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Operational Risk

Bending the tail of the dragon

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AGENDA

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- Operational Risk
- Model Assumption

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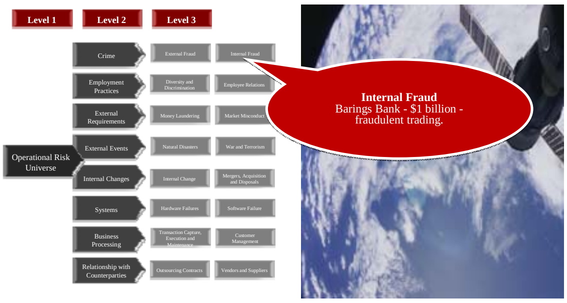
- Model Assumption
- Case Study

Operational Risk Universe

Level 1	Level 2	Level 3	
Operational Risk Universe	Crime	External Fraud	Internal Fraud
	Employment Practices	Diversity and Discrimination	Employee Relations
	External Requirements	Money Laundering	Market Misconduct
	External Events	Natural Disasters	War and Terrorism
	Internal Changes	Internal Change	Mergers, Acquisition and Disposals
	Systems	Hardware Failures	Software Failure
	Business Processing	Transaction Capture, Execution and Maintenance	Customer Management
	Relationship with Counterparties	Outsourcing Contracts	Vendors and Suppliers

Operational Risk Universe



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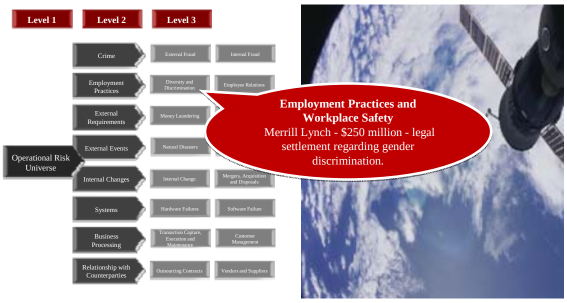
Operational Risk Universe



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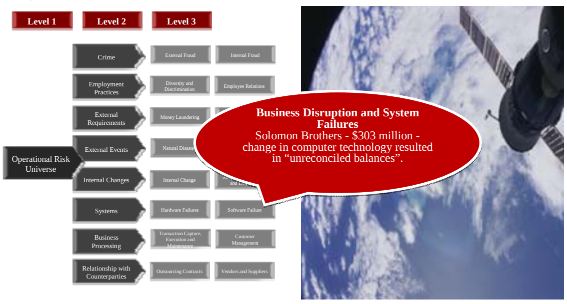
Operational Risk Universe



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Operational Risk Universe

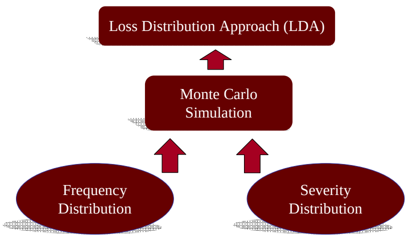
More than 100 losses exceeding $100 Million over the last decade in the financial industry

Operational Risk Loss Characteristic

High frequency low Impact operational risk events

Low frequency high impact operational risk events

Operational Risk Quantification



Questions we hope to answer…

1. How much is the capital calculation affected by the choice of model for operational risk assessment?
2. Is it necessary to make use of more sophisticated models?

Data Availability

- Internal data not collected.
- Insufficient internal data.
- Insufficient internal data / external data.
- Internal data / external data.
- Internal data / consortium data / publicly available data.

Scenario Analysis Model

- Expert opinions on annual frequency and on percentile values of the severity.
- Distribution assumption for the severity and frequency distribution.
- Find the inverse function of the distribution.
- Solve system of equations.
- Weibull distribution

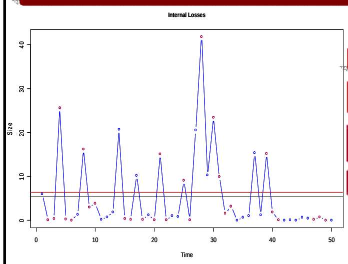
$$\begin{cases} -\beta\left(\log\left(1-q_1\right)\right)^{1/\alpha} = x_{q_1} \\ -\beta\left(\log\left(1-q_2\right)\right)^{1/\alpha} = x_{q_2} \end{cases} \Rightarrow \quad \alpha = \frac{\log(\log(1-q_2)/\log(1-q_1))}{\log(x_{q_2}/x_{q_1})}, \quad \beta = -\left(\frac{x_{q_1}^{\alpha}}{\log(1-q_1)}\right)^{1/\alpha}$$

Parametric Model

- Parametric fit based on internal data / external data.
- Distribution assumption for the severity and frequency distribution.
- Maximum likelihood estimation.
- For example, the lognormal distribution.

Underreporting Model

- Underreporting means that not all losses in the company are reported.

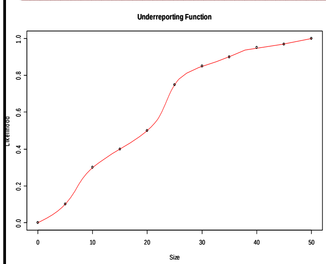


- Occurred losses $(X_i)_{1\le i\le 50} \sim g$
- Indicator function $I(i) = \begin{cases} 1 \text{ if } X_i \text{ is reported} \\ 0 \text{ otherwise} \end{cases}$
- Total number of reported: $N = \sum_{i=1}^{50} I(i)$
- Reported losses $(Y_j)_{1\le j\le 27} \sim f$

Ref: [1] & [2]

Underreporting Model

- An underreporting function encodes the likelihood that a loss of particular size is reported.



- Underreporting function $u(x)$
- Define g as a function of f and u

$$g(x) = \frac{f(x)\{u(x)\}^{-1}}{\int f(w)\{u(w)\}^{-1}dw}$$

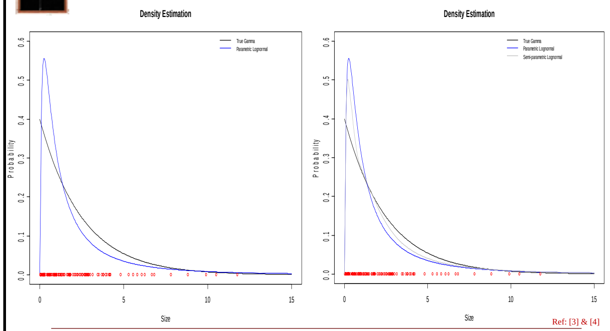
- The probability of observing an operational risk loss $\int_0^\infty g(w)u(w)dw$
- Assume Poisson distribution, then

$M \in Po(\lambda)$, and

$N \in Po\left(\lambda \int_0^\infty g(w)u(w)dw\right)$

Ref: [1] & [2]

Semiparametric Model



Ref: [3] & [4]

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Semiparametric Model

- When data are limited, the model is close to a parametric model.
- As the number of losses increases, the model becomes more non-parametric.
  - Distribution Assumption. Estimate the distribution parameters on the data $(X_i)_{1\le i\le 100}$
  - Transform losses to the bounded support $[0,1]$ with $Z_i = F_\theta(X_i)$
  - KDE: $\hat{g}(z) = \frac{1}{100}\sum_{i=1}^{100} K_h(Z_i - z), K(z) = \frac{3}{4}(1-z^2)I_{\{|z|<1\}}, K_h(\cdot) = \frac{1}{h}K\left(\frac{\cdot}{h}\right)$
  - Semi-parametric density. $\hat{f}(x) = f_\theta(x)\hat{g}(x)$

Ref: [3] & [4]

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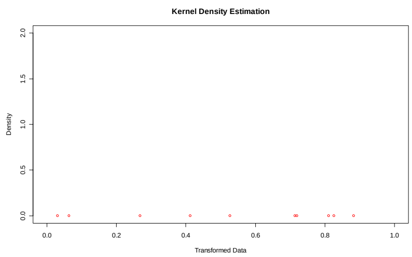
Semiparametric Model



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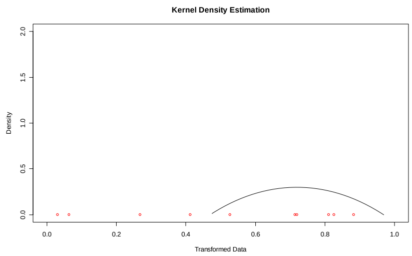

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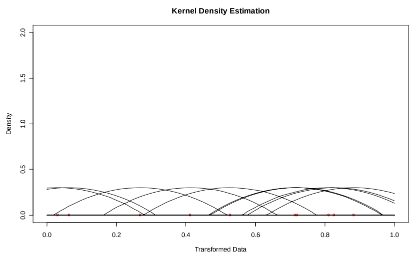

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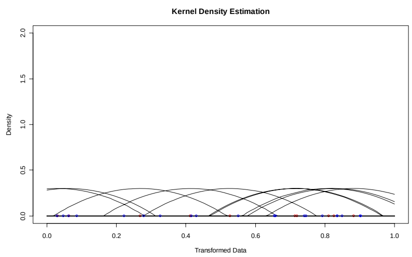

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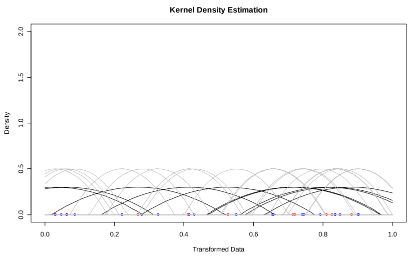

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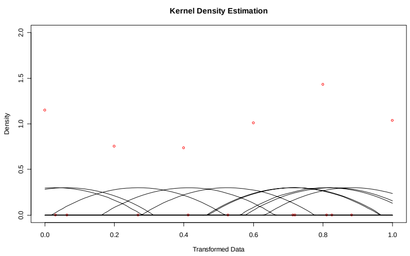

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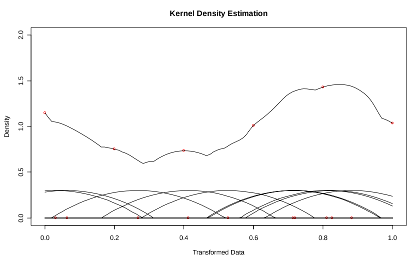

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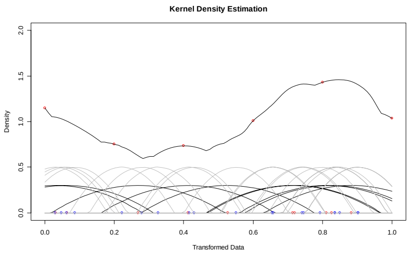

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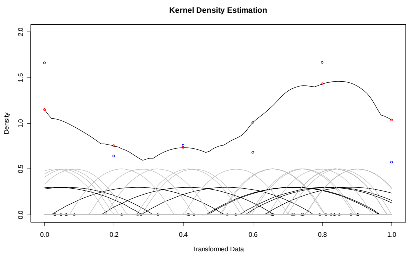

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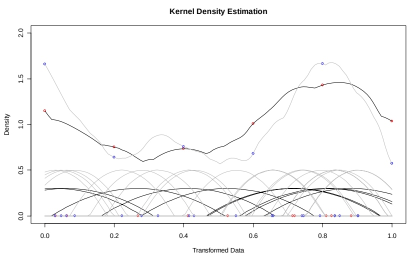

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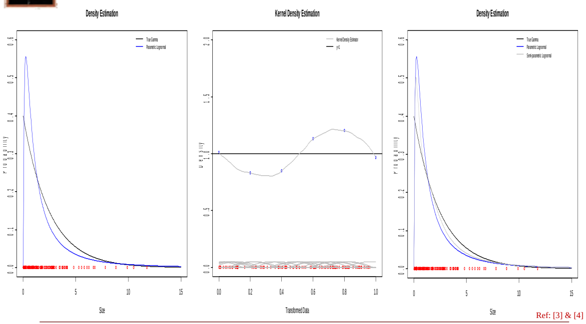

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Mixing Model

- Include prior knowledge from external data.
- Correct the external global start with internal observed data.
  - Distribution Assumption: Estimate the distribution parameters on the data $(Y_j)_{0\le j\le M}$
  - Transform losses to the bounded support [0,1] with $Z_i = F_{\hat{\theta}}(X_i)$
  - Kernel density estimation $\hat{g}(z) = \frac{1}{100}\sum_{i=1}^{100} K_h(Z_i - z),\ K(z) = \frac{3}{4}(1-z^2)I_{\{|z|\le 1\}}$
  - Semi-parametric density: $\hat{f}_M(x) = f_{\hat{\theta}}(x)\hat{g}(x)$

Ref: [5]

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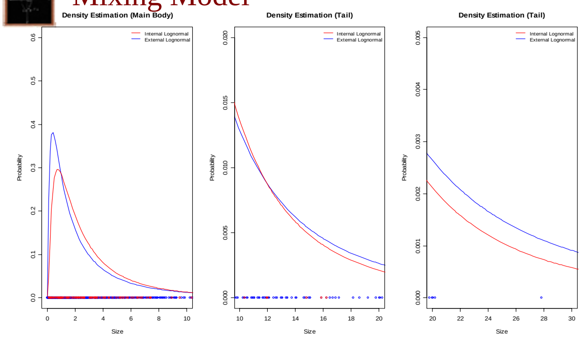
Mixing Model



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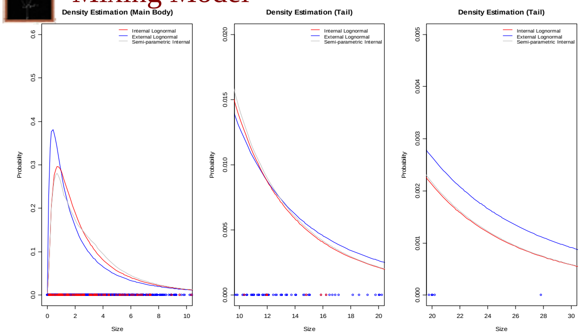
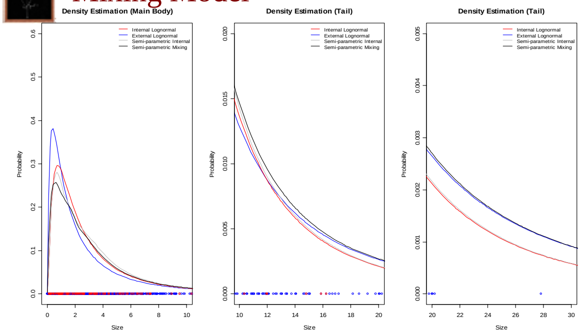


Mixing Model



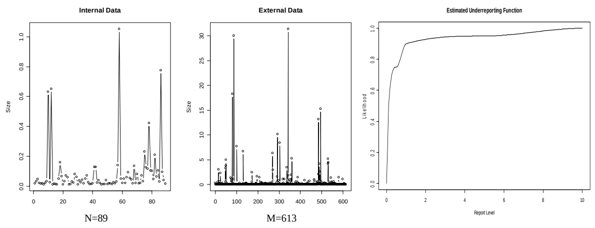
Mixing Credibility Model

- Distribution Assumption. Estimate distribution parameters on the data $(X_i)_{1\le i\le N}$ and $(Y_j)_{1\le j\le M}$
- Transform losses to the bounded support [0,1] with $Z_i = F_{\hat\theta_x}(X_i)$ and $V_j = F_{\hat\theta_y}(Y_j)$
- Kernel Density Estimation $\hat{h}(u)$ - Internal and $\hat{g}(u)$ - External
- Credibility Density Estimation, $\hat{f}_C(u) = (1 - z_u)\hat{g}(u) + z_u\hat{h}(u),\ z_u = COV(h(u), \hat{h}(u)) / V(\hat{h}(u))$
- Semi-parametric density, $\hat{f}_M(x) = f_{\hat\theta_x}(x)\,\hat{f}_C(x)$

Ref: [6] and [7]

Case Study

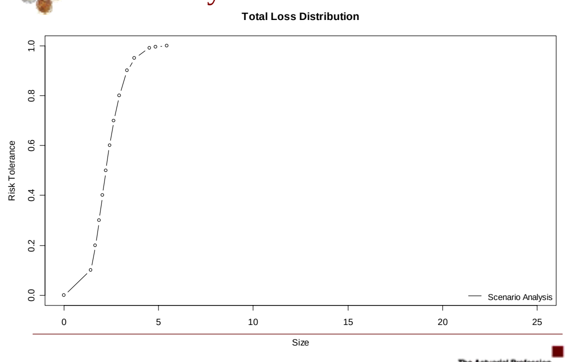
- Scenario Analysis $x_{0.5} = 0.05$, $x_{0.95} = 1$, $\lambda = 30$



Case Study

Summary Severity Assumption		
Model 1	Scenario Analysis	Expert Opinion
Model 2	Parametric Lognormal	Internal Data
Model 3	Semi-Parametric Lognormal	Internal Data
Model 4	Parametric Lognormal Underreporting	Internal Data, Expert Opinion
Model 5	Parametric Lognormal	External Data
Model 6	Mixing	Internal Data, External Data
Model 7	Mixing Credibility	Internal Data, External Data

Case Study



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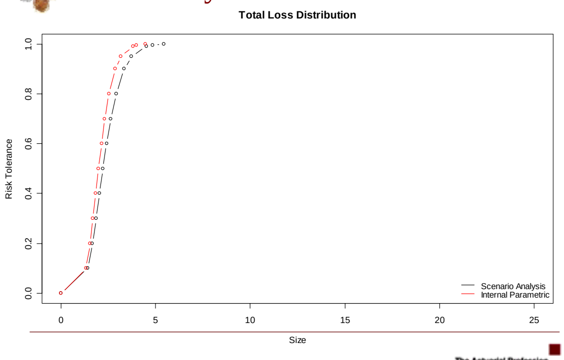

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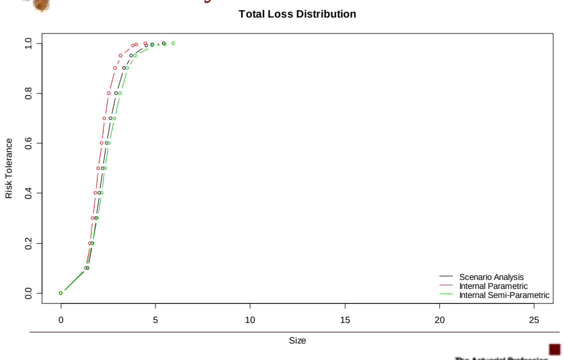

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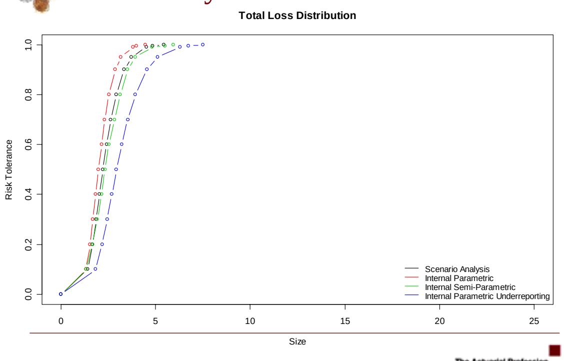

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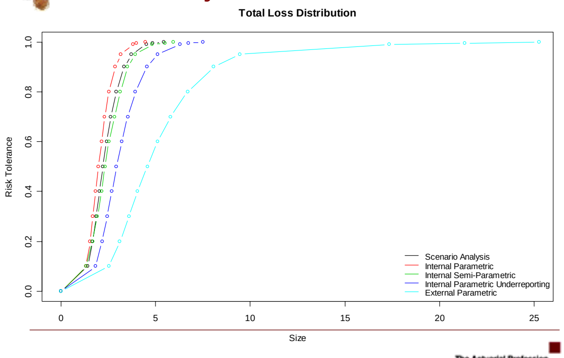

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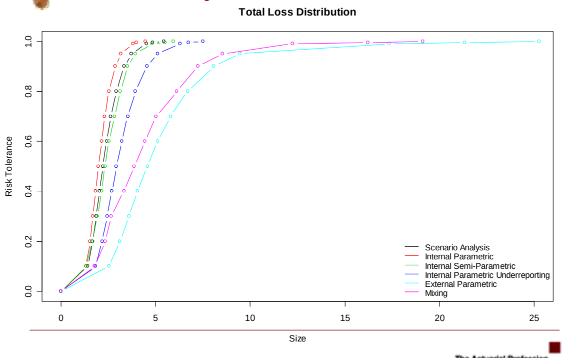

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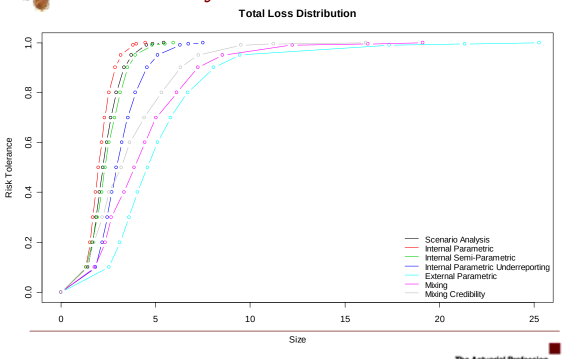

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Questions we hope to answer…

1. How much is the capital calculation affected by the choice of model for operational risk assessment?

Very much! The model choice depends on the information available.

2. Is it necessary to make use of more sophisticated models?

'All models are wrong, but some models are useful'
- Box (1979).

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Thanks for your attention.

Questions?

References

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[1] Guillen, M., Gustafsson, J., Nielsen, J.P. and Pritchard, P. (2007). Using external data in operational risk. *The Geneva Papers, Vol. 32, 178-189.*

[2] Buch-Kromann, T., Englund, M., Gustafsson, J., Nielsen, J.P. and Thuring, F. (2007). Non-parametric estimation of operational risk losses adjusted for under-reporting. *Scandinavian Actuarial Journal, Vol. 4, 293-304.*

[3] Wand, M.P., Marron, J.S. and Ruppert, D. (1991). Transformation in Density Estimation (with comments). *Journal of the American Statistical Association, Vol. 94, 1231-1241*.

[4] Silverman, B.W. (1986). *Density Estimation for Statistics and Data Analysis*. Chapman & Hall, London.

[5] Gustafsson , J. and Nielsen, J.P. (2008). A Mixing Model for Operational Risk. *Journal of Operational Risk, Vol. 3, No. 3*

[6] Gustafsson, J., Nielsen, J.P., Pritchard, P. and Roberts, D. (2006). Quantifying Operational Risk Guided by Kernel Smoothing and Continuous credibility: A Practioner's view. *The Journal of Operational Risk, Vol. 1, No. 1, 43-56.*

[7] Gustafsson, J., Nielsen, J.P., Pritchard, P. and Roberts, D. (2006). Quantifying Operational Risk Guided by Kernel Smoothing and Continuous credibility. *The ICFAI Journal of Financial Risk Management, Vol. III, No. 3, 30-44.*