The Actuarial Profession
making financial sense of the future

One-year reserve and premium risk

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Outline

- One-year risk: background and motivation
- Analytical methods
- Modelling of the one-year risk in simulation models
- Example
- Conclusion

One-year risk: background and motivation

Solvency II (SCR) requirement[1]:

> „The Solvency Capital Requirement corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%, i.e. ruin would occur once every 200 years. The Solvency Capital requirement is calculated using Value-at-Risk techniques, either in accordance with the standard formula, or using an internal Model: all potential losses, including adverse revaluation of assets and liabilities, **over the next 12 months** are to be assessed. The Solvency Capital Requirement reflects the true risk profile of the undertaking, taking account of all quantifiable risks, as well as the net impact of risk mitigation techniques.“

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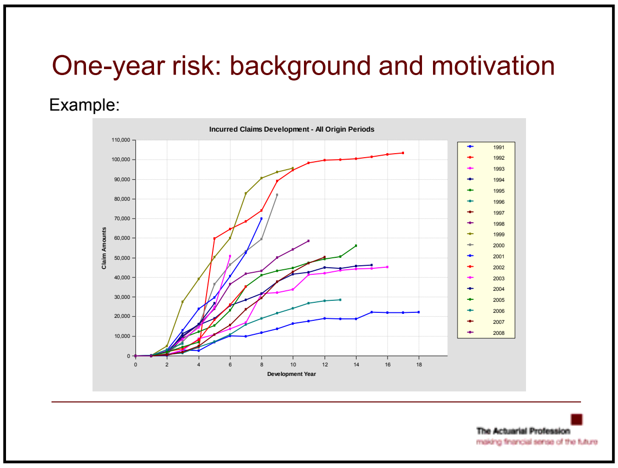
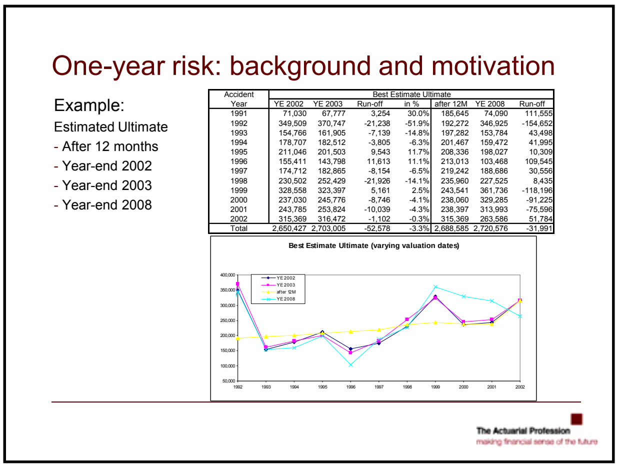
[1] Recast directive



One-year risk: background and motivation

Example:



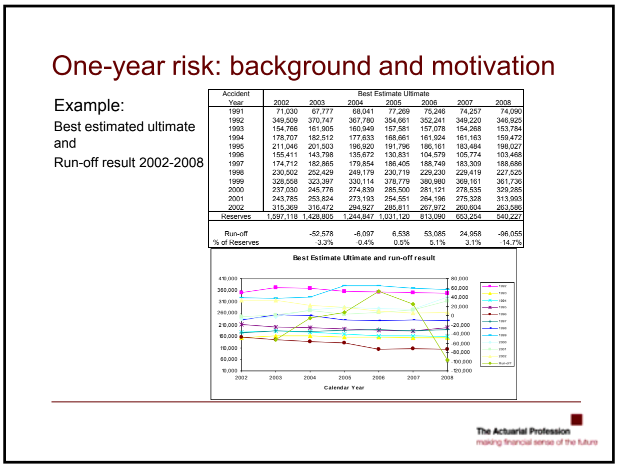
One-year risk: background and motivation

Example:

Estimated Ultimate

- After 12 months
- Year-end 2002
- Year-end 2003
- Year-end 2008

Accident	Best Estimate Ultimate						
Year	YE 2002	YE 2003	Run-off	in %	after 12M	YE 2008	Run-off
1991	71,030	67,777	3,254	30.0%	185,645	74,090	111,555
1992	349,509	370,747	-21,238	-51.9%	192,272	346,925	-154,652
1993	154,766	161,905	-7,139	-14.8%	197,282	153,784	43,498
1994	178,707	182,512	-3,805	-6.3%	201,467	159,472	41,995
1995	211,046	201,503	9,543	11.7%	208,336	198,027	10,309
1996	155,411	143,798	11,613	11.1%	213,013	103,468	109,545
1997	174,712	182,865	-8,154	-6.5%	219,242	188,686	30,556
1998	230,502	252,429	-21,926	-14.1%	235,960	227,525	8,435
1999	328,558	323,397	5,161	2.5%	243,541	361,736	-118,196
2000	237,030	245,776	-8,746	-4.1%	238,060	329,285	-91,225
2001	243,785	253,824	-10,039	-4.3%	238,397	313,993	-75,596
2002	315,369	316,472	-1,102	-0.3%	315,369	263,586	51,784
Total	2,650,427	2,703,005	-52,578	-3.3%	2,688,585	2,720,576	-31,991



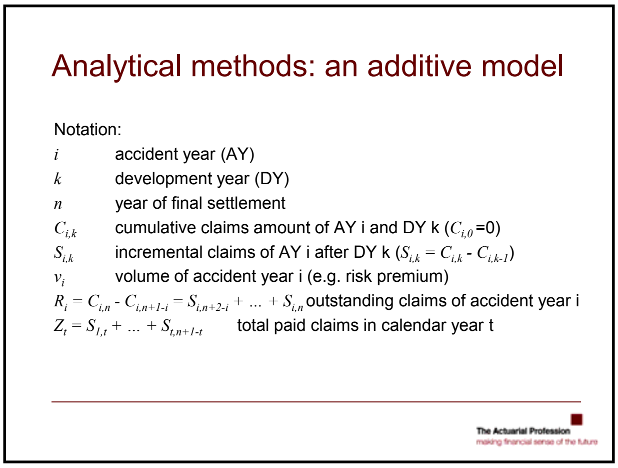
One-year risk: background and motivation

Example:

Best estimated ultimate and

Run-off result 2002-2008

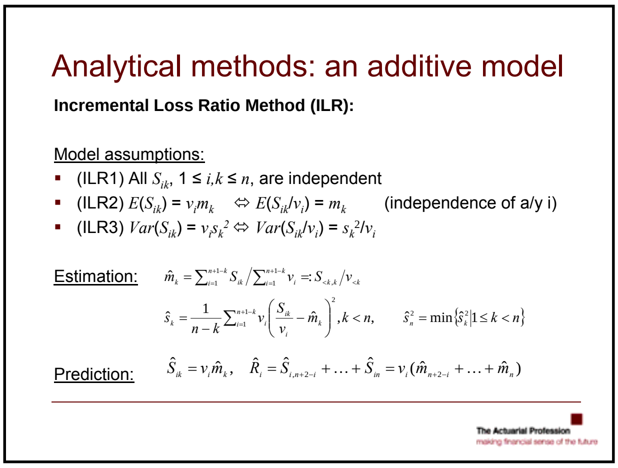
Accident	Best Estimate Ultimate						
Year	2002	2003	2004	2005	2006	2007	2008
1991	71,030	67,777	68,041	77,269	75,246	74,257	74,090
1992	349,509	370,747	367,780	354,661	352,241	349,220	346,925
1993	154,766	161,905	160,949	157,581	157,078	154,268	153,784
1994	178,707	182,512	177,633	168,661	161,924	161,163	159,472
1995	211,046	201,503	196,920	191,796	186,161	183,484	198,027
1996	155,411	143,798	135,672	130,831	104,579	105,774	103,468
1997	174,712	182,865	179,854	186,405	188,749	183,309	188,686
1998	230,502	252,429	249,179	230,719	229,230	229,419	227,525
1999	328,558	323,397	330,114	378,779	380,980	369,161	361,736
2000	237,030	245,776	274,839	285,500	281,121	278,535	329,285
2001	243,785	253,824	273,193	254,551	264,196	275,328	313,993
2002	315,369	316,472	294,927	285,811	267,972	260,604	263,586
Reserves	1,597,118	1,428,805	1,244,847	1,031,120	813,090	653,254	540,227
Run-off		-52,578	-6,097	6,538	53,085	24,958	-96,055
% of Reserves		-3.3%	-0.4%	0.5%	5.1%	3.1%	-14.7%



Analytical methods: an additive model

Notation:

i	accident year (AY)
k	development year (DY)
n	year of final settlement
$C_{i,k}$	cumulative claims amount of AY i and DY k ($C_{i,0}$ =0)
$S_{i,k}$	incremental claims of AY i after DY k ($S_{i,k} = C_{i,k} - C_{i,k-1}$)
v_i	volume of accident year i (e.g. risk premium)
$R_i = C_{i,n} - C_{i,n+1-i} = S_{i,n+2-i} + \ldots + S_{i,n}$	outstanding claims of accident year i
$Z_t = S_{1,t} + \ldots + S_{t,n+1-t}$	total paid claims in calendar year t



Analytical methods: an additive model

Incremental Loss Ratio Method (ILR):

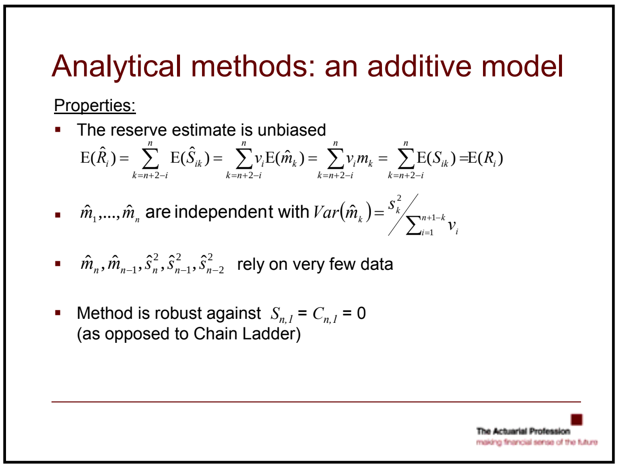
Model assumptions:

- (ILR1) All S_{ik}, $1 \le i,k \le n$, are independent
- (ILR2) $E(S_{ik}) = v_i m_k \Leftrightarrow E(S_{ik}/v_i) = m_k$ (independence of a/y i)
- (ILR3) $Var(S_{ik}) = v_i s_k^2 \Leftrightarrow Var(S_{ik}/v_i) = s_k^2/v_i$

Estimation: $\hat{m}_k = \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} v_i =: S_{<k,k} / v_{<k}$

$$\hat{s}_k = \frac{1}{n-k} \sum_{i=1}^{n+1-k} v_i \left(\frac{S_{ik}}{v_i} - \hat{m}_k \right)^2, k < n, \qquad \hat{s}_n^2 = \min\{\hat{s}_k^2 | 1 \le k < n\}$$

Prediction: $\hat{S}_{ik} = v_i \hat{m}_k , \quad \hat{R}_i = \hat{S}_{i,n+2-i} + \ldots + \hat{S}_{in} = v_i (\hat{m}_{n+2-i} + \ldots + \hat{m}_n)$



Analytical methods: an additive model

Properties:

- The reserve estimate is unbiased
  $$\mathrm{E}(\hat{R}_i) = \sum_{k=n+2-i}^{n} \mathrm{E}(\hat{S}_{ik}) = \sum_{k=n+2-i}^{n} v_i \mathrm{E}(\hat{m}_k) = \sum_{k=n+2-i}^{n} v_i m_k = \sum_{k=n+2-i}^{n} \mathrm{E}(S_{ik}) = \mathrm{E}(R_i)$$
- $\hat{m}_1, \ldots, \hat{m}_n$ are independent with $Var(\hat{m}_k) = s_k^2 \Big/ \sum_{i=1}^{n+1-k} v_i$
- $\hat{m}_n, \hat{m}_{n-1}, \hat{s}_n^2, \hat{s}_{n-1}^2, \hat{s}_{n-2}^2$ rely on very few data
- Method is robust against $S_{n,1} = C_{n,1} = 0$
  (as opposed to Chain Ladder)

Analytical methods: an additive model

Example:

Triangle of incremental claims $s_{i,k}$

Accident						Development Year								
Year	v_i	1	2	3	4	5	6	7	8	9	10	11	12	R_i
1	185.762	29	1.012	8.032	-395	13.734	9.868	-1.037	6.278	5.565	8.637	4.178	4.293	0
2	192.303	120	1.964	4.294	7.321	174.511	15.074	12.247	17.238	47.564	17.397	10.821		4.444
3	197.230	207	2.308	6.454	18.614	6.498	9.028	10.008	46.463	1.400	5.393			12.383
4	201.577	48	2.984	32.743	13.851	10.484	20.428	9.306	9.664	19.250				23.667
5	208.449	50	2.196	26.870	9.313	10.243	24.483	37.688	18.316					44.271
6	213.044	136	1.807	2.896	9.636	8.623	11.388	16.065						66.427
7	219.254	160	1.235	4.756	9.705	18.061	15.851							83.783
8	236.063	82	3.054	27.545	19.279	25.164								107.877
9	242.977	755	14.872	71.200	36.087									150.313
10	237.884	362	3.683	18.432										162.642
11	238.133	450	8.804											185.483
12	315.427	190												251.527
m_k	2.688.102	0,001	0,019	0,095	0,065	0,162	0,075	0,070	0,099	0,095	0,055	0,040	0,023	1.092.818
s_k^2		0,174	66,152	1748,610	396,693	17421,540	145,944	764,377	1259,192	2257,611	203,079	107,797	0,174	

$$\text{e.g.}\quad \hat{m}_{10} = \frac{8,637 + 17,397 + 5,393}{185,762 + 192,303 + 197,230} = 0.055, \qquad \hat{S}_{4,10} = v_4 \cdot \hat{m}_k = 201,577 \cdot 0.055 = 11,011$$

$$Var\left(\hat{S}_{4,10}\right) = Var\left(v_4 \cdot \hat{m}_{10}\right) = v_4^2 \cdot \frac{s_{10}^2}{\sum_{i=1}^{3} v_i} = 10,621,733$$

$$\hat{R}_4 = \hat{S}_{4,10} + \hat{S}_{4,11} + \hat{S}_{4,12} = v_4 \cdot \left(\hat{m}_{10} + \hat{m}_{11} + \hat{m}_{12}\right) = 201,577 \cdot (0.055 + 0.040 + 0.023) = 23,667$$

Analytical methods: an additive model

Change in Reserves (I)

At the end of CY n the total reserve estimate is

$$\hat{R}^{(n)} = \sum_{i=2}^{n} v_i (\hat{m}_{n+2-i}^{(n)} + \hat{m}_{n+3-i}^{(n)} + \ldots + \hat{m}_n^{(n)}) \text{ with } \hat{m}_k^{(n)} =: S_{<k,k} / v_{<k}$$

In CY n + 1 we have (without new business) payments

$Z_{n+1} = S_{2,n} + S_{3,n-1} + \ldots + S_{n,2}$ and an updated reserves estimate of

$$\hat{R}^{(n+1)} = \sum_{i=3}^{n} v_i (\hat{m}_{n+3-i}^{(n+1)} + \ldots + \hat{m}_n^{(n+1)}) \text{ with } \hat{m}_k^{(n+1)} = S_{\le k,k} / v_{\le k}$$

Analytical methods: an additive model

Change in Reserves (II)

The total change in reserves Δ_{n+1} in calendar year n+1 is

$$\Delta_{n+1} = Z_{n+1} + \hat{R}^{(n+1)} - \hat{R}^{(n)}$$

$$= \sum_{i=2}^{n} \left(S_{i,n+2-i} - v_i \hat{m}_{n+2-i}^{(n)} + \sum_{k=n+3-i}^{n} v_i (\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)}) \right)$$

Using $$\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)} = \frac{v_{n+2-k}}{v_{\le k}} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) \qquad (*)$$

and inverting the sums this leads to

$$\Delta_{n+1} = \sum_{k=2}^{n} \frac{v_{n+2-k} v_+}{v_{\le k}} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) \quad \text{with } v_+ = v_1 + \ldots + v_n$$

Analytical methods: an additive model

Side calculation:

$$\begin{aligned}
\Delta_{n+1} &= Z_{n+1} + \hat{R}^{(n+1)} - \hat{R}^{(n)} \\
&= \sum_{i=2}^{n} \left(S_{i,n+2-i} - v_i \hat{m}_{n+2-i}^{(n)} + \sum_{k=n+3-i}^{n} v_i (\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)}) \right) \\
&= \sum_{i=2}^{n} \left(S_{i,n+2-i} - v_i \hat{m}_{n+2-i}^{(n)} \right) + \sum_{i=2}^{n} \sum_{k=n+3-i}^{n} v_i (\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)}) \\
&= \sum_{k=2}^{n} \left(S_{n+2-k,k} - v_{n+2-k} \hat{m}_k^{(n)} \right) + \sum_{k=2}^{n} \sum_{i=n+3-k}^{n} v_i (\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)}) \\
&= \sum_{k=2}^{n} v_{n+2-k} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) + \sum_{k=2}^{n} v_{>k} (\hat{m}_k^{(n+1)} - \hat{m}_k^{(n)}) \\
&= \sum_{k=2}^{n} v_{n+2-k} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) \left(1 + \frac{v_{>k}}{v_{\le k}} \right) \quad \text{cp. (*)} \\
&= \sum_{k=2}^{n} v_{n+2-k} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) \frac{v_+}{v_{\le k}} \quad \text{with } v_+ = v_1 + \ldots + v_n
\end{aligned}$$

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Analytical methods: an additive model

The reserve risk

So the total volatility of the change in reserves is

$$Var(\Delta_{n+1}) = Var\left(\sum_{k=2}^{n} \frac{v_{n+2-k} v_+}{v_{\le k}} \left(\frac{S_{n+2-k,k}}{v_{n+2-k}} - \hat{m}_k^{(n)} \right) \right)$$

....

$$= \sum_{k=2}^{n} \left(\frac{v_{n+2-k} v_+}{v_{\le k}} \right)^2 \left(\frac{s_k^2}{v_{n+2-k}} + \frac{s_k^2}{v_{<k}} \right) = \sum_{k=2}^{n} \frac{v_+^2}{v_{\le k} v_{<k}} v_{n+2-k} s_k^2$$

Estimated by inserting $\hat{s}_k^2$

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Analytical methods: an additive model

Conclusion

(1) Because of $v_+ > v_{\le k}$ we have $Var(\Delta_{n+1}) > Var(Z_{n+1})$
e.g. CY volatility is bigger than volatility in 1-year cashflow only

(2) With new business (inclusion of premium risk), the summation starts at
$k = 1$, $v_+ = v_1 + \ldots + v_{n+1}$

.

(3) Chain Ladder is more difficult due to the missing independence between the increments

$\hat{s}_k^2$

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Analytical methods: Merz, Wüthrich

Chain-Ladder Model (CL):

- Merz and Wüthrich[1] developed formulae for the (conditional) mean squared prediction error of the total one year run-off result (CDR(I+1)):
- For a single accident year i:

$$\hat{msep}_{\hat{CDR}_i(I+1)|\mathrm{D}}(0) = \left(\hat{C}_{i,J}^{I}\right)^2 \left[\frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^{I})^2}{C_{i,I-i}} + \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^{I})^2}{S_{I-i}^{I}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{S_j^{I+1}} \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_j^{I})^2}{S_j^{I}} \right]$$

- Total of all accident years:

$$\hat{msep}_{\sum_{i=1}^{I}\hat{CDR}_i(I+1)|\mathrm{D}}(0) = \hat{msep}_{\hat{CDR}_i(I+1)|\mathrm{D}}(0) + 2\sum_{k>i>0} \hat{C}_{i,J}^{I}\,\hat{C}_{k,J}^{I} \left[\frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^{I})^2}{S_{I-i}^{I}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{S_j^{I+1}} \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_j^{I})^2}{S_j^{I}} \right]$$

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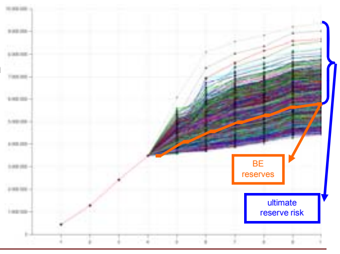
[1] e.g. Merz, Wüthrich (2008). Modelling the Claims Development Results for Solvency Purposes. ASTIN 2008

Modelling of the one-year risk in simulation models

Simulation model (reserve risk) – 1st step:

The best estimate loss reserves $R_{BE(t=0)}$ at time t=0 are estimated based on a loss reserving algorithm A.

- Algorithm A is also underlying the simulation model that we use to simulate the distribution of possible outcomes of the outstanding claims $R_{t=0}$ $E(R_{t=0}) = R_{BE(t=0)}$.
- So-called actuarial judgement cannot be reflected in algorithm A.

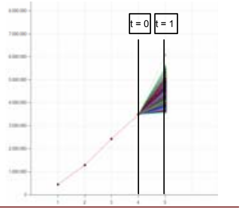


Modelling of the one-year risk in simulation models

Simulation model (reserve risk) – 2nd step:

- Next we look at the simulated results of the first year, conditional on the observations at time t=0.
- If C_{ij} are the claims in accident year i at development j, the second step means that we simulate the new diagonal of our claims development triangle.

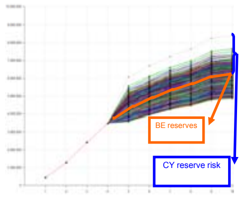
Anfall-	Abwicklungsjahr						
jahr	1	2	3	...	i-1	i	i+1
i - 9	$C_{i-9,1}$	$C_{i-9,2}$	$C_{i-9,3}$	...	$C_{i-9,i-1}$	$C_{i-9,i}$	**$C_{i-9,i+1}$**
i - 8	$C_{i-8,1}$	$C_{i-8,2}$	$C_{i-8,3}$	...	$C_{i-8,i-1}$	**$C_{i-8,i}$**	
i - 7	$C_{i-7,1}$	$C_{i-7,2}$	$C_{i-7,3}$	...	**$C_{i-7,i-1}$**		
⋮	⋮	⋮	⋮				
i - 1	$C_{i-1,1}$	$C_{i-1,2}$	**$C_{i-1,3}$**				
i	$C_{i,1}$	**$C_{i,2}$**					



Modelling of the one-year risk in simulation models

Simulation model (reserve risk) – 3rd step:

- The last step is to calculate the best-estimate claims reserves $R_{BE(t=1)}$ for every simulation from step **2** based on the same algorithm A from step **1**. We call this step “**re-reserving**”.
- Algorithm A could be a generalised Chain-Ladder Method or a Bornhuetter-Ferguson Method.
- $R_{BE(t=1)}$ is a random variable and describes the distribution of outstanding claims at time t=1.
- $R_{BE(t=0)} – (R_{BE(t=1)} + Z_{t=1})$ describes the one year run-off result or the one year reserve risk, with $Z_{t=1}$ being the claim payments in calendar year t=1.



Modelling of the one-year risk in simulation models

Simulation model (premium risk):

1st step:

- We simulate the distribution of the best estimate ultimate based on an “ultimate” premium risk model.
- Attritional claims, large claims and natural catastrophe claims are modelled separately based on frequency / severity models.

2nd step:

- The proportion of claims that are paid or reported in the first year, $C_{i+1,1}$, can be simulated using a Beta distribution.

Modelling of the one-year risk in simulation models

Simulation model (premium risk):

3rd step:

In the last step we use the same claims reserving algorithm A from the reserve risk model to estimate the best-estimate claim reserves $R_{BE(t=1)}$ at time t=1 for every simulation from step **2**.

Accident			Development Year				
Year	1	2	3	…	i-1	i	i+1
i - 9	$C_{i-9,1}$	$C_{i-9,2}$	$C_{i-9,3}$	…	$C_{i-9,i-1}$	$C_{i-9,i}$	$\mathbf{C_{i-9,i+1}}$
i - 8	$C_{i-8,1}$	$C_{i-8,2}$	$C_{i-8,3}$	…	$C_{i-8,i-1}$	$\mathbf{C_{i-8,i}}$	
i - 7	$C_{i-7,1}$	$C_{i-7,2}$	$C_{i-7,3}$	…	$\mathbf{C_{i-7,i-1}}$		
⋮	⋮	⋮	⋮				
i - 1	$C_{i-1,1}$	$C_{i-1,2}$	$\mathbf{C_{i-1,3}}$				
i	$C_{i,1}$	$\mathbf{C_{i,2}}$					
i+1	$\mathbf{C_{i+1,1}}$						

Comparison of analytical and simulation models

Analytical models:

- Models only give us first two moments, not a full distribution of possible outcomes;
- Calculations can be done in a spreadsheet, no simulation software needed;

Simulation models:

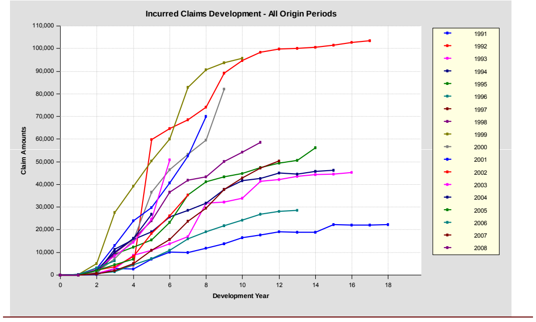
- The model provides the full distribution of possible outcomes, including the underlying stochastic cash-flows.
- The model can easily be generalised to use the Bornhuetter-Fergusons Method for re-reserving.
- Tailfactors can easily be taken into account by fitting a parametric curve to the development factors.
- The model can easily be generalised to also estimate the one-year premium risk in a consistent way.
- The communication of results can easily be supported graphically (Solvency II: „use test" !)
- The validation of results might be easier because more intuitive than analytical methods (Solvency II: „ORSA" !)

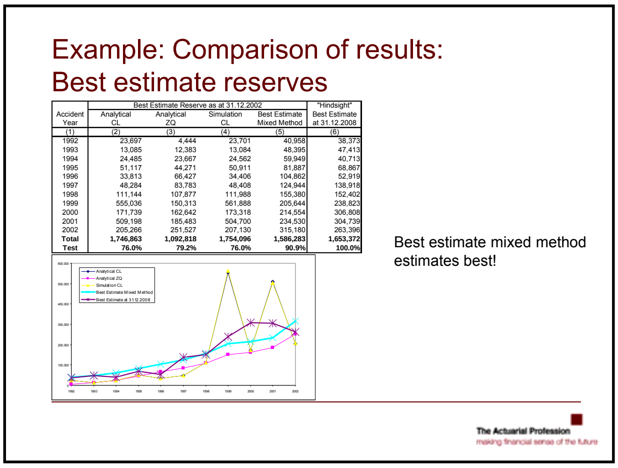
Example

We will compare the results of two analytical methods with the results of the simulation model and empirical observations

- **Analytical methods:**
  - Additive model (ZQ) (no tail)
  - Chain-Ladder Model (CL) (no tail)
- **Simulation methods:**
  - Chain-Ladder Model (no tail)
  - Mixed method of CL and Bornhuetter-Ferguson (with and without tail)

Example

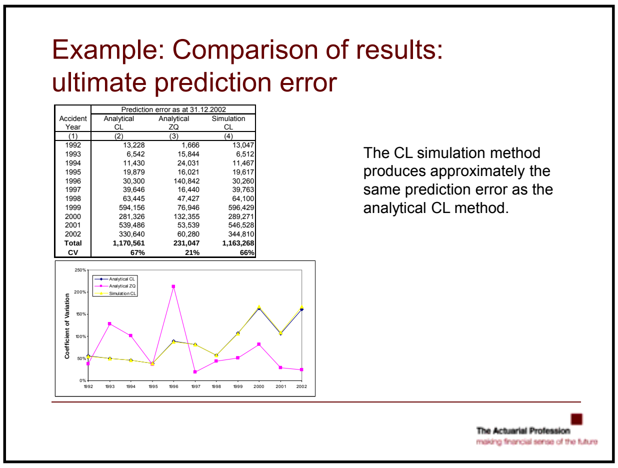




Example: Comparison of results: Best estimate reserves

Accident Year	Best Estimate Reserve as at 31.12.2002 Analytical CL	Analytical ZQ	Simulation CL	Best Estimate Mixed Method	"Hindsight" Best Estimate at 31.12.2008
(1)	(2)	(3)	(4)	(5)	(6)
1992	23,697	4,444	23,701	40,958	38,373
1993	13,085	12,383	13,084	48,395	47,413
1994	24,485	23,667	24,562	59,949	40,713
1995	51,117	44,271	50,911	81,887	68,867
1996	33,813	66,427	34,406	104,862	52,919
1997	48,284	83,783	48,408	124,944	138,918
1998	111,144	107,877	111,988	155,380	152,402
1999	555,036	150,313	561,888	205,644	238,823
2000	171,739	162,642	173,318	214,554	306,808
2001	509,198	185,483	504,700	234,530	304,739
2002	205,266	251,527	207,130	315,180	263,396
Total	**1,746,863**	**1,092,818**	**1,754,096**	**1,586,283**	**1,653,372**
Test	**76.0%**	**79.2%**	**76.0%**	**90.9%**	**100.0%**

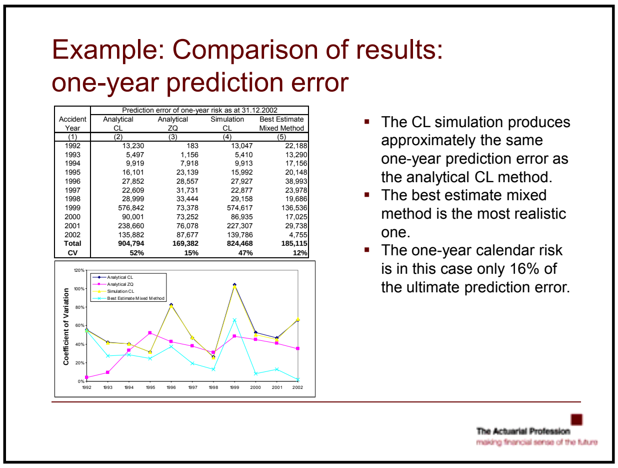
Best estimate mixed method estimates best!



Example: Comparison of results: ultimate prediction error

Accident Year	Prediction error as at 31.12.2002 Analytical CL	Analytical ZQ	Simulation CL
(1)	(2)	(3)	(4)
1992	13,228	1,666	13,047
1993	6,542	15,844	6,512
1994	11,430	24,031	11,467
1995	19,879	16,021	19,617
1996	30,300	140,842	30,260
1997	39,646	16,440	39,763
1998	63,445	47,427	64,100
1999	594,156	76,946	596,429
2000	281,326	132,355	289,271
2001	539,486	53,539	546,528
2002	330,640	60,280	344,810
Total	**1,170,561**	**231,047**	**1,163,268**
CV	**67%**	**21%**	**66%**

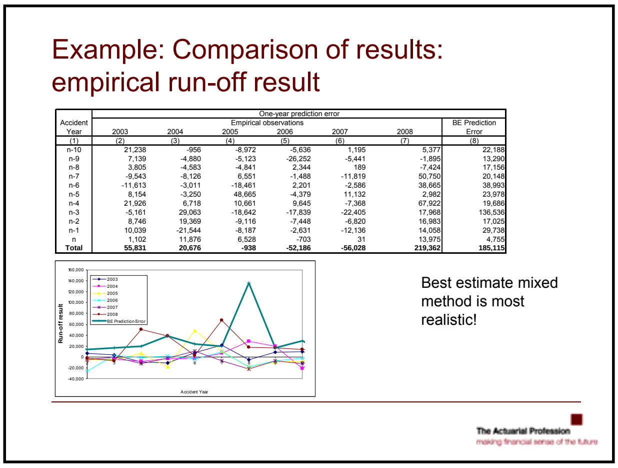
The CL simulation method produces approximately the same prediction error as the analytical CL method.



Example: Comparison of results: one-year prediction error

Accident Year	Prediction error of one-year risk as at 31.12.2002 Analytical CL	Analytical ZQ	Simulation CL	Best Estimate Mixed Method
(1)	(2)	(3)	(4)	(5)
1992	13,230	183	13,047	22,188
1993	5,497	1,156	5,410	13,290
1994	9,919	7,918	9,913	17,156
1995	16,101	23,139	15,992	20,148
1996	27,852	28,557	27,927	38,993
1997	22,609	31,731	22,877	23,978
1998	28,999	33,444	29,158	19,686
1999	576,842	73,378	574,617	136,536
2000	90,001	73,252	86,935	17,025
2001	238,660	76,078	227,307	29,738
2002	135,882	87,677	139,786	4,755
Total	**904,794**	**169,382**	**824,468**	**185,115**
CV	**52%**	**15%**	**47%**	**12%**

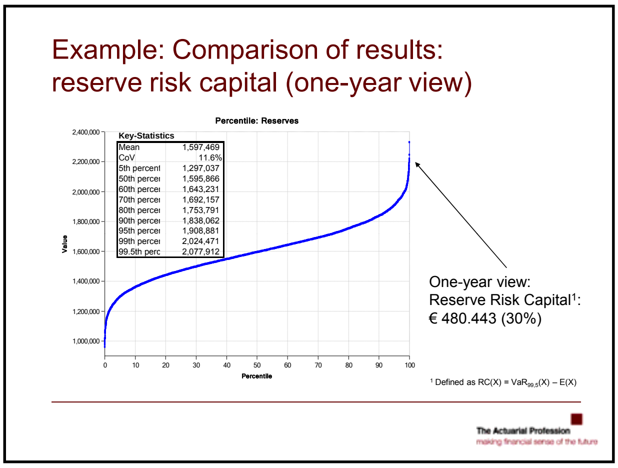
- The CL simulation produces approximately the same one-year prediction error as the analytical CL method.
- The best estimate mixed method is the most realistic one.
- The one-year calendar risk is in this case only 16% of the ultimate prediction error.



Example: Comparison of results: empirical run-off result

Accident Year	One-year prediction error						
	Empirical observations						BE Prediction Error
	2003	2004	2005	2006	2007	2008	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
n-10	21,238	-956	-8,972	-5,636	1,195	5,377	22,188
n-9	7,139	-4,880	-5,123	-26,252	-5,441	-1,895	13,290
n-8	3,805	-4,583	-4,841	2,344	189	-7,424	17,156
n-7	-9,543	-8,126	6,551	-1,488	-11,819	50,750	20,148
n-6	-11,613	-3,011	-18,461	2,201	-2,586	38,665	38,993
n-5	8,154	-3,250	48,665	-4,379	11,132	2,982	23,978
n-4	21,926	6,718	10,661	9,645	-7,368	67,922	19,686
n-3	-5,161	29,063	-18,642	-17,839	-22,405	17,968	136,536
n-2	8,746	19,369	-9,116	-7,448	-6,820	16,983	17,025
n-1	10,039	-21,544	-8,187	-2,631	-12,136	14,058	29,738
n	1,102	11,876	6,528	-703	31	13,975	4,755
Total	**55,831**	**20,676**	**-938**	**-52,186**	**-56,028**	**219,362**	**185,115**

Best estimate mixed method is most realistic!



Example: Comparison of results: reserve risk capital (one-year view)

Percentile: Reserves

Key-Statistics	
Mean	1,597,469
CoV	11.6%
5th percent	1,297,037
50th percei	1,595,866
60th percei	1,643,231
70th percei	1,692,157
80th percei	1,753,791
90th percei	1,838,062
95th percei	1,908,881
99th percei	2,024,471
99.5th perc	2,077,912

One-year view:
Reserve Risk Capital[1]:
€ 480.443 (30%)

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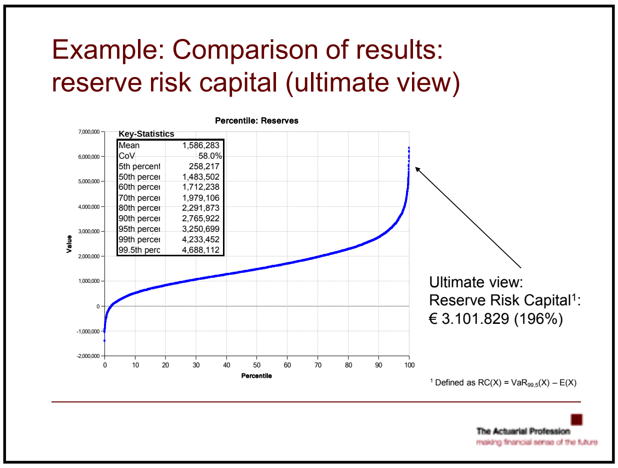
[1] Defined as RC(X) = $VaR_{99,5}(X) – E(X)$



Example: Comparison of results: reserve risk capital (ultimate view)

Percentile: Reserves

Key-Statistics	
Mean	1,586,283
CoV	58.0%
5th percent	258,217
50th percei	1,483,502
60th percei	1,712,238
70th percei	1,979,106
80th percei	2,291,873
90th percei	2,765,922
95th percei	3,250,699
99th percei	4,233,452
99.5th perc	4,688,112

Ultimate view:
Reserve Risk Capital[1]:
€ 3.101.829 (196%)

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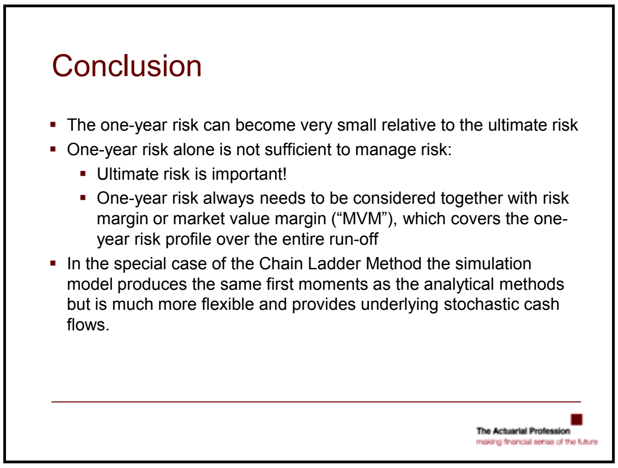
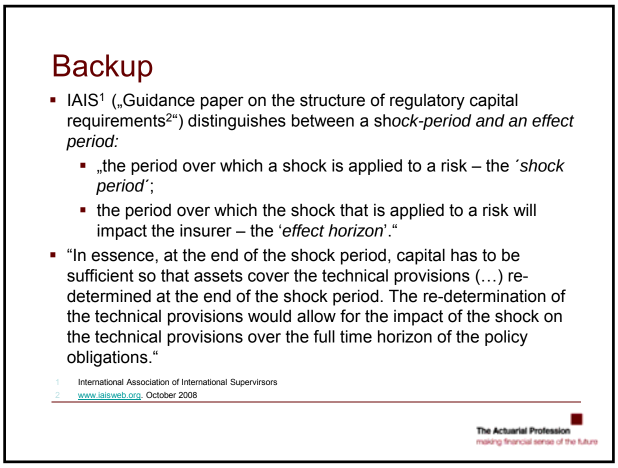
[1] Defined as RC(X) = $VaR_{99,5}(X) – E(X)$



Conclusion

- The one-year risk can become very small relative to the ultimate risk
- One-year risk alone is not sufficient to manage risk:
  - Ultimate risk is important!
  - One-year risk always needs to be considered together with risk margin or market value margin ("MVM"), which covers the one-year risk profile over the entire run-off
- In the special case of the Chain Ladder Method the simulation model produces the same first moments as the analytical methods but is much more flexible and provides underlying stochastic cash flows.



Backup

- IAIS[1] („Guidance paper on the structure of regulatory capital requirements[2]") distinguishes between a sh*ock-period and an effect period:*
  - „the period over which a shock is applied to a risk – the ´*shock period*´;
  - the period over which the shock that is applied to a risk will impact the insurer – the '*effect horizon*'."
- "In essence, at the end of the shock period, capital has to be sufficient so that assets cover the technical provisions (…) re-determined at the end of the shock period. The re-determination of the technical provisions would allow for the impact of the shock on the technical provisions over the full time horizon of the policy obligations."

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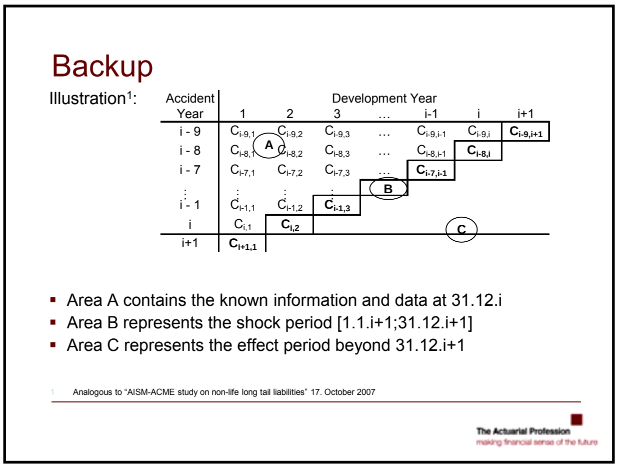
[1] International Association of International Supervirsors

[2] www.iaisweb.org. October 2008



Backup

Illustration[1]:

Accident Year	Development Year 1	2	3	…	i-1	i	i+1
i - 9	$C_{i-9,1}$	$C_{i-9,2}$	$C_{i-9,3}$	…	$C_{i-9,i-1}$	$C_{i-9,i}$	$\mathbf{C_{i-9,i+1}}$
i - 8	$C_{i-8,1}$	$C_{i-8,2}$	$C_{i-8,3}$	…	$C_{i-8,i-1}$	$\mathbf{C_{i-8,i}}$	
i - 7	$C_{i-7,1}$	$C_{i-7,2}$	$C_{i-7,3}$	…	$\mathbf{C_{i-7,i-1}}$		
⋮	⋮	⋮	⋮				
i - 1	$C_{i-1,1}$	$C_{i-1,2}$	$\mathbf{C_{i-1,3}}$				
i	$C_{i,1}$	$\mathbf{C_{i,2}}$					
i+1	$\mathbf{C_{i+1,1}}$						

- Area A contains the known information and data at 31.12.i
- Area B represents the shock period [1.1.i+1;31.12.i+1]
- Area C represents the effect period beyond 31.12.i+1

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[1] Analogous to "AISM-ACME study on non-life long tail liabilities" 17. October 2007

Backup

- Most publications on claims reserving consider only the ultimate risk, i.e. let R_0 be the claims reserves of the opening balance and C_∞ the claims payments over the entire run-off, then we can describe the ultimate reserve risk by $RR_\infty = R_0 - C_\infty$
- AISM-ACEME[1]:
  "Only a few members were aware of the inconsistency between their assessment on the ultimate cost and the Solvency II framework which uses a one year horizon. … the use of innovative actuarial methodologies is required to replace the classical ones which are inappropriate." (2007, press release)

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[1] Association Internationale des Sociétés d'Assurance Mutuelle – Association of European Cooperative and Mutual Insurers

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Chain-Ladder simulation method

Simulation model (1/2):

- The chain ladder bootstrap considers the development factors $F_{ik:}$

$$E\left[F_{ik} \mid C_{ik}\right] = f_k \qquad Var\left[F_{ik} \mid C_{ik}\right] = \frac{\sigma_k^2}{C_{ik}}$$

- The scaled Pearson residuum is defined as:

$$r_{ik} = r_{PS}(F_{ik}, \hat{f}_k, w_{ik}, \hat{\sigma}_k) = \frac{\sqrt{w_{ik}}\,(F_{ik} - \hat{f}_k)}{\hat{\sigma}_k}$$

- The chain ladder model is a recursive model, where the forecast is simulated step by step. The starting point are the cumulated claims

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Chain-Ladder simulation method

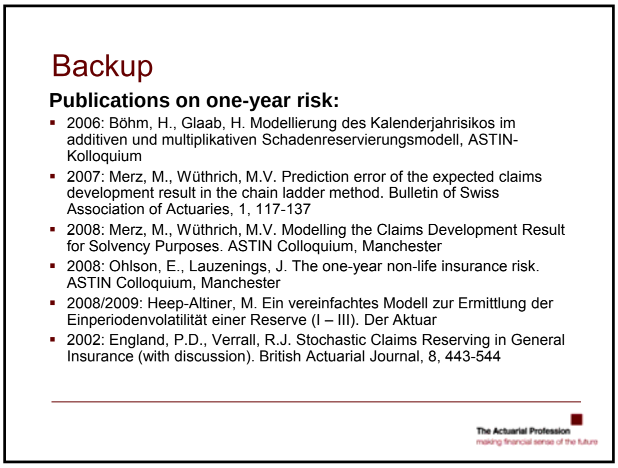
Simulation model (2/2):

- The forecast in the first step is obtained for every bootstrap iteration by sampling from the underlying process distribution, i.e. for i = 2, 3, …,I and k = I+2-i:

$$C_{i,k+1}^{*} \mid C_{ik} \sim Normal(\tilde{f}_k C_{ik}, \hat{\sigma}_k^2 C_{ik})$$

- The forecast in the second step is obtained by sampling from:

$$C_{i,k+1}^{*} \mid C_{ik}^{*} \sim Normal(\tilde{f}_k C_{ik}^{*}, \hat{\sigma}_k^2 C_{ik}^{*}), \mathrm{i} = 3, 4, \ldots, \mathrm{I} \text{ und } \mathrm{k} = \mathrm{I} + 3 - \mathrm{i}, \mathrm{I} + 4 \ \ \mathrm{i}, \ldots, \mathrm{I}$$



Backup

Publications on one-year risk:

- 2006: Böhm, H., Glaab, H. Modellierung des Kalenderjahrisikos im additiven und multiplikativen Schadenreservierungsmodell, ASTIN-Kolloquium
- 2007: Merz, M., Wüthrich, M.V. Prediction error of the expected claims development result in the chain ladder method. Bulletin of Swiss Association of Actuaries, 1, 117-137
- 2008: Merz, M., Wüthrich, M.V. Modelling the Claims Development Result for Solvency Purposes. ASTIN Colloquium, Manchester
- 2008: Ohlson, E., Lauzenings, J. The one-year non-life insurance risk. ASTIN Colloquium, Manchester
- 2008/2009: Heep-Altiner, M. Ein vereinfachtes Modell zur Ermittlung der Einperiodenvolatilität einer Reserve (I – III). Der Aktuar
- 2002: England, P.D., Verrall, R.J. Stochastic Claims Reserving in General Insurance (with discussion). British Actuarial Journal, 8, 443-544

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