Pensions and Financial Engineering

Moshe A. Milevsky, Ph.D.
Associate Professor of Finance, York University
Schulich School of Business, Toronto, Canada

Executive Director, The IFID Centre
at the Fields Institute

Agenda

- I do not plan to discuss whether equities belong in pension funds or whether liabilities should be discounted at lower (or perhaps higher) interest rates.
- Main Question: What is an appropriate contribution rate for DB plans with embedded optionality?
- Focus on actuarial methodology for valuing these pension options and attempt to reconcile with a financial engineering approach.
- Discuss a unique pension experiment that was conducted in the State of Florida
- Analogy with the valuation of incentive (employee) stock options: Current models.
- Relation between illiquidity and irrationality.


COMPANIES & FINANCE UK & IRELAND

Actuarial antiquarians out of touch with risk

JOHN PLENDER ON MONDAY

But still sold on moonshine

Euroglow

Bugged

U.S. Social Security Projections: Assets of $1,378 Billion (Jan/03)

Year	Low Cost	Med Cost	High Cost
2001	$1,379	$1,372	$1,363

Projections assuming three different scenarios:
Low Cost (Best), Medium Cost (Average), High Cost (Worst)

Source: SoA Pension Section News March 2004

U.S. Social Security Projections: Assets of $1,378 Billion (Jan/03)

Year	Low Cost	Med Cost	High Cost
2001	$1,379	$1,372	$1,363
1999	$1,424	$1,407	$1,350
1997	$1,295	$1,225	$1,148
1995	$1,284	$1,068	$845

Projections assuming three different scenarios:
Low Cost (Best), Medium Cost (Average), High Cost (Worst)

Source: SoA Pension Section News March 2004

U.S. Social Security Projections: Average Discrepancy

Year	Low Cost	Med Cost	High Cost
1992- 1994	+5%	-20%	-46%
1995- 1997	-8%	-18%	-28%
1998- 2001	-1%	-1%	-3%
1992- 2002	0%	-11%	-24%

Projections assuming three different scenarios:
Low Cost (Best), Medium Cost (Average), High Cost (Worst)

The lesson:

- The real-world (a.k.a. physical or statistical) measure is a very tricky thing to estimate.
- Let me explain with a thought experiment.

Q#1: Fair Actuarial Premium?

- Age 100 pure endowment policy pays $20 in one year, conditional on survival.
- Probability of survival (IAM2000) is 75%.
- Opportunity cost of funds is 10%.
- Fair actuarial premium (no loading) is:

$$\$13.636 = \frac{(0.75)(\$20)}{1.10}$$

Q#2: Fair Actuarial Premium?

- You must pay $20, in one year, if the temperature in Buenos Aires exceeds 45c, during the year. You pay nothing otherwise.
- Meteorologist estimate the probability of this event is 75%.
- Opportunity cost of funds is 10%.
- Fair insurance premium (no loading) is:

$$\$13.636 = \frac{(0.75)(\$20)}{1.10}$$

Traditional Actuarial Pricing: Law of Large Numbers

Today… …Tomorrow

Probability of Survival: p
Interest Rate: R

The current price (present value) of $1 conditional on survival is:

$$\alpha = \frac{p}{1+R}$$

Yes…Margins, Fees & Profits

$$\alpha = \frac{p}{1+R}$$

$$\alpha = \frac{p+m}{1+R-f} + c$$

Q#3: Fair Actuarial Premium?

- You are obligated to pay $20, in exactly one year, if the Dow Jones Industrial Average exceeds 11,000 by the end of the year.
- Stock market experts estimate the probability of event is 75%.
- Cost of funds is 10%; fair premium is?

$$\$13.636 = \frac{(0.75)(\$20)}{1.10}$$

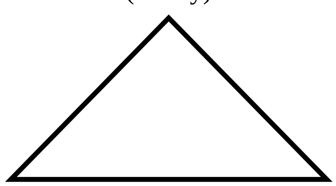
- WRONG! Why?

Manufacturing & Replication

- If the underlying contingent-claim can be perfectly replicated using the underlying security, then the market price of this risk is zero and economic value of this liability is the capital market manufacturing cost.
- This manufacturing cost may be higher or lower than the expected (or quantile of) loss from this exposure.

What is the Claim Worth?

Value
(Utility)



Cost
(Manufacturing)

Price
(Market)

Pricing in Complete Markets: Two Securities & Two States of Nature

Assume you observe the following market prices:

$$\begin{pmatrix} \text{today..} \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \text{tomorrow..} & \\ \text{good} & \text{bad} \\ 2 & 2 \\ 3 & 0.5 \end{pmatrix}$$

These securities can be purchased, or sold (short), without any restrictions.

An investment bank would like to *manufacture* a new security that only pays \$1 in the good state, and nothing in the bad state. What is it worth?

$$\begin{pmatrix} \text{today..} \\ x \end{pmatrix} \Rightarrow \begin{pmatrix} \text{tomorrow..} & \\ \text{good} & \text{bad} \\ 1 & 0 \end{pmatrix}$$

What is the capital market price of this instrument?

The bank can *manufacture* (create, hedge, synthesize) this security with the help of simple linear algebra. Notice: They can hold *B* units of the first security (bond) and Δ units of the second security (stock) so that:

$$\begin{pmatrix} \text{Good} & \text{Bad} \\ 3\Delta+2B & 0.5\Delta+2B \\ =1 & =0 \end{pmatrix}$$

This leads to a portfolio of Δ= +0.4, and *B* =- 01 units to manufacture the *redundant* contingent claim.

The capital market price of the security must be:

$$x=(1)(0.4)+(1)(-0.1)=0.3$$

We never mentioned the real world probability of the good or bad state.

Why?

Financial Engineering Valuation:

The market price of any traded contingent-claim can be expressed as…

$$X = \frac{(\pi)Xu + (1-\pi)Xd}{R}$$

…which is a weighted average of the contingent payout divided by the risk-free interest rate.

Q-Measure vs. P-Measure

Question: What is the lowest probability (equity premium) value that would induce you to invest in the risky asset (stocks) vs. the safe asset (bonds)?

Answer: We must solve:

$$\pi Xu + (1-\pi)Xd = XR$$

Which implies:

$$\pi = \frac{R-d}{u-d}$$

This is the risk neutral probability, also known as the Q-measure, which is distinct from the real world P-measure.

- If I were risk averse- which I am- I would only invest in the risky asset, if the probability of the good state is greater than the risk neutral probability π.

Risk Neutral:

$$S = \frac{\pi Su + (1-\pi)Sd}{R}$$

Risk Premium:

$$S = \frac{pSu + (1-p)Sd}{R + \lambda\sigma}$$

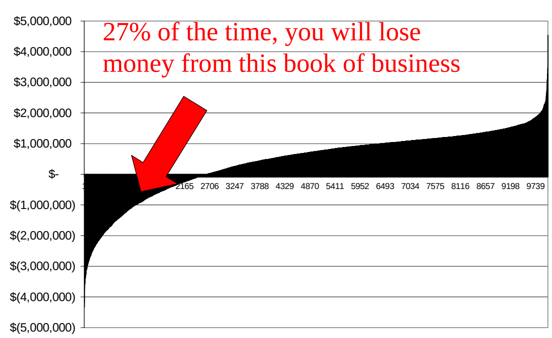
Certainty Equivalent:

$$S = \frac{pSu + (1-p)Sd - Z}{R}$$

Example, if p =0.8 > π = 0.60, then:

$\lambda\sigma$ = 0.5 or Z = 50.

Present Value of P&L



Actuary vs. Financial Engineer: The 'worth' of a European-style put option

r = 0.06, μ = 0.1098, σ = 0.1871

Valuation Method	T = 5 years	T = 10 years
RNV	4.93	3.46
QRM(90)	5.39	0.00
QRM(95)	14.75	2.33
CTE(90)	16.81	6.62
CTE(95)	23.79	13.05

QRM(x) is the sum needed, in a risk-free account, to pay the benefit x% of the time. CTE(x) is the sum needed to pay for the worst (1-x)% of the cases.

www.NationalUnderwriter.com LIFE & HEALTH

National Underwriter

THE WEEK IN LIFE & HEALTH

MARCH 29, 2004
VOL. 108, NO. 12
$5.00

Coventry Now Buying Unwanted VA Contracts

BY ALLISON BELL

COVENTRY FIRST L.L.C., Fort Washington, Pa., has started buying unwanted variable annuities.

The program appears to be the first well-publicized, systematic effort to buy contracts that are still in the accumulation phase.

"We've thought about doing this for several years," says Coventry CEO Alan Buerger. "We have registered reps and insurance agents asking about this on a regular basis."

Coventry, a life settlements company, prefers contracts in which investment performance has been so poor that the value of the death benefits now exceeds the value of the tax benefits.

Selling a VA contract to another party might be a better deal than letting the contract lapse, Coventry says, because the market value of the death benefits is often higher than the surrender value.

U.S. life settlement companies now buy about $5 billion in life policy face value each year, and VA purchase volume could amount to about 10% of life settlement volume, Buerger estimates.

VA resale prices will depend on the carrier, the contract terms and the seller's age, but typical holders might get the current value of their VA principal along with a price equal to about 13% of the death benefit value, Buerger says.

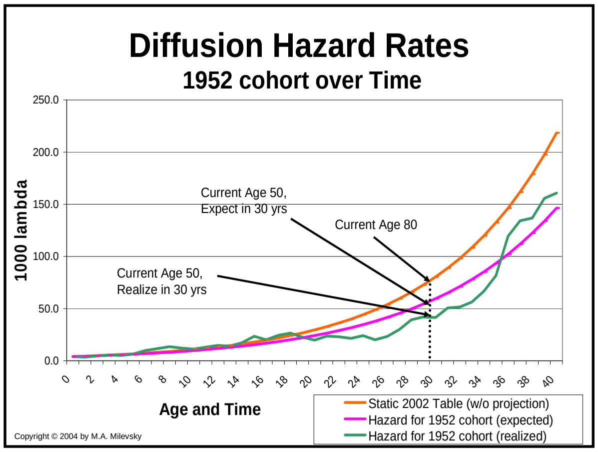
The Lesson

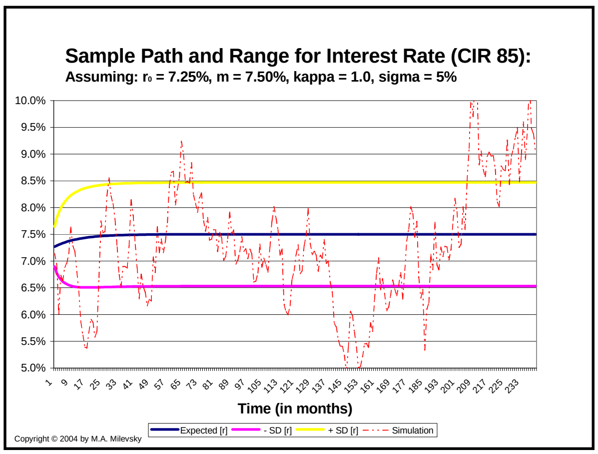
- The financial engineer focuses on the cost of the replicating strategy needed to create the given payout.
- By chance this cost happens to correspond to an expectation under the so called risk neutral measure.
- Implications: We will find many options embedded within pension and insurance contracts the appear mispriced.
- The required contribution rates would appear to be higher in order to fund these options
- Lets get closer to pension annuities…

Basic Insurance Pricing

$$ {}_t p_x = \exp\{-\int_0^t \lambda_{(x+s)} ds\} $$

$$ \bar{a}_x = \int_0^\infty ({}_t p_x) e^{-(r_t)t} dt $$





An Actuarial Finance Model for Pricing Mortality Contingent Claims

$$({}_t p_x) = E^Q[\exp\{-\int_x^{x+t} h_s ds\}]$$

$$\lambda_{t+x} = \frac{d \ln[{}_t p_x]}{dt}$$

$$\bar{a}_x = \int_0^\infty E^Q[\exp\{-\int_0^t (r_s + h_{x+s}) ds\}] dt$$

The Mortality Variables:

$$E^{Q}[h_t] \longleftrightarrow \lambda_t$$

Financial Economic question

Mortality Forward Curve

$$E^{P}[h_t]$$

Bio-statistical question

If a mortality risk premium exists…

$$E^{Q}[h_t] \neq E^{P}[h_t]$$

Main Point:

- *The expectation is with respect to the Q-measure, which may or may not be the same as the physical P-measure, since mortality risk may not be entirely diversifiable.*
- *Can you hedge pension annuity payouts with life insurance products? If not, then there should be a risk-premium in pricing.*
- *Once again the focus is on replication and a manufacturing strategy.*

Three Pension Examples

1. Option to convert a DC pension account into a DB plan (Florida)
2. Option to purchase a life annuity at a fixed price (UK).
3. Non-reduction guarantee available on variable annuity payouts (Canada)

- They all lead to divergent valuations.

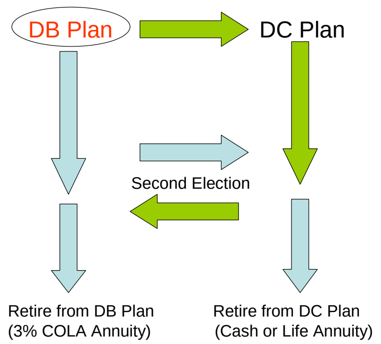
Florida Pension Conversion

- In early 2002, over 650,000 employees of The State of Florida were given the choice to convert their traditional Defined Benefit (DB) pension plan into a self-managed Defined Contribution (DC) account.
- Each employee electing to participate was given an initial DC balance equal to the Accumulated Benefit Obligation (ABO).
- They could choose from a limited set of mutual funds and some institutional investment products.

Florida Pension Election

- In order to mitigate some of the investment risk, the State legislature provided the option to make a second election and possibly return to the Defined Benefit plan at any time prior to retirement.
- The strike price of this option, or the cost of re entry to the DB plan, is the value of the accumulated benefit obligation (ABO) at the time of the second election. (For existing employees.)
- A massive education campaign conducted in the State of Florida, spearheaded by Financial Engines (a.k.a. Nobel laureate Prof. Bill Sharpe)

The Florida Decision Tree



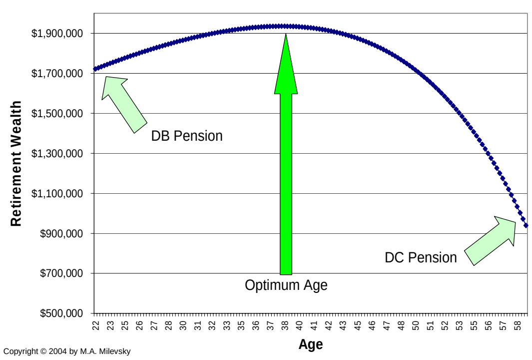
Deterministic Analysis:

Retirement Wealth, assuming
the Option is Exercised at Time *s*

$$W_T(s) = B_T + (C_s - B_s)e^{\mu(T-s)}$$

…where μ denotes the force of interest
inside the individual DC account.

Retirement Wealth:
Assuming 2nd Election at Age...



Definition of the Option Value

- We can define the option value as the (expected) percentage increase in retirement wealth as a result of the second election.

$$v(s) := \frac{W_T(s) - \max[C_T, B_T]}{\max[C_T, B_T]}$$

How much more has to be contributed to the DC plan, in order to generate the same retirement benefit?

Dynamics of DC Account:

$$C_s = C_0 e^{\mu s} + \int_0^s \left(\hat{c} I_0 e^{gt}\right) e^{\mu(s-t)}\, dt$$

$C_0 :=$ Initial Account Balance

$\hat{c} :=$ Plan Contribution Rate.

$I_0 :=$ Initial Salary.

$g :=$ Salary Growth Rate.

Value of DC Account

$$C_0 e^{\mu s} + \frac{\hat{c} I_0 (e^{gs} - e^{\mu s})}{g - \mu}$$

$$or\ \ (C_0 + \hat{c} I_0 s) e^{\mu s} \quad when \quad \mu = g$$

Dynamics of the A.B.O.

$$B_s = \hat{b}(s+\tau)(I_0 e^{gs})(a_x e^{-\rho(T-s)})$$

…where τ denotes the years of service at s=0, a_x is the indexed annuity factor at retirement, and ρ denotes the plan valuation rate.

Note the implicit assumptions about survivorship and termination.

The value of the A.B.O. at any future time *s*, can be written as…

$$B_s := B_0 e^{\rho s} + \int_0^s b_t e^{\rho(s-t)} dt$$

where

$$b_s = B_s' - rB_s = \left(\frac{1}{s+\tau} + g\right) B_s$$

Locate the 'switching time' that maximizes the value of retirement wealth:

$$W_T(s) = B_T + (C_s - B_s)e^{\mu(T-s)}$$

The First Order Condition is:

$$W_T'(s) = (C_s' - B_s')e^{\mu(T-s)} - \mu(C_s - B_s)e^{\mu(T-s)} = 0$$

The F.O.C. is satisfied, and
the second derivative is negative,
at the unique value of *s*, where

$$b_s - c_s = (\mu - \rho) B_s$$

The statement is somewhat obvious when $\mu = \rho$:

Exercise the option as soon
as the contributions to the DB plan
exceed the contributions to the DC plan.

But, when $\mu > \rho$ you should switch a bit later,
and when $\mu < \rho$ you should switch a bit earlier…

…which after some algebra leads to the
smallest value of *s* for which…

$$\phi(s) \geq \mu - \rho - g$$

where

$$\phi(s) := \frac{1}{s+\tau}\left(1 - \frac{\hat{c}e^{\rho(T-s)}}{\hat{b}a_x}\right)$$

The phi function does not depend on μ or g,
which will help us later in the stochastic model.

Another way to think…

- One can define a threshold rate of return μ, needed to justify staying in the DC plan.
- If the instantaneous rate of return in the DC plan is lower than this threshold, the participant should switch (back) to the DB plan since the 'implied' return is higher.
- As the participant ages, the threshold return grows…

The Threshold Investment Return (mu),
Required to Justify Staying in the DC Plan.

Entry Age	*Initial Service*	*Age at which the second election is being contemplated*					
		30	**35**	**40**	**45**	**50**	**55**
50	**5**					15.7%	17.1%
	10					14.3%	15.6%
	15					13.8%	14.9%
	20					13.5%	14.5%
	25					13.4%	14.2%
40	**5**			-5.0%	10.0%	13.8%	14.9%
	10			3.9%	10.1%	13.5%	14.5%
	15			6.8%	11.4%	13.3%	14.2%
30	**5**	-51.3%	-5.4%	6.8%	11.4%	13.3%	14.2%
	10	-19.3%	0.6%	8.3%	11.6%	13.2%	13.9%

Risk Neutral Expectations:

- Assuming the market is complete, a rational Floridian will locate the optimal stopping time *s*, that will maximize the risk-neutral expected payoff of the pension.
- Justification: One can always borrow to invest in assets that are identical to those available within the DC plan.
- We then replace μ with a risk-free rate.

Simplifying Assumption:

- Under Geometric Brownian Motion (GBM) asset dynamics, if we assume a constant risk-free rate, a constant actuarial valuation rate, and a non-stochastic wage profile, the optimal stopping time collapses to the same condition:

$$\phi(s) \geq r - \rho - g$$

Hint: Compute the expectation of a zero-strike Arithmetic Asian option.

What is the value of the option? and when should you switch?

Service	Age 42	Age 37	Age 32	Age 27	Age 22
22yrs	0.0% (s = 0)				
17yrs	0.0% (s = 0)	0.5% (s = 2)			
12yrs	0.0% (s = 0)	1.3% (s = 3)	4.6% (s = 7)		
7yrs	0.3% (s = 1)	2.8% (s = 5)	7.0% (s = 8)	12.3% (s = 13)	
2yrs	1.6% (s = 3)	5.5% (s = 6)	10.6% (s = 10)	16.6% (s = 14)	23.4% (s = 20)

Assuming: 4.75% salary growth, and 8% valuation and risk-free rate.

Florida 401(a) Plan Update: May 2004

- The Florida DC plan:
  - 35,000 participants.
  - $575 million invested.
  - 40% allocated to balanced funds.
- Of the 650,000 employees 5% of eligible participants selected the DC plan.
- 80,000 new employees each year.
- 20% of new hires chose DC plan.

Reconciliation?

- Embedded options are being offered within pension plans that appear to be under-priced if we use a No Arbitrage type methodology.
- A similar problem is encountered in the valuation of incentive (executive) stock options.
- Should we use the Black-Scholes capital market value to expense against earnings?
- Remember that they too are illiquid and non-traded…

The Value of an Illiquid Option:
Relative to the Black-Scholes price

Fraction of individual wealth already Invested in company stock:	**10%**	**25%**	**50%**	**75%**
Value of stock options to moderately risk averse employee:	**95%**	**88%**	**81%**	**76%**
Value of stock options to highly risk averse employee:	**75%**	**50%**	**27%**	**15%**

Source: J.E. Ingersoll (2002) "*Subjective & Objective Evaluation Of Incentive Stock Options*"

Conclusion…

- Take-away points:

1. …
2. …
3. …

How to Reach Me:

- moshe@milevsky.net
- Tel: (416) 348-9710 ext: 3010
- Fax: (416) 348-9385

The Individual Finance and Insurance Decisions (IFID) Centre

www.ifid.ca