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NOTES ON THE TRANSACTIONS OF THE FACULTY OF ACTUARIES

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J. L. Anderson. *Notes on the effect of changes in rates of interest on the bonus-earning power of an office paying a uniform compound reversionary bonus.* Although the methods of dealing with a rise in the rate of interest are not excluded (and they are obviously of a complementary type), the subject of the paper is essentially the problem of a fall in that rate. The case is taken of an office whose premiums are based on interest at 4% and a bonus of 40*s*.% compounding annually, it being supposed that the net rate of interest is reduced to 3%. It is assumed that bonuses will be reduced to 38*s*.% for five years, 33*s*.% for the next five years, and 28*s*.% thereafter. The theoretically more correct ideas of (1) a bonus based on calendar year and duration, (2) an extra large bonus distributing all appreciation at the first opportunity, followed necessarily by a fall to below the previous level, (3) a contribution system, are also considered, but the plan first mentioned is, of course, the only one a working actuary would be likely to adopt. An essential assumption is that, considering the life assurance fund as closed, investments are so arranged that in each sub-group (identical age, class, and duration) there are sufficient investments maturing to meet liabilities in the year when the group reserve falls below its amount at the time of the present valuation; but, as A. T. Haynes and R. J. Kirton pointed out in the discussion, this does not provide for the intermediate rise and fall. The injustices of this plan are moderate and tolerable; there is some degree of overpayment on policies of short duration and vice versa, the underpayment being most marked in the case of whole-life assurances of long duration. The case of a further fall (due to tax) in the net rate of interest to $2\frac{1}{2}$% is also considered. An appendix gives formulae. It is assumed that mortality and expenses will work out according to the assumptions made in calculating the premiums.

J. Murray Laing. *Life assurance—past and present.* An interesting review of developments in life assurance during the past forty years. Among the subjects alluded to are the increases in numbers of policies, amounts assured, etc.; the extent and the effects of changes in the rate of interest, the rate of income tax, the rate of mortality, and the distribution of assets; the changes in the types of assurance favoured and, in particular, the growth in popularity of endowment assurances; changes in policy conditions and the treatment of extra risks; and the great saving of labour effected by the use of mechanical devices and improved valuation methods.

A. Fraser. *Valuation of widows' funds where there is appreciation in value of investments.* The object is to show that, whereas before allowing for new entrants the surplus, bringing appreciation into account and, consequently, valuing at a lower rate of interest, may be larger than when book values are used, the position may be reversed when allowance is made for new entrants. The figures are derived from the valuation of the widows' fund of one of the large banks. There is a supplementary note arising from the fact that it was desired to obtain the $3\frac{1}{4}$% reserves by interpolation; and, as it was found that the geometrical mean between the 3% and 4% reserves was closer to the known $3\frac{1}{2}$% reserve than the arithmetical mean, the geometrical mean was used to obtain an approximation to the $3\frac{1}{4}$% reserve also. It is pointed out by G. J. Lidstone, to whom the note was sent, that geometrical interpolation is in fact only a particular case of logarithmic interpolation; and a test made by Lidstone with model office sums assured showed that in certain cases second difference logarithmic interpolation might give very good results.

G. J. LIDSTONE. *Note on logarithmic interpolation.* This was submitted in connexion with Fraser's note, referred to above. If λ denotes the natural logarithm of a function V, by Leibniz's theorem we have

$$V''' = V\lambda''' + 2V'\lambda'' + V''\lambda',$$

and so on, and as V''' and $V\lambda'''$ are proportional to the errors resulting from direct and logarithmic interpolation respectively, logarithmic interpolation will give the best results provided that each of the sequences $V, V', V'', \ldots$ and $\lambda', \lambda'', \lambda''', \ldots$ preserves a uniform sign or alternates regularly in sign. In the particular case considered here, the condition is that λ', λ'' and λ''' should be positive for a second-difference interpolation.

J. G. ANDERSON. *William Morgan and X-rays.* Correcting a remark in W. P. Elderton's lecture on Morgan (*T.F.A.* Vol. XIV), the writer points out that in Morgan's first paper to the Royal Society in 1785 he described experiments in which he had produced X-rays and that the simple apparatus which he used was the first X-ray tube. (Since Anderson's note there has been an article by E. Ashworth Underwood in the *Proceedings of the Royal Society of Medicine*, Vol. XXXVIII, which gives an illustration of the apparatus.)

CORRESPONDENCE. A letter from A. C. Murray gives figures worked out by R. K. Lochhead showing the percentages of the total assets held in various classes by ten life offices for the years 1935, 1940 and 1944.