60.0%	20.0%	3.00	2.00
Reinsurer L	1,200,000	A-	0.089%	55.0%	20.0%	2.85	2.33
Reinsurer M	400,000	A	0.053%	55.0%	20.0%	2.85	2.33
Reinsurer N	800,000	BBB	0.250%	58.0%	20.0%	2.95	2.14
Reinsurer O	1,200,000	BB	0.860%	60.0%	20.0%	3.00	2.00
Reinsurer P	1,600,000	A-	0.089%	55.0%	20.0%	2.85	2.33

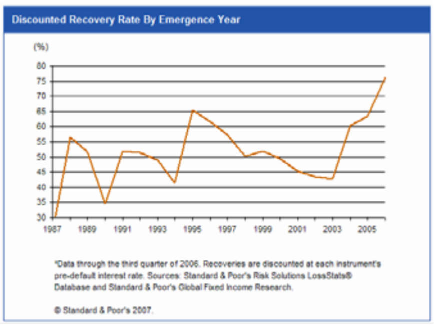
- Exposure (assumed to be Constant) – Separate for Prior and Current Year
- Credit Rating
  - Probability of Default (duration)
  - Loss Given Default – Variability – Yes or No

Modelling Reinsurance Credit Risk Loss
Annual Default Rates (Corporate Debt data)

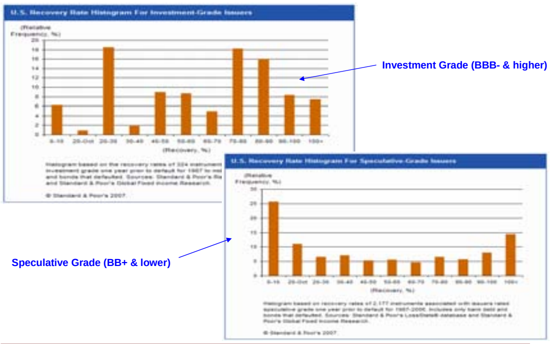
- Default Rates are very cyclical
- There is no obvious relationship between the pattern of Insurance industry defaults and those of other industry groupings.



Modelling Reinsurance Credit Risk Loss
Recovery Rates (Corporate Debt data)



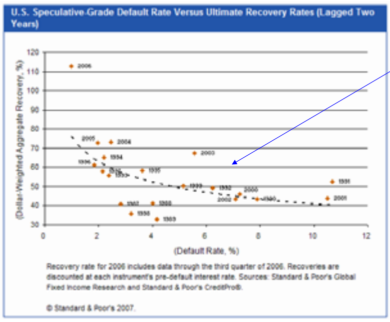
Modelling Reinsurance Credit Risk Loss
Recovery Rates (Corporate Debt data)



Investment Grade (BBB- & higher)

Speculative Grade (BB+ & lower)

Modelling Reinsurance Credit Risk Loss
Default Rate vs Recovery Rate (Corporate Debt data)



Inverse relationship between Probability of Default and Recovery Rate

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Numerical Examples

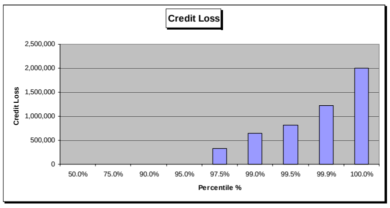
Results – 16 Reinsurers (Constant Exposure)

OUTPUTS

	CreditLoss	% Exposure	No.
EC (VaR)	**792,265**	**8.1%**	
EC (TVaR)	**1,067,136**	**10.7%**	
Minimum	**0**	0.0%	0
Maximum	**1,994,984**	19.9%	4
Expected	**19,710**	**0.2%**	**0.0**
Standard Deviation	**113,337**	1.1%	0.2

Economic Capital VaR (@ 99.5%)
= 40.2 x Expected Loss

- 1 year Default Probabilities
- 25,000 simulations
- Asset Correlation of 25%



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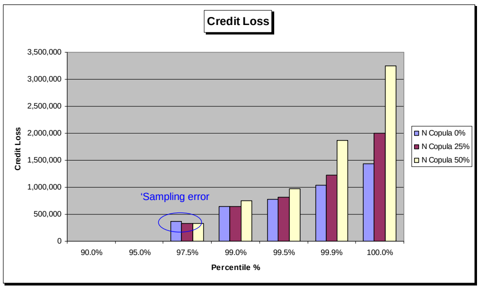
Numerical Examples

Results – 16 Reinsurers (Constant Exposure)



- 25,000 simulations

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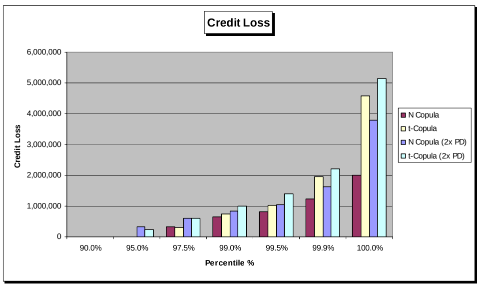
Numerical Examples

Results – 16 Reinsurers (Constant Exposure)



- 25,000 simulations
- Asset Correlation of 25%

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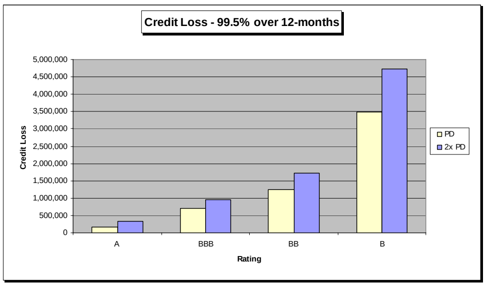
Numerical Examples
Results – 16 Reinsurers, identical rating (Constant Exposure)



- 25,000 simulations
- Asset Correlation of 25%

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Modelling Issues
Issues

- Assumptions for:
  - Probability of Default - “Stressed levels” (‘Willingness to Pay’)
  - Loss Given Default
  - Asset (or Default Correlation)
  - → and how they evolve over time
- Dependencies:
  - PD and LGD
  - Insurance loss process and Default Rate
- Multi-variate Normal distribution:
  - May be reasonable for non-financial corporate sector
  - Could be issue for the insurance sector:
    - Interdependence within the industry - reinsurance
    - Shared exposures to aggregate industry losses (Large Cats, Systemic issues)
  - Multi-variate t-distribution → ‘Fatter’ Tails (perhaps more realistic)

Modelling Issues
Issues

- Monte Carlo Sampling error:
  - Problem for highly rated portfolios and for high loss percentiles (~Capital)
    - Probability of Default = 0.05% → Average one default per 2,000 simulations
  - Especially for very lumpy exposures
  - Error term decreases as $N^{-0.5}$ (N – No. of simulations)
  - Need to either:
    - Run a very large number of simulations
    - Use Monte Carlo acceleration methods (i.e. 'Variance Reduction techniques')
- VaR as a Risk Measure:
  - Linked to the Monte-Carlo sampling error (Especially lumpy exposures)
  - TVaR a better risk measure
- Parameter Uncertainty
- Model Risk

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Conclusions
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- Reinsurance Credit Risk is difficult risk to model accurately
  - More complex than Insurance Risk
  - Varying degrees of risk relationships – Asset and Liability side
  - Data (or lack of) – Corporate Debt data vs Reinsurance
  - Modelling algorithms
- Very easy to underestimate the risk of extreme losses (~ Capital)
  - Perhaps because 90% – 97.5% of losses are zero (typical portfolio)
  - Optimistic parameter selection / simple models
  - Tail Dependencies not appreciated
- Parameter uncertainty and model risk need to be better understood