Institute
and Faculty
of Actuaries

Winner's Curse: Application of Game Theory to Insurance Pricing

Daniel Kendrick

2 December 2016

Agenda

Winner’s Curse

- What is Winner’s Curse?
- Examples of Winner’s Curse
- Winner’s Curse in insurance.

Application of Game Theory to Insurance Pricing

- Market with equal level of knowledge
- Market with variable levels of knowledge
- Conclusions.

Q&A

Institute and Faculty of Actuaries

Institute and Faculty of Actuaries

Winner’s Curse

Background and examples taken from:
“Winner's curse. The unmodelled impact of competition. Report of the Winner's Curse GIRO Working Party.“
https://www.actuaries.org.uk/documents/winners-curse-unmodelled-impact-competition-report-winners-curse-giro-working-party-0

2 December 2016

Winner's Curse

What is Winner's Curse?

Wisdom of Crowds

- Where parties are given access to certain data and asked to estimate a quantity from that data a range of estimates will arise
- The average of the parties' estimates can be a good estimator of the unknown quantity; the so-called "Wisdom of Crowds".

Winner's Curse

- There are some circumstances where the estimate that matters is not the mean estimate, but the extreme estimate
- Examples include the highest price at auction or the lowest price for a quoted insurance policy
- In such situations the "winner" is likely to have been "cursed" by either paying too much for the goods at auction or obtaining insufficient premium for the insured risk.

Institute and Faculty of Actuaries

Winner’s Curse

Examples of Winner’s Curse

Bidding on a jar of coins

- A glass jar was filled with 5p coins and auctioned to the members of a sizeable group
- Bidders were allowed to handle the jar and compare it to a £2 roll of forty 5p coins before placing a bid
- Whether the bids were audible or sealed, the jar was sold for more than its cash value on virtually every occasion
- The strategies taken across the bidders clearly aren’t optimal – what strategy should be taken to make a profit?

Actuarially relevant examples

- **Bulk annuity buy-out**: Companies looking to transfer their DB pension schemes liabilities will seek multiple quotes
- **Personal lines insurance**: Individuals will compare multiple quotes, made even easier with price comparison websites
- **Commercial real-estate loans**: Companies seeking such a loan will approach multiple lenders to seek the best rates.

Winner’s Curse

Winner’s Curse in Insurance

Almost all areas of insurance are vulnerable to the winner’s curse due to some systemic features:

- High **uncertainty** of future cashflows and more particularly the difficulty of modelling and measuring those cashflows
- The **nature of the bidding process** - in general the lowest bid wins, even when that bid is significantly out of line with the next lowest. However, there may be an impact from brand loyalty / company reputation
- The **“common-value”** property with the nature of the cashflows being the same for all bidders, although the actual values may differ
- The “**price-focus”** of the buyer - in many markets customers place a low value on the non-financial aspects of the insurance purchased
- The high level of **competition** - number of competitors participate in certain segments, paired with low cost of switching.

Institute and Faculty of Actuaries

Institute
and Faculty
of Actuaries

Application of Game Theory to Insurance Pricing

Market With Equal Level of Knowledge

2 December 2016

Application of Game Theory to Insurance Pricing

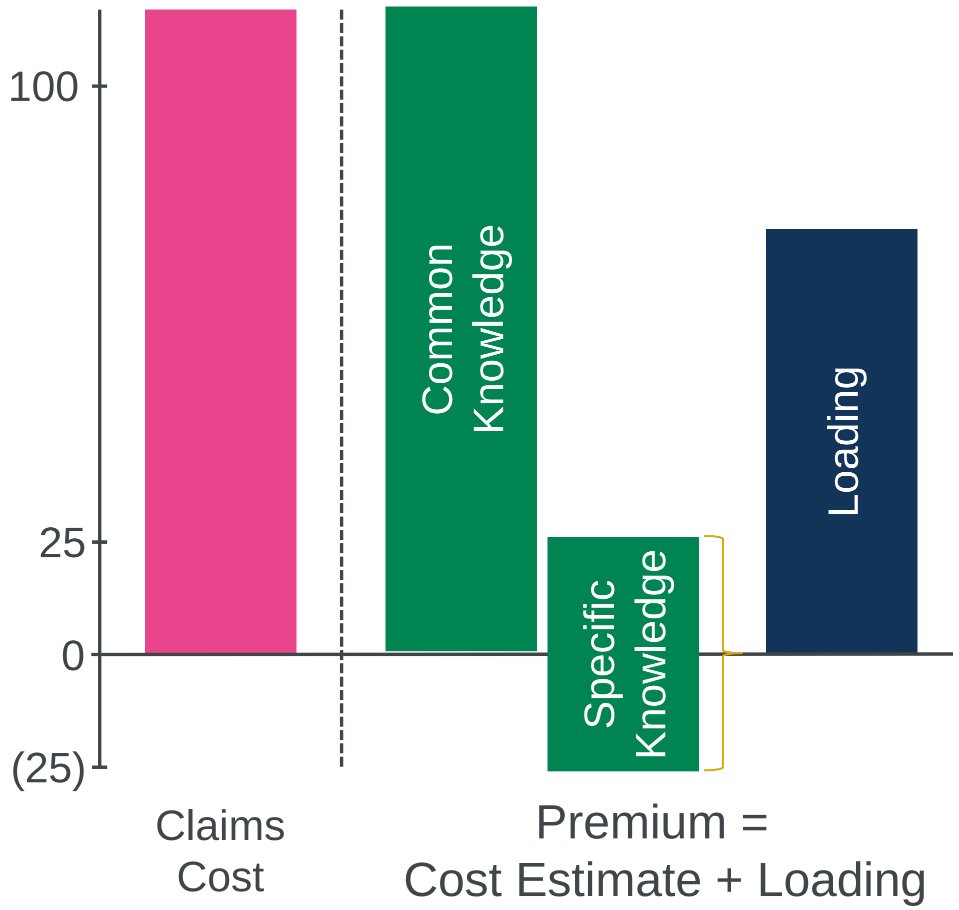
Scenario 1 – Two Insurers with Equal Level of Knowledge

Suppose a market of two insurers have a common estimation error of £50 in mean claims

What loading should be applied to the cost estimate for this uncertainty?

Insurance Pricing Model

Model Set Up - Components



What are we modelling?

- The model considers a single insurance policy
- An identical policy is available from a number of providers
- The theory can be applied at any level of pricing cell.

Components of the Premium

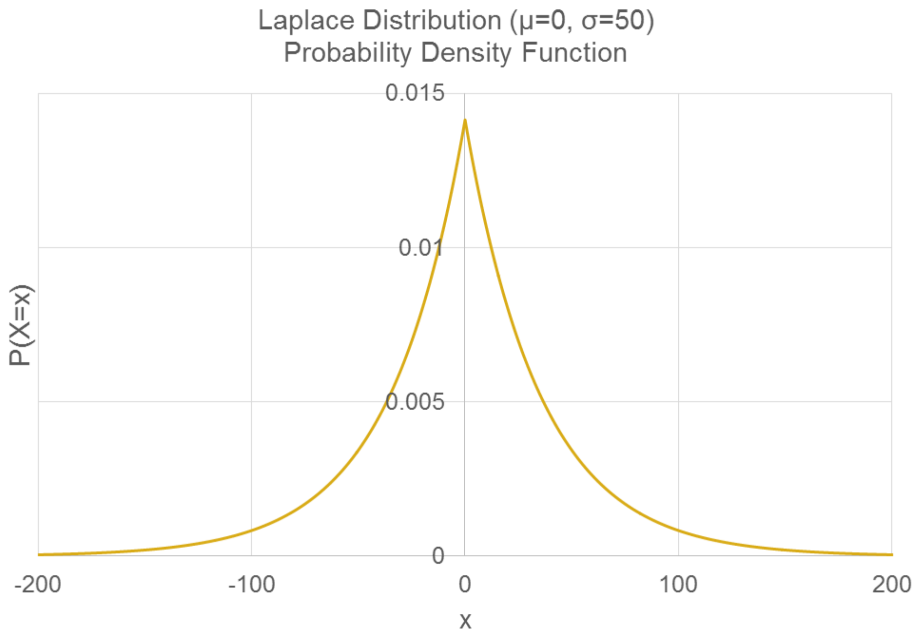
- Insurers compete by submitting sealed premium quotes
- The common knowledge is the same for all insurers. The expectation of the claims cost and common knowledge is the same
- The specific knowledge is modelled as a random variable with expectation of zero and a given estimation error
- The loading is a constant addition to the cost estimate.

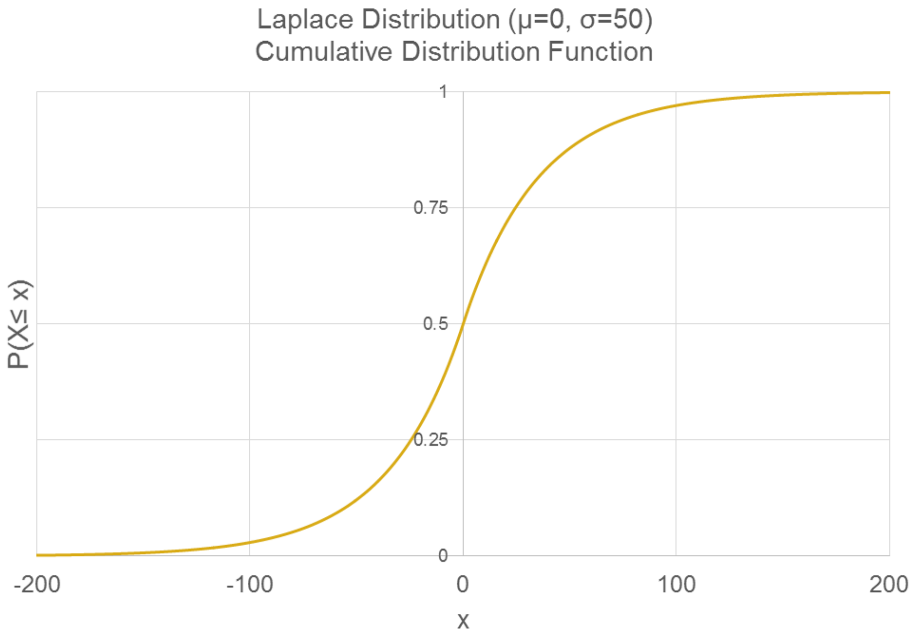
Institute and Faculty of Actuaries

Insurance Pricing Model

Model Set Up – Specific Knowledge

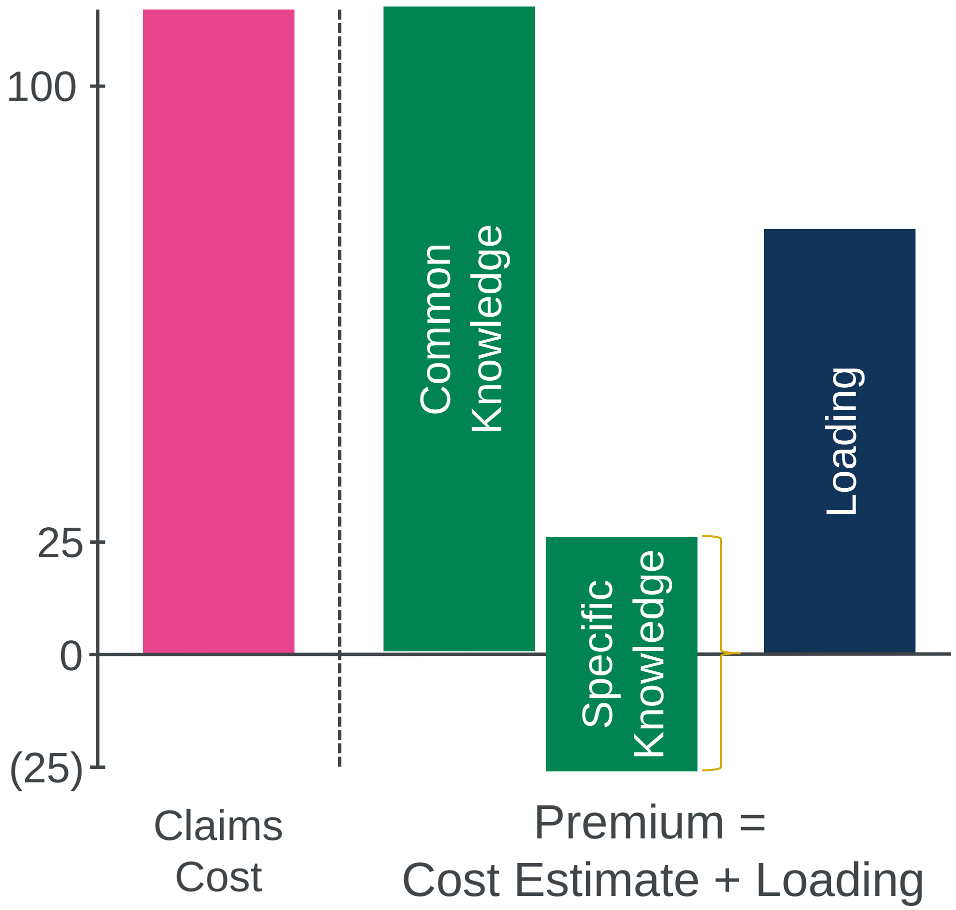
- The specific knowledge component of the premium represents internal company data, e.g. own claims experience
- For each premium quoted, the specific knowledge is taken as an observation from a chosen distribution with mean zero and given estimation error
- The results in this presentation have been derived using a Laplace distribution. The PDF and CDF are shown below:





Insurance Pricing Model

Model Set Up – Simplifying Assumptions



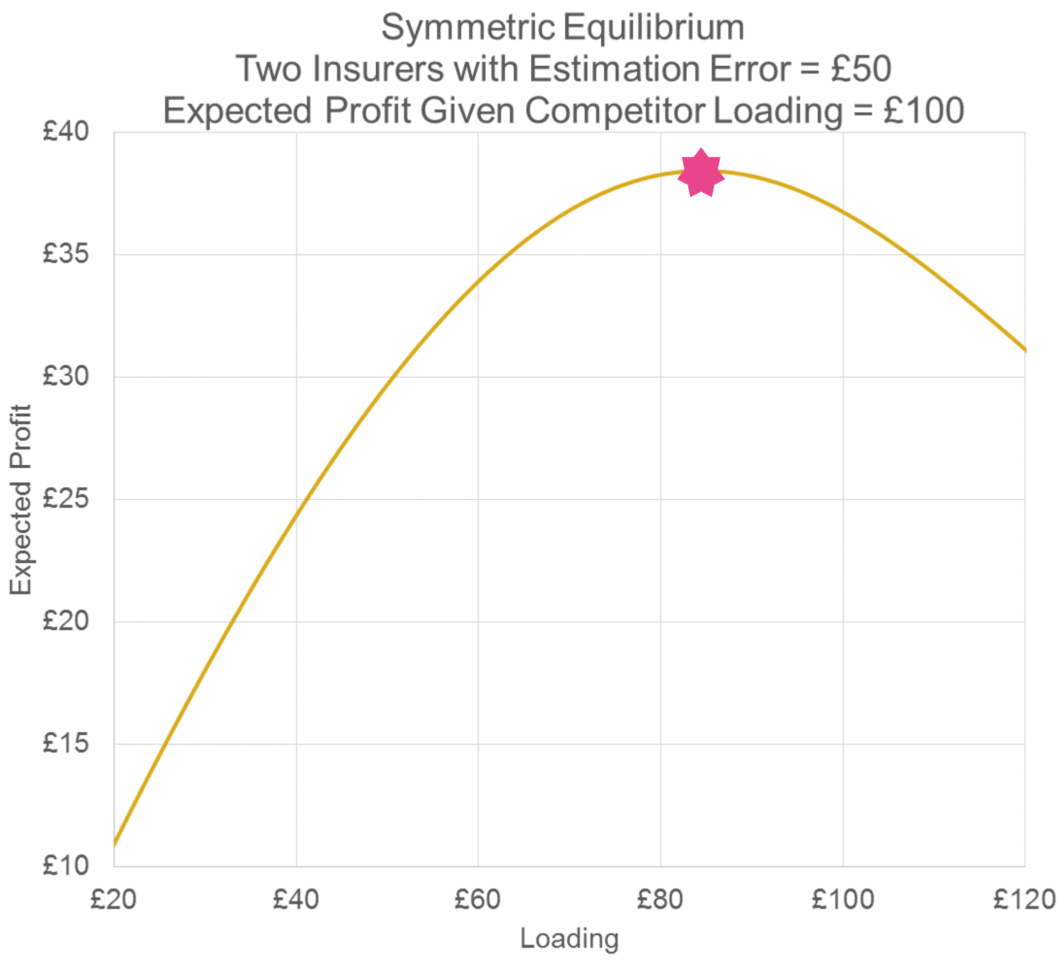
Assumptions

- The policyholder buys the policy from the cheapest insurer
- Insurers optimise their expected profit
- The claims cost is the same regardless of which insurer settles it
- Insurers cannot split their cost estimate between common knowledge and specific knowledge
- Each insurer knows the distribution of the specific knowledge and the loadings for its competitors, but not their premium
- Cost estimates are not shared.

Institute and Faculty of Actuaries

Symmetric Equilibrium

Two Insurers – Expected Profit



Your competitor sets its loading to £100. What is your optimal strategy in response?

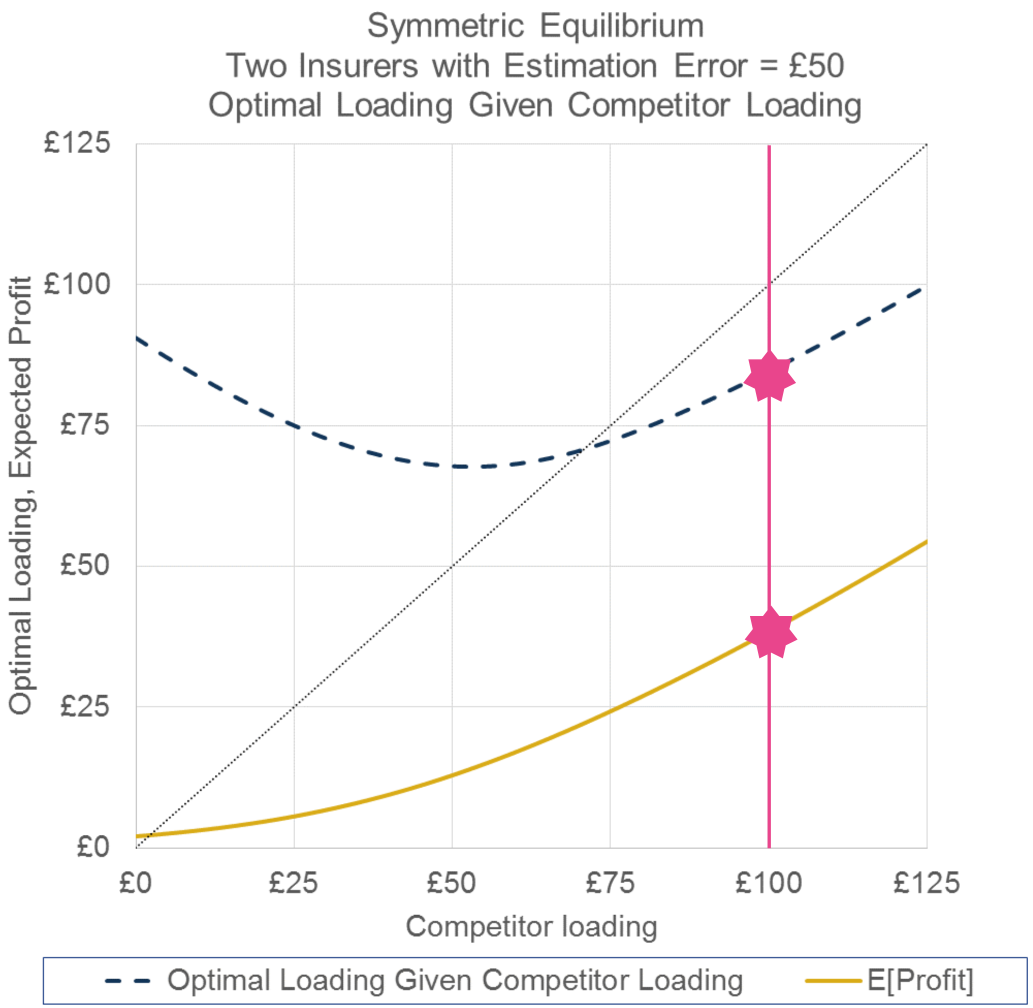
What does the graph show?

- The graph shows your expected profit for different loadings, given your competitor's loading is £100
- Your expected profit is maximised when your loading is set to £84.60, undercutting your competitor by £15.40
- Based on this pair of loadings, the probability of having the cheapest quote is 61%
- Your expected profit is £38.42, as shown by the pink star.

Institute and Faculty of Actuaries

Symmetric Equilibrium

Two Insurers – Optimal Loading



We now consider your strategy for varying competitor loadings.

What does the graph show?

- The blue, dashed curve represents your expected profit maximising loading, given your competitor's loading
- The gold, solid curve represents your expected profit. It is strictly increasing, even though you decrease your loading when your competitor sets a small loading
- The pink stars correspond to the previous graph – when your competitor's loading is £100 your optimal loading is £84.60, with expected profit £38.42.

Institute and Faculty of Actuaries

What is a ‘Nash Equilibrium’?

The Nash Equilibrium is a concept of game theory where the optimal outcome of a game is one where no player has an incentive to deviate from his chosen strategy after considering an opponent's choice.

Overall, an individual can receive no incremental benefit from changing actions, assuming other players remain constant in their strategies.

John Forbes Nash Jr. (June 13, 1928 – May 23, 2015)

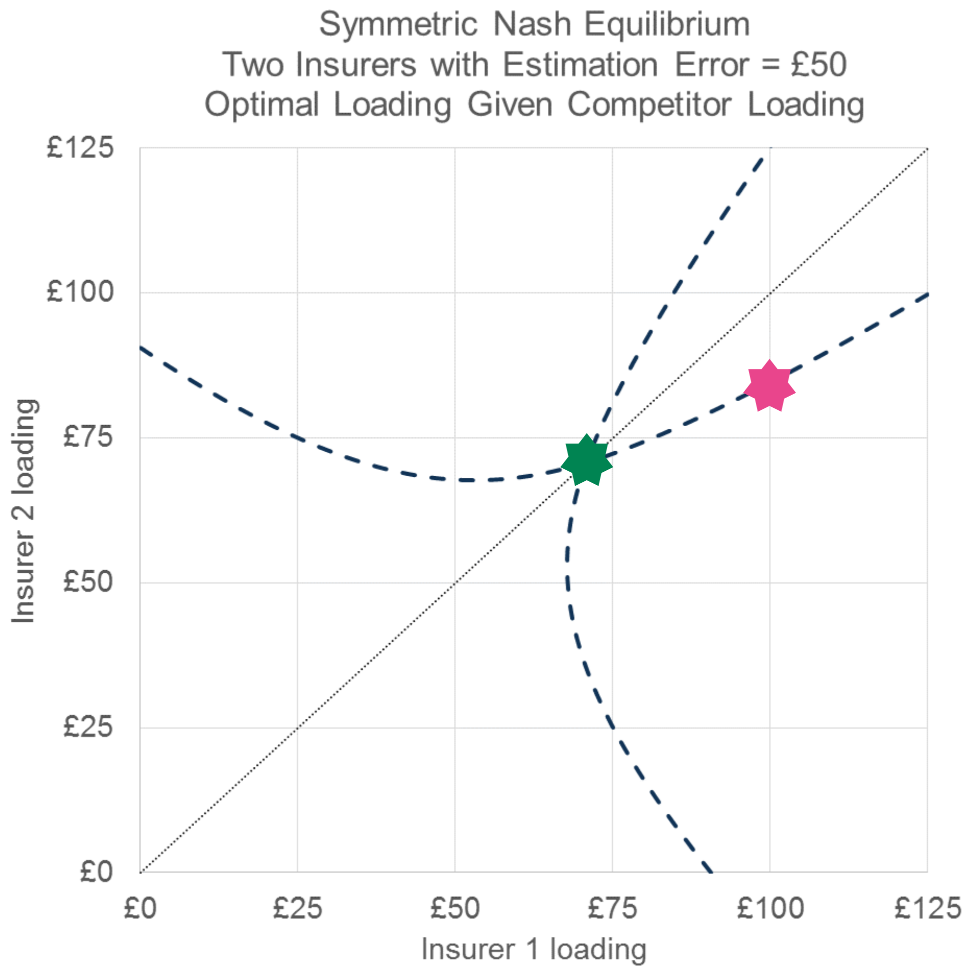
Winner of the 1994 Nobel Memorial Prize in Economic Sciences (along with John Harsanyi and Reinhard Selten)

Source: http://www.investopedia.com/terms/n/nash-equilibrium.asp

Institute and Faculty of Actuaries

Symmetric Nash Equilibrium

Two Insurers – Nash Equilibrium Loading



We now consider the strategies for both insurers.

What does the graph show?

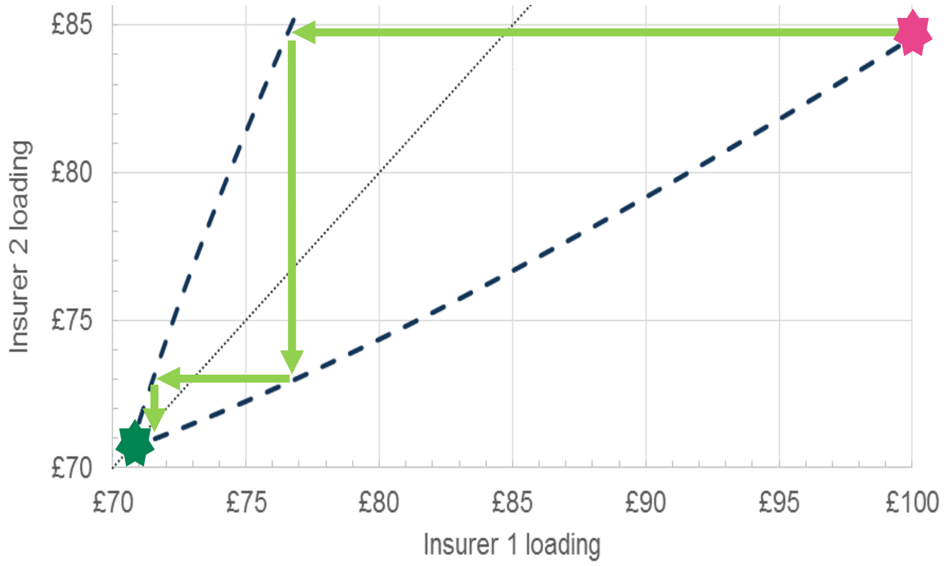
- The blue, dashed curves show the optimal loadings for both competitors. Given both insurers have a common estimation error the graph is symmetric
- The curves intersect when both insurers apply a loading of £70.71, shown by the green star. The expected profit is £22.10
- The strategy in which both insurers applying a loading of £70.71 is a Nash equilibrium
- From this Nash equilibrium neither insurer has an incentive to change their loading, as doing so would lead to a reduced expected profit
- The Nash equilibrium is unique.

Institute and Faculty of Actuaries

Symmetric Nash Equilibrium

Two Insurers – Approaching the Nash Equilibrium

How do we reach the Nash equilibrium?



Loading	Step 1	Step 2	Step 3	Step 4	Step 5
Insurer 1	£84.60	£84.60	£72.88	£72.88	£70.97
Insurer 2	£100.00	£76.51	£76.51	£71.47	£71.47

E[Profit]	Step 1	Step 2	Step 3	Step 4	Step 5
Insurer 1	£38.42	£24.35	£25.07	£22.46	£22.48
Insurer 2	£26.53	£29.45	£23.06	£23.19	£22.22

What does the graph show?

- In the first graph we found if your competitor's loading was £100, your optimal loading was £84.50
- The graph above shows strategies, as each insurer responds to their competitors change in loadings
- After five steps, the loadings are close to the Nash equilibrium loading of £70.71.

Institute and Faculty of Actuaries

Application of Game Theory to Insurance Pricing

Scenario 2 – Multiple Insurers with Common Knowledge

Suppose a market of *n* insurers have a common estimation error of £50 in mean claims

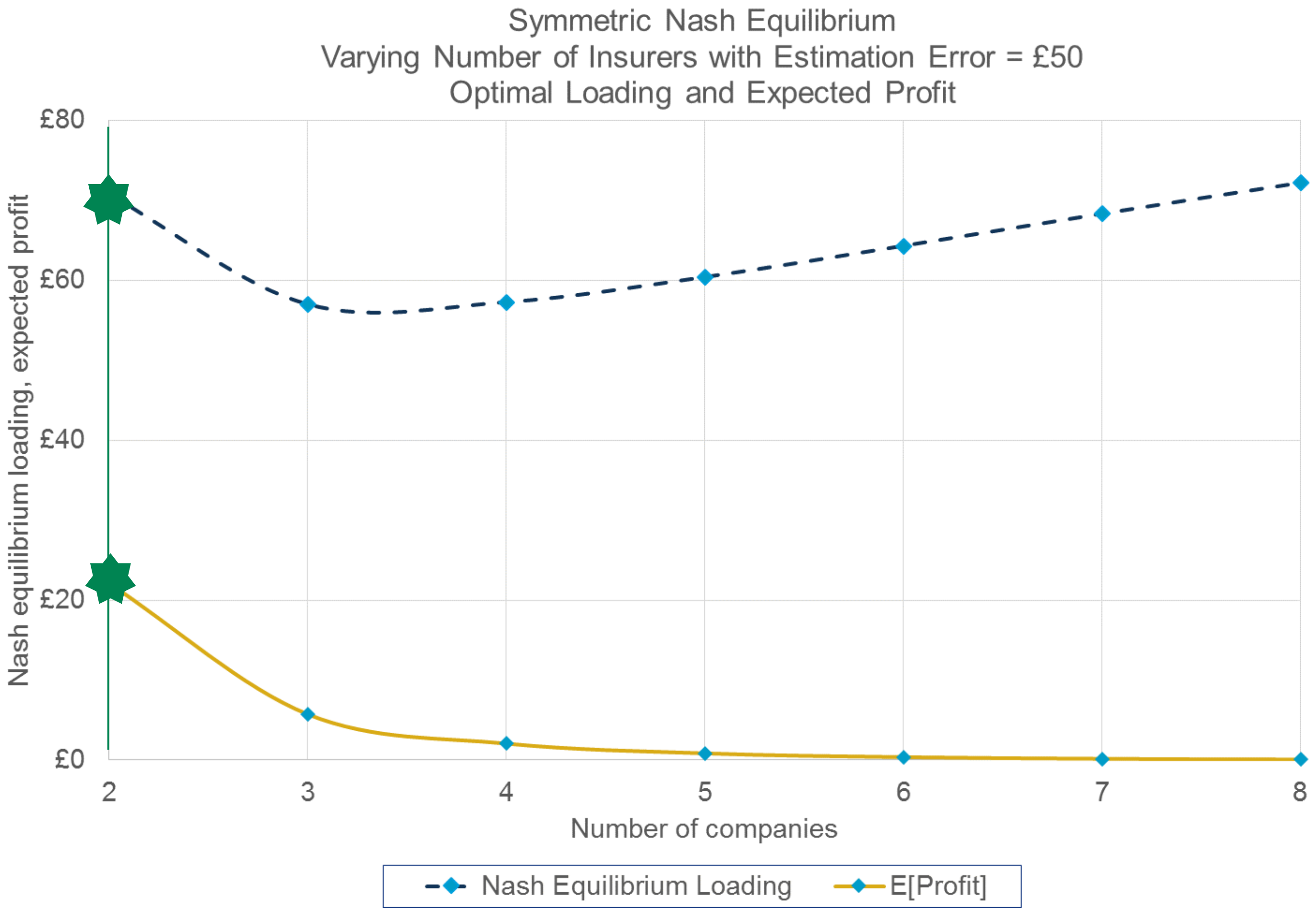
What is the Nash equilibrium loading for this uncertainty?

What is the impact on expected profits?

Institute and Faculty of Actuaries

Symmetric Nash Equilibrium

Varying Market Size – Nash Equilibrium Loading and Expected Profit



Let's increase the number of insurers.

What does the graph show?

- The green stars corresponds to the previous graph, where there are two insurers
- The expected profit is a rapidly decreasing function of the number of companies
- The Nash equilibrium loading initially decreases, but is an increasing function of the number of companies thereafter
- This implies as the number of companies in the market increases, all insurers should increase their prices.

Institute and Faculty of Actuaries

Institute
and Faculty
of Actuaries

Application of Game Theory to Insurance Pricing

Market With Variable Levels of Knowledge

2 December 2016

Application of Game Theory to Insurance Pricing

Scenario 3 – Multiple Insurers With Variable Levels of Knowledge

Suppose there are four insurers in two groups:

Group A: Two insurers with a common estimation error of £50 in mean claims

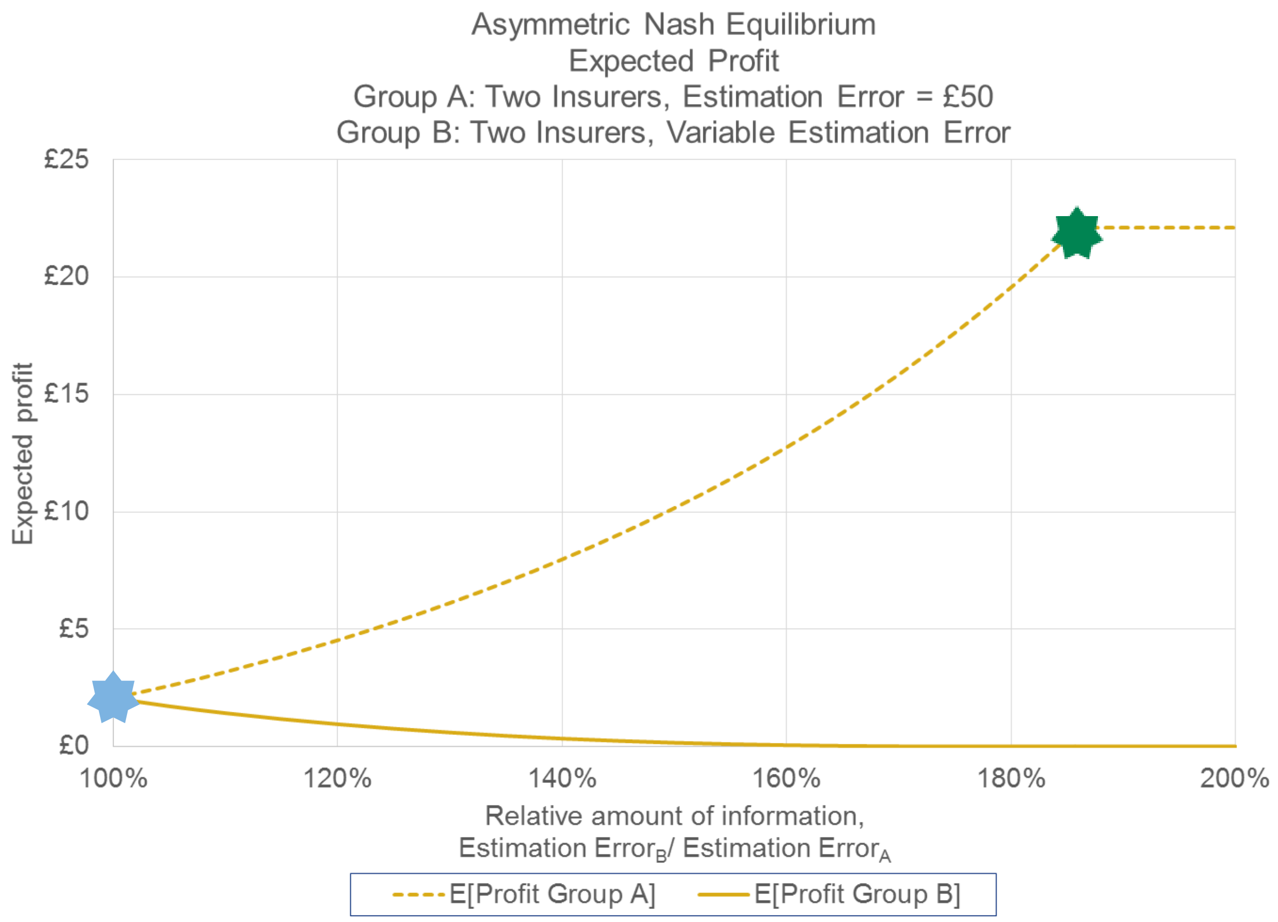
Group B: Two insurers with a common, variable estimation error in mean claims

How do expected profits behave under Nash equilibrium strategies?

Institute and Faculty of Actuaries

Asymmetric Nash Equilibrium

Four Insurers, Two Less Informed – Nash Expected Profits and Market Entry



What does the graph show?

- If group B's estimation error is £50 we have the symmetric Nash equilibrium for four insurers, shown by the blue star
- As group B's estimation error increases:
  - E[Profit Group A] increases
  - E[Profit Group B] tends to zero
- If group B's estimation error exceeds £92.85 (relative information = 186%), E[Profit Group B] < 0 and hence the two insurers should leave the market
- In this case the market returns to the symmetric Nash equilibrium for two insurers, shown by the green star.

Institute and Faculty of Actuaries

Application of Game Theory to Insurance Pricing

Scenario 4 – Market Entry Point

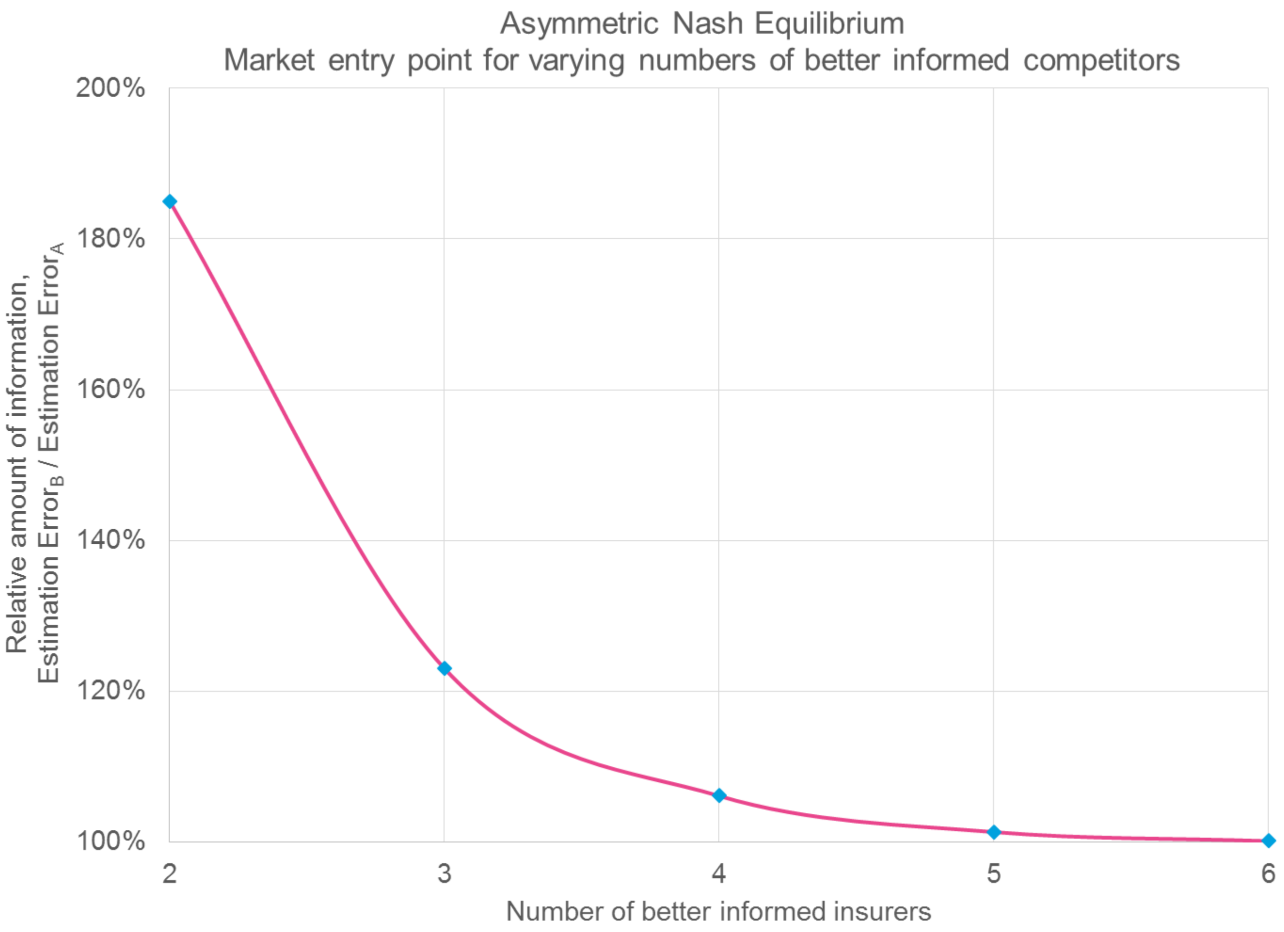
Suppose a market of *n* insurers have a common estimation error of £50 in mean claims

At what level of estimation error is it worth an additional company entering the market?

Institute and Faculty of Actuaries

Asymmetric Nash Equilibrium

Market Entry Point



What does the graph show?

- The graph shows the ratio Estimation Error$_B$ / Estimation Error$_A$ at the market entry point, assuming Nash equilibrium loadings are applied
- When there are two better informed insurers, your estimation error must be within 185% of existing companies to join the market
- If the two better informed insurers have an estimation error of £50 in mean claims, your estimation error must be less than £92.50
- The ratio approaches 100% quickly as the number of better informed insurers increases.

Institute and Faculty of Actuaries

Institute
and Faculty
of Actuaries

Application of Game Theory to Insurance Pricing

Conclusions

2 December 2016

Conclusions

Idealised Models - Is There a Place For Them?

Insights from the Idealised Model

- For a given number of insurers, the optimal loading isn't necessarily an increasing function of your competitors' loadings
- As the number of insurers grows, the Nash equilibrium loading initially decreases, but increases thereafter
- As the estimation error of a less well informed insurer increases, the Nash equilibrium strategy tends towards that of a smaller market without the less well informed insurer
- The relative amount of information needed to enter the market increases very quickly.

Limitations of Idealised Models

- Underlying these results are assumptions which do not hold in reality…
- Does this mean the model doesn't provide any real insights?

Institute and Faculty of Actuaries

Conclusions

What Do The Results Tell Us?

- Don’t be one of ten identical insurers – differentiate your products
  - Focus on niche areas where competition is lower, or build features into products that cannot easily be replicated
- Although you don’t know your competitors estimation error, you may have a view on the relative level of information
  - A lower estimation error leads to a lower loading than your competitors, and hence on average lower premiums
- Think about your loadings from both a pricing and future business planning perspective
  - The loading is a function of your claim estimate, but also the strategy of other companies in the industry
- Consider how to respond to an increase in competition within the industry
  - New entrants are likely to have less information – you should be able to increase your loading in response
- In practice, it’s unlikely we’ll be able to build a model which generates a loading strategy given a view of the market…
  - But never say never…..

Institute and Faculty of Actuaries

Questions

Comments

The views expressed in this presentation are those of invited contributors and not necessarily those of the IFoA. The IFoA do not endorse any of the views stated, nor any claims or representations made in this presentation and accept no responsibility or liability to any person for loss or damage suffered as a consequence of their placing reliance upon any view, claim or representation made in this presentation.

The information and expressions of opinion contained in this publication are not intended to be a comprehensive study, nor to provide actuarial advice or advice of any nature and should not be treated as a substitute for specific advice concerning individual situations. On no account may any part of this presentation be reproduced without the written permission of the IFoA *or authors*.

Institute and Faculty of Actuaries

Application of Game Theory to Insurance Pricing

Further Reading

2 December 2016

Further Reading

Pricing: The Impact of Uncertainty (Andrew D Smith and Keith Chandler, 1994)

- This note gives an early introduction of the concept of winner's curse in insurance, using a simple example
- https://www.actuaries.org.uk/documents/winners-curse-unmodelled-impact-competition-report-winners-curse-giro-working-party (see Appendix 4)

Nash's Nobel (Andrew D Smith, 2002)

- This note explains the concept of a Nash equilibrium, outlining the game theory context and the key results for which Nash gained his Nobel
- https://www.actuaries.org.uk/documents/nashs-nobel

Winner's Curse: The Unmodelled Impact of Competition (Winner's Curse GIRO Working Party, 2009)

- This paper outlines the theory of the winner's curse, provides examples of its operation in practice and illustrates where it may be a key feature of insurance markets
- https://www.actuaries.org.uk/documents/winners-curse-unmodelled-impact-competition-report-winners-curse-giro-working-party-0

Game Theory in General Insurance: How to outdo your adversaries while they are trying to outdo you (Game Theory GIRO Working Party, 2012)

- This paper explores the application of some of the existing game theory to the problem of strategically managing non-life insurance products where current methods of setting price do not typically consider reactions of competitors
- https://www.actuaries.org.uk/documents/brian-hey-prize-game-theory-general-insurance

Institute and Faculty of Actuaries

Technical Appendices:

Model Set Up

Further Reading

2 December 2016

Institute and Faculty of Actuaries

Application of Game Theory to Insurance Pricing

Model Set Up

2 December 2016

Insurance Pricing Model

Parameters

- n - number of insurers in the market
- X - aggregate claims on an insurance policy, modelled as random variables
- $\{ Y - Z_i : 1 \leq i \leq n \}$ – estimate of expected claims. We will think of Z_i as being a zero mean observation error
  - $\{ Z_i : 1 \leq i \leq n \}$ has cumulative distribution function $F_i(z)$ and associated density $F_i'(z)$.
- $\{ \lambda_i : 1 \leq i \leq n \}$ – constant loading for insurer i.

Set Up

- Insurers compete for the business by submitting sealed premium quotes of the form

$$Y - Z_i + \lambda_i$$

Assumptions

- The policyholder buys insurance from the cheapest insurer. There is no customer loyalty or brand effect to overcome this
- The claims cost X is the same regardless of which insurer settles the claim, ignoring investment returns, expenses, capital requirements etc.
- Insurers optimise their own expected profit, with no regard for other properties of the profit probability distribution
- The random variables X, Y and $\{ Z_i : 1 \leq i \leq n \}$ are independent, and their probability distributions are known to all insurers
- The variables X and Y have the same mean E[X] = E[Y]. The variables Z_i have $E[Z_i] = 0$
- Each insurer sees their cost estimate $Y - Z_i$, but cannot separate the two terms
- Each insurer knows the loadings $\{\lambda_j : j \neq i \}$ for its competitors

Insurance Pricing Model

Win condition

- Insurer i wins the business if their premium quote is the lowest, that is, if $Y - Z_i + \lambda_i \leq Y - Z_j + \lambda_j$ for all $j \neq i$. As Y appears on each side, the insurer i win condition is

$$Z_i - \lambda_i \geq Z_j - \lambda_j, \quad \forall j \neq i$$

Expected Profit

- In the case that insurer i wins the auction, the profit is the premium minus the claims, $Y - Z_i + \lambda_i - X$. To find the expected profit, we need to calculate the mean of this expression, but counting the cases where insurer i wins the auction:

$$\mathbb{E}\left[(Y - Z_i + \lambda_i - X) I\left\{Z_j - \lambda_j \leq Z_i - \lambda_i, \forall j \neq i\right\}\right] = -\int_{-\infty}^{\infty} z \left\{\prod_{j \neq i} F_j(z + \lambda_j)\right\} F_i'(z + \lambda_i) \mathrm{d}z$$

- A profit maximising insurer will seek λ_i to maximise the expected profit, given $\{\lambda_j : j \neq i\}$.

Winner's Curse

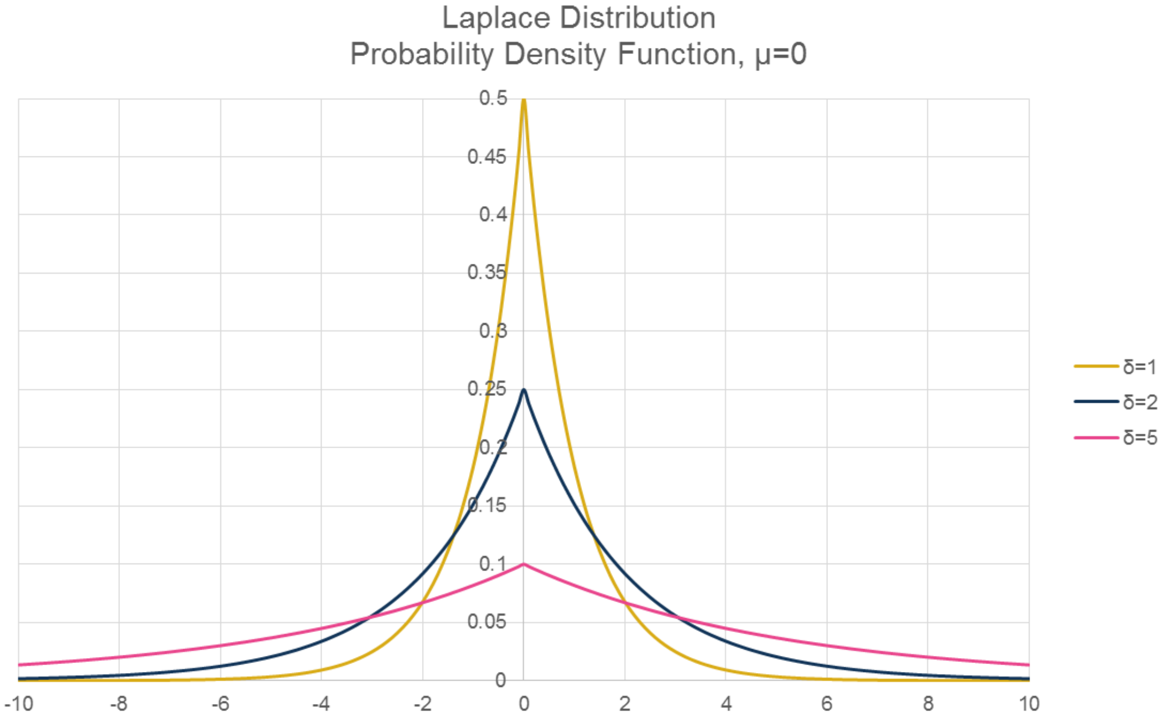
- The difference between the planned margin λ_i and the expected profit is the impact of winner's curse.

Institute and Faculty of Actuaries

Insurance Pricing Model

Laplace Distribution

- We illustrate the optimisation with an example based on Laplace distributions, given the ability to derive explicitly the expected profit and its derivative
- Let us suppose that $\{ Z_i : 1 \le i \le n \}$ have a Laplace distribution with location parameter $\mu = 0$ and scale parameter $\delta_i > 0$



$$\mathbb{E}[Z_i] = \mu = 0 \qquad Var[Z_i] = 2\delta_i^2$$

$$F_i(z+\lambda_i) = \begin{cases} \frac{1}{2}\exp(\frac{z+\lambda_i}{\delta_i}) & z+\lambda_i \le 0 \\ 1 - \frac{1}{2}\exp(\frac{-z-\lambda_i}{\delta_i}) & z+\lambda_i \ge 0 \end{cases}$$

$$F_i'(z+\lambda_i) = \begin{cases} \frac{1}{2\delta_i}\exp(\frac{z+\lambda_i}{\delta_i}) & z+\lambda_i \le 0 \\ \frac{1}{2\delta_i}\exp(\frac{-z-\lambda_i}{\delta_i}) & z+\lambda_i \ge 0 \end{cases}$$

Institute and Faculty of Actuaries