The Actuarial Profession
making financial sense of the future

Optimisation in a capital scarce world

Edinburgh, October 2009

Gareth Haslip
Colin Kerley

Outline

- Plan to talk about ways to investigate the structure of an “optimal” portfolio
- Start with risk / return definitions
- Look at investment portfolios
- Extend the idea to portfolios of liabilities
- And possibly mix the two

Introduction

- The problem : we want to maximise the return from our investment of risk capital subject to a defined level of risk
- We'd like to talk about some possible approaches to this problem

Applications

- Selecting the optimal investment strategy for a given risk budget
- Optimising reinsurance portfolios or insurance linked security fund allocation
- Minimising regulatory capital requirements for a target required level of asset return

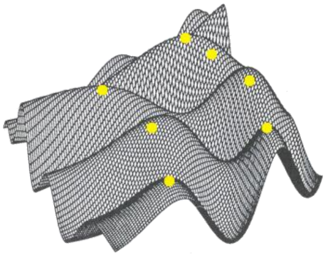
Definitions : Risk and Return

- Return :
  - Assets : income + capital gains or losses
  - Liabilities : premium - expenses - losses
- Risk :
  - Many definitions of risk
  - StDev / PML / VaR / TVaR
  - We will focus on TVaR as it has some attractive properties as a risk measure

Why TVaR ?

- Let f(x) be the distribution of possible returns from the proposed portfolio
  - VaR looks at a single point on the distribution, say the 99%
  - TVaR is the average of all losses for f(x) given they are greater than a certain point
- Coherent risk measure
  - Lots of good properties including sub-additivity
- CoTVaR gradient
  - For hill climbers can calculate “risk gradient” from co-TVaR

General Problem



- Optimisation problem
- Maximise return function subject to constraints
  - or minimise risk subject to constraints
- Lots of algorithms to do optimisation
  - Analytic solution
  - Random search
  - Hill Climbers
  - Linear programming
  - Genetic Algorithm

Simple Example : Investments

The Actuarial Profession
making financial sense of the future

Simple example : investments

- Case study from a paper
- Consider 3 asset classes : S&P 500, US Gov Bonds, US Small Cap
- Over a one month time horizon we want to choose the amount to invest in each class to minimise the risk – given a certain minimum target return
- Underlying assumptions
  - Returns normally distributed
  - Gaussian copula defines the dependencies

The Actuarial Profession
making financial sense of the future

Simple example : investments

- Definitions
  - r1,r2,r3 returns from each asset class
  - w1,w2,w3 chosen weights for our portfolio
  - Portfolio return R = w1*r1 + w2*r2 + w3*r3
- For a given return we want to find weights to minimise the risk of the portfolio subject to some constraints
  - w1+w2+w3=1
  - for this problem all weight must be > 0
- We will assume we can generate return distributions via monte-carlo simulation so have access to vectors r(i) with n samples

The Actuarial Profession
making financial sense of the future

Random Search

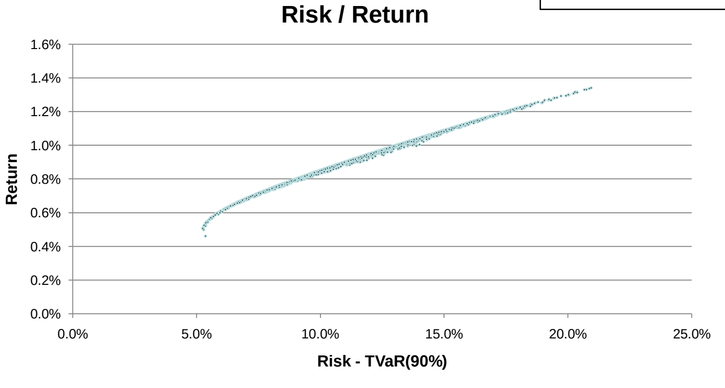
- Start with the simplest numeric algorithm : random weight selection
  - “brute force and ignorance”
- Method
  - Choose random weights subject to constraints
  - Generate the return distribution for the portfolio
  - If the return exceeds the target threshold then look at the risk
  - If the risk is the smallest so far, remember the results
  - Repeat until bored

The Actuarial Profession
making financial sense of the future

Results

- Or just look at all random outputs

	Mean	StDev
S & P 500	*1.0%*	*5.7%*
US Bonds	*0.4%*	*2.2%*
Small Cap	*1.4%*	*8.7%*



The Actuarial Profession
making financial sense of the future

Linear programming

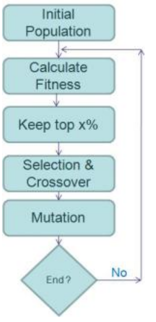
- Constrained maximisation where risk is measured using TVaR can be expressed as a linear programming problem
- Using Monte Carlo simulation output with n samples the problem is translated into a linear system with n + q + 1 variables
  (q = # asset classes)
  - E.g. for 50K simulations we have to maximise a linear constrained system with over 50K variables…

Linear programming

- …luckily modern computing power can handle large linear systems with ease
  - Optimal solution can be found within minutes and is guaranteed to be the true global maxima
  - Additional constraints can easily be added with virtually no additional overhead, e.g.
    - Restrictions on the movement in book value
    - Min / max allocations to each asset class
    - Rating agency capital requirement

Genetic Algorithms

- Have been used successfully for a wide variety of optimisation problems
- Basic recipe
  - Create a population of individuals – all candidates for a solution
  - Define a "gene" that specifies how fit an individual is for solving the solution
  - Repeatedly create new generations of individuals where those with the highest fitness are more likely to have their genes passed on to the next generation
  - Each generation genes are altered via mutation & crossover



Some initial results

TVaR%	Method	VaR	TVaR	Time (mn)	Weights: S&P 500	Weights: Bonds	Weights: Small Cap
1%	Random	13.26%	15.279%	1.89	0.3891	0.1374	0.4735
1%	Linear	13.23%	15.264%	0.62	0.3927	0.1367	0.4706
1%	GA	13.24%	15.268%	1.84	0.3359	0.1555	0.5085
5%	Random	9.06%	11.614%	1.89	0.4256	0.1231	0.4513
5%	Linear	9.06%	11.599%	0.67	0.4364	0.1199	0.4437
5%	GA	9.06%	11.603%	1.83	0.4013	0.1333	0.4654
10%	Random	6.77%	9.706%	1.89	0.4221	0.1251	0.4528
10%	Linear	6.76%	9.701%	0.68	0.4529	0.1136	0.4335
10%	GA	6.76%	9.704%	1.82	0.4312	0.1254	0.4434

- Used the three methods to solve the problem
  - We wanted to find the portfolio with the minimum risk subject to a minimum return threshold
  - Both random & GA can be run forever but we set them to run until they came to a solution within x% of the "true" solution

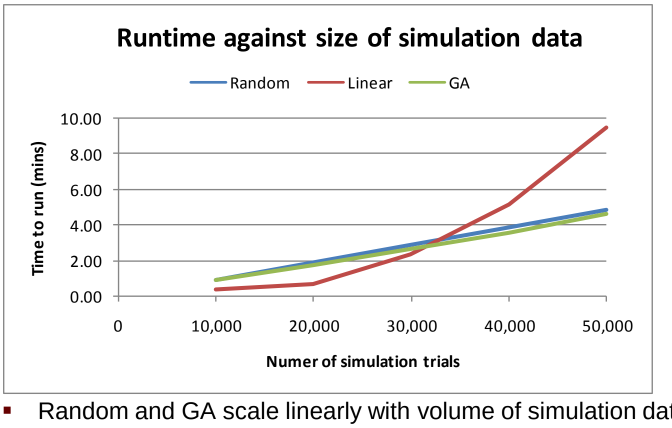
Some initial results (2)

TVaR%	Method	VaR	TVaR	Time (mn)	S&P 500	Weights Bonds	Small Cap
1%	Random	13.19%	15.301%	4.90	0.4444	0.1177	0.4379
1%	Linear	13.18%	15.298%	9.50	0.4447	0.1177	0.4376
1%	GA	13.18%	15.302%	4.63	0.4183	0.1409	0.4408
5%	Random	11.57%	11.571%	4.91	0.4746	0.1059	0.4195
5%	Linear	8.97%	11.564%	9.50	0.4418	0.1188	0.4394
5%	GA	8.98%	11.566%	4.63	0.4203	0.1270	0.4527
10%	Random	6.80%	9.685%	4.94	0.4372	0.1204	0.4423
10%	Linear	6.80%	9.684%	9.50	0.4492	0.1160	0.4349
10%	GA	6.80%	9.685%	4.62	0.4311	0.1229	0.4460

- Models run against 50k simulations of output

The Actuarial Profession
making financial sense of the future

Optimisation Speed

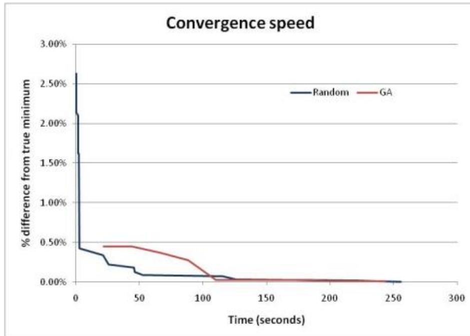


- Random and GA scale linearly with volume of simulation data
- Linear scales $o(n^2)$ – stratified sampling is recommended

The Actuarial Profession
making financial sense of the future

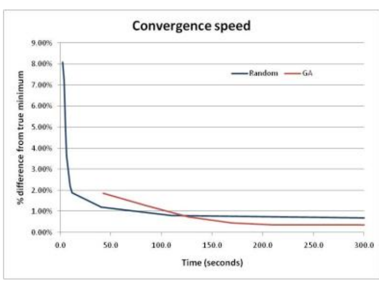
Observations

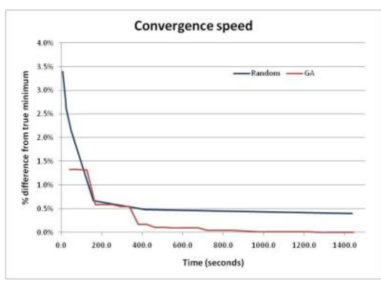
- Linear method will find an optimal solution
- Random and GA can come arbitrarily close to a good solution but time is a problem
- Disappointing the GA does not perform much better



Investments – six assets

- We ran the same optimisation exercise with six assets
- Again, the linear approach found the best solution
- However clear difference between the Random and GA
  - Random finding it hard to get close to a good answer





The Actuarial Profession
making financial sense of the future

Good algorithm

- Linear programming works well for these sort of problems
- Some issues though
  - The constraints need to be linear
  - Careful in definition of TVaR if the underlying risk distributions are not continuous
  - Algorithm scales $o(n^2)$ with respect to sample size

Problem 2 :Insurance world

Insurance world

- No linearity in risk
  - Risk not 100% correlated
  - The payoff distribution from a class of business is a function of how much you have invested and where
- Complex payoff functions
  - Excess of loss
  - Multi year structured deals
- Dependencies odd
  - Primary vs xl
  - Cat models

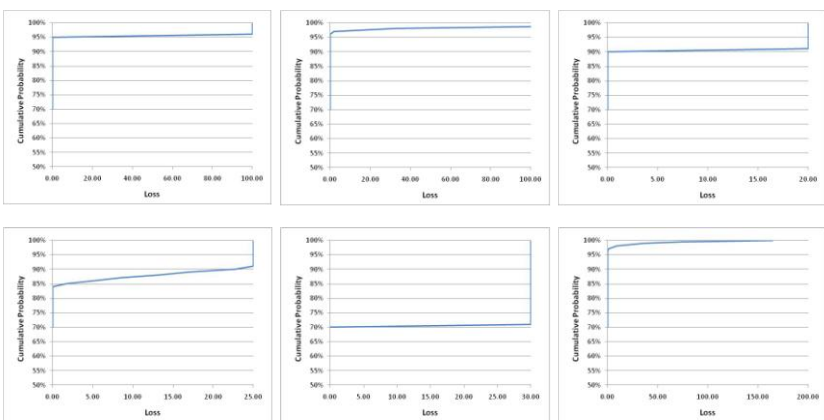
Possible solutions

- Linear approach works well
  - Only restriction is linear constraints – but probably not a major problem for many standard applications
- Use GA
  - Slower
  - Allows non-linear constraints
  - Not restricted to TVaR as risk measure
- Random should be a last resort

A Reinsurance case study

- We have a pool of risk capital and six property cat reinsurance treaties to participate in
- We can participate up to 100% in each risk
- Must find a portfolio mix that maximises our expected profit given our risk capital limit
  - All the treaties are exposed to US Hurricane risk
  - Mixture of ILW's and Cat XL
  - RI Premiums consistent with the market

The portfolio

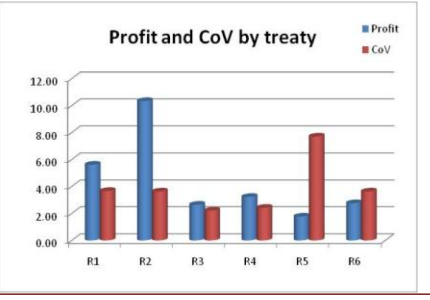


- Mixture of binary / high xs payoff – not very smooth

The Actuarial Profession
making financial sense of the future

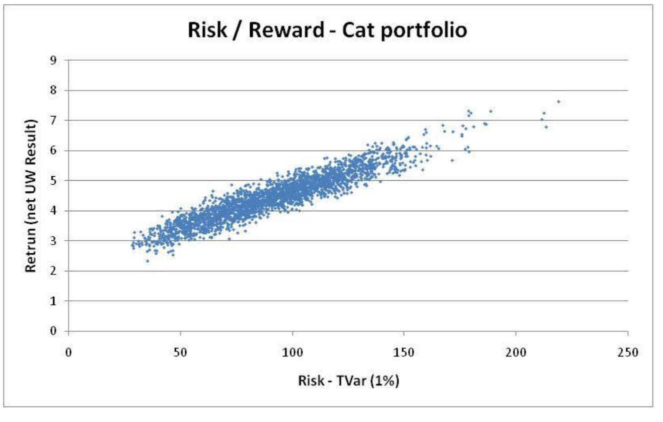
Cat Portfolio returns

Risk	Type	Premium	Losses	Profit	StDev	CoV	Prob (Loss)
R1	ILW	10.00	4.52	5.62	20.59	3.7	4.8%
R2		14.99	4.11	10.33	37.51	3.6	2.5%
R3	ILW	4.50	1.89	2.64	5.85	2.2	9.3%
R4	ILW	6.25	3.06	3.23	7.80	2.4	13.6%
R5	ILW	10.50	8.72	1.77	13.62	7.7	29.1%
R6		10.00	4.52	5.62	20.59	3.7	4.6%



The Actuarial Profession
making financial sense of the future

Random - scatterplot



The Actuarial Profession
making financial sense of the future

Results

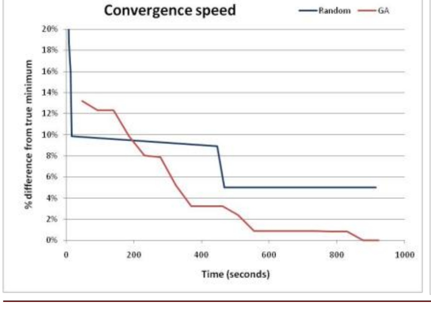
- Target max return on portfolio subject to TVaR < 100 (risk capital target)
- Time ran for Random set to be the same as the GA
- GA run time set so answer about 1% near optimal value

TVaR%	Method	TVaR	Return	Time (mn)	w1	w2	w3	w4	w5	w6
0.5%	Random	98.51	9.63	20.10	43.4%	2.3%	96.0%	97.9%	61.5%	6.4%
0.5%	Linear	100.00	10.53	11.10	51.4%	0.0%	100.0%	100.0%	100.0%	0.0%
0.5%	GA	99.64	10.12	21.51	45.1%	2.2%	79.9%	98.6%	97.3%	0.3%
1.0%	Random	92.37	9.63	20.20	43.4%	2.3%	96.0%	97.9%	61.5%	6.4%
1.0%	Linear	100.00	10.53	11.12	51.4%	0.0%	100.0%	100.0%	100.0%	0.0%
1.0%	GA	99.54	10.38	21.48	45.7%	0.4%	99.2%	98.6%	97.3%	8.7%
2.5%	Random	99.01	11.28	20.10	43.0%	3.2%	94.2%	99.7%	66.6%	59.5%
2.5%	Linear	100.00	11.73	11.14	36.0%	20.0%	100.0%	100.0%	100.0%	0.0%
2.5%	GA	99.86	11.70	21.20	31.9%	6.1%	100.0%	99.1%	100.0%	60.1%

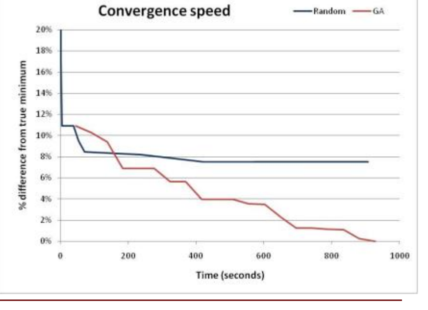
Convergence

- We can see the GA outperforming the Random approach very clearly now
- Random does not get close to an “optimal” solution

TVaR 0.5%

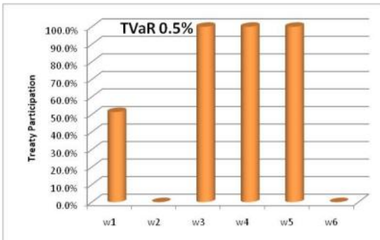


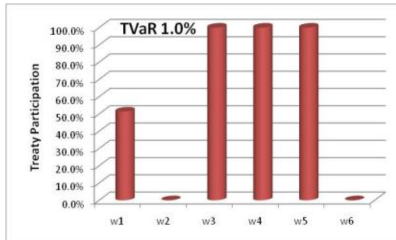
TVaR 1.0%

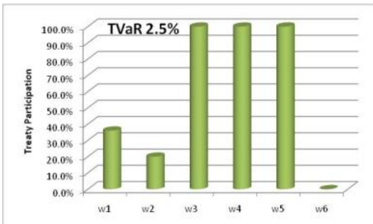


The Actuarial Profession
making financial sense of the future

Portfolio Composition





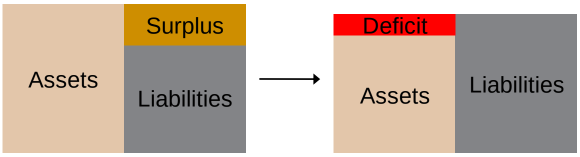


- Portfolio mix relatively stable with change of risk measure
- Optimisation – algorithms often find problems in the question
- Need human judgement – we can keep our jobs

Problem 3 : Asset Liability Management

Understanding Market Risk

- Market risk is often misunderstood for insurers
- It is **not** about how the value of assets change due to movements in the financial markets
  - It is about how the surplus (net asset value) changes in response to market movements
- Worse case scenario is where market movements can decrease assets and increase liabilities simultaneously
- Managing market risk is about managing the sensitivity of the surplus process to movements in market variables



Asset Liability Optimisation

- Aim of asset allocation for insurers is to maximise expected outperformance of assets over liabilities
  - Subject to constraints on the potential downward movement in the surplus process
- We can use the same optimisation framework to solve the asset allocation problem in this setting
- Procedure is very similar to before except that an additional Monte Carlo output vector is required
  - The % change in the discounted value of the liabilities at the end of the time period under consideration (e.g. end of year for Solvency II / SST)
  - Important that the interest rate scenarios applied to generate asset returns are ordered consistently with the liability simulation
  - Only works for non-life insurance where liabilities are independent of asset allocation

Asset Liability Optimisation

- Optimisation method is applied to surplus process:

S = Initial Assets * (w1*r1 + w2*r2 + w3*r3) – 100% x Initial Liability x r_l

- Idea is that there is a fixed -100% holding in the liabilities and then the optimisation algorithm is applied as before
- This allows asset allocation strategies to be developed in the context of Solvency 2 and the Swiss Solvency Test definitions of market risk
- For example, develop an asset strategy that minimises the regulatory market risk capital requirement subject to achieving a target level of return
  - Allows insurers to control market risk budget and concentrate on applying capital to insurance risk

Conclusions

- Linear Programming approach seems to work best for these optimisation problems
  - But only works for TVaR as a risk measure, not VaR
  - And again, constraints need to be linear
- Powerful tool for risk management…for both assets and liabilities

The Actuarial Profession
making financial sense of the future

Questions ?