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Practical use of GLMs in Reserving

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16 June 2016

Overview

1. Fitting a reserving GLM
2. GLMS in reserving - model structure and covariates
3. Case study

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Fitting a reserving GLM

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Example data

Workers compensation, New Jersey Manufacturing Group

Accident year		Incremental paid losses in development year ($000)									
		1	2	3	4	5	6	7	8	9	10
1988	1	41821	34729	20147	15965	11285	5924	4775	3742	3435	2958
1989	2	48167	39495	24444	18178	10840	7379	5683	4758	3959	
1990	3	52058	47459	27359	17916	11448	8846	5869	5391		
1991	4	57251	49510	27036	20871	14304	10552	7742			
1992	5	59213	54129	29566	22484	14114	10000				
1993	6	59475	52076	26836	22332	14756					
1994	7	65607	44648	27062	22655						
1995	8	56748	39315	26748							
1996	9	52212	40030								
1997	10	43962									

Conventional chain ladder model

Accident year		Age-to-age factor for development year								
		1	2	3	4	5	6	7	8	9
1988	1	1.830	1.263	1.165	1.100	1.048	1.037	1.028	1.025	1.021
1989	2	1.820	1.279	1.162	1.083	1.052	1.038	1.031	1.025	
1990	3	1.912	1.275	1.141	1.079	1.057	1.036	1.032		
1991	4	1.865	1.253	1.156	1.092	1.062	1.043			
1992	5	1.914	1.261	1.157	1.085	1.056				
1993	6	1.876	1.241	1.161	1.092					
1994	7	1.681	1.245	1.165						
1995	8	1.693	1.278							
1996	9	1.767								

Average age-to-age factor for development year								
1	2	3	4	5	6	7	8	9
1.815	1.261	1.158	1.088	1.055	1.039	1.030	1.025	1.021

Accident year		Forecast cumulative paid losses to and including development year ($000)										Estimated outstanding claims ($000)
		1	2	3	4	5	6	7	8	9	10	
1988	1										**144781**	
1989	2									**162903**	166301	3398
1990	3								**176346**	180731	184501	8155
1991	4							**187266**	192924	197721	201845	14579
1992	5						**189506**	196828	202774	207817	212151	22645
1993	6					**175475**	185209	192364	198176	203104	207340	31865
1994	7				**159972**	174108	183766	190866	196632	201522	205725	45753
1995	8			**122811**	142227	154795	163381	169693	174820	179168	182904	60093
1996	9		**92242**	116312	134700	146603	154735	160713	165569	169686	173225	80983
1997	10	**43962**	79788	100608	116513	126809	133843	139014	143214	146775	149836	105874
												373346

ODP Mack model ($E[Y_{kj}] = \alpha_k \beta_j$) gives same liability estimate

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Fitting with a GLM

- Fit incremental payment data as a cross-classified model (ODP model)
  - Same liability estimate as the chain ladder
  - ODP formulation

j or k	$ln\,\hat{\alpha}_k$		$ln\,\hat{\beta}_j$	
	Estimate	Standard error	Estimate	Standard error
1	10.657	0.0316	0.000	
2	10.795	0.0299	-0.205	0.0228
3	10.899	0.0289	-0.747	0.0282
4	10.989	0.0281	-1.017	0.0328
5	11.039	0.0278	-1.452	0.0421
6	11.016	0.0285	-1.833	0.0547
7	11.008	0.0295	-2.140	0.0715
8	10.891	0.0327	-2.348	0.0931
9	10.836	0.0367	-2.513	0.1267
10	10.691	0.0510	-2.664	0.1993

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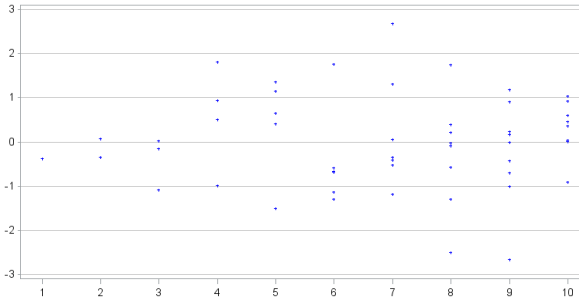
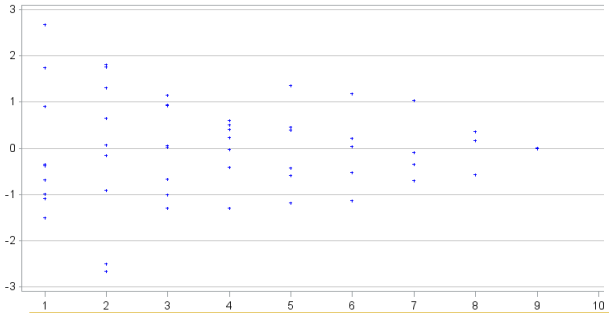
Residual diagnostics

Calendar year residuals



Development year residuals



Actual/expected heat map

	Development									
Accident	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	98%	100%	100%	104%	113%	87%	96%	92%	100%	100%
2	99%	99%	106%	103%	95%	95%	99%	102%	100%	
3	96%	108%	107%	91%	90%	102%	92%	104%		
4	97%	103%	96%	97%	103%	111%	111%			
5	95%	107%	100%	100%	97%	100%				
6	98%	105%	93%	101%	104%					
7	109%	91%	95%	104%						
8	106%	90%	105%							
9	103%	97%								
10	100%									

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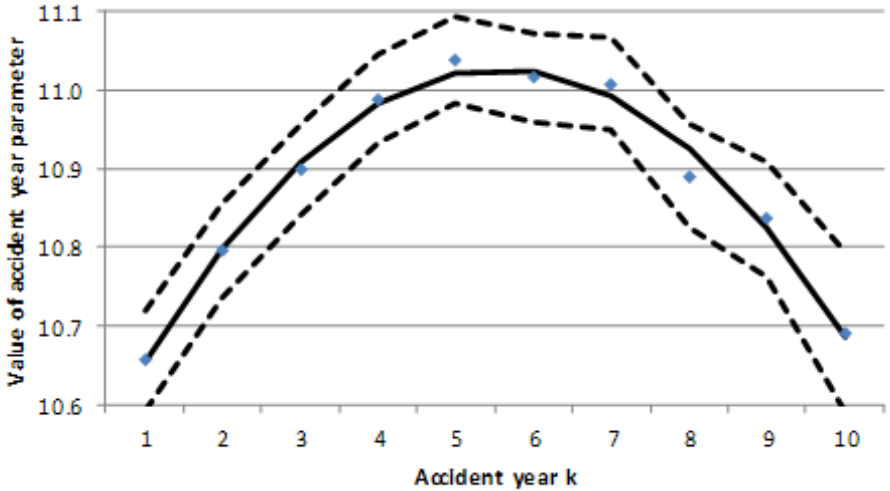
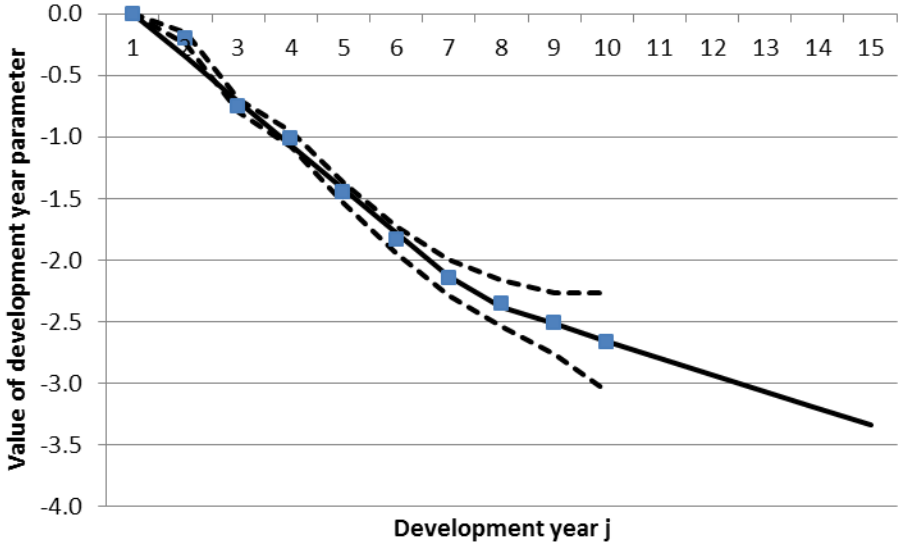
Simplifying the model

- Reducing the number of parameters



Quadratic – fits well



Spline with a knot at 7.5.
Good apart from j=2

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Updated model

j or k	$ln\,\hat{\alpha}_k$		$ln\,\hat{\beta}_j$	
	Estimate	Standard error	Estimate	Standard error
1	10.657	0.0316	0.000	
2	10.795	0.0299	-0.205	0.0228
3	10.899	0.0289	-0.747	0.0282
4	10.989	0.0281	-1.017	0.0328
5	11.039	0.0278	-1.452	0.0421
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10	10.691	0.0510	-2.664	0.1993

Parameter	Estimate
Accident year parameters	
a_0	10.469
a_1	0.200
a_2	-0.018
Development year parameters	
b_1	-0.358
b_2	0.236
c	0.155

$$E[Y_{kj}] = \exp(a_0 + a_1 k_i + a_2 k_i^2 + b_1(j_i - 1) + b_2 \max(0, j_i - 7.5) + cJ_{i2})$$

J_{i2} = 1 if i^{th} observation relates to development year 2

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Not there yet…

Updated heat map

Accident year	Development year									
	1	2	3	4	5	6	7	8	9	10
1988	99%	101%	98%	111%	112%	84%	97%	96%	100%	97%
1989	99%	99%	102%	109%	93%	90%	99%	106%	99%	
1990	95%	107%	102%	96%	88%	97%	92%	107%		
1991	97%	103%	94%	104%	102%	107%	113%			
1992	97%	108%	99%	108%	97%	98%				
1993	97%	104%	89%	106%	101%					
1994	110%	92%	93%	112%						
1995	102%	87%	99%							
1996	105%	98%								
1997	101%									

Payment pattern altered?

for development year:

1. a distinct area of blue in the earlier accident years;
2. a distinct area of pink in the earlier accident years;
3. a possible progression from pink to blue with increasing accident year;
4. a preponderance of pink over the whole set of accident years.

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Final model

- $E[Y_{kj}] = \exp(x_i^T \beta) = \exp(a_0 + a_1 k_i + a_2 k_i^2 + b_1(j_i - 1) + b_2 \max(0, j_i - 7.5) + c_1 J_{i2} + c_2 J_{i4} + d_1 J_{i1} K_{i,1-6} + d_2 J_{i2} K_{i,1-6} + d_3 J_{i3} k)$
  - Where $K_{i,1-6}$ = 1 if ith observation relates to accident years 1-6

- 10 parameters vs 19 for ODP Mack
- Coefficient of variation of prediction indicates an improvement:
  - 3.5% for ODP Mack
  - 2.9% for final model

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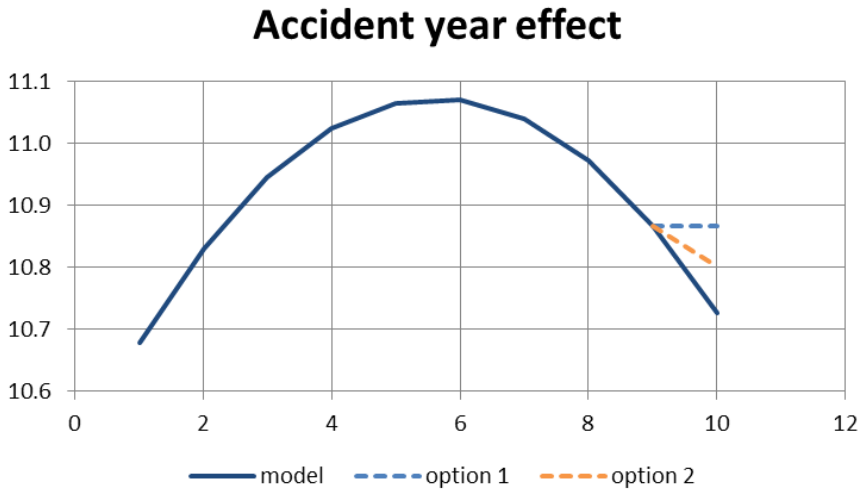
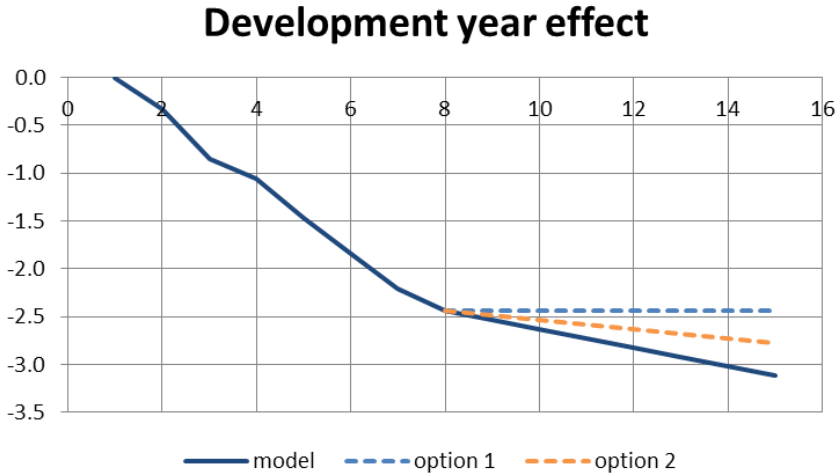
Incorporating judgement into projections





(for accident year 10)

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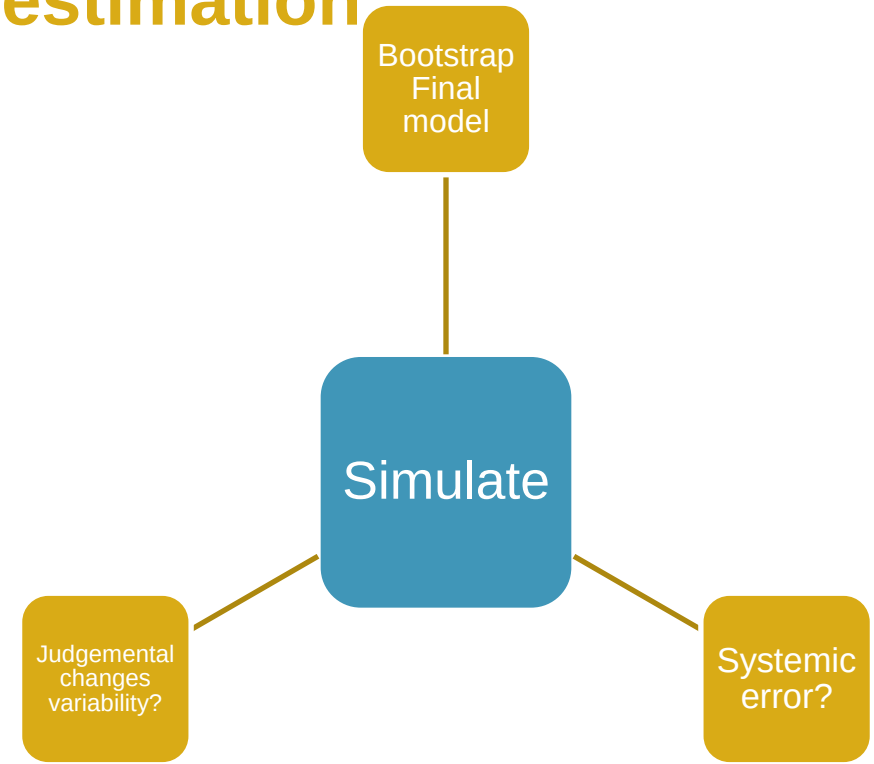
Uncertainty estimation



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GLMs in Reserving

Model structure and covariates. Some observations from experience

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Additional model covariates?

Data for finalisation numbers, reported claim numbers

- Permits separate frequency and size models
- Taylor & Xu (2016), empirical study of US Schedule P data:
  - Chain ladder, Payments Per Claim Finalised [PPCF], Payments Per Claim Incurred [PPCI]
  - 80% of time PPCF and PPCI outperformed chain ladder
  - Claim count data more important for longer tailed line of business

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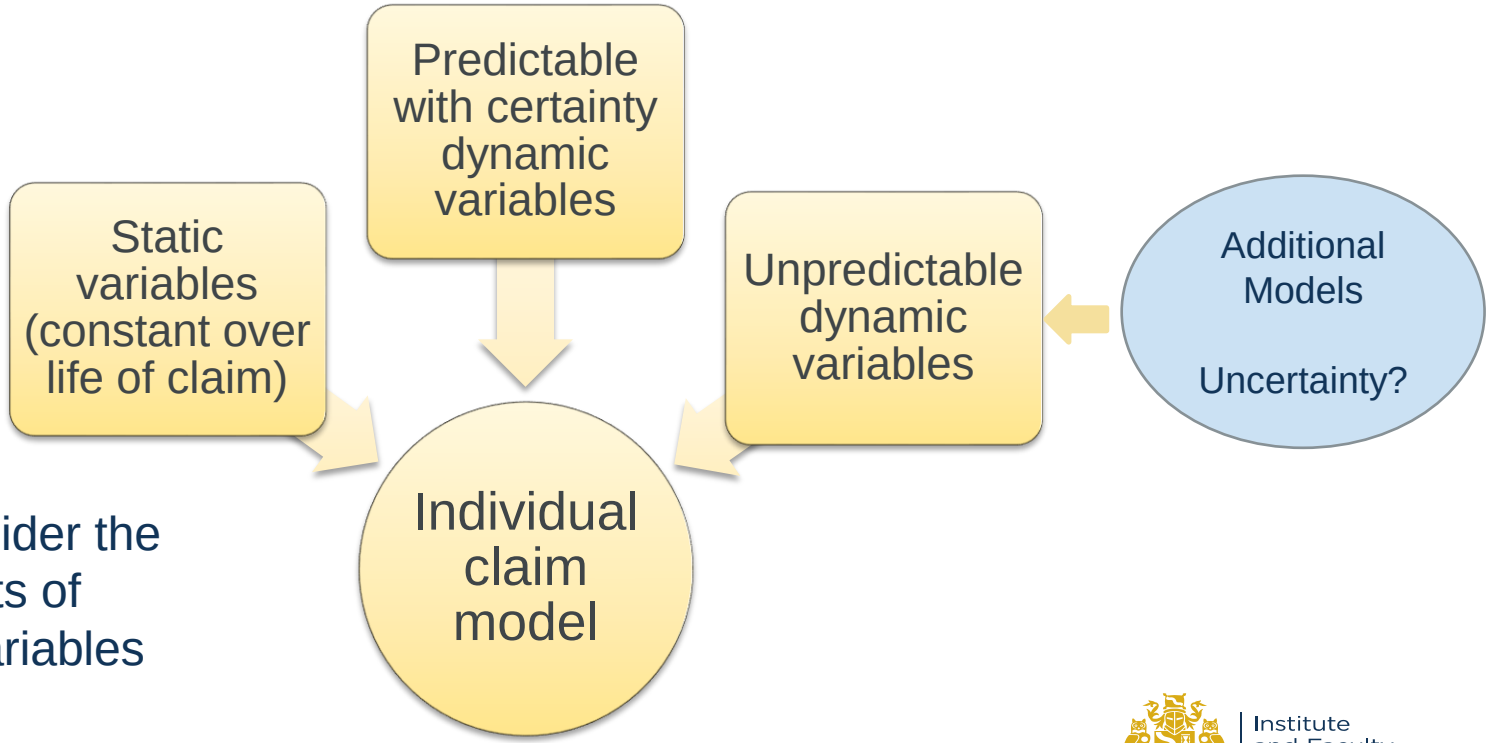
Other covariates for individual models?



Always consider the costs/benefits of additional variables

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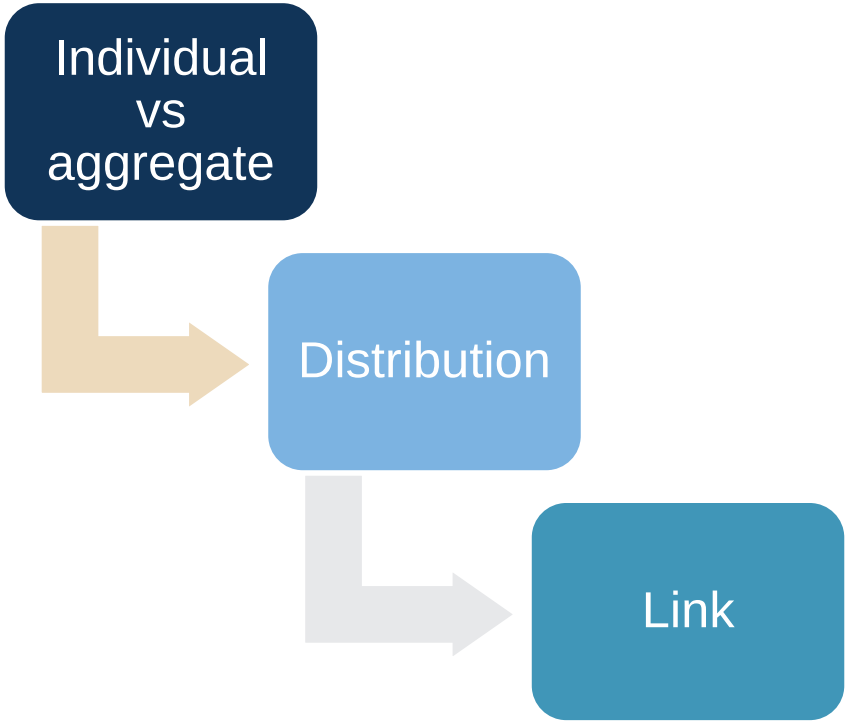
GLM Model structure



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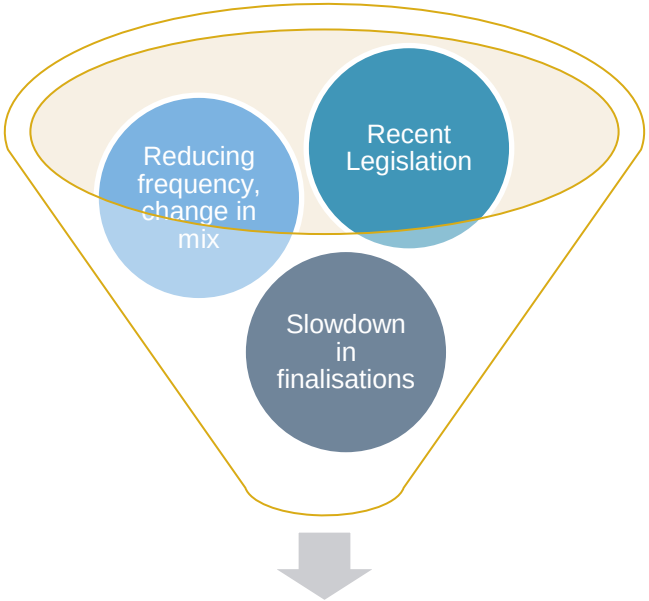
Case study

Motor bodily injury reserving – central estimate

Background



Reserves?

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Data

Accident quarter	Average claim size values (in 30/6/06 values) in development quarter								
	0	1	2	3	4	5	6	7	8
	$	$	$	$	$	$	$	$	$
Sep-02	4,214	3,924	7,255	17,142	23,051	27,608	29,819	34,735	38,020
Dec-02	9,799	3,713	7,670	13,351	17,125	18,590	27,397	28,418	39,862
Mar-03	1,430	2,972	3,210	4,968	6,600	9,476	10,249	18,004	14,679
Jun-03	2,851	1,854	2,767	3,163	5,514	7,195	10,772	20,490	23,987
Sep-03	128	1,679	3,051	4,887	7,192	10,227	13,946	16,479	24,030
Dec-03	1,099	1,604	3,280	3,692	5,992	6,098	11,952	19,127	32,322
Mar-04	-	2,354	4,415	2,621	3,786	8,802	13,667	22,697	25,314
Jun-04	495	2,846	2,746	3,923	4,563	12,713	14,161	25,069	36,179
Sep-04	408	1,296	2,186	4,267	6,125	10,688	19,161	32,930	
Dec-04	815	1,190	3,882	5,058	5,845	12,976	18,057		
Mar-05	1,783	1,966	3,818	4,653	6,900	10,265			
Jun-05	896	1,899	3,287	3,927	7,244				
Sep-05	1,732	2,347	3,743	6,130					
Dec-05	2,367	2,567	3,900						
Mar-06	1,266	2,274							
Jun-06	2,367								

Operational time in middle of development quarter								
0	1	2	3	4	5	6	7	8
%	%	%	%	%	%	%	%	%
0	1	5	10	16	23	32	40	46
0	1	4	8	13	21	29	37	43
0	1	3	6	11	17	24	30	35
0	1	3	7	12	18	24	30	36
0	1	4	8	13	18	23	29	36
0	2	5	10	15	20	25	32	38
0	1	4	8	13	18	25	31	38
0	1	5	9	14	20	27	34	41
0	1	5	9	15	20	26	33	
0	2	5	9	15	20	26		
0	2	5	9	14	20			
0	2	5	10	15				
0	2	5	10					
0	2	6						
0	2							
0								

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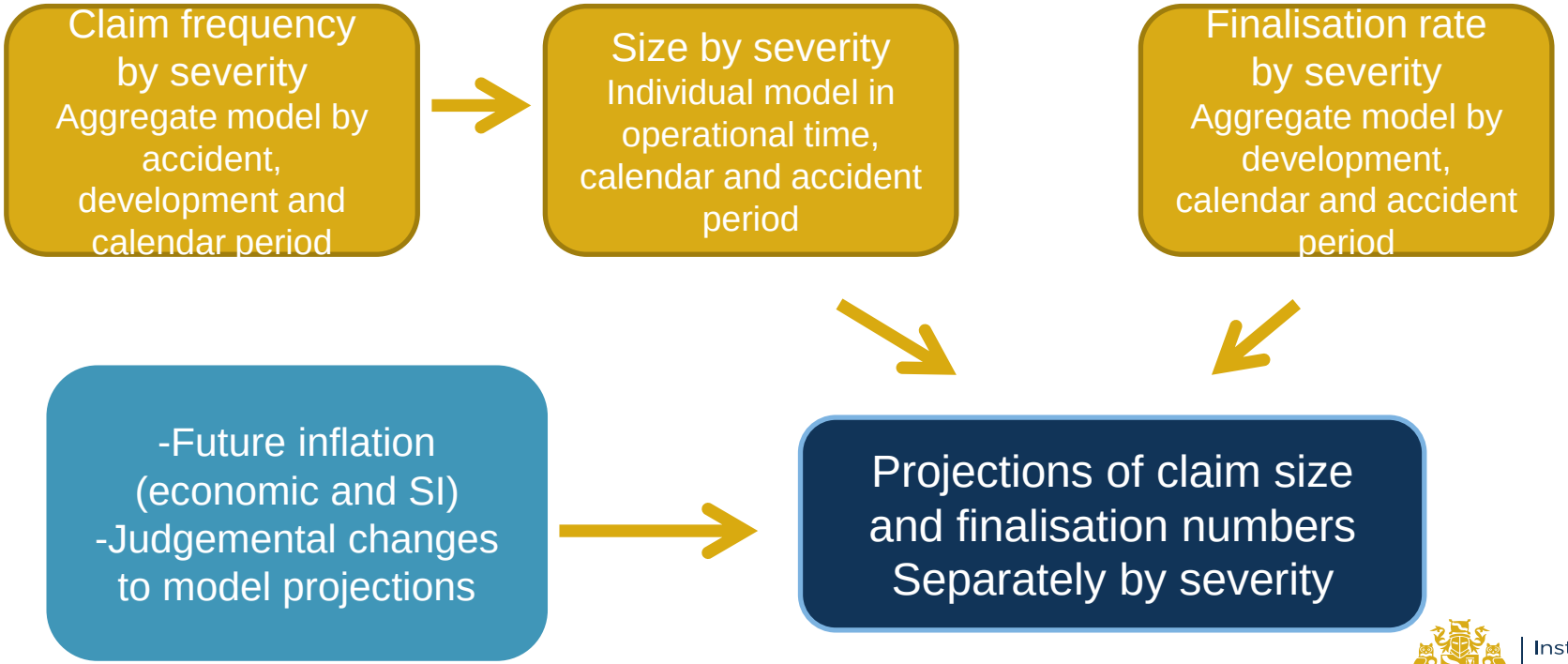
Model structure – PPCF model in operational time



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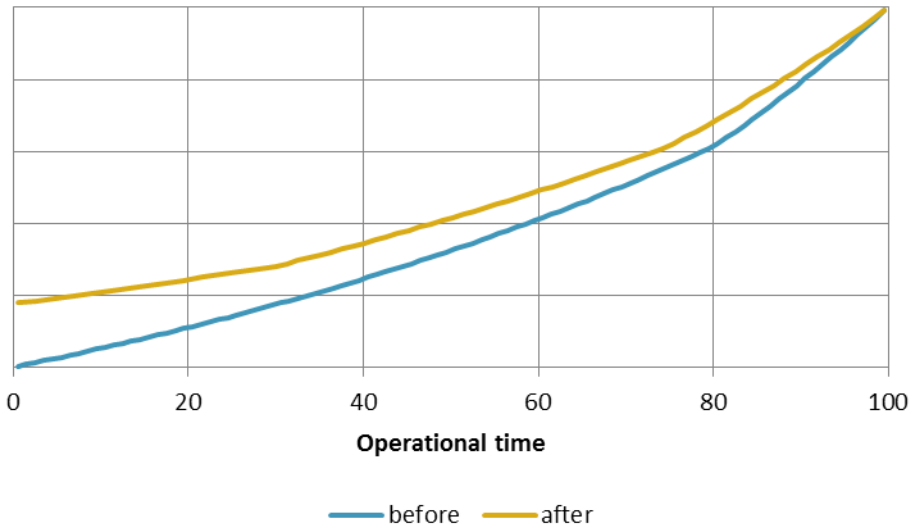
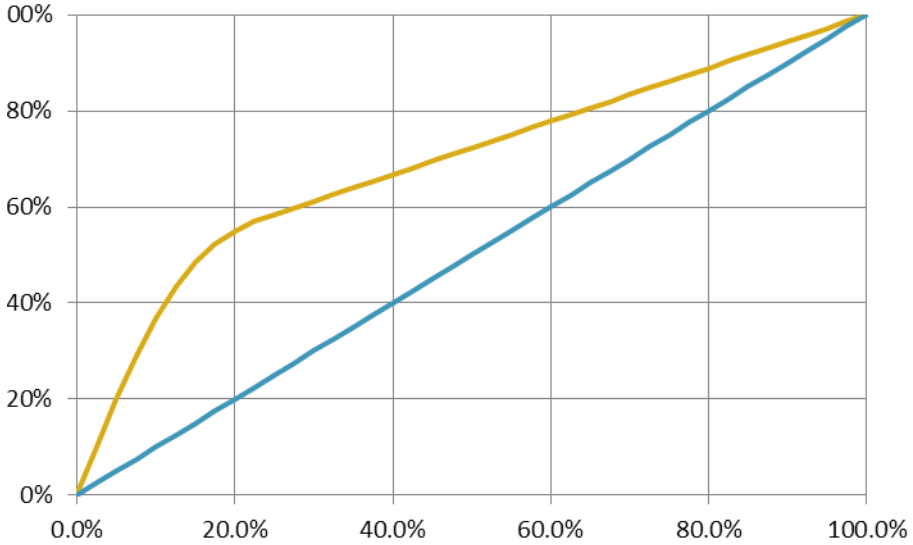
Modelling the removal of small claims

Change in Claim size



Mapped operational time



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Concluding comments

GLMs are useful!

References

References are specific to some things discussed in this paper. Plenty of other material out there, including a lot of UK-based work which you will know about already

- General overview of stochastic reserving – example on fitting a GLM drawn from this
  - Stochastic Loss Reserving using Generalized Linear Models, G Taylor and G McGuire, CAS monograph series. http://www.casact.org/pubs/monographs/index.cfm?fa=taylor-monograph03
- Advantages of using claims count data
  - An empirical investigation of the value of finalisation count information to loss reserving. G Taylor and J Xu (2016) Variance, in press.
- Case studies on loss reserving with GLMs (which was the source for the case study here)
  - Loss reserving with GLMs: a case study. Taylor G & McGuire G (2004). Casualty Actuarial Society 2004 Discussion Paper program, pp 327-392
  - McGuire G (2007). Individual claim modelling of CTP data. Institute of Actuaries of Australian XIth Accident Compensation Seminar, Melbourne, Australia. Available at http://actuaries.asn.au/Library/6.a_ACS07_paper_McGuire_individual claim modelling of CTP data.pdf

Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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